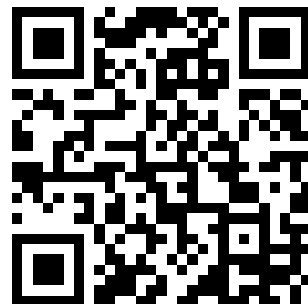
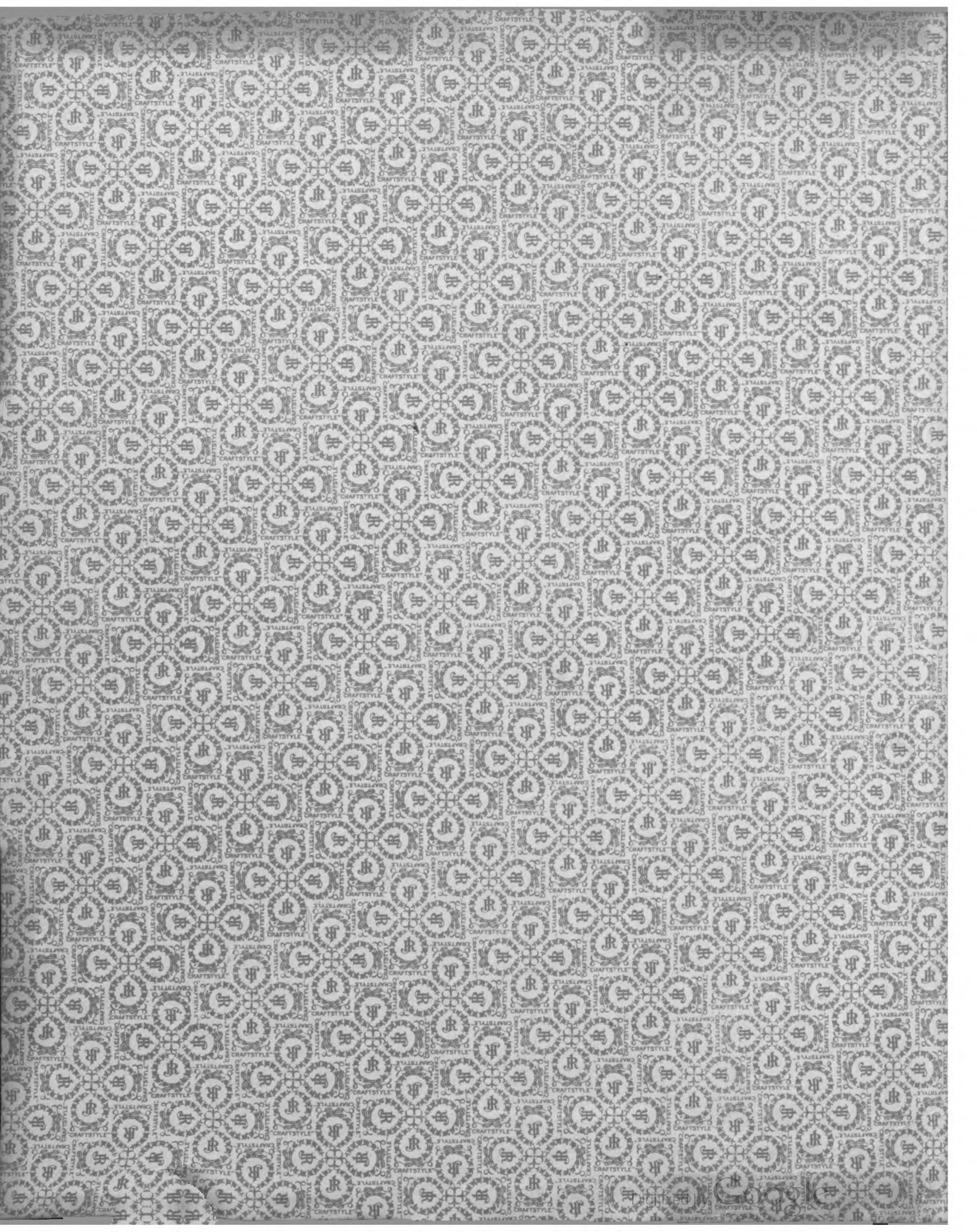

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APPENDIX 3

REPORT 1904

MANUAL OF TIDES—PART IV B

COTIDAL LINES FOR THE WORLD

By ROLLIN A. HARRIS

PREFACE

Part IV A, "Outlines of Tidal Theory," appeared as Appendix 7, Coast and Geodetic Survey Report for 1900. The aim of that appendix is to obtain through theoretical considerations, rational so far as they go, a first approximation to the times of the principal ocean tides.

Part IV B aims at a system of cotidal lines which, while conforming to all reliable data, shall reasonably well meet the requirements of the theory alluded to, and so, for most regions, constitute an essentially true representation of the tides.

The labor involved in the construction of these lines has been so great that many matters relating to the subject have, of necessity, either remained undeveloped or, if appearing in the accompanying text, have been but scantily treated.

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MANUAL OF TIDES—PART IV B. COTIDAL LINES FOR THE WORLD.

By ROLLIN A. HARRIS.

CHAPTER I.

ON THE COMBINATION OF LONG WAVES.

1. The combination of long waves under several conditions has been briefly considered in Chapters III and VIII, Part IV A, and some similar questions for ordinary waves have been treated in Chapter II, Part I, and Chapter IV, Part IV A. A few questions relating to this subject and which have important bearings upon the construction of cotidal lines will be considered in this chapter.

2. *The combination of two progressive waves.*

Suppose the space origin to be situated at any convenient point and suppose one progressive wave to move toward $+x$ and the other, at some angle to this direction. Suppose the time to be reckoned from Greenwich or any other given meridian. Let T' denote the time of high water of the first wave at $x=0$ and T'' that of the second wave at the origin. Then the vertical displacement may be written

$$\begin{aligned}\zeta &= A' \cos (at - lx - aT') + A'' \cos (at - ly - aT'') \\ &= A' \cos (\theta - lx - \epsilon') + A'' \cos (\theta - ly - \epsilon'')\end{aligned}\tag{1}$$

where the axis of y is generally oblique to the axis of x . When the direction of the second system is the same as the first, or opposite to it, y should be replaced by x or by $-x$.

If $A''=A'$ the lines of no tide are given by the equation

$$lx + \epsilon' = ly + \epsilon'' \pm (2\nu + 1)\pi.$$

If $A'' \neq A'$, then there must be some rise and fall at every point of the surface covered by either or both of these wave systems.

From $\frac{\partial \zeta}{\partial \theta} = 0$ we have for finding the time of the resultant high or low water

$$\tan \theta = \frac{A' \sin (lx + \epsilon') + A'' \sin (ly + \epsilon'')}{A' \cos (lx + \epsilon') + A'' \cos (ly + \epsilon'')}\tag{2}$$

If y coincides with x , we then have from $\frac{\partial \zeta}{\partial \theta} = 0$,

$$\tan lx = \frac{A' \sin (\theta - \epsilon') + A'' \sin (\theta - \epsilon'')}{A' \cos (\theta - \epsilon') + A'' \cos (\theta - \epsilon'')} \quad (3)$$

which determines the position of high or low water for this case at any given time.

The time is referred to the time of high water of the first wave at $x=0$ if $\epsilon'=0$, and to the time of high water of the second wave at $y=0$ if $\epsilon''=0$. The amplitude of the tide at any point x, y is obtained by substituting for θ in (1) its value from (2).

3. *The combination of a stationary and a progressive wave, both lying in the same direction.*

Suppose the space origin to be situated at a loop of the stationary wave and suppose the progressive wave to move toward $+x$. Suppose the time to be reckoned from Greenwich or any other given meridian. Let T' denote the time of high water of the stationary wave at $x=0$ and T'' that of the progressive wave at $x=0$. Then the vertical displacement may be written

$$\begin{aligned} \zeta &= A' \cos lx \cos (at - aT') + A'' \cos (at - lx - aT'') \\ &= A' \cos lx \cos (\theta - \epsilon') + A'' \cos (\theta - lx - \epsilon'') \end{aligned} \quad (4)$$

From $\frac{\partial \zeta}{\partial \theta} = 0$ we have for finding the time of the resultant high or low water

$$\tan \theta = \frac{A' \cos lx \sin \epsilon' + A'' \sin (lx + \epsilon'')}{A' \cos lx \cos \epsilon' + A'' \cos (lx + \epsilon'')} \quad (5)$$

and for the position of high or low water at any given time

$$\tan lx = \tan (\theta - \epsilon'') + \frac{A' \sin (\theta - \epsilon')}{A'' \cos (\theta - \epsilon'')} \quad (6)$$

The time is referred to the time of high water of the stationary wave at $x=0$ if $\epsilon'=0$, and of the progressive wave at $x=0$, if $\epsilon''=0$. The amplitude of the tide at any point x is obtained by substituting for θ in (4) its value from (5).*

In case the progression is toward $-x$, replace lx by $-lx$.

Since for natural bodies of water the end boundaries at the loops of a stationary wave are seldom such as to turn back the approaching particles simultaneously because of irregularities in the shore line, extensive shoaling, or more especially breaks in the walls, it is evident that progressive waves must often accompany stationary ones.† Here no attempt will be made to determine what the relative positions and amplitudes of the two waves must be. In fact it must generally happen that the amplitude of the progressive wave diminishes as we go from the locality giving rise to it, to points from which it appears to come (Cf. § 8). Although such combinations are quite common in the oceans, waves from other causes frequently so obscure them that it is seldom worth while to actually perform the computations necessary for making the combination.

Some fairly good instances may be cited and which will be referred to in the description of the cotidal lines of the several localities: The northeastern corner of the Indian Ocean, the Red Sea, the Gulf of Suez, the Pacific Ocean west of the South

* The angle lx can be found by aid of Table 15, in Part III.

† Cf. lemmas 14 and 21, § 70, Part IV A.

Shetland Islands. See Figs. 6, 7, and 28, also Fig. 23 of Part IV A. Other instances are the Bay of Bengal, Adriatic Sea, English Channel, Irish Channel, Baffin Bay, Long Island Sound, Gulf of California, and Gulf of Georgia. In most of these cases the progressive wave is rather small, it being due to the shoaling and irregularities of the body of water and not to an opening in an end boundary. But, as will be noted in Chapter III, the earth's rotation often sensibly modifies the tide in the vicinity of a nodal line, apparently increasing the range of the progressive wave on one side of the channel and diminishing it upon the other.

The general tendency of the progressive wave is to obliterate the nodal line of the stationary one; the larger its amplitude, the less crowding up there generally is of the cotidal lines at the nodal line.

This combination of waves applies only to regions of water where the oscillatory motion is nearly rectilinear. Progressive waves in the ocean are generally directed toward openings in the shore line, especially if the rise and fall of the tide be there considerable. Consequently it seldom happens that in the ocean the particles oscillate in the same direction in both stationary and progressive waves.

4. *The combination of one stationary wave with another lying transversely to it.*

Suppose the space origin to be situated at a loop of each wave. Suppose the time to be reckoned from Greenwich or any other given meridian. Let T' denote the time of high water, at $x=0$, of the stationary wave whose motion is parallel to x , and T'' the time of high water, at $y=0$, of the stationary wave whose motion is parallel to y . Then the vertical displacement may be written

$$\begin{aligned}\zeta &= A' \cos lx \cos (at - aT') + A'' \cos ly \cos (at - aT'') \\ &= A' \cos lx \cos (\theta - \epsilon') + A'' \cos ly \cos (\theta - \epsilon'')\end{aligned}\quad (7)$$

From $\frac{\partial \zeta}{\partial \theta} = 0$ we have for finding the time of the resultant high or low water

$$\tan \theta = \frac{A' \cos lx \sin \epsilon' + A'' \cos ly \sin \epsilon''}{A' \cos lx \cos \epsilon' + A'' \cos ly \cos \epsilon''}\quad (8)$$

[If both stationary waves lie in the same direction, replace y by x in (7) and (8). The amplitude of the tide at any point x is then obtained by substituting for θ in (7) its value from (8).]

By assigning a value to θ (i. e., to at), this equation gives a relation between x and y which is the equation of the cotidal line for the assumed time.

From (8) we have

$$\frac{dy}{dx} = \frac{\tan \theta (A' \sin lx \cos \epsilon') - A' \sin lx \sin \epsilon'}{-\tan \theta (A'' \sin ly \cos \epsilon'') + A'' \sin ly \sin \epsilon''}\quad (9)$$

which gives the direction of the cotidal line whose characteristic is θ at any permissible point x, y . This ratio appears indeterminate at $x=0, y=0$ and of course at such points as $x=\mu\pi, y=\nu\pi$, μ and ν denoting integers. The value is, however, ± 1 .

For the lines $x=0$ or $x=\mu\pi$, $\frac{dy}{dx}$ becomes zero, and for $y=0$ or $y=\nu\pi$ it becomes infinite.

Hence:

The cotidal lines are normal to the real or virtual boundaries of a square oscillating area excepting the two lines which coincide with the diagonals of the square.

Upon substituting in (9) the value (8) of $\tan \theta$ we have

$$\frac{dy}{dx} = \frac{\tan lx}{\tan ly} \frac{\sin(\epsilon'' - \epsilon')}{\sin(\epsilon'' - \epsilon')} = \frac{\tan lx}{\tan ly} \quad (10)$$

which gives the direction of the cotidal line through *any* point in the area. This does not depend upon the amplitudes or the phases of the two waves. Hence if we have a drawing covering the square completely with cotidal lines based upon a simple assumption like $A'' = A' = 1$ and $\epsilon'' - \epsilon' = \pm 90^\circ$, ϵ' or $\epsilon'' = 0$, the cotidal lines for any other assumptions can be immediately traced therefrom as soon as we have identified or computed one point in each line.

At the center of the square, where $lx = 90^\circ$, $ly = 90^\circ$, there is no rise and fall and so all cotidal lines must radiate from this point. The value of $\frac{dy}{dx}$ at this point is

$$\frac{A'}{A''} \frac{\tan \theta \cos \epsilon' - \sin \epsilon'}{\cos \epsilon'' + \sin \epsilon''} \quad (11)$$

The fact that two stationary waves of different phases are combined together shows that high water must occur at the middle points of the sides of the square area at times given by the waves when considered separately. And it is readily seen that the times of high water change successively through all values of a period as we proceed around the square. The distribution of the radiating cotidal lines will, however, be quite uneven, as expression (10) shows.

Cotidal lines, whose numbers progress through all hours (i. e., form a complete cycle of values) around a no-tide point, may be called *amphidromic*, or the locality concerned may be said to be amphidromic.

It has already been noted that the center of gravity of the surface of a deep lake is a no-tide point, and that around this point the cotidal lines are what is here called amphidromic. For a more extended body of water the corrected equilibrium theory still gives a no-tide point, about which the cotidal lines are amphidromic. But the determination of the point and the radiating lines is more difficult than in the case of a small, deep body of water. See § 40, Part I; §§ 49, 50, Part II; and §§ 3, 4, 92, Part IV A.

If a large island lie near the theoretical no-tide point of a body having equilibrium tides the cotidal lines will, because some wave motion is always present, probably radiate from its shores rather than from the exact theoretical point. E. g., the island of Crete, Fig. 19.

Amphidromic cotidal lines depending upon still other causes will be noted in §§ 5, 12, 13, 15, and in the description of cotidal maps.

From expressions (3), (4) Part I, we see that the amplitude of the resultant oscillation (7) is

$$\sqrt{A'^2 \cos^2 lx + A''^2 \cos^2 ly + 2 A' A'' \cos lx \cos ly \cos(\epsilon' - \epsilon'')} \quad (12)$$

and the angle (or negative phase) corresponding to ϵ is

$$\tan^{-1} \frac{A' \cos lx \sin \epsilon' + A'' \cos ly \sin \epsilon''}{A' \cos lx \cos \epsilon' + A'' \cos ly \cos \epsilon''} \quad (13)$$

The amplitude squared and equated to a constant R^2 is the equation of a line of equal range. Its direction is given by the equation

$$\frac{dy}{dx} = -\frac{A' \sin lx}{A'' \sin ly} \frac{A' \cos lx + A'' \cos ly \cos (\epsilon' \sim \epsilon'')}{A'' \cos ly + A' \cos lx \cos (\epsilon' \sim \epsilon'')} \quad (14)$$

For $x=0$ or $x=\mu\pi$, this ratio becomes zero, and for $y=0$ or $y=\nu\pi$, it becomes infinite. Hence: The lines of equal rise and fall are normal to the real or virtual boundaries of a square oscillating area. Cf. § 26, Part IV B, where $\epsilon' \sim \epsilon'' = 60^\circ$, and § 26, Part IV A, where $\epsilon' = \epsilon''$.

5. *The combination of a progressive wave with a stationary wave lying transversely to it.*

Suppose the space origin is so taken that $x=0$ at a loop of the stationary wave and suppose the progressive wave to move toward $+y$, a direction transverse to the motion in the stationary wave. Suppose the time to be reckoned from Greenwich or any other given meridian. Let T' denote the time of high water of the stationary wave at $x=0$, and T'' that of the progressive wave at $y=0$. Then the vertical displacement may be written

$$\begin{aligned} \zeta &= A' \cos lx \cos (at - aT') + A'' \cos (at - ly - aT''), \\ &= A' \cos lx \cos (\theta - \epsilon') + A'' \cos (\theta - ly - \epsilon''). \end{aligned} \quad (15)$$

From $\frac{\partial \zeta}{\partial \theta} = 0$ we have, for finding the time of the resultant high or low water.

$$\tan \theta = \frac{A' \cos lx \sin \epsilon' + A'' \sin (ly + \epsilon'')}{A' \cos lx \cos \epsilon' + A'' \cos (ly + \epsilon'')} \quad (16)$$

The time is referred to the time of high water of the stationary wave at $x=0$ if $\epsilon'=0$; and of the progressive wave at $y=0$ if $\epsilon''=0$.

By assigning a value to θ (i. e. to $a t$), this equation gives a relation between x and y , which is the equation of the cotidal line for the assumed time.

From (16) we have

$$\frac{dy}{dx} = \frac{-\tan \theta (A' \sin lx \cos \epsilon') + A' \sin lx \sin \epsilon'}{\tan \theta (A'' \sin ly + \epsilon'') + A'' \cos (ly + \epsilon'')} \quad (17)$$

which gives the direction of the cotidal line whose characteristic is θ at any permissible point x, y .

. . . All cotidal lines intersect the line $x=0$ or $x=\mu\pi$ perpendicularly.

At a point defined by the equations

$$A' \cos lx = \pm A''. \quad (18)$$

$$\epsilon' = ly + \epsilon'' + \left(\frac{2\nu + 1}{\text{or } 2\nu} \right) \pi \quad (19)$$

there is no rise and fall of tide because ζ then becomes zero. At this point $\tan \theta$ is indeterminate, and all cotidal lines radiate from it. Along the nodal line of the stationary wave the tide is due to the progressive wave alone; therefore the cotidal lines divide

the nodal line into equal parts or segments whose lengths each represent the distance over which the progressive wave moves in one hour. Along the line

$$A' \cos lx = \pm A''$$

the value of the cotidal hour at any point (y) is the arithmetical mean between the hour there given by the progressive wave and the hour of the stationary wave at $x=0$; the times at the given point are always so taken as to not differ more than 6 hours from each other.

Referring to the coordinates of the no-tide point, it will be seen that such a point can not exist unless the amplitude of the stationary wave at the loop exceeds the amplitude of the progressive wave.

CHAPTER II.

ADDITIONAL LEMMAS.

6. In Chapter VII, Part IV A, a number of lemmas are given which seemed to be of chief importance in attempting a partial explanation of the tides. It is convenient to here lay down other lemmas more or less related to those there given, assigning to them numbers which shall form a continuation of those already used.

(26) Two areas which have a considerable stretch of latent or imaginary boundary in common will generally oscillate together and form parts of one and the same system. (See § 29, Part IV A.)

Many of the areas forming parts of the systems shown in Fig. 23, Part IV A, will serve as illustrations.

If, however, the dimensions and locations of the areas are such that the virtual work of the tidal forces upon the system is always small, no considerable tides will result even if the free periods of the areas agree well with the period of the forces; e. g., a short and shallow canal one wave length long (two half-wave lengths placed end-to-end).

(27) If two hypothetical systems, each capable of independent oscillation in a period of 12 hours, overlap, the two motions will generally coexist with little interference; but if their individual periods differ from 12 hours more than does the period of the two when combined together, synchronization will generally take place.

This lack of interference is evident in the case of two systems in a square area whose edges are $\frac{1}{2} \lambda$ in length. (§ 26, Part IV A.)

Consider next an L-shaped figure whose extreme length in either direction is $\frac{1}{2} \lambda$.

Now if the L is slender, it is probable that the two oscillations will synchronize because the free period of both branches of the L differs but little from the free period of either branch. To see this, imagine it to be high water at the angle of the L on account of an oscillation in one branch, and suppose little or no oscillation exists in the other branch. This implies a slope, and so acceleration, along the direction of the second branch (eq. (92), Part IV A); that is, when it is high water in the angle of the L the surface slopes away from the angle in both directions. After several periods the whole body will oscillate as one system. Of course a better oscillation would be obtained by taking the extreme dimensions of the L-shaped figure a little greater than $\frac{1}{2} \lambda$ so that the virtual lengths of the two trapezoidal areas would each be $\frac{1}{2} \lambda$ in length. (Cf. § 43, Part IV A.)

If the L is no longer slender but broad, then although the oscillation in one branch produces a simultaneous slope, and so acceleration, in the other branch, yet if we draw such lines of motion as a synchronization of the two systems into one would imply, it becomes evident that the periods of the resulting trapezoidal areas will be sensibly less

than the period of the forces. Since the surface slope belonging to the oscillation in one branch produces a simultaneous slope in the other branch, the resulting stationary oscillation in the second branch, if there be any, must synchronize with that in the first branch. But, by hypothesis, the dimensions are such that one system can not be formed out of these two branches. The transverse acceleration due to the first branch may affect an existing and sustained oscillation in the second branch, especially if the width of the first branch approaches the value $\frac{1}{4} \lambda$.

Synchronization may therefore generally be expected in a slender L-shaped figure provided either its external dimensions or virtual lengths equal $\frac{1}{2} \lambda$, but in a broad L-shaped region only when its external dimensions exceed $\frac{1}{2} \lambda$ by such an amount that the virtual length of either arm equals $\frac{1}{2} \lambda$.

Other cases of overlapping systems can be studied in a somewhat similar manner.

Examples of overlapping areas are shown in Fig. 23, Part IV A. It is probable that the North Indian system does slightly accelerate the half-wave area extending from Mozambique Channel to Báluchistán and India, also that the South Atlantic system accelerates the North Atlantic system. In each of these two cases the effect of the smaller system upon the larger may be safely neglected. The two Pacific systems must be practically independent of each other, and the same is true of the South Atlantic and South Indian systems.

7. (28) High water occurs remarkably late along the inner side of capes which guard the entrance to a bay or broad passage.

Several cases in which an inner body of water derives its tides through a strait from an outer body have been briefly considered in §§ 102-113, Part IV A. In most of these cases it can be readily seen that in the strait and in its immediate vicinity the time of tide must change rapidly, and so the time of high water on the inner side of capes partially inclosing the bay or broad passage must occur remarkably late.

It is here proposed to call attention to the lateness of the tide at the inner side of the capes in comparison with the time of tide in the near-by channel or axis, the bay being broad enough or so situated with reference to the incoming water that transverse propagation is possible near the capes.

In the first place it may be noted that if the distance between the capes is so great that a wave can be transmitted at approximately the rate due to depth, the crest of the wave will be convex inward and retarded at the capes on account of the lesser depths at and off the capes than in mid-channel. In this connection see § 36, Part IV A.

But even if the depth were as great at the capes as in mid-channel, the tide would still be retarded. At the time of high water between the capes, the neighboring inner waters are at a less elevation. Now, whatever these inward accelerations and resulting velocities may be, the phase of the horizontal velocity in the direction taken by the incoming flood must be in advance of the phase of the horizontal velocity in the transverse direction. For, the longitudinal velocity is generally near its maximum value at the time of high water between the capes, because of the assumed motion transmitted from without, whereas the transverse velocity depending upon the transverse slope or acceleration can hardly have begun—provided the bay is large enough to permit of transverse progressive wave motion. As a matter of fact the incoming flood streams are not all directed parallel to the axis of the bay, but spread out considerably toward either shore of the bay. But high water along the inner side of the cape can not occur until the

flood current along this shore reaches its maximum velocity. Hence we may expect delays of various amounts according to the sharpness of the capes and trend of the shores.

Examples: Cape Chidley, Labrador, Fig. 12; Cape Charles, Virginia, Fig. 14; northern part of New Zealand, Fig. 40.

In most bays, even if wide and shallow, there is no great opportunity for transverse progressions because the shores generally are such that transverse stationary waves would be formed. In such case no considerable delay will occur.

The most conspicuous delays, however, are due to incomplete wave eddies, § 14. Examples are Shantung Promontory and each end of Formosa Island.

8. (29) If a progressive wave is developed at an opening in a rigid boundary of an oscillating area or over a submerged boundary, an *antecedent wave* is necessitated; that is, there must exist a progressive wave moving toward the opening or shoal.

For, the periodic, horizontal, and vertical displacements of water in the opening or over the shoal will entrain displacements far out at sea, whose phases must agree with those in the opening or over the shoal as this part of the coast is approached; e. g., the waters south of the Sunda Islands, and west of Cape Horn.

(30) A stationary wave in a strait involves no antecedent wave.

Three cases of stationary wave motion may be noticed. See §§ 35, 102, 103, Part IV A.

Case 1.—Where one of the bodies connected by the strait has no tide.

At the end of the strait joining the tided body, high water occurs at the time of elongation of the particles in the strait toward that body. Hence no progressive antecedent wave can partake of this motion; e. g., no progressive wave approaches the strait of Gibraltar from the west.

Case 2.—Where the two connected bodies have tides of opposite phase.

At the time of high water in either body the particles in the strait will be at elongation toward that body. For example, no considerable progressive wave approaches the strait between the Färöe Islands and Iceland from the south; the same is true for North Channel.

Case 3.—Where the two connected bodies have tides of like phases.

At the time of high water in either body, as well as throughout the strait, the particles in the strait will be at their elongations toward the middle of the strait, or, rather away from the tided bodies. Many narrow channels between islands are examples of this.

9. (31) If the lines of motion of the water particles off a given part of the shore can be seen to be normal to and directed toward this part of the shore while the lines of motion off a neighboring stretch of shore are parallel with or oblique to the shore line, then, other things being equal, high water will occur earlier in the first-mentioned locality than in the second.

From the first locality a stationary wave will extend seaward, and this, combined with whatever local progression there may be, causes the tide to occur remarkably early as judged from its time of occurrence at neighboring places where the motion of the off-lying particles is oblique or parallel to the shore line. (Cf. lemmas 12, 15, and 16.) Several cases may be noticed.

Case 1.—Stationary wave connected with an oscillating area.

Instances of stationary waves along the shore line are very numerous. In fact, they almost always exist to a greater or less degree immediately at the shore line. They, however, become conspicuous in localities where they cause the tide to occur not only earlier than would otherwise have been expected, but nearly simultaneously over a considerable stretch of coast because the offshore deep-water tide is very nearly simultaneous over an extended region. (Cf. lemma 15 and §§ 99, 112, Part IV A.)

Examples: The northwestern coast of Australia, the outer coast of Scotland north of Ireland, the Atlantic coast of the United States from Rhode Island to Florida, and the eastern coast of Nova Scotia. For many miles out from any of these shores the water is less than 100 fathoms deep, and yet no great amount of progression occurs.

Case 2.—Tide wave progressing toward a large island or a corner of a continent.

As the wave approaches the land the tidal streams will no longer be normal to the direction of its crest at an earlier epoch; but will be deflected as soon as the shore line is felt. In one limited locality, however, the streams may have the direction normal to the crest. This point is therefore characterized by earlier tides. Examples: On the eastern coast of Madagascar, southwestern coast of Sumatra, southeastern coast of Kiusiu, and eastern coast of Formosa, Figs. 7, 36.

Lemma 22 should now be dispensed with.

Case 3.—On and just off the open coast not far from the mouth of an estuary or bay up which the tide progresses.

Consider first a point on the coast some distance from the tidal inlet: High water will occur early on account of the stationary character of the wave, the flood setting shoreward and slack water occurring soon after the time of high water.

As the inlet is approached, the flood just offshore is not directed normally to the shore line of the inlet, but gradually approaches parallelism therewith; hence, the incoming particles are not reflected back from the shore line and do not produce any considerable stationary wave, and so the hastening of the time of tide from this cause does not occur to the extent that it occurred at a point of the coast farther away from the inlet.

This explains why the cotidal lines may be somewhat convex outward off the mouths of tidal rivers, estuaries, etc. In some instances shoals and channels among islands may resemble a tidal estuary or bay in delaying the time of tide at a near-by point of the shore line, the essential thing being a transmitting medium. Examples: The mouth of the Gulf of Cambay, the mouth of Delaware Bay, of Rio de la Plata, off the Golden Gate, off the Loo Choo Islands, and off Spencer Gulf.

Case 4.—At the capes or mouth of an estuary or bay up which body the tide progresses. The set of the currents in the main or outside tidal wave is supposed to be parallel to the general shore line.

Suppose that two capes or bends in the shore line mark the mouth of the tidal estuary or bay. By placing flood arrows just off these capes, the one following the general direction of the outer coast and the other that of the bank of the estuary, the resultant arrow will show that the flood stream must be directed toward the cape on the far side of the estuary and away from the cape on the near side. High water will occur unexpectedly early on the far cape or unexpectedly late on the near one.

The maps of cotidal lines for the North Sea show several instances where the cotidal lines are suddenly broken or recurved at or off the far capes or forelands. The

coast south of the Gulf of Cambay, India, and in the neighborhood of Point Arena, California; the eastern coast of Kiusiu, Japan, and of Basilan, Philippine Islands, are other examples.

Case 5.—A broad reflective bay the direction of whose axis is transverse to the direction of the currents in the main body.

In a broad reflective bay leading off abruptly from the general shore line parallel to which the off-shore tidal currents set, high water may occur earlier along the axis of the bay than at certain shore points nearer to the mouth.

For, the off-shore flood has the general direction of the shore line, and the flood well in the bay must be comparatively weak and be directed toward the head of the bay. The wave being practically stationary along the axis, it will be high water at the head of the bay as early as at its mouth. This tongue of water, having remarkably early tide, gradually produces high water along the shores of the bay in the same manner as a stationary wave in off-shore deep water generally controls the tide at the shore. These small delays may be due to the trend of the shore line of the bay, to shoals, to tidal rivers branching from it, etc. The fact that the off-shore tidal current is supposed to set in the general direction of the shore prevents the water at and outside of the mouth of the bay from all flowing toward the bay and filling it practically instantaneously as a perfectly stationary off-shore wave would have done. That is, the paths of these particles differ somewhat from what the paths would have been had the tide in the bay been simply a stationary wave connected with some oscillating area of the ocean whose direction of motion coincided with the direction of the axis of the bay. Cf. lemmas 9, 28.

The Bay of Biscay tide occurs earlier at the eastern part of the northern coast of Spain than at the western part of the coast. High water occurs earlier at Cadiz than at Cape St. Vincent. It occurs earlier in the Cromarty Firth than along the coast extending eastward. The locality of earliest tide on Cape Cod Peninsula is about the middle of the western coast. See Figs. 15, 18, and 21.

10. (32) In going up a tidal river the rate of propagation at any point is usually somewhat less than that given by Lagrange's formula \sqrt{gh} , where h denotes the mean depth of the cross section.

Reasons for this will be considered in Part V.

CHAPTER III.

MATTERS CONCERNING AMPHIDROMIC REGIONS.

11. *The deflecting force of the earth's rotation.*

From equations (116), (117), Part IV A, we have as the components of the force which a unit particle moving on the earth's surface is capable of exerting because of the earth's rotation,

$$\text{Southward force} = 2k_1 v_e \cos \theta = -2k_1 v_w \cos \theta \quad (20)$$

$$\text{Eastward force} = -2k_1 v_s \cos \theta = 2k_1 v_n \cos \theta \quad (21)$$

$$\therefore \text{Total force} = 2k_1 v \cos \theta. \quad (22)$$

The direction (from the south via east) of action of this force is given by the equation

$$\tan \psi = -\frac{v_s}{v_e} = \frac{v_n}{v_w} \quad (23)$$

and the direction (χ) of the path of the moving particles by

$$\tan \chi = -\frac{v_e}{v_n} = \frac{v_s}{v_w}. \quad (24)$$

The force therefore acts at right angles to the instantaneous path of the particle and is therefore a deflecting force. The moving unit particle has a tendency to crowd, or slightly move relatively to the earth's surface, to the right in the northern hemisphere and to the left in the southern, the force representing this tendency being $2k_1 v \cos \theta$.

This deduction from Laplace's equations of motion is known as Ferrel's law. It was demonstrated and published by Ferrel in 1859, in Runkle's Mathematical Monthly. The paper containing the demonstration has been republished by the Signal Service of the United States Army in Professional Paper No. VIII.

In order to show that the earth's axial rotation must affect the tides, recourse is often had to the case of a body moving in a north-and-south direction at a given latitude but gradually altering its local direction as other latitudes are reached. However, this restriction to north-and-south motions is wholly unnecessary, as equation (22) shows; and, moreover, the results of even these motions are not always understood and correctly presented.

Ferrel thus states the law, and comments upon it:

*Hence in whatever direction a body moves on the surface of the earth, there is a force arising from the earth's rotation, which deflects it to the right in the northern hemisphere, but to the left in the southern. This is an extension of the principle upon which the theory of the trade winds is based, and which has been heretofore supposed to be true only of bodies moving in the direction of the meridian.**

* Runkle's Mathematical Monthly, Vol. I (1859), p. 307.

It is of interest to quote here from Prof. G. H. Darwin's recent book on tides. Of course, the author's purpose is illustration rather than comprehensiveness or extreme accuracy; but the statements are somewhat misleading in that they impress one with the necessity of north-and-south motion in order that a deflecting force shall arise. Moreover the second sentence is misleading even for north-and-south motion. In fact, the east-and-west acquired component velocity relative to the earth's surface is not simply the difference between the absolute velocities of two points fixed on the surface of the rotating earth. From D'Alembert's principle we readily obtain

$$\frac{d}{dt} \sum m \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right) = \sum m (xY - yX), \quad (25)$$

which equals zero because gravity, being the only impressed force considered, passes through the axis of rotation. For one particle we have

$$m \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right) = \text{constant} = mr^2 \frac{d\phi}{dt} = r \cdot mr \frac{d\phi}{dt} = \text{moment of momentum.} \quad (26)$$

That is, the total eastward velocity times r remains the same for all latitudes. In these equations the coördinate axes are fixed in space and r denotes the distance to the axis of rotation.

When, in the northern hemisphere, water moves from north to south it passes from a place where the surface of the earth is moving slower, to where it is moving quicker. Then, as the water goes to the south, it carries with it only the velocity adapted to the northern latitude, and so gets left behind by the earth. Since the earth spins from west to east, a southerly current acquires a westward trend. Conversely, when water is carried northward of its proper latitude, it leaves the earth behind and is carried eastward. Hence the water can not oscillate northward and southward, without at the same time oscillating eastward and westward.*

That a particle moving upon the earth's surface with an eastward velocity tends toward the equator, while a particle having a westward velocity tends toward the poles can be readily seen from the fact that the ellipticity of the earth's meridian is chiefly, directly or indirectly, due to the motion of rotation. Therefore an addition to the absolute eastward velocity increases the ellipticity while a diminution in this velocity lessens the ellipticity.†

Since the earth rotates 360° in one sidereal day, its angular velocity (k_1) is 0.000 072 921 radian per second, and so the deflecting force becomes

$$0.000\ 145\ 842\ v \cos \theta \quad (27)$$

poundals. This force divided by g gives

$$0.000\ 004\ 533\ 18\ v \cos \theta \quad (28)$$

where v is expressed in feet per second, or

* The Tides and Kindred Phenomena in the Solar System (American edition), p. 176.

† The following references relate to the subject of the earth's deflecting force: Ferrel, A Popular Treatise on the Winds, Ch. II. Lamb, Hydrodynamics, p. 322. Under Maclaurin, this manual, Part I, § 95.

$$0.000\ 014\ 872\ 6\ v\ \cos\ \theta \quad (29)$$

where v is expressed in meters per second. This is the gradient or transverse slope of a narrow stream arising from the earth's deflecting force.

12. *Amphidromic regions in straits and canals.*

In §§ 4, 5 amphidromic regions due to two systems of free waves in the same body have been briefly considered. Mention is there made of the fact that small, deep bodies of water which obey the corrected equilibrium theory constitute such regions. This section deals with those dependent upon the deflecting force of the earth's rotation.

If a channel or strait of sufficient length be so narrow that its free transverse oscillation period be several times less than twelve hours, the flood and ebb streams will give rise to the transverse slope just indicated.

This transverse oscillation may be combined with longitudinal oscillations, either stationary or progressive, in ways analogous to those given in Chapter I. Of course, this treatment will be only approximate because the character of such motions, when influenced by the earth's rotation, has never been fully ascertained. However, if the transverse motions are small, this treatment must be nearly correct. Allowance must be made for the variation in velocity at different parts of the channel, as the value of the transverse slope depends directly upon the velocity of the stream.

In a strait connecting two bodies of water whose high waters differ in time of occurrence by six hours, and so indicate the existence of a nodal line across the strait, the alternating transverse slope will reduce the nodal line to a no-tide point around which the cotidal lines will be amphidromic; the progression, i. e., sequence of tidal hours, will be counterclockwise in the Northern Hemisphere and clockwise in the Southern. Similarly for a stationary oscillation in a canal more than $\frac{1}{4}\lambda$ in length.

Examples of this are North Channel, Strait of Korea, Norton Sound, arm of sea between Holland and England (Figs. 20, 22, 34, 36).

In a narrow canal or tidal river up which the tide wave is propagated at the rate due to depth, the range of tide will be increased upon that side toward which the flood stream crowds and decreased upon the opposite side. The times of the tide will, however, not be altered.

In a canal-like body having both a stationary and a progressive wave, the no-tide point will either disappear or be transferred from the position given by the stationary wave. The effects can be seen by combining all waves, as in Chapter I. Suppose high water of the progressive wave falls upon the nodal line of the stationary wave at the time of maximum flood stream; then for a small amplitude of the progressive wave the no-tide point will be moved from the center of the stream toward one side, and for a large amplitude the no-tide point will cease to exist, but a crowding up of the cotidal lines will occur on the one side and a spreading out on the other.

Examples are the English Channel, Irish Channel, Gulf of California, Gulf of Georgia, and Adriatic Sea (Figs. 19, 20, 31, 32).

13. *Dependent landlocked wave eddies.*

In § 115, Part IV A, a brief account is given of ordinary eddies; i. e., eddies in which the flow is everywhere steady for a time at least. We are now to consider

eddies in bodies of water which are largely surrounded by land, and which are so large and shallow that various phases of the tidal streams exist simultaneously. Comparison may be made with cases 2 and 5 of the section just referred to.

The motion may be assumed to begin at a point of the coast of the embayment (or body of water covered by the wave eddy) where the rise and fall is comparatively large. This can be inferred from the rise and fall of the neighboring waters, upon which the derived wave eddy depends. The wave proceeds thence along the shores of the embayment, but gradually decreasing in amplitude, until it again joins the main or outside body of water or passes into some other arm of the sea. Through the greater part of its course the rate of the shore wave is about that due to depth; but there may be much crowding together or spreading apart of the cotidal lines where the wave finally joins the main body. Generally speaking, the nearer this shore wave comes to agreeing in phase with the main body where the latter is finally reached, the more likely are such wave eddies to exist. The point of beginning of the shore wave is often a cape or headland where the coast suddenly recedes from the general direction of the tidal motion in the outside body. (Of. Lemma 19.)

Examples are the southern part of the Gulf of St. Lawrence, Fox Channel, southern part of the Gulf of Pechili, the waters between Borneo and Malacca Strait, and the main body of the North Sea. The first three are twelve-hour eddies, the fourth a twenty-four-hour eddy, and the North Sea may be regarded as an incomplete eddy, since the final progression passes into the Skagerrak. There may, however, be a small wave from the Naze toward Stavanger, thus completing the twenty-four hours. (See Figs. 7, 13, 21, 22, 26, 36.)

Suppose that for some distance the main body of water rises and falls simultaneously, but that the amplitude decreases from the cape at which the progression of the eddy is supposed to originate toward, but not generally as far as the other cape or corner of the embayment. This diminution is generally due to the failure of any considerable transverse wave motion to set out from the tide of the main body across the central or some other part of the opening of the embayment because of the deflecting influence or shielding effects of near-by capes and lands. It is sometimes due to there being a nodal line of the main body abreast the center of the opening. In this case an island situated near the center of the opening will facilitate the formation of the wave eddy.

Consider the low water of the shore wave a short time after leaving the first cape. The surface slope from the region of the main body where the amplitude is small, toward the locality of low water just mentioned, will then have its maximum value. The maximum radial velocity will, because the motion is oscillatory, occur three hours later, and so high water from this cause will occur six hours later; i. e., will coincide in time with the high water of the progressive transverse shore wave. The causes which produce transverse motion at the cape do not exist to any great extent near the region of small rise and fall, and so the apparent transverse progression in the embayment is governed by the progression along the shore. The cotidal lines radiate from some point or locality where the range of tide disappears or becomes very small. If the circuit requires 24 hours instead of 12, it is probable that the range of tide nearly disappears over a region of considerable size; because radial flows in nearly opposite directions for opposite points of the eddy are in this case necessitated at one and the same time.

In all of these cases the cotidal lines may be spoken of as amphidromic.

Each of the wave eddies referred to in this section is dependent upon the tide of an outside or main body of water with which it is connected. But dependent wave eddies are not always landlocked. Suppose the tide wave to be progressing from open water toward the coast through shallow waters. Suppose a broad gulf in the shore line converts the incoming wave into a stationary wave of large range at the head of the gulf and of small range $\frac{1}{4}\lambda$ seaward from this point.* Now suppose that from one of the capes marking the head of the gulf the adjacent shore line so recedes that a progressive wave sets along the coast and even outward.† For a while it will have a greater range than has the incoming tide. But as it must finally join the latter there will be much crowding up of the cotidal lines soon after leaving the cape; consequently there will be little or no progression at the rate due to depth, as was assumed to characterize the shore wave of a landlocked eddy during a portion of its course.

Two such eddies are situated off the eastern coast of Patagonia (see Fig. 29).

In all these cases the effect of the earth's rotation, although of course present, is probably insignificant, notwithstanding the fact that the order of the cotidal lines is generally counterclockwise in the northern hemisphere and clockwise in the southern.

14. *Incomplete eddies, and islands.*

Incomplete eddies more or less landlocked may be formed in a similar way. Referring to Fig. 36 it will be noticed that at Shantung Promontory the range of tide is only 4 feet, while at the cape on opposite Korean coast it is 7.7 feet. The shore wave follows the Korean coast and governs the time of tide in the shallow sea between the Yellow Sea and Pechili Strait. Shantung Promontory is a turning point for the tide around which the cotidal lines are crowded together. Lemma 28 applies to this case, but it also applies to many capes at which the range of tide is even greater than in the offing. The southwestern corner of the Falkland Islands is a turning point for an imperfect wave eddy, whose sustaining shore line extends from Staten Island to Port Santa Cruz. Cape Chidley and Ungava Bay form another example.

In channels of moderate width incomplete eddies may result from the deflecting force of the earth's rotation. Examples are given at the close of § 12.

If near one end of an island in the ocean there is a much larger tide than prevails off the other coasts of the island, the tide wave may appear to progress completely around it and the tidal hours form a complete cycle of values. The cotidal lines are made to radiate, as it were, from the island from two causes—the first, that given under lemma 28, depending upon the curvature and trend of the shore line, and the second that given in § 36, Part IV A, and depending upon the shoaling around the island. Examples are New Zealand and Iceland.

Generally, however, the wave which passes around both ends of an island is so nearly alike in range and in phase that although some of the cotidal lines radiate from the island (both causes just mentioned being operative) only certain tidal hours will be represented. Examples of this are the Falkland Islands, New Caledonia, Formosa, and Franz Josef Land.

* Cf. § 34, Part IV A.

† Lemma 19.

15. *Amphidromic regions in the ocean.*

If an ocean be conceived to oscillate in more than one system it may happen that around a certain point or region the theoretical hours of the tide; i. e. those given by the tidal systems, form a cycle of values.

In § 4 the cotidal lines for two systems in a square body of water have been discussed. An irregular progression from the points representing the middle of the loops of one system to similar points of the other system was shown to exist—the direction of the progression being such that the time difference between any two adjacent loops is less than six hours. But if a nodal line of one system, whatever its form, crosses a nodal line of the other there must be a progression about a no-tide point, or point where the nodal lines intersect; for, these lines may always be regarded as right lines for some distance from their intersection, and the particles of each system there oscillate as if near the nodal lines of two rectangular areas. Upon referring to Fig. 23, Part IV A, it is evident that one such progression must occur along the coast of Somali Land and another along the coast of California.

Antecedent waves which move toward openings in the coast line (or toward suddenly receding parts of it) at approximately the rate due to depth, often govern the time of tide in certain parts of amphidromic regions, especially near the nodal lines and the free boundaries of the oscillating areas, where the range of the direct tide is small. Such a progression, due to the openings and shallow seas into and around the British Isles, to Denmark Strait, and to the opening between Cape Farewell and Labrador, extends across the northern part of the North Atlantic Ocean (Figs. 6, 12, 18, and 20).

The southwesterly progression toward the Fiji Islands and New Caledonia influences the time of tide in the northwestern half of the amphidromic region whose no-tide point is situated northwest of the Society Islands.

According to lemma 25, between two not too distant nonsimultaneous regions, the time of tide will gradually change from the value belonging to the one to that of the other; i. e., the tide wave will seem to progress, but not as a free wave at the rate due to depth. That this must be true is self-evident; but how the change takes place between two regions belonging to different systems has not been ascertained.

If two bodies of water having tides of their own, which differ in time by six hours, are connected by a strait, the wave of the strait is stationary and there will be no progression from the one to the other, and a nodal line will extend across the strait.* If now on one side of the strait a progression exists it will obliterate a portion of the nodal line, and a very rapid progression between the two bodies will be observed. The horizontal motions of the stationary and progressive waves may be supposed to agree in phase at the nodal line. Similarly, for a progression which may exist on the other side of the strait. If both progressions are in the same direction the cotidal lines will simply bunch up in the strait. If the progressions take opposite directions on opposite shores the strait will constitute an amphidromic region. The cotidal lines will either radiate from a no-tide point, or, if the amplitudes of the progressive waves become zero not far from the shores, they will branch off from the nodal line. In either case there will be at each end of the strait a region of nearly simultaneous tide, while along the shores the time of tide changes rapidly.

* Cf. §§ 35, 102, Part IV A.

Examples: The broad strait between Iceland and the Färöe Islands has at its Iceland side a southerly progression due to the considerable tide passing through Denmark Strait and following the northern and eastern coasts of Iceland; at its Färöe side it has a northward progression due to the fact that these islands form a part of the broken boundary adjoining a good rise and fall.* In a similar way is explained the amphidromic region between the Färöe and Shetland islands.

As noted in the last section, islands or capes accompanied by gradual shoaling of near-by waters often have the effect of turning or deflecting toward themselves progressive waves. The reasons for this are obvious in case of a solitary wave advancing at rates due to depths. The phenomenon is analogous to refraction.† Other means of accomplishing deflections are given under lemma 28.

Amphidromic regions in the ocean generally have one or two broad spaces or sectors of practically simultaneous tides. Over such sectors none of the three causes just mentioned as giving rise to apparent progressions are more than barely discernible. They are regions in which progressions from other quarters of the ocean lose themselves or in which progressions to distant quarters begin to appear. According to circumstances the range of the tide may be considerable or it may be small.

That the deflecting force of the earth's rotation materially assists in the production of amphidromic regions in the ocean is doubtful; moreover, the one in the Indian Ocean lies under the Equator and the one around Tahiti lies comparatively near to it.

16. *The principal amphidromic regions of various kinds.*

Location	Position of no-tide point or center		Kind
	Latitude	Longitude	
Indian Ocean	1 33 S.	64 52 E.	Ocean eddy
S. E. of Newfoundland	40 00 N.	40 00 W.	Ocean eddy
Off Holland	52 27½ N.	3 14½ E.	Channel eddy ‡
S. W. of Norway	57 37 N.	5 15 E.	Wave eddy
North Channel	55 18 N.	6 00 W.	Channel eddy ‡
Bet. Shetland and Färöe Is.	61 39 N.	5 25 W.	Broad-strait eddy
Bet. Färöe and Iceland	63 06 N.	10 18 W.	Broad-strait eddy
Fox Channel	69 09 N.	79 00 W.	Wave eddy
Off Patagonia	45 18 S.	63 37 W.	Wave eddy
Off Patagonia	40 55 S.	60 45 W.	Wave eddy
Bet. California and Hawaii	30 25 N.	141 25 W.	Ocean eddy
Norton Sound	63 53 N.	163 41 W.	Channel eddy ‡
Strait of Korea	35 32 N.	130 45 E.	Channel eddy ‡
Gulf of Pechili	38 24 N.	119 47 E.	Wave eddy
West of Borneo	0 21 S.	107 20 E.	Wave eddy
N. W. of Society Is.	14 7½ S.	153 13 W.	Ocean eddy
S. E. of New Zealand	51 30 S.	172 10 W.	Ocean eddy
Mediterranean Sea	Isle of Crete		Equilibrium eddy
Gulf of St. Lawrence	Magdalen Islands		Wave eddy
W. end of Java Sea	Billiton Island		Wave eddy

* Lemma 14.

† See § 36, Part IV A.

‡ That is an eddy due to the deflecting force of the earth's rotation acting upon a stationary wave which possesses a nodal line and occurs in a channel, strait, or sound whose width is a moderately small fraction of λ , § 12.

CHAPTER IV.

COTIDAL LINES.

17. *Remarks on cotidal charts.*

Definite suggestions concerning cotidal lines were made by Dr. Thomas Young about a century ago.* In the Philosophical Transactions for 1831, Sir John W. Lubbock marked upon two charts, one for Great Britain and one for the world, the Greenwich and local times of the tide at the time of new or full moon. In a few instances he indicated the positions of cotidal lines. Dr. William Whewell drew cotidal lines for Great Britain, the coasts of Europe, and the world at large, although attempting little in the Pacific Ocean.†

A few years later, Sir Geo. B. Airy gave in his *Tides and Waves* a chart for Great Britain and one for the world based chiefly upon the charts of Whewell.

A map of the world showing the establishments, Greenwich times of tide, and spring ranges is given in a book entitled "*The Tides*" published by the Society for Promoting Christian Knowledge, London, 1857. A chart of the coasts of Europe, showing the Greenwich times of tide, the spring ranges, also the character of the tide waves, whether progressive or stationary, accompanies the text. No attempt is there made at drawing the cotidal lines.

In the U. S. Coast Survey Reports for 1854 and 1857, Superintendent Bache gives a sketch of the cotidal lines for the Atlantic Coast; a similar sketch is given for the Pacific Coast in the Report for 1855. In the Report for 1856 a sketch is given for the diurnal wave of the Gulf of Mexico, and in the Report for 1862, maps for both diurnal and semidiurnal waves around the Gulf coast of Florida.

Many other maps of cotidal lines for limited areas have been published at various times. The most recent cotidal chart of the world is one by Berghaus, constructed in 1889-90, and which accompanies his *Physikalischer Atlas*; it is reproduced as Fig. 25, Part IV A. He also gives a chart for western Europe, reproduced as Fig. 26, Part IV A, one for the East Indies and China, and one for the West Indies and the Atlantic coast of the United States. Van der Stok, in his book entitled "*Wind and Weather, Currents, and Tidal Streams in the East Indian Archipelago*," gives two maps around this archipelago, which are reproduced as Figs. 29, 30, Part IV A.

Generally the cotidal charts have been constructed for the purpose of showing the Greenwich lunar time of high water on the days of full and change of the moon; that

* Lectures on Natural Philosophy, Vol. 1; Nicholson's Journal, Vol. 35 (1813), pp. 145-217; Encyclopedia Britannica (Eighth Ed.), article "Tides;" Miscellaneous Works, Vol. II.

† Phil. Trans., 1833, 1836, 1848.

is, the vulgar establishment is converted into lunar hours and to it the west longitude, expressed in hours, is added or from it the east longitude is subtracted. If harmonic analyses were sufficiently numerous, the establishment obtained by dividing M_2^0 by 30 could be used to advantage; the chief drawback to this proceeding would be the fact that the mean time of high water is affected by the higher harmonics M_4 , M_6 , M_8 , . . .

In the charts accompanying this paper, the mean or corrected establishment has been used. While this may, from a scientific point of view, be less desirable than the M_2 establishment, it has the advantage of referring to actual high water instead of to component high water. The difference may be considerable in rivers and other shallow bodies of water.

The Roman numerals denote the Greenwich lunar time of mean high water. *The side of the line upon which a numeral is written indicates the direction in which the wave appears to progress.* Of course in most oceans and deep bodies of water simple progressions of free waves at rates due to depth seldom occur by themselves; and reasons for this fact have already been given. For convenience the word "progression" will be constantly used in connection with any sequential change however irregular or complicated this may be.

At most localities the vulgar establishment is about one-fourth of a lunar hour greater than the corrected. The approximate cotidal hours from the values of M_2 intervals are given in § 97, Part IV A; § 19, Part IV B. The values of the true mean intervals, which alone are supposed to be used in the accompanying charts, have not always been determined. Many are given in the Coast and Geodetic Survey Tide Tables.

The Arabic numerals scattered over the cotidal maps indicate mean ranges of the tide in feet at the points or localities to which they refer. If the values are estimated from spring and neap ranges they are generally given to whole feet only; if, however, a somewhat closer estimate is attempted, common, and not decimal, fractions are used. The latter fractions generally signify that the mean ranges have been determined either by computation from harmonic constants (according to rules given in Part III) or have been deduced directly from observations. If the diurnal wave is so large that only one high water and one low water occurs daily when the moon is near extreme declination, the diurnal components are omitted from the computation of the mean semidaily range of tide, and the resulting ranges are bracketed.

Charts of ocean depths are given as Figs. 19, 20, Part IV A, but in actually constructing cotidal lines many detailed charts are required. A few of the more interesting localities are given as Figs. 31-39, Part IV A.

In shallow bodies there may be progressions at rates due to the depths. Table 50, Part II, may be adapted to such cases. Table 51, Part IV A, is for deeper bodies.

Time and data are wanting for constructing cotidal lines for the diurnal tides, but this work may be undertaken in the future. It is hardly necessary to say that the accompanying charts have cost much labor, and still leave much to be done.

18. Sources of information.

Much data relating to tides can be found in §§ 79-97, Part IV A. A table containing the more important harmonic constants from analyses recently available is given in § 19. It forms a continuation of the table found in § 97, Part IV A. A brief list

for general reference is given here. Other references intended for particular regions or localities will appear in connection with the descriptions of the cotidal lines. Many sources of minor importance, although used in constructing the charts, will not be referred to.

Tide Tables for the British and Irish Ports, by the Admiralty.

Tide Tables, by the Coast and Geodetic Survey.

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Proceedings of the Royal Society of London, especially Vol. 39 (1885), Vol. 45 (1889), Vol. 71 (1902).

Reports of the British Association for the Advancement of Science.

Annales Hydrographiques.

Reports of the Survey of India, 1886-89, 1892-95, 1900-1901.

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Comptes-Rendus des Séances de l'Association Géodésique Internationale for 1900, Vol. II, special report.

J. P. Van der Stok: Wind and Weather, Currents, and Tidal Streams in the East Indian Archipelago. Batavia, 1897.

W. Bell Dawson: Reports of the Survey of Tides and Currents in Canadian Waters.

Archives Néerlandaises des Sciences Exactes et Naturelles, 2d Series, Vol. 6.

Algemeene Dienst van den Waterstaat. Verzamelingstabel der Waterhoogten, for the month of April, 1894.

Vandstands-Observationer, Udgivet af den norske Gradmaalings-Kommission. Christiania, 1882.

Resultater af Vandstands-Observationer paa den Norske Kyst Udgivet af den Norske Gradmaalings-Kommission. Christiania, 1904.

Narrative of the Surveying Voyages of His Majesty's Ships Adventure and Beagle, Appendix to Vol. II. London, 1839.

Voyage autour du Monde, de l'Uranie, Vol. II. Paris, 1826.

Voyage autour du Monde sur la Frigate Vénus. Paris, 1844.

Data relating to tides and tidal streams in nearly all parts of the world may be found in the various Pilots, also upon many of the Charts, both of which are published by the British Admiralty. Somewhat similar matter may be found in the Pilots and upon the Charts issued by the U. S. Hydrographic Office; also, for the United States and dependencies, in the Coast Pilots issued by the Coast and Geodetic Survey. Many references to particular sources of information are given in the preface to the Coast and Geodetic Survey Tide Tables.

19. *Intervals, ranges, cotidal hours, etc., derived from harmonic constants.*

The following table forms a continuation of the one given under § 97, Part IV A, where a brief description may be found.

No.	Station.	Geographic position.				M ^g .		S ₂ .	S ^g .	N ₂ .	N ^g .	K ₁ .
		Latitude.	Longitude.		M ₂ .	De- grees.	Lunar hours.					
			Arc.	Time.								
	EAST COAST OF AMERICA.	° ' "	° ' "	h. m. Ft.	°	h.	Ft.	°	Ft.	°	Ft.	
		North.	West.									
154	Nassau, Bahamas	25 05	77 21	5 09 1.24	213.4	7.11	0.21	237	0.30	191	0.28	
155	Great Harbor, Culebra Island	18 18	65 17	4 21 0.29	241.2	8.04	0.04	266	0.05	223	0.25	
156	San Juan	18 29	66 07	4 24 0.49	246.3	8.21	0.07	267	0.11	232	0.27	
157	Ponce, P. R.	17 59	66 40	4 27 0.03	280.0	9.33	0.02	264	0.01	160	0.24	
165	Colon	9 18	79 51	5 19 0.27	8.2	0.27	0.03	195	0.37	
170	Iles du Salut	5 17	52 35	3 30 2.59	118	3.93	0.92	142	0.33	
		South.										
180	Rio de Janeiro	22 55	43 09	2 53 1.03	82.7	2.76	0.59	75	0.16	
184	Montevideo	34 53	56 12	3 45 0.19	34.2	1.14	0.04	318	0.06	354	0.05	
	WEST COAST OF AMERICA.	North.										
251	Sergius Narrows	57 25	135 38	9 03 4.93	11.7	0.39	1.66	45	1.06	348	1.58	
251.5	Hooniah	58 07	135 47	9 03 5.97	14.2	0.47	2.03	48	1.12	341	1.71	
252	Port Althorp	58 07	136 17	9 05 3.01	353.5	11.78	1.13	35	1.48	
252.5	Granite Cove	58 12	136 24	9 06 4.01	5.9	0.20	1.29	38	0.77	337	1.45	
255	Kokinhenic I	60 18	145 03	9 40 1.12	11.9	0.40	0.28	51	0.26	348	0.41	
256	Pete Dahl Slough	60 23	145 24	9 42 3.52	12.7	0.42	1.05	46	0.64	358	1.57	
259	Orca, Prince William Sound	60 34	145 41	9 43 4.52	357.7	11.92	1.61	40	0.88	335	1.53	
260	Orca, Cape Whitshed	60 28	145 55	9 44 4.42	8.4	0.28	1.56	44	0.80	344	1.51	
261	Camp April	60 32	146 00	9 44 4.54	356.0	11.87	1.53	32	0.91	331	1.47	
262	Valdez Arm	61 07	146 27	9 46 4.51	353.7	11.79	1.52	25	0.86	327	1.66	
267	Peterson Bay	54 24	162 38	10 51 1.92	354.8	11.83	0.73	18	0.37	342	1.36	
268	Tigalda Bay	54 05	165 10	11 01 0.38	60.1	2.00	0.28	5	0.20	47	1.09	
269	Unalga Bay	54 00	166 10	11 05 0.61	105.2	3.51	0.13	304	0.29	67	1.06	
270	Dutch Harbor	53 54	166 32	11 06 0.86	111.5	3.72	0.07	350	0.31	62	1.06	
271	Kashega Bay	53 26	167 05	11 08 0.71	95.5	3.18	0.11	91	1.13	
285	Port Clarence	65 14	166 24	11 06 0.47	213.4	7.11	0.03	346	0.14	133	0.25	
	EAST COAST OF ASIA.											
300	Pitlekaj	67 03	173 30	11 34 0.10	4.8	0.16	0.03	69	
		East.										
309	Tomari	43 46	145 29	9 42 0.94	107.9	3.60	0.43	149	0.85	
317	Kiritappu, Yezo	43 03	145 10	9 41 0.92	105.4	3.51	0.43	144	0.68	
328.5	Ohatake, Nippon	41 25	141 10	9 25 0.89	108.5	3.62	0.42	144	0.59	
333	Orinohama	38 23	141 26	9 26 1.11	114.3	3.81	0.52	152	0.77	
336	Hirataka	36 51	140 48	9 23 0.97	115.4	3.85	0.48	145	0.72	
337	Chosi Kawaguchi	35 44	140 50	9 23 0.67	132.8	4.43	0.23	173	0.63	
338	Nagasaki	35 42	140 51	9 23 1.11	118.8	3.96	0.51	149	0.73	
339	Katsura	35 10	140 17	9 21 1.19	138.3	4.61	0.52	170	0.78	
339.5	Otohamo	34 55	139 56	9 20 1.10	143.2	4.77	0.53	180	0.69	
341	Shinagawa	35 37	139 45	9 19 1.60	159.0	5.30	0.74	191	0.71	
341.5	Kanagawa	35 28	139 39	9 19 1.45	140.0	4.67	0.80	181	0.83	
342.1	Uraga nishi Uraga	35 15	139 44	9 19 1.24	145.9	4.86	0.60	177	0.78	
342.2	Aburatsubo	35 09	139 37	9 18 1.21	143.4	4.78	0.53	173	0.77	
342.3	Hashirimitsu	35 15	139 44	9 19 1.36	150.3	5.01	0.66	181	0.77	
342.4	Sagami Daiichi Kaiho	35 19	139 46	9 19 1.46	148.7	4.96	0.69	180	0.81	
342.6	Yokosuka	35 18	139 39	9 19 1.43	152.3	5.08	0.69	181	0.77	
342.7	Shimoda	34 40	138 57	9 16 1.29	162.4	5.41	0.59	186	0.79	
342.8	Tago	34 48	138 47	9 15 1.33	166.9	5.56	0.69	186	0.74	
357	Hii	33 55	135 06	9 00 1.52	180.8	6.03	0.73	203	0.70	
357.1	Osaki, Inland Sea	34 07	135 09	9 01 1.49	185.7	6.19	0.73	207	0.77	
357.2	Wakanoura Dejima, Inland Sea	34 11	135 11	9 01 1.40	185.2	6.17	0.68	225	0.75	
357.3	Osaka, Inland Sea	34 39	135 26	9 02 1.01	214.8	7.16	0.62	227	0.88	

K ₁ ^o .	O ₁ .	O ₁ ^o .	P ₁ .	P ₁ ^o .	S ₂ M ₂ .	N ₂ M ₂ .	O ₁ K ₁ .	P ₁ K ₁ .	K ₁ + O ₁ .	S ₂ - M ₂ ^o .	M ₂ ^o - N ₂ ^o .	K ₁ ^o - O ₁ ^o .	† (K ₁ ^o + O ₁ ^o).		Cotidal hour.		No.
													De- grees.	Lunar hours.	Semi- diur- nal.	Diur- nal.	
°	Fl.	°	Fl.	°					Fl.	°	°	°	°	h.	h.	h.	
120	0.21	124	0.09	122	0.17	0.24	0.75	0.32	0.49	24	22	-4	122	8.13	0.26	13.28	154
162	0.19	155	0.14	0.17	0.76	0.44	25	18	7	158.5	10.57	0.39	14.92	155
163	0.24	161	0.14	0.22	0.89	0.51	21	14	2	162	10.80	0.61	15.20	156
186	0.18	175	0.67	0.33	0.75	0.42	-16	120	11	180.5	12.03	5.72	157
158	0.20	160	0.11	0.54	0.57	-173	-2	159	10.60	5.59	15.92	165
199	0.23	181	0.36	0.70	0.56	24	18	190	12.67	7.43	16.17	170
161	0.33	101	0.57	2.06	0.49	-8	60	131	8.73	5.64	11.61	180
318	0.02	256	0.21	0.32	0.40	0.07	-76	40	62	287	19.13	4.89	22.88	184
130	1.03	111	0.34	0.22	0.65	2.61	33	24	19	120.5	8.03	9.44	17.08	251
130	0.98	111	0.34	0.19	0.57	2.69	34	33	19	120.5	8.03	9.52	17.08	251.5
119	0.65	99	0.31	0.44	2.13	41	20	109	7.27	8.86	16.35	252
126	0.94	114	0.32	0.19	0.65	2.39	32	29	12	120	8.00	9.30	17.10	252.5
157	0.34	178	0.25	0.23	0.83	0.75	39	24	-21	167.5	11.17	10.07	20.84	255
137	0.84	121	0.30	0.18	0.54	2.41	33	15	16	129	8.60	10.12	18.30	256
130	0.98	115	0.36	0.19	0.64	2.51	42	23	15	122.5	8.17	9.64	17.89	259
130	1.06	118	0.35	0.18	0.70	2.57	36	24	12	124	8.27	10.01	18.00	260
124	0.98	110	0.34	0.20	0.67	2.45	36	25	14	117	7.80	9.60	17.53	261
123	0.97	111	0.34	0.19	0.58	2.63	31	27	12	117	7.80	9.56	17.57	262
124	0.77	97	0.38	0.19	0.57	2.13	23	13	27	110.5	7.37	10.68	18.22	267
146	0.63	134	0.74	0.53	0.58	1.72	-55	13	12	140	9.33	1.02	20.35	268
148	0.72	131	0.21	0.53	0.68	1.78	-161	38	17	139.5	9.30	2.59	20.38	269
152	0.72	142	0.08	0.36	0.68	1.78	-122	49	10	147	9.80	2.82	20.90	270
151	0.74	129	0.15	0.65	1.87	-5	22	140	9.33	2.31	20.46	271
115	0.12	287	0.06	0.30	0.48	0.37	133	80	-172	201	13.40	6.21	0.50	285
.....	0.30	11.73	300
160	0.70	142	0.46	0.82	1.55	64	18	151	10.07	5.90	0.37	309
152	0.62	161	0.47	0.91	1.30	41	-9	156.5	10.43	5.83	0.75	317
144	0.48	142	0.47	0.81	1.07	35	2	143	9.53	6.20	0.11	328.5
156	0.66	152	0.47	0.86	1.43	38	4	154	10.27	6.38	0.84	333
159	0.56	156	0.49	0.78	1.28	30	3	157.5	10.50	6.47	1.12	336
174	0.44	166	0.34	0.70	1.07	40	8	170	11.33	7.05	1.95	337
161	0.59	150	0.46	0.81	1.32	30	11	155.5	10.37	6.58	0.99	338
166	0.56	154	0.44	0.72	1.34	31	12	160	10.67	7.26	1.32	339
172	0.60	155	0.48	0.87	1.29	37	17	163.5	10.90	7.44	1.57	339.5
169	0.61	164	0.46	0.86	1.32	32	5	166.5	11.10	7.98	1.78	341
164	0.63	160	0.55	0.76	1.46	41	4	162.5	10.80	7.35	1.48	341.5
163	0.61	154	0.48	0.78	1.49	31	9	158.5	10.57	7.54	1.25	342.1
170	0.59	154	0.44	0.77	1.36	30	16	162	10.80	7.48	1.50	342.2
169	0.64	158	0.49	0.83	1.41	31	11	163.5	10.90	7.69	1.58	342.3
165	0.58	158	0.47	0.72	1.39	31	7	161.5	10.77	7.64	1.45	342.4
164	0.61	137	0.48	0.79	1.38	29	27	150.5	10.03	7.76	0.71	342.6
169	0.64	163	0.46	0.81	1.43	24	6	166.0	11.07	8.14	1.80	342.7
182	0.50	170	0.52	0.68	1.24	19	12	176	11.73	8.31	2.48	342.8
177	0.55	172	0.48	0.79	1.25	22	5	174.5	11.63	9.03	2.63	357
182	0.63	170	0.49	0.82	1.40	21	12	176	11.73	9.17	2.71	357.1
189	0.57	165	0.49	0.76	1.32	40	24	177	11.80	9.15	2.78	357.2
195	0.65	180	0.61	0.74	1.53	12	15	187.5	12.50	10.13	3.47	357.3

No.	Station.	Geographic position.					M ² .		S ₂ .	S ₂ ⁰ .	N ₂ .	N ₂ ⁰ .	K ₁ .
		Latitude.	Longitude.		M ₂ .	De- grees.	Lunar hours.						
			Arc.	Time.									
	EAST COAST OF ASIA—continued.	° /	° /	h. m.	Fl.	°	h.	Fl.	°	Fl.	°	Fl.	
357.4	Kobe, Inland Sea	34 41	135 11	9 01	1.02	216.0	7.20	0.55	220			0.84	
357.5	Akashi, Inland Sea	34 39	134 59	9 00	3.44	256.7	8.50	1.31	245			0.83	
357.6	Shikama, Inland Sea	34 47	134 41	8 59	0.89	319.5	10.65	0.37	306			1.02	
362.05	Setoda, Inland Sea	34 18	133 05	8 52	3.48	36.7	10.22	1.21	343			0.91	
362.10	Kure, Inland Sea	34 14	132 32	8 50	3.40	277.1	9.24	1.20	309			0.99	
362.15	Ujima, Inland Sea	34 21	132 29	8 50	3.26	283.4	9.45	1.60	300			0.93	
362.20	Etauchi, Inland Sea	34 15	132 28	8 50	3.25	278.1	9.27	1.43	311			0.97	
362.25	Nasakejima, Inland Sea	33 57	132 28	8 50	3.13	262.8	8.76	1.48	298?			1.05	
362.30	Ohatake, Inland sea	33 58	132 10	8 49	2.92	260.9	8.70	1.32	290			0.95	
362.35	Aohama, Kiusiu	33 57	131 02	8 44	3.54	254.1	8.47	1.74	287			0.95	
362.40	Kakachi, Inland Sea	33 41	131 31	8 46	3.08	258.4	8.61	1.33	286			0.97	
362.45	Kaminoseki, Inland Sea	33 50	132 06	8 48	2.66	257.6	8.59	1.18	289			0.86	
362.50	Okikamuro Shima, Inland Sea	33 51	132 22	8 49	2.94	262.5	8.75	1.10	291			0.97	
362.55	Aoshima, Shikoku, Inland Sea	33 44	132 29	8 50	3.03	255.2	8.51	1.23	284			1.01	
362.60	Gokoshima, Shikoku, Inland Sea	33 55	132 41	8 51	3.15	270.3	9.01	1.19	303			0.98	
362.62	Mitarai, Shikoku, Inland Sea	34 10	132 52	8 51	3.59	289.3	9.64	1.26	323			0.96	
362.65	Kurushima, Shikoku, Inland Sea	34 07	132 59	8 52	3.54	288.7	9.62	1.35	328			1.05	
362.70	Kuroshima, Niigori Syo	33 59	133 20	8 53	3.71	290.8	9.69	1.33	356			1.05	
362.75	Awashima, Shikoku	34 16	133 38	8 55	3.54	328.3	10.94	1.35	3			1.03	
362.80	Naoshima	34 27	134 00	8 56	2.22	319.3	10.64	0.73	355			0.99	
362.85	Konoura	34 26	134 14	8 57	1.60	317.5	10.58	0.49	336			1.01	
362.90	Jeshima	34 40	134 31	8 58	0.95	339.9	11.33	0.38	309			0.90	
362.95	Aiketa, Shikoku	34 14	134 24	8 58	1.14	348.1	11.60	0.29	333			0.90	
363	Murotsu, Awaji	34 32	134 53	9 00	0.72	334.7	11.16	0.26	286			0.86	
363.05	Anaga, Awaji	34 16	134 40	8 50	1.00	338.1	11.27	0.24	341			0.90	
363.10	Fukura, Awaji	34 15	134 43	8 59	1.48	192.3	6.41	0.77	214			0.73	
363.15	Swaya, Awaji	34 36	135 01	9 00	0.37	219.9	7.33	0.36	237			0.78	
363.20	Dōnoura, Shikoku	34 13	134 35	8 58	0.95	218.9	7.30	0.56	232			0.77	
363.25	Kitodomari, Shikoku	34 14	134 35	8 58	0.58	270.1	9.00	0.31	260			0.85	
363.30	Azuro Kameura				1.17	350.7	11.69	0.29	340			0.88	
363.35	Magasaki, Shikoku	34 14	134 39	8 59	0.90	329.9	11.00	0.20	347			0.89	
363.40	Tosadomari, Shikoku	34 11	134 38	8 59	1.10	204.6	6.82	0.62	225			0.75	
363.50	Fukuoka, Kiusiu	33 36	130 22	8 41	1.88	272.4	9.08	0.80	306			0.48	
363.60	Fuyasikemura, Kiusiu	33 47	130 27	8 42	1.58	280.7	9.36	0.75	303			0.69	
363.70	Kanekasemura, Kiusiu	33 53	130 30	8 42	1.51	336.1	11.20	0.59	344			0.45	
363.80	Moji, Kiusiu	33 57	130 39	8 43	2.44	261.5	8.72	1.08	297			0.53	
363.90	Hedomari, Shemonseki Str	33 57	130 52	8 43	1.26	289.8	9.66	0.58	321			0.38	
363.95	Omishima, Nippon	34 24	131 13	8 45	0.63	316.5	10.55	0.34	340			0.32	
365	Kosigahama, Nippon	34 28	131 24	8 46	0.53	326.1	10.88	0.28	338			0.30	
379	Maizuru, Nippon	35 27	135 19	9 01	0.24	67.1	2.24	0.05	94			0.23	
379.5	Wajima, Nippon	37 24	136 53	9 08	0.20	75.2	2.51	0.07	106			0.16	
381	Nanao, Nippon	37 03	136 57	9 08	0.21	77.9	2.60	0.08	115			0.19	
384.5	Putami	37 57	138 14	9 13	0.20	81.5	2.72	0.08	100			0.18	
385	Ebisu	38 05	138 25	9 14	0.18	73.9	2.46	0.07	117			0.18	
385.5	Kamo	38 48	139 48	9 19	0.17	88.4	2.95	0.08	128			0.18	
386.05	Fukaura	40 41	139 59	9 20	0.17	97.8	3.26	0.08	137			0.16	
386.10	Kodomari	41 07	140 17	9 21	0.25	89.4	2.98	0.07	126			0.13	
386.15	Asadoko	40 57	140 48	9 23	0.59	104.9	3.50	0.31	140			0.20	
386.20	Moura	41 01	141 12	9 25	0.58	105.0	3.50	0.20	134			0.18	
386.25	Ominato	41 15	141 09	9 25	0.66	103.9	3.46	0.30	134			0.18	
386.40	Suttsu	42 47	140 16	9 21	0.16	116.4	3.88	0.06	158			0.15	
386.45	Iwanai	42 59	140 33	9 22	0.18	118.4	3.95	0.07	155			0.17	

K ₁ °.	O ₁ .	O ₁ °.	P ₁ .	P ₁ °.	S ₂ . M ₂ .	N ₂ . M ₂ .	O ₁ . K ₁ .	P ₁ . K ₁ .	K ₁ +O ₁ .	S ₂ -M ₂ °.	M ₂ -N ₂ °.	K ₁ -O ₁ °.	‡ (K ₁ °+O ₁ °).		Cotidal hour.		No.
													De- grees.	Lunar hours.	Semi- diurnal.	Diurnal.	
°	Fl.	°	Fl.	°					Fl.	°	°	°	°	h.	h.	h.	
192	0.67	183			0.54		0.80		1.51	10		9	187.5	12.50	10.18	3.48	357.4
212	0.62	194					0.75		1.45	12		18	203	13.53	11.56	4.53	357.5
220	0.85	205			0.42		0.83		1.87	14		15	212.5	14.17	1.67	5.19	357.6
224	0.80	206			0.35		0.88		1.71	36		18	215	14.33	1.35	5.46	362.5
206	0.70	199			0.38		0.71		1.69	32		7	202.5	13.50	0.37	4.67	362.10
217	0.80	201			0.49		0.86		1.73	17		16	209	13.93	0.62	5.10	362.15
198	0.78	192			0.44		0.80		1.75	33		6	195	13.00	0.44	4.17	362.20
196	0.75	189			0.47		0.71		1.80	35		7	192.5	12.83	11.93	4.00	362.25
195	0.70	199			0.45		0.74		1.65	29		1	198.5	13.23	11.88	4.41	362.30
197	0.71	196			0.49		0.75		1.66	32		1	196.5	13.10	11.74	4.37	362.35
202	0.68	189			0.43		0.70		1.65	28		13	195.5	13.03	11.84	4.26	362.40
197	0.61	191			0.44		0.71		1.47	31		6	194	12.93	11.79	4.13	362.45
200	0.72	193			0.37		0.74		1.69	28		7	196.5	13.10	11.93	4.28	362.50
193	0.73	183			0.41		0.72		1.74	20		10	188	12.53	11.68	3.70	362.55
204	0.79	193			0.38		0.81		1.77	33		11	198.5	13.23	0.16	4.38	362.60
203	0.80	201			0.35		0.83		1.76	34		2	202	13.47	0.79	4.62	362.62
212	0.76	201			0.38		0.72		1.81	39		11	206.5	13.77	0.75	4.90	362.65
232	0.71	207			0.36		0.68		1.76	65		25	219.5	14.63	0.81	5.75	362.70
225	0.76	224			0.38		0.74		1.79	35		1	224.5	14.97	2.02	6.05	362.75
222	0.74	177			0.33		0.75		1.73	36		45	199.5	13.30	1.71	4.37	362.80
221	0.67	197			0.31		0.66		1.68	18		24	209	13.93	1.63	4.98	362.85
221	0.70	209			0.40		0.78		1.60	31		12	215	14.33	2.36	5.36	362.90
221	0.69	215			0.25		0.77		1.59	15		6	218	14.53	2.63	5.56	362.95
221	0.78	206			0.36		0.91		1.64	49		15	213.5	14.23	2.16	5.23	363
236	0.68	211			0.22		0.76		1.58	3		25	223.5	14.90	2.29	5.92	363.05
194	0.55	177			0.52		0.75		1.28	22		17	185.5	12.37	9.43	3.39	363.10
205	0.68	196					0.87		1.46	17		9	200.5	13.37	10.33	4.37	363.15
198	0.59	194			0.59		0.77		1.36	13		4	196	13.07	10.33	4.10	363.20
224	0.64	195			0.53		0.75		1.49	10		29	209.5	13.97	0.93	5.00	363.25
220	0.67	215			0.25		0.76		1.55	11		5	217.5	14.50			363.30
263	0.77	198?			0.20		0.87		1.66	17					2.02		363.35
215	0.56	187			0.56		0.75		1.31	20		28	201	13.40	9.84	4.42	363.40
254	0.46	244			0.43		0.96		0.94	34		10	249	16.60	0.40	7.92	363.50
257	0.43	254			0.47		0.62		1.12	22		3	255.5	17.03	0.66	8.33	363.60
	0.44	280			0.39		0.98		0.89	8					2.50		363.70
211	0.45	226			0.44		0.85		0.98	35		15	218.5	14.57	0.00	5.85	363.80
266	0.46	27			0.46		1.21		0.84	31		1	266.5	17.77	0.04	9.05	363.90
320	0.38	301			0.54		1.19		0.70	24		19	310.5	20.70	1.80	11.95	363.95
321	0.38	303			0.53		1.27		0.68	12		18	312	20.80	2.11	12.03	365
328	0.14	306			0.21		0.61		0.37	27		22	317	21.13	5.22	12.11	379
329	0.17	314			0.35		1.06		0.33	31		15	321.5	21.43	5.38	12.30	379.5
328	0.20	320			0.38		1.05		0.39	37		8	324	21.60	5.47	12.47	381
320	0.17	267?			0.40		0.94		0.35	18					5.50		384.5
336	0.15	329			0.39		0.83		0.33	43		7	332.5	22.17	5.23	12.94	385
336	0.19	328			0.47		1.06		0.37	40		8	332	22.13	5.63	12.81	385.5
338	0.17	329			0.47		1.06		0.33	39		9	333.5	22.23	5.93	12.90	386.05
352	0.13	326			0.28		1.00		0.26	37		26	339	22.60	5.63	13.25	386.10
167	0.11	155			0.53		0.55		0.31	35		12	161	10.73	6.12	1.35	386.15
153	0.11	147			0.50		0.61		0.29	29		6	150	10.00	6.08	0.58	386.20
157	0.14	142			0.45		0.78		0.32	30		15	149.5	9.97	6.04	0.55	386.25
342	0.16	346			0.38		1.07		0.31	42		4	344	22.97	6.53	13.62	386.40
331	0.14	322?			0.39		0.82		0.31	37		9	326.5	21.77	6.58	12.40	386.45

No.	Station.	Geographic position.				M ₂ ⁰ .			S ₂ .	S ₂ ⁰ .	N ₂ .	N ₂ ⁰ .	K ₁ .
		Latitude.	Longitude.		M ₂ .	De- grees.	Lunar hours.						
			Arc.	Time.									
	EAST COAST OF ASIA—continued.	° ' "	° ' "	h. m.	Fl.	°	h.	Fl.	°	Fl.	°	Fl.	
386.50	Otaru	43 12	140 00	9 20	0.17	111.0	3.70	0.07	144	0.18	
386.55	Raigishi	43 20	140 24	9 22	0.15	111.8	3.73	0.08	139	0.17	
386.60	Hamamashi	43 36	141 23	9 26	0.15	107.6	3.59	0.06	153	0.16	
386.65	Rumoye	43 56	141 30	9 27	0.14	125.4	4.18	0.07	157	0.17	
386.70	Tomamae	44 19	141 30	9 27	0.15	132.9	4.43	0.08	159	0.15	
386.75	Wakanai	45 25	140 40	9 23	0.07	80.6	2.69	0.07	156	0.19	
386.80	Oshidomari	45 14	141 14	9 25	0.09	136.0	4.53	0.06	156	0.17	
388.2	Mimitsu	32 20	131 37	8 46	1.71	178.9	5.96	0.67	200	0.71	
388.4	Kagoshima	31 36	130 34	8 42	2.54	205.7	6.86	1.20	231?	0.82	
388.6	Yamakawa	31 12	130 38	8 43	2.44	205.1	6.83	1.16	233	0.84	
388.8	Misumi	32 37	130 27	8 42	3.94	256.6	8.55	1.70	290	0.87	
389.2	Wakatsu	33 07	130 20	8 41	5.27	261.9	8.73	2.23	301	0.82	
389.4	Fake saki shima	32 57	130 14	8 41	5.25	259.2	8.64	2.37	296	0.92	
389.6	Haedomari	32 47	130 22	8 41	4.36	259.4	8.65	1.81	299	0.90	
389.8	Kuchinotsu	32 36	130 11	8 41	3.18	242.9	8.10	1.42	287	0.82	
390.2	Ikenoura	32 23	130 21	8 41	3.45	253.1	8.44	1.47	283	0.88	
390.4	Ushibuka	32 12	130 01	8 40	2.75	226.1	7.54	1.21	252	0.93	
390.7	Tomioka	32 31	130 02	8 40	2.93	219.0	7.30	1.36	255	0.84	
390.8	Senzokushima, Zogoseto	32 34	130 20	8 42	3.83	249.9	8.33	1.63	292	0.98	
392.2	Kogozaki	33 06	129 40	8 39	2.73	238.1	7.94	1.24	270	0.79	
392.4	Sasebo	33 10	129 43	8 39	2.84	246.8	8.23	1.31	275	0.82	
392.6	Ainoura	33 11	129 39	8 39	2.90	230.9	7.70	1.28	256	0.81	
392.8	Wamakatsu (Goto Island)	32 53	129 0	8 36	2.73	250.1	8.34	1.21?	283	0.78	
394.1	Kamoize	34 40	129 22	8 37	1.54	253.9	8.46	0.76	274	0.27	
394.2	Nishidomari	34 39	129 28	8 38	1.12	249.5	8.32	0.59	282	0.69	
394.3	Shimayama	34 19	129 18	8 37	2.16	255.9	8.53	0.95	280	0.37	
394.4	Hirugaura	34 19	129 16	8 37	2.17	259.1	8.64	1.04	288	0.40	
394.45	Tsu Shima Sound	34 17	129 21	8 37	3.07	275.6	8.19	1.51	313	0.92	
394.5	Izuhara	34 11	129 17	8 37	1.72	251.0	8.37	0.86	288	0.24	
394.6	Kazamoto	33 51	129 41	8 39	1.96	270.0	9.00	0.91	298	0.51	
394.7	Gonoura	33 44	129 41	8 39	2.17	271.9	9.06	1.05	303	0.51	
394.8	Kurotojima	33 23	129 33	8 38	2.69	250.7	8.36	1.16	272	0.64	
395	Fukushima	33 24	129 48	8 39	2.38	277.7	9.26	1.17	305	0.63	
395.5	Hoshigaura	33 23	129 40	8 38	2.18	258.4	8.61	1.14	287	0.58	
400	Kasari	28 27	131 30	8 46	1.82	199.2	6.64	0.78	231	0.65	
400.2	Kuji	28 14	129 15	8 37	1.71	211.3	7.04	0.84	234	0.70	
400.4	Unten	26 40	128 00	8 32	1.72	196.2	6.54	0.77	228	0.64	
400.6	Sesoko	26 38	127 53	8 32	1.85	197.0	6.57	0.65	231	0.61	
400.8	Naka	26 23	127 40	8 31	1.86	197.5	6.58	0.85	222	0.68	
401	Tamsui	25 11	121 24	8 06	3.36	321.7	10.72	0.87	6	0.72	
401.2	Kiirun	25 09	121 45	8 07	0.73	277.3	9.24	0.13	285	0.64	
401.4	So-ō	24 35	121 52	8 07	1.80	185.0	6.17	0.44	209	0.64	
402	Miyakojima	24 50	125 11	8 21	1.65	215.9	7.20	0.69	240	0.55	
402.2	Punauke	24 20	123 44	8 15	1.49	199.9	6.66	0.64	216	0.70	
402.4	Taketomijima	24 09	124 05	8 16	1.47	197.4	6.58	0.65	217	0.62	
403	Gyo-o-to	23 37	119 31	7 58	2.94	332.4	11.08	0.74	20	0.79	
403.2	Santakuto	23 38	119 31	7 58	3.99	323.9	10.80	1.15	6	0.89	
403.4	Hatto retto	23 21	119 31	7 58	2.43	134.5?	4.48?	0.36	276?	0.81	
403.6	Toko, Formosa	22 28	120 27	8 02	0.59	242.5	8.06	0.24	233	0.58	
414	Sosaingpho	35 28	129 25	8 38	0.54	209.1	6.97	0.28	246	0.09	
414.2	Tsauliang Hai	35 08	129 02	8 36	1.34	232.7	7.76	0.64	261	0.13	
414.4	Douglas Inlet	35 02	128 48	8 35	1.85	240.0	8.00	0.93	268	0.28	

K ₁ °.	O ₁ .	O ₁ °.	P ₁ .	P ₁ °.	S ₂ . M ₂ .	N ₂ . M ₂ .	O ₁ . K ₁ .	P ₁ . K ₁ .	K ₁ +O ₁ .	S ₂ °. M ₂ °.	M ₂ °-N ₂ °.	K ₁ °-O ₁ °.	i (K ₁ °+O ₁ °)		Cotidal hour.		No.
													De- grees.	Lunar hours.	Semi- diurnal.	Diurnal.	
°	Fl.	°	Fl.	°					Fl.	°	°	°	°	h.	h.	h.	
340	0.16	322			0.41		0.89		0.34	33		18	331	22.07	6.37	12.74	386.50
344?	0.15	324			0.53		0.88		0.32	27		20	334	22.27	6.36	12.90	386.55
339	0.18	321			0.40		1.12		0.34	46		18	330	22.00	6.16	12.57	386.60
326	0.17	334			0.50		1.00		0.34	32		8	330	22.00	6.73	12.55	386.65
334	0.17	324			0.50		1.14		0.32	26		10	329	21.93	6.98	12.48	386.70
360	0.16	322			1.00		0.84		0.35	75		38	341	22.73	5.31	13.35	386.75
344	0.18	338			0.67		1.06		0.35	20		6	341	22.73	7.11	13.31	386.80
185	0.57	168			0.39		0.80		1.28	21		17	176.5	11.77	9.19	3.00	388.2
184	0.61	178			0.47		0.74		1.43	25		6	181	12.07	10.16	3.37	388.4
195	0.70	178			0.48		0.83		1.54	28		17	186.5	12.43	10.11	3.71	388.6
202	0.64	189			0.43		0.74		1.51	33		13	195.5	13.03	11.85	4.33	388.8
215	0.60	193			0.42		0.73		1.42	39		22	204	13.60	0.05	4.92	389.2
208	0.69	197			0.45		0.75		1.61	37		11	202.5	13.50	11.96	4.82	389.4
209	0.69	194			0.42		0.77		1.59	40		15	201.5	13.43	11.97	4.75	389.6
210	0.69	193			0.45		0.84		1.51	44		17	201.5	13.43	11.42	4.75	389.8
210	0.60	196			0.43		0.68		1.48	30		14	203	13.53	11.76	4.85	390.2
195	0.65	187			0.44		0.70		1.58	26		8	191	12.73	10.87	4.06	390.4
199	0.65	183			0.46		0.77		1.49	36		16	191	12.73	10.63	4.06	390.7
207	0.67	183			0.43		0.68		1.65	42		24	195	13.00	11.63	4.30	390.8
201	0.60	192			0.45		0.76		1.39	32		9	196.5	13.10	11.29	4.45	392.2
214	0.63	198			0.46		0.77		1.45	28		16	206	13.73	11.58	5.08	392.4
200	0.65	193			0.44		0.80		1.46	25		7	196.5	13.10	11.05	4.45	392.6
204	0.58	196			0.44		0.74		1.36	33		8	200	13.33	11.74	4.73	392.8
216	0.16	200			0.49		0.59		0.43	20		7	212.5	14.17	11.84	5.55	394.1
217	0.11	248			0.53		1.22		0.20	32		-31	232.5	15.50	11.69	6.87	394.2
201	0.31	203			0.44		0.84		0.68	24		-2	202	13.47	11.91	4.85	394.3
197	0.31	197			0.48		0.77		0.71	29		1	198.5	13.23	0.02	4.60	394.4
185	0.54	167			0.49		0.59		1.46	37		18	176	11.73	0.57	3.10	394.45
211	0.18	194			0.50		0.75		0.42	37		17	202.5	13.50	11.75	4.88	394.5
242	0.44	235			0.46		0.86		0.95	28		7	238.5	15.90	0.35	7.25	394.6
227	0.42	228			0.48		0.82		0.93	31		-1	227.5	15.17	0.41	6.52	394.7
220	0.50	212			0.43		0.94		1.24	21		8	216	14.40	11.73	5.77	394.8
230	0.51	224			0.49		0.81		1.14	27		6	227	15.13	0.61	6.48	395
235	0.52	216			0.52		0.90		1.10	29		19	225.5	15.03	11.98	6.40	395.5
191	0.47	181			0.43		0.72		1.12	32		10	186	12.40	9.87	3.63	400
201	0.54	196			0.49		0.77		1.24	23		5	198.5	13.23	10.42	4.61	400.2
195	0.51	192			0.45		0.80		1.15	33		3	193.5	12.90	10.01	4.37	400.4
202	0.52	184			0.35		0.85		1.13	34		18	193	12.87	10.04	4.34	400.6
195	0.50	193			0.46		0.74		1.18	24		2	194	12.93	10.06	4.41	400.8
240	0.58	217			0.26		0.81		1.30	44		23	228.5	15.23	2.62	7.13	401
124?	0.48	205			0.18		0.75		1.12	8					1.12		401.2
217	0.57	207			0.24		0.89		1.21	24		10	212	14.13	10.05	6.01	401.4
235	0.51	212			0.42		0.93		1.06	24		23	223.5	14.90	10.85	6.55	402
208	0.54	197			0.43		0.77		1.24	16		11	202.5	13.50	10.41	5.25	402.2
209	0.56	196			0.44		0.90		1.18	20		13	202.5	13.50	10.31	5.23	402.4
271	0.76	245			0.25		0.96		1.55	48		26	258	17.20	3.11	9.23	403
264	0.84	231			0.29		0.94		1.73	42		33	247.5	16.50	2.83	8.53	403.2
268?	0.72	250			0.15		0.80		1.53			18	259	17.27		9.30	403.4
294	0.53	256			0.41		0.91		1.11	-9		38	275	18.33	0.05	10.30	403.6
35	0.07	346			0.52		0.78		0.16	37		49	10.5	0.70	10.34	16.07	414
130	0.06	144			0.48		0.46		0.19	28		-14	137	9.13	11.16	0.53	414.2
150	0.15	136			0.50		0.54		0.43	28		14	143	9.53	11.42	0.95	414.4

No.	Station.	Geographic position.				M ₂ °.			S ₂ .	S ₂ °.	N ₂ .	N ₂ °.	K ₁
		Latitude.	Longitude.		M ₂ °.	De- grees.	Lunar hours.						
			Arc.	Time.									
EAST COAST OF ASIA—continued.													
		° ' "	° ' "	<i>h. m.</i>	<i>Fl.</i>	°	<i>h.</i>	<i>Fl.</i>	°	<i>Fl.</i>	°	<i>Fl.</i>	
		<i>North.</i>	<i>East.</i>										
414.6	Masanpho.....	35 12	128 34	8 34	2.10	244.8	8.10	1.00	269				0.28
414.8	Sylvia Basin.....	35 04	128 33	8 34	2.12	239.2	7.97	1.04	252				0.30
415	Koje Do.....	34 50	128 42	8 35	1.96	244.0	8.13	0.77	273				0.34
415.2	Cargodo Bluff.....	34 50	128 35	8 34	2.68	251.2	8.37	0.85	311				0.67
415.4	Shadwell Gulf.....	34 53	128 27	8 34	2.29	244.6	8.15	1.15	290				0.24
415.6	Sylvia Basin, Kojé Do.....	34 55	128 30	8 34	2.04	230.4	7.68	1.09	254				0.26
415.8	Daryang Do.....	34 57	128 20	8 33	3.24	241.8	8.06	1.24	271				0.58
416	Sinko Do.....	34 51	128 13	8 33	2.73	251.7	8.39	1.20	277				0.57
416.2	Willie's Gulf.....	34 44	127 45	8 31	3.34	251.5	8.39	1.55	278				0.68
416.4	North of Herschel Island.....	34 37	127 33	8 30	3.65	267.7	8.42	1.75	289				0.72
416.6	Herschel Island.....	34 28	127 28	8 30	3.43	260.1	8.67	1.58	283				0.82
416.8	Mandarin (Goalen Island).....	34 21	126 53	8 28	3.00	282.2	9.41	1.35	303				0.86
417	Mandarin Island, Long Reach.....	34 22	126 47	8 27	3.51	262.8	8.76	2.03	309				0.77
417.2	Chrichton Har.....	34 08	126 38	8 27	2.79	295.4	9.85	1.31	328				0.71
417.4	Green Islands.....	34 27	126 25	8 26	3.22	307.8	10.26	1.33	355				0.93
417.6	East of Thistle Island.....	34 24	126 19	8 25	3.01	341.5	11.38	0.67	166?				2.60
417.8	Thistle Island.....	34 24	126 08	8 25	3.70	3.7	0.12	1.34	43				0.97
418	Montreal Island.....	34 20	126 04	8 24	3.04	345.6	11.52	1.12	12				0.95
418.2	Amherst Island.....	34 32	126 02	8 24	3.57	5.1	0.17	1.26	38				1.01
418.3	Mokpho.....	34 45	126 22	8 25	3.94	53.0	1.77	1.32	98				1.03
418.4	Pigum Do.....	34 44?	125 56	8 24	3.83	17.4	0.58	0.71	55				1.00
418.6	North Twin Island.....	34 51	126 02	8 24	4.50	43.4	1.45	1.49	74				0.88
418.8	Fire Island.....	35 03	126 05	8 24	4.67	44.5	1.48	1.93	81				0.83
419	Kokuntau Islands.....	35 49	126 25	8 26	7.40	85.2	2.84	2.36	118				1.22
419.2	Won-san Islands.....	36 22	126 26	8 26	7.86	91.2	3.04	2.94	128				1.06
420.8	Gets-nai-tau Island.....	38 03	124 49	8 19	3.48	101.4	5.38	1.15	212				1.28
421	Dau-chen Island.....	38 37	125 00	8 20	5.04	226.2	7.54	1.42	270				1.60
421.2	Piö sem.....	38 40	125 10	8 21	5.48	243.8	8.13	1.81	280				1.31
421.4	Ping-yang Inlet.....	38 38	125 35	8 22	6.68	261.7	8.72	2.05	308				1.22
427	Wei-hai-wei.....	37 30	122 11	8 00	2.06	312.6	10.09	0.55	2				0.77
436	Swatow, China.....	23 23	116 30	7 47	1.35	23	0.77	0.32	86	0.21	358		0.94
438	Whampoa.....	23 05	113 26	7 34	2.18	32	1.07	0.67	64	0.38	16		1.07
445	Nau-chau, Kwangsi.....	21 00	110 38	7 23	2.50	303	10.10	1.05	345				1.34
446	Port Beaumont.....				3.12	322	10.73	1.15	19				1.28
OCEANICA.													
590	Boloengan, Borneo.....	2 50	117 22	7 49	0.93	336	11.20	0.49	291	0.12	228		0.55
		<i>South.</i>											
591	Samarinda, Borneo.....	0 30	117 08	7 49	1.39	209	6.07	0.86	261				0.58
592	Batoe Panggal, Borneo.....	0 32	117 06	7 48	1.20	208	6.03						
593	Moera Djawa, Borneo.....	0 37	117 18	7 49	1.61	198	6.60	1.05	256	0.14	152		0.55
594	Bay of Balik Papan, Borneo.....	1 16	116 48	7 47	1.89	153	5.10	1.64	204	0.20	125		0.57
598	Macassar.....	5 08	119 24	7 58	0.27	70	2.33	0.36	194	0.09	347		0.91
600	Donggala.....	0 40	119 44	7 59	1.55	159	5.30	1.30	208	0.12	108		0.73
		<i>North.</i>											
602	Tontoli.....	1 00	120 53	8 04	1.38	161	5.37	1.16	199	0.20	131		0.47
634	Iloilo, Point Gimalik.....	10 40	122 35	8 10	1.35	332.6	11.09	0.64	18	0.21	306		1.14
634.5	Cebu.....	10 18	123 54	8 16	1.37	334.3	11.14	0.75	22	0.22	324		0.97
635	Tacloban.....	11 15	125 00	8 20	0.53	220.6	7.35	0.13	269	0.14	199		0.50
635.5	Santa Elena.....	11 21	124 59	8 20	0.49	312.3	10.41	0.34	30				0.72
636	Santa Rita Island.....	11 26	124 57	8 20	1.18	347.8	11.59	0.76	50	0.18	357		0.79
636.5	Catbalogan.....	11 47	124 52	8 19	1.50	341.1	11.37	0.90	36	0.24	315		0.91
637	Calbayoc.....	12 04	124 35	8 18	1.11	342.7	11.42	0.74	42				0.77

K ₁ °	O ₁	O ₁ °	P ₁	P ₁ °	S ₂ M ₂	N ₂ M ₂	O ₁ K ₁	P ₁ K ₁	K ₁ +O ₁	E ₂ M ₂	M ₂ -N ₂	K ₁ -O ₁	↓ (K ₁ +O ₁)		Cotidal hour.		No.
													De- grees.	Lunar hours.	Semi- diurnal.	Diur- nal	
°	Fl.	°	Fl.	°					Fl.	°	°	°	°	h.	h.	h.	
170	0.12	143			0.48		0.43		0.40	24		27	156.5	10.43	11.59	1.86	414.6
143	0.11	124			0.49		0.37		0.41	13		19	133.5	8.90	11.49	0.33	414.8
170	0.17	145			0.39		0.50		0.51	29		25	157.5	10.50	11.55	1.02	415
191	0.35	151			0.32		0.52		1.02	60		40	171	11.46	11.80	2.83	415.2
166	0.13	147			0.50		0.54		0.37	45		19	156.5	10.42	11.58	1.86	415.4
179	0.11	107			0.52		0.42		0.37	24		72	143	9.53	11.11	0.96	415.6
176	0.52	147			0.38		0.90		1.10	29		29	161.5	10.77	11.51	2.22	415.8
165	0.37	151			0.46		0.65		0.94	25		15	158.5	10.57	11.84	2.02	416
165	0.42?	151			0.45		0.62		1.10	26		14	158	10.53	11.87	2.01	416.2
184	0.46	171			0.48		0.64		1.18	21		13	177.5	11.83	0.42	3.33	416.4
174	0.53	157			0.46		0.65		1.35	23		17	165.5	11.03	0.17	2.53	416.6
182	0.53	170			0.45		0.62		1.39	21		12	176	11.73	0.94	3.26	416.8
182	0.70	157			0.58		0.91		1.47	46		25	169.5	11.30	0.31	2.85	417
189	0.57	179			0.47		0.80		1.28	33		10	184	12.27	1.40	3.82	417.2
201	0.67	179			0.41		0.72		1.40	47		22	190	12.67	1.83	4.24	417.4
60					0.22										2.96		417.6
225	0.74	209			0.36		0.75		1.71	39		16	217	14.47	3.70	6.05	417.8
211	0.67	203			0.37		0.71		1.62	26		8	207	13.80	3.12	5.40	418
217	0.77	209			0.35		0.76		1.78	33		8	213	14.20	3.77	5.80	418.2
246	0.81	227			0.34		0.79		1.81	45		19	236.5	15.77	5.35	7.35	418.3
235	0.80	213			0.19		0.80		1.80	38		22	224	14.93	4.18	6.53	418.4
240	0.71	238			0.33		0.81		1.59	31		2	239	15.93	5.08	7.53	418.6
252	0.55	216			0.41		0.66		1.38	37		36	234	15.60	5.08	7.20	418.8
256	0.95	241			0.32		0.78		2.17	33		15	248.5	16.57	6.41	8.14	419
265	0.77	249			0.37		0.73		1.53	37		17	257.5	17.17	5.61	8.74	419.2
304	0.84	274			0.33		0.66		2.12	51		30	289	19.27	9.06	10.95	420.8
316	0.93	292			0.28		0.58		2.53	44		24	304	20.27	11.21	11.94	421
316	0.93	294			0.33		0.71		2.24	36		22	305	20.33	11.78	11.98	421.2
331	0.95	305			0.31		0.78		2.17	46		26	318	21.20	0.35	12.83	421.4
300	0.58	271			0.27		0.75		1.29	49		29	285.5	19.03	1.94	10.88	427
292	0.76	254	0.27	285	0.24	0.18	0.81	0.29	1.70	63	25	38	273	18.20	4.99	10.42	436
354	0.82	310	0.35	15	0.31	0.17	0.77	0.33	1.89	32	16	44	332	22.13	5.50	14.56	438
345	1.21	276			0.41		0.90		2.55	42		69	310.5	20.70	2.72	13.32	445
313	1.21	291			0.37		0.95		1.49	57		22	302	201.30			446
319	0.24	254	0.10	264	0.54	0.13	0.44	0.18	0.79	45	108	65	286.5	19.10	3.38	11.28	590
300	0.71	271	0.11	237	0.58		1.22	0.19	1.29	52		29	255.5	19.03	11.15	11.21	591
	0.60	267													11.13		592
318	0.44	285	0.38	302	0.65	0.09	0.80	0.69	0.99	55	46	33	301.5	20.10	10.78	12.28	593
293	0.49	257	0.44	159	0.86	0.11	0.86	0.77	1.06	51	28	45	280	18.67	9.32	10.89	594
300	0.56	278	0.34	296	1.33	0.33	0.62	0.37	1.47	124	83	22	289	19.27	6.36	11.30	598
277	0.35	239	0.22	291	0.84	0.68	0.48	0.30	1.08	49	51	38	258	17.20	9.32	9.22	600
285	0.46	227	0.19	328	0.84	0.14	0.98	0.93	0.93	38	30	58	256	17.07	9.30	9.00	602
326	1.01	294			0.47	0.16	0.80		2.15	45	27	32	310	20.67	2.92	12.50	634
330	0.92	294	0.31	332	0.55	0.16	0.95	0.32	1.89	48	10	36	312	20.80	2.87	12.53	634.5
301	0.59	276			0.25	0.26	1.18		1.09	48	22	25	288.5	19.23	11.52	10.90	635
316	0.66	270			0.69		0.92		1.38	78		46	293	19.53	2.08	11.20	635.5
332	0.79	296			0.64	0.15	1.00		1.58	62	9	36	314	20.93	3.26	12.60	636
335	0.88	291			0.60	0.16	0.97		1.79	55	26	44	313	20.87	3.05	12.55	636.5
337	0.73	302			0.67		0.95		1.50	59		35	319.5	21.30	3.12	13.00	637

No.	Station.	Geographic position.				M ₂ ^o .			S ₂ ^o .	S ₂ ^o .	N ₂ ^o .	N ₂ ^o .	K ₁ .
		Latitude.	Longitude.		M ₂ .	De- grees.	Lunar hours.						
			Arc.	Time.									
OCEANICA—continued.		° ' "	° ' "	h. m.	Fl.	°	h.	Fl.	°	Fl.	°	Fl.	
		North.	East.										
637.5	Halsey Harbor.....	11 48	119 57	8 00	0.78	311.1	10.37	0.33	4	0.22	289	0.98	
641	Olongapo.....	14 49	120 17	8 01	0.56	292.9	9.76	0.20	325	0.11	283	0.90	
642	Santa Cruz.....	15 46	119 54	8 00	0.38	271.0	9.03	0.06	324	0.09	274	0.84	
643	Bolinao.....	16 24	119 54	8 00	0.32	278.3	9.28	0.12	313			0.85	
644	Sual.....	16 04	120 06	8 00	0.29	275.9	9.20	0.09	311	0.10	250	0.89	
645	Tabaco.....	13 22	123 44	8 15	1.75	174.7	5.82	0.77	199	0.29	150	0.53	
		South.	West.										
665	Gambier Island.....	23 08	135 00	9 00	0.89	86	2.87	0.30	38			0.07	
			East.										
680.5	Wellington.....	41 17	174 46	11 39	1.60	137.1	4.57	0.09	325	0.35	104	0.08	
681	Port Chalmers.....	45 50	172 30	11 30	2.39	99.0	3.30	0.27	96	0.47	70	0.08	
681.5	Port Darwin.....	12 23	130 37	8 42	6.56	144	4.90	3.44	193	1.04	121	1.91	
682	Cooktown.....	15 27	145 15	9 41	1.87	282	9.40	0.79	258	0.45	239	0.29	
682.5	Cairns Harbor.....	16 55	145 47	9 43	1.96	282	9.40	1.12	245	0.66	269	0.87	
683	Brisbane Bar.....	27 31	153 00	10 12	2.20	290	9.67	0.58	315	0.46	288	0.59	
683.5	Ballina.....	28 52	153 33	10 14	1.08	262	8.73	0.28	275	0.20	254	0.45	
684	Newcastle.....	32 57	151 44	10 07	1.60	249	8.30	0.39	265	0.35	235	0.51	
689.5	Princess Royal Harbor.....	35 08	118 00	7 52	0.16	339	11.30	0.26	342	0.07	17	0.62	
INDIAN OCEAN.													
810	Navanar.....	22 44	69 43	4 39	6.04	24.4	0.81	1.89	55	1.26	11	1.53	
811	Hanstal.....	22 56	70 21	4 41	6.85	45.6	1.52	1.93	85	1.19	26	1.50	
		North.											
845	Suez.....	29 58	32 32	2 10	1.85	342.4	11.41	0.45	7	0.60	313	0.16	
		South.											
863	Diego Saurez.....	12 25	49 20	3 17	2.10	111	3.70	0.95	155			0.39	
864	Tamatave.....	18 10	49 28	3 18	0.72	49	1.63	0.30	67			0.07	
866	Mayotte.....	12 47	45 20	3 01	3.71	121	4.03	1.74	163			0.56	
868	Réunion Island.....	21 20	55 28	3 42	0.46	70	2.63	0.23	77			0.16	
WEST COAST OF AFRICA AND EUROPE.													
		North.											
910	Dakar.....	14 40	17 25	1 10	1.54	224	7.47	0.56	264			0.20	
925	Toulon.....	43 05	5 55	0 24	0.20	246	8.20	0.09	250	0.05	226	0.10	
943	Rochelle.....	46 09	1 09	0 05	5.82	92.3	3.08	2.11	126	1.22	72	0.21	
		West.											
957	Hull.....	53 44	0 20	0 01	7.56	175.8	5.80	2.34	228	1.25	164	0.56	
		East.											
983	Hook of Holland.....	51 56	4 05	0 16	2.54	72	2.40	0.65	131	0.43	44	0.25	
984	Ymuiden.....	52 28	4 33	0 18	2.20	113	3.77	0.58	180	0.35	89	0.25	
985	Helder.....	52 57	4 46	0 19	1.74	171	5.70	0.50	238	0.26	151	0.18	
1000	Christiania.....	59 55	10 44	0 43	0.37	128	4.27	0.12	86	0.10	92		
1001	Oscarsborg.....	59 41	10 37	0 42	0.47	129	4.30	0.16	80	0.12	89		
1002	Arendal.....	58 27	8 46	0 35	0.28	100	3.33	0.09	68	0.08	64		
1003	Stavanger.....	58 59	5 44	0 23	0.48	282.5	9.42	0.22	332	0.10	264		
1004	Bergen.....	60 24	5 08	0 21	1.44	297.5	9.92	0.52	334	0.28	270	0.11	
1005	Trondhjem.....												
1006	Bodoe.....	67 17	14 23	0 58	2.84	356.5	11.88	0.98	35	0.57	334	0.34	
1007	Fineide.....	67 17	15 30	1 02	1.74	57.0	1.90	0.50	106	0.34	36	0.26	
1008	Kabelvaag.....	68 13	14 30	0 58	2.98	3.5	0.12	1.08	44	0.61	340	0.34	
1009	Narvik.....												
1010	Bredvik.....												
1011	Vardoe.....	70 20	31 06	2 04	3.29	163.5	5.45	0.92	208	0.72	130	0.38	
1030	Teplitz Bay.....	81 47	58 04	3 52	0.47	168.4	5.61	0.17	230			0.09	

K ₁ °.	O ₁ .	O ₂ °.	P ₁ .	P ₂ °.	S ₂ . M ₂ .	N ₂ . M ₂ .	O ₁ . K ₁ l.	P ₁ . K ₁ .	K ₁ + O ₁ .	S ₂ ° - M ₂ °.	M ₂ ° - N ₂ °.	K ₁ ° - O ₁ °.	† (K ₁ ° + O ₁ °).		Cotidal hour.		No.
													De- grees.	Lunar hours.	Semi- diur- nal.	Diur- nal.	
°	Fl.	°	Fl.	°					Fl.	°	°	°	°	h.	h.	h.	
318	0.98	276	0.42	0.28	1.00	1.96	53	22	42	297	19.80	2.37	11.80	637.5
316	0.81	276	0.27	309	0.36	0.20	0.90	0.30	1.71	32	10	40	296	19.73	1.74	11.71	641
313	0.72	267	0.16	0.24	0.86	1.56	53	-3	46	290	19.33	1.03	11.33	642
313	0.68	275	0.38	0.80	1.53	35	38	294	19.60	1.28	11.60	643
325	0.72	274	0.31	0.34	0.81	1.61	35	26	41	294.5	19.63	1.20	11.63	644
203	0.39	190	0.44	0.17	0.74	0.91	24	25	13	196.5	13.10	9.57	4.85	645
84	0.03	276	0.34	0.43	0.10	-48	-92	230	153.30	5.87	6.33	665
81	0.10	36	0.03	67	0.06	0.22	1.25	0.38	0.18	-172	33	45	58.5	3.90	4.92	16.25	680.5
84	0.09	59	0.03	95	0.11	0.20	1.12	0.38	0.17	-3	29	25	71.5	4.77	3.80	16.30	681
313	1.14	313	0.44	1	0.52	0.16	0.60	0.23	3.05	49	23	23	224.5	14.97	8.10	6.27	681.5
171	0.30	113	0.42	0.24	1.03	0.59	-24	43	58	142	9.47	11.72	23.79	682
190	0.41	166	0.57	0.34	0.47	1.28	-37	13	24	178	11.87	11.68	2.15	682.5
176	0.32	139	0.26	0.21	0.54	0.91	25	2	37	157.5	10.50	11.47	0.30	683
155	0.31	128	0.14	149	0.26	0.19	0.69	0.31	0.76	13	8	27	141.5	9.43	10.50	23.20	683.5
120	0.29	88	0.15	116	0.24	0.22	0.57	0.29	0.80	16	14	32	104	6.93	10.18	20.81	684
330	0.42	312	17	332	1.62	0.44	0.68	0.27	1.04	3	-38	18	327	21.40	3.43	13.53	689.5
63	0.68	66	0.28	72	0.31	0.21	0.44	0.18	2.21	31	13	-3	64.5	4.30	8.16	23.65	810
81	0.75	75	0.38	84	0.28	0.17	0.50	0.25	2.25	39	20	6	78	5.20	8.84	0.52	811
190	0.04	216	0.05	112	0.24	0.32	0.25	0.31	0.20	25	29	-26	203	13.53	9.24	11.36	845
55	0.26	63	0.45	0.67	0.65	44	-8	59	3.93	0.42	0.65	863
56	0.10	73	0.42	1.43	0.17	18	-17	64.5	4.30	10.33	1.00	864
49	0.30	61	0.47	0.54	0.86	42	-12	55	367.00	1.01	0.65	866
159	0.10	103	0.50	0.62	0.26	-2	56	131	8.73	10.93	5.03	868
328	0.13	226	0.36	0.65	0.33	40	102	277	18.47	6.30	16.30	910
186	0.06	120	0.04	178	0.45	0.25	0.60	0.40	0.16	4	20	66	153	10.20	7.80	9.80	925
67	0.23	321	0.09	58	0.36	0.21	1.10	0.43	0.44	34	20	106	14	0.93	3.00	0.85	943
282	0.43	119	0.31	0.17	0.77	0.99	52	11	163	200.5	13.37	5.88	13.39	957
345	0.35	182	0.13	327	0.26	0.17	1.40	0.52	0.60	59	28	163	263.5	17.57	2.13	17.30	983
350	0.36	185	0.13	334	0.26	0.16	1.44	0.52	0.61	67	24	165	267.5	17.83	3.47	17.53	984
356	0.25	196	0.08	0	0.29	0.15	1.39	0.44	0.43	67	20	160	276	18.40	5.38	18.08	985
.....	0.32	0.27	-42	36	3.55	1000
.....	0.34	0.26	-49	40	3.60	1001
.....	0.32	0.29	-32	36	2.75	1002
.....	0.46	0.21	50	18	9.04	1003
170	0.10	18	0.04	152	0.36	0.19	0.91	0.36	0.21	36	28	152	94	6.27	9.57	5.92	1004
.....	1005
208	0.13	32	0.10	194	0.35	0.20	0.38	0.29	0.47	39	22	176	120	8.00	10.91	7.03	1006
250	0.08	102	0.07	236	0.29	0.20	0.31	0.27	0.34	49	21	148	176	11.73	0.87	10.70	1007
212	0.13	54	0.09	202	0.36	0.20	0.38	0.26	0.47	40	24	158	133	8.87	11.15	7.90	1008
.....	1009
.....	1010
286	0.10	92	0.11	282	0.28	0.22	0.26	0.29	0.48	44	34	-166	9	0.60	3.38	22.53	1011
11	0.05	354	0.36	0.56	0.14	62	17	2.5	0.17	1.74	20.30	1030

20. *Note on the measurement of tides at sea.*

The measuring of the rise and fall of the tide at sea has seldom been undertaken because of the difficulties connected with such operations.

By anchoring a boat from either end and frequently measuring the depth of the water (about 20 fathoms) over an even bottom, Captain Hewett, R. N., ascertained that there was practically no rise and fall at a point between Holland and England. This has been taken to be a no-tide point in Fig. 22, and it agrees well with the location inferred from the tide along the coasts.*

More recently, attempts have been made to ascertain the rise and fall in shallow bodies of water by means of pressure gauges. The gauges have generally been placed upon the bottom. It has been suggested that a suitable anchorage and several guys might be arranged which would allow the gauge to be floated just below the action of the storm waves, thus reducing the total pressure of the superincumbent water column. If the gauge is not too far below the surface, the pressure may be exerted upon a tube of mercury closed at one end.

In the Surveyor (Sydney, Australia), Vol. 16 (1903), pp. 25-28, Mr. G. H. Halligan describes a gauge designed to be submerged in small depths, and in which the pressure is measured by a column of mercury. Capt. Adolf Mensing, of the Imperial German Navy, has designed an elaborate self-registering tide gauge which works in depths not exceeding 100 fathoms. The instrument does not measure the total pressure, but the difference between the total pressure and the pressure for an assumed depth, which does not quite equal the actual depth of water.

In 1903 Captain Cust of the *Triton* secured tidal curves by means of a pneumatic tide gauge in the North Sea, on Brown Ridge, Swarte Bank, and the northwest corner of Dogger Bank.†

It seems to be worth while to point out the difficulty of getting a good instrumental range when the pressure is exerted upon a column of air contained in a pipe closed at the upper end. Let l denote the length of the pipe and y the distance from the upper end of the pipe downward to the surface of the water which enters it, i. e., y denotes the length of the air column. Let n denote the number of atmospheres which measure the pressure at the mouth of the pipe. Consequently, when the pressure is n atmospheres, the length of the air column will be $l/n (=y)$ and the depth of the mouth of the pipe below the surface will be

$$(n-1) \text{ 33 feet } (=z). \quad (30)$$

$$\begin{aligned} \therefore dy &= -\frac{l}{n^2} dn, \\ dz &= 33 \, dn; \\ dy &= -\frac{1}{33} \frac{l}{n^2} dz = -\frac{33 \, l}{(z+33)^2} dz. \end{aligned} \quad (31)$$

This shows how very small is the change in the length of the air column when the depth is altered by a given amount dz .

* Report B. A. A. S., 1841, II, pp. 32-35.

† Report on Admiralty Surveys for the year 1903, p. 5.

The most reliable means for measuring the tide away from the coast appears to be a sounding apparatus consisting essentially of a piano wire attached to a heavy weight composed of material of little value, such as a box or bag of stones or gravel. The weight when once cast is to remain immovable on the bottom and is not to be recovered. The wire when drawn taut will indicate whether or not the vessel is directly over the weight. The aim of the observers on board is to so maneuver the boat that the wire shall become approximately vertical as many times as possible throughout the period of observation, and at each such time to note the depth of the water. There certainly can be no serious difficulty in measuring tides in a few hundred fathoms where the surface of the water is reasonably calm. For, the tension can be made sufficiently great for eliminating the errors caused by the sag of the wire resulting from its own weight combined with the impulse of the tidal current.

By aid of this apparatus and suitable floats it is probable that permanent currents can be measured at sea in any depth of water.*

21. Note concerning the fundamental systems.

The principal systems upon which the semidiurnal ocean tides depend are shown in Fig. 23, and described in §§ 72-78 of Part IV A. These constitute a rational scheme with reference to which observed facts can be arranged and, in a measure, interpreted; in fact, they make it possible to estimate the time of tide in various parts of the ocean itself.

By aid of the cotidal lines and ranges as now shown in Figs. 6-41, it would doubtless be possible to modify the systems in some details. This, however, would require considerable time and study; and the results obtained by aid of the modified systems could hardly differ much from those depending upon the original. The following are a few suggestions:

The half-wave area extending from Mozambique Channel to Baluchistan and India has in § 73, Part IV A, been described as if belonging to the South Atlantic system. It will be noted in § 26 that it is mainly a dependent area having tides sustained by the South Atlantic and South Indian systems and modified by the North Indian system.

The southern nodal line of the North Atlantic systems should probably have been drawn more nearly parallel to the coast of South America, thus bringing its eastern end nearer to the Cape Verde Islands.

By referring to Figs. 35, 36, 37, and 39, it appears that the nodal line from Japan might be extended southward to the equator, and that the northern shores of Celebes, Gilolo, and New Guinea might be regarded as forming a portion of the southern boundary of the North Pacific system.

It seems probable that the South Pacific system should be considered as comprising little, if anything, besides the L-shaped figure shown on the chart of systems. With this view of the case the nodal line south of the Hawaiian Islands and belonging to the northern branch should be moved southwesterly, or toward the Fiji Islands. The southwesterly progression toward these islands probably helps to explain the tides in the vicinity of this line. It is probable that the nodal line near New Zealand should be moved southeasterly.

* See Science, Vol. 19 (1904), pp. 704-706.

22. Remarks on tidal problems.

While I believe that all candid readers of Parts IV A and IV B will recognize therein a partial and approximate explanation of the tides, I also believe that more comprehensive modes of treatment will be desired and undertaken by the analyst. It is here proposed to show what are some of the difficulties to be encountered.

For many years after the time of Laplace, nearly all writers on tidal theory followed closely in his footsteps and laid great stress upon the tides in an hypothetical sheet of water which either covers a rotating globe or constitutes a zonal sea, the depth being assumed to depend upon the latitude but not upon the longitude. More recently it has been recognized that an important step toward the complete solution of tidal problems is the determination of the free periods of the bodies of water upon a rotating globe. The hopelessness of this undertaking can be easily seen upon recalling the fact that the mode and free period of a plane rectangular body of water rotating about a vertical line have never been determined. The free oscillation of a circular sheet rotating about a central line can, however, be found by aid of Bessel's functions.* Laplace succeeded in finding solutions of his tidal equation for a rotating globe covered with water. But the possible modes of free oscillation for an ocean covering a globe uniformly has only recently been investigated by Mr. S. S. Hough in the *Philosophical Transactions*. If the effect of rotation be ignored, spherical sheets of water bounded by meridians or parallels of latitude or both can be treated by aid of general spherical harmonics. These results are, to say the least, not very encouraging.

The real tidal problem presented to us by nature is concerned not so much with the possible free oscillations of an ocean as a whole as with those oscillations which may exist across certain parts of the ocean and whose free periods are nearly equal to the period of the tidal forces.

Consider for the moment an oscillation between two opposing walls placed in a broad tank of water at such a distance apart as to best respond to the impressed periodic forces. If one or both of the lateral boundaries be wanting, the deflecting force caused by rotation at a moderate rate about any vertical axis can have little to do with the oscillation because its effect can not accumulate. Even in the case of a rectangular body of water bounded on all sides by rigid walls, the deflecting force due to rotation would not, excepting for certain critical widths, seriously alter the mode and period of oscillation. The approximate effect upon a narrow rectangular body can be seen from § 11, making $\theta = 0$ for the case of a plane sheet of water.

In nearly all cases of tidal oscillations rigid lateral boundaries are incomplete. The problems seeming to require first attention are those relating to the free oscillations of such bodies disregarding in the first instance the deflecting force of the earth's rotation. It is probable that similar problems relating to plane sheets of water if capable of satisfactory solution could be readily extended to sheets upon a sphere. A somewhat analogous subject is that of the vibration of a stretched membrane where a portion of the rigid boundary is wanting; but the analogy is obviously incomplete, because we are in this latter case concerned only with the membrane itself. The problem of an open organ pipe is in some respects more nearly analogous to the one in question, for account must be taken of the motion of the air outside of the pipe as well

*Lamb: *Hydrodynamics*, §§ 201-203.

as of the air within. It seems probable that experiments, if carefully made by aid of suitable apparatus, can throw much light upon the movements of the water particles, especially along the free boundaries of imperfectly inclosed areas.

Many tidal problems of minor importance require attention. One intimately connected with the problem just referred to is the law of transition of the times of the tide between two nonsimultaneous areas in the open ocean (lemma 25). Another is the transition or sequence of the times experienced in passing through a strait connecting two independently tided bodies; or bodies whose tides are not independent of each other. Another problem relates to the possibility of equilibrium tides in a gulf or partially inclosed sea.*

Questions relating to the resistances in liquid motion and to the nature of flow at various depths will be considered in Part V.

The resistance referred to in Chapter VI, Part IV A, is assumed to be small and to vary as the first power of the velocity of the moving particles. This assumption generally permits the character and period of the motion to remain the same as it would have been had no resistance (and so no sustaining force) been involved. The amount being kept in abeyance leaves the absolute value of the amplitude of tide undetermined. Even if the amount of resistance were known, the fact that the free period of the body differs somewhat from the period of the forces will directly affect the amplitude of the tide.

Many important conclusions respecting tides in canals, where friction (proportioned to the velocity) is taken into account, have been reached by Airy.† For simplicity such matters have not been brought into Parts IV A and IV B, although a complete explanation of the tides in shallow bodies of water must depend largely upon them. For instance, Airy shows that in a canal stopped by a barrier the tide can not consist of a stationary wave alone, but that the resistance encountered necessitates a wave progressing inward and up the canal. Observations show that, however suddenly terminated a shallow bay or river may be, the stationary wave is always accompanied by a progressive one, although it is hard to find examples in which other causes are not also operative. It is not probable, however, that progressions found in deep bodies often owe their existence to the frictional resistance experienced by the bodies themselves.

The effect upon the tides of the attraction of the water in its disturbed state can be safely assumed to be small. (See Fig. 6.)

* See Bulletin of the Philosophical Society of Washington, Vol. 14, pp. 93-99.

† Tides and Waves, Arts. 315-346.

CHAPTER V.

THE SEMIDIURNAL TIDES IN THE INDIAN OCEAN.

23. The Indian Ocean north of the thirtieth degree of south latitude is, with one exception, but little influenced by the tides of other waters. The exception is due to the fact that there is a good rise and fall around southern Africa and in Mozambique Channel, where the tide depends upon two systems of oscillations which are determined by boundaries largely outside of the North Indian region. These systems, styled South Atlantic and South Indian, are described in Chapter VII, Part IV A. It may be noted here that observations indicate about 1.5 as the Greenwich lunar time of high water in Mozambique Channel, and that this is about the theoretical time of the tide, for it is a mean between XII or 0 and III; see Fig. 1, which is taken from the chart of semidiurnal systems in the Part just referred to. Extending from Mozambique Channel to Baluchistan and India is a half-wave area whose time of tide, as will be noted later, is largely governed by the tide in the channel.

24. *The North Indian system.*

This system consists of the canal-like whole-wave area extending from the north-western coast of Australia to the coast of Somaliland and Arabia, and of a dependent fractional area, viz., the Bay of Bengal. Since the eastern part of the whole-wave area has much greater depth than the western, there are some advantages in regarding the whole strip as two half-wave areas.

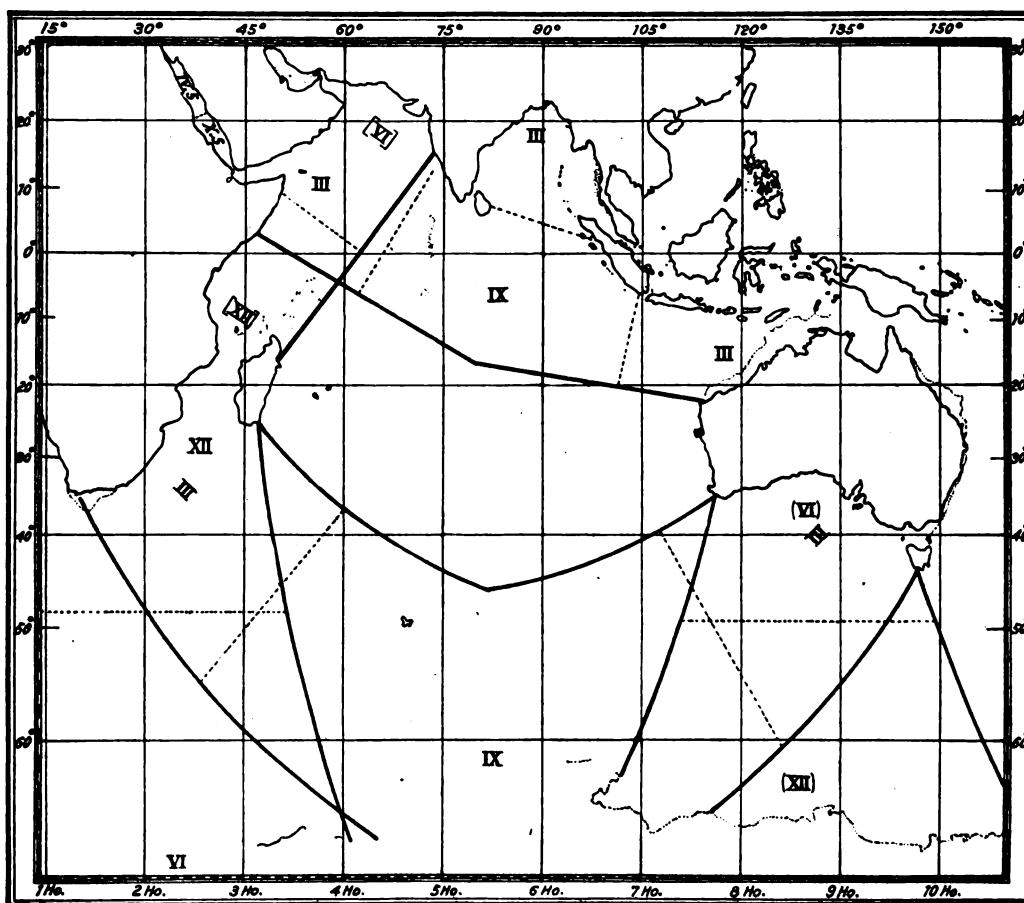
First of all it is required to ascertain the theoretical times of high water at the loops (i. e., ends and middle) of this strip by applying thereto arrows which denote the intensity and direction of the tidal forces at as many points as we find it necessary to take. In this instance the two points where the axis or central line crosses the two nodal lines will probably be sufficient. The positions of these points are about latitude $12\frac{1}{2}^{\circ}$ south, longitude 103° east, and $3\frac{1}{2}^{\circ}$ north, $67\frac{3}{4}^{\circ}$ east, respectively. The trend of the axis of the eastern half of the canal is about south 75° east and of the western half, north $58^{\circ} 40'$ west. The free period of this body of water being reasonably close to a half lunar day for a binodal mode of oscillation, therefore the elongation of the particles, and the high or low water, must happen when the virtual work of the impressed periodic forces becomes zero.* For, as will appear from the inspection of Fig. 2, the length and position of the canal are such that the forces tend to incite a considerable oscillation; that is, their effects are not continually neutralized. At various assumed Greenwich lunar hours reduced to the local time by adding the east longitude, project the force arrows upon the axis of the canal.† Considering first the canal as uniform throughout its length, and supposing the forces to act upon equal masses undergoing equal but opposite displacements at the two nodes, the virtual work at any

* § 63, Part IV A.

† §§ 2, 65, Part IV A.

given time may be represented simply by the sum of the forces applied to the two nodes provided we regard those as positive which urge the water, say, toward the ends, and as negative those urging it from the ends toward the center. In Fig. 2 the forces being applied only at the two nodes, are given equal weight; those acting on the western area are written to the left of the ordinate, and those acting on the eastern are placed to the right. Forces directed westerly are represented by broken lines and easterly by full lines. The curve in Fig. 2 may be regarded as representing the virtual work in the

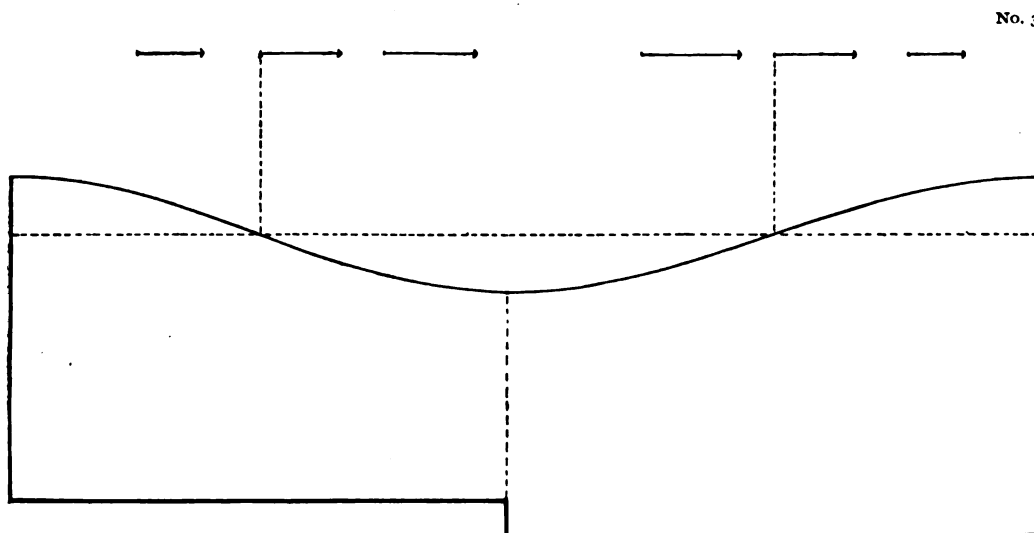
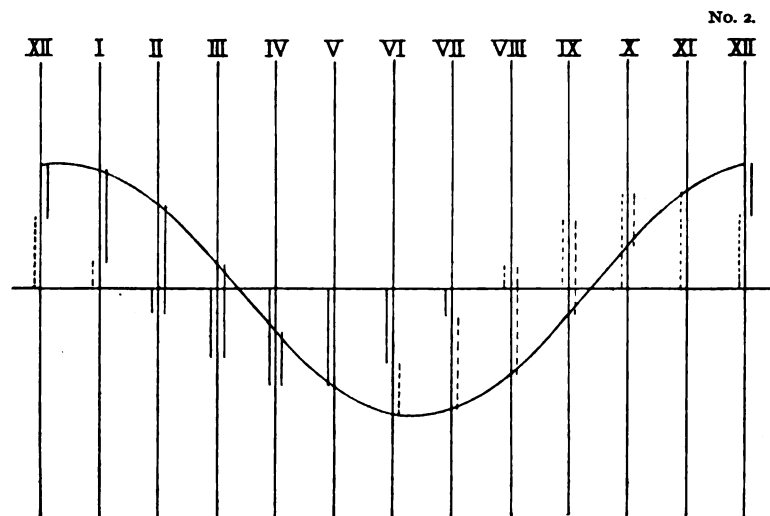
No. 1.



Semidiurnal systems for the Indian Ocean.

present instance for all hours because we can assume that at various times the magnitude of the arbitrary time element is so taken that at a given point the absolute length of the corresponding virtual displacement remains the same for all times; in other words, that the numerical values of the virtual displacements are independent of the time and depend only on the positions of the points considered. They are comparatively large at nodal lines and zero at the loops. Fig. 2 shows that high water at the ends should occur at III.36, Greenwich lunar time, and so at the middle at IX.36. Fig. 3 shows the surface of the water and the configuration of the forces at III.36 both for the nodal

points and for points halfway between nodes and loops. It will be noted that the forces in the eastern half are then equal to those in the western and act in the same direction. But if the points are given a slight horizontal virtual displacement, as by putting them in positions which they occupy just previous to the time of elongation, it



will be noted that for a uniform canal the displacements are equal and opposite in the two half-wave lengths. Hence the sum of the products of the impressed periodic forces by the virtual displacements must be zero at the hour III.36.*

If two half-wave rectangular areas of different depths and widths meet end-to-end at a loop and are otherwise completely surrounded by rigid walls, the above rule for

* It may be noted here that the factor $\cos \alpha_{v,\mu}$ should be annexed to each of the parts of eq. (314) Part IV A, it having been accidentally omitted in the copying or printing.

finding the time of tide by means of the virtual work can be carried out in almost as simple a manner as in the case where both are uniform; for, considering each area as being divided into the same number of elementary slices, the product mass \times force \times displacement, in the two areas, is proportional to depth \times width \times force. In other words, instead of giving equal weights to the two sets of values, as was done in Fig. 2, one set is to be given greater weight than the other. Again, and this probably has more bearing upon the actual strip under consideration, it is reasonable to suppose that, because of the numerous straits eastward from Java, and the obliquity of the north-western coast of Australia to the general direction of the strip and the shoaling along this coast, a greater percentage of the forces acting upon the eastern half-wave area will be lost than of those acting upon the western. In other words, the set of values in Fig. 2 belonging to the western part should have a slightly greater weight than the set belonging to the eastern. For this reason it appears that III is probably about as good a value for the times of high water of the stationary wave at the two ends as can be readily estimated from theoretical considerations.

Immediately connected with the whole-wave area is the Bay of Bengal. We can consider the water extending from the head of the bay to some distance southeast of Ceylon as a dependent area somewhat canal-like in form with a nodal line extending from Ceylon to the western coast of Sumatra at a distance from the virtual head of the bay of $\frac{1}{4}\lambda$, or one-quarter wave length. This area synchronizes with the remainder of the system of which it forms a part, and the two together constitute the north Indian system.*

25. *The waters north and northwest of Australia.*

The eastern half of the whole-wave area has, besides the stationary wave, whose high waters occur at III and IX, a progressive wave, due largely to straits or openings between the islands. Of special importance is the strait south of Timor Island. From this opening to the Gulf of Carpentaria, especially where the water is shallow, the tide is chiefly progressive, as shown by the cotidal lines. The progression due to this and other openings is felt halfway or more to the African coast. On account of the great depth of the Banda Sea and of the shortness and considerable depths of the passages around Timor Island and vicinity, the tide is nearly simultaneous over this sea. It is somewhat later and a trifle smaller than the tide around Timor. The maximum eastward velocity through Ombay Passage north of Timor must occur between the time of mean sea level rising (for western Timor) and high water. The progressive wave due to this short strait, approaching from the west, must have its maximum velocity, or high-water phase, northwest of Timor at about the time of maximum velocity in the strait; consequently it must be in advance of what it would be were it to reach the Ombay Passage at the time of high water of the stationary wave. In other words, the tide just west or south of this boundary, pierced by short straits, is a little earlier than it would have been had the straits been broad and the area beyond shallow. It seems reasonable, therefore, to suppose that the progressive wave at the eastern extremity of the one-wave area (say about along the meridian of Kupang) should be about half an hour in advance of the stationary wave. Assuming that the

* § 76, Part IV A.

amplitude of the progressive wave along the axis of the eastern part of the whole-wave area is equal to the amplitude of the stationary wave at the loops and that the former is half a lunar hour in advance of the latter at the loops, we have, by § 3, the distribution of the cotidal lines shown on the map along the central line of the area.

At first sight it seems strange that the amplitude of this progressive wave could be more than a small part of the amplitude of the stationary wave at the loops. But it should be noted that the energy coming through the tidal forces into the oscillating system is mainly consumed in overcoming the resistances experienced throughout its various parts. If, now, a part of this energy be lost to the system because of breaks in the boundary near a loop or because of extensive shoaling, the resulting progressive wave traversing a region small in comparison with the whole system may, for some considerable distance westward from the opening in the boundary, have an amplitude comparable with that of the remaining stationary wave, and the system will oscillate nearly as it would in the case where no energy were thus lost; but, of course, the amplitude will be somewhat diminished.

It is reasonable to suppose that the southern extremities of the cotidal lines west of Australia and numbered IX to II turn eastward, somewhat as shown upon the map, because of the progression directed toward Timor Island.

26. *The half-wave area.*

This area extends from Mozambique Channel to Baluchistan and India, being largely a dependent one whose tide is governed by the rise and fall in and just south of the channel. It has already been remarked that observation and theory give about I.5 as the time of high water in the channel.

The bracketed values XII or 0 and VI, shown in Fig. 1, are intended to refer only to the South Atlantic system; that is, they indicate the theoretical time of tide as it would be without the North and South Indian systems, and not the times of the actual tide.

Assume now the existence of the South Indian system without the North Indian; the tides in this area will, as was assumed before, synchronize with those in the channel, and so instead of 0 and VI we shall have about I.5 and VII.5.

If, on the other hand, a wall were built across the channel, the theoretical times of tide produced in this half-wave area would then be II.1 and VIII.1.

But the L-shaped region extending from Mozambique Channel toward India, thence to northwestern Australia, can not have twelve hours as its free period because the virtual length of either of the two trapezoids into which the region can be divided is too small, being in each case less than that of the original rectangle. It is, therefore, fair to assume that the Mozambique-India half-wave area does not synchronize with the Australia-Arabia whole-wave area; for, although each has a free period approximating twelve hours, the period of the combination regarded as an L-shaped figure is much shorter.*

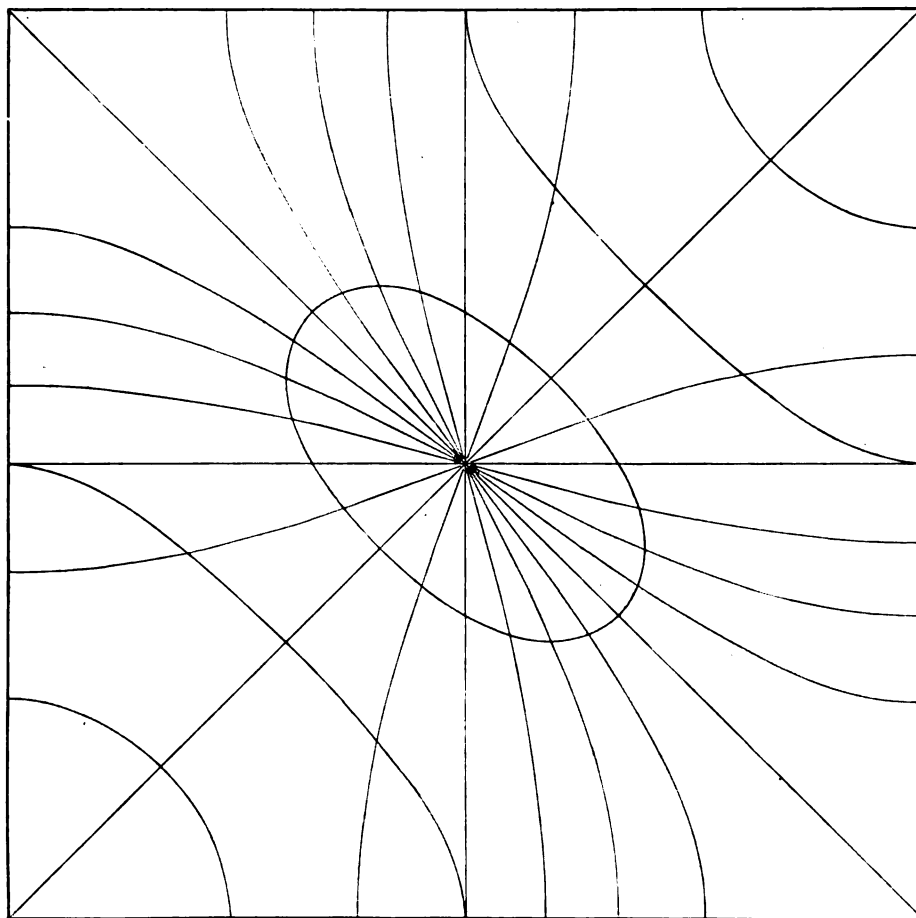
We can, therefore, by lemma 27, suppose that the oscillations of the north Indian system and that of the Mozambique-India area are nearly independent of each other, although it is probable that the latter is accelerated a few minutes by the former because

* § 43, Part IV A or lemma 5.

the loop in the Arabian Sea marked III, Fig. 1, lies unequally with respect to the nodal line off Somaliland.

Before proceeding further, let us consider the case of two systems of oscillation in a square area. For simplicity, let the two amplitudes be taken as equal. Suppose the phase of the north-and-south oscillation to be 60° in advance of that of the east-and-west oscillation. Fig. 4 shows the arrangement of the resulting cotidal lines for each half hour, also the lines along which the range of the tide is constant. It is to be noted

No. 4.

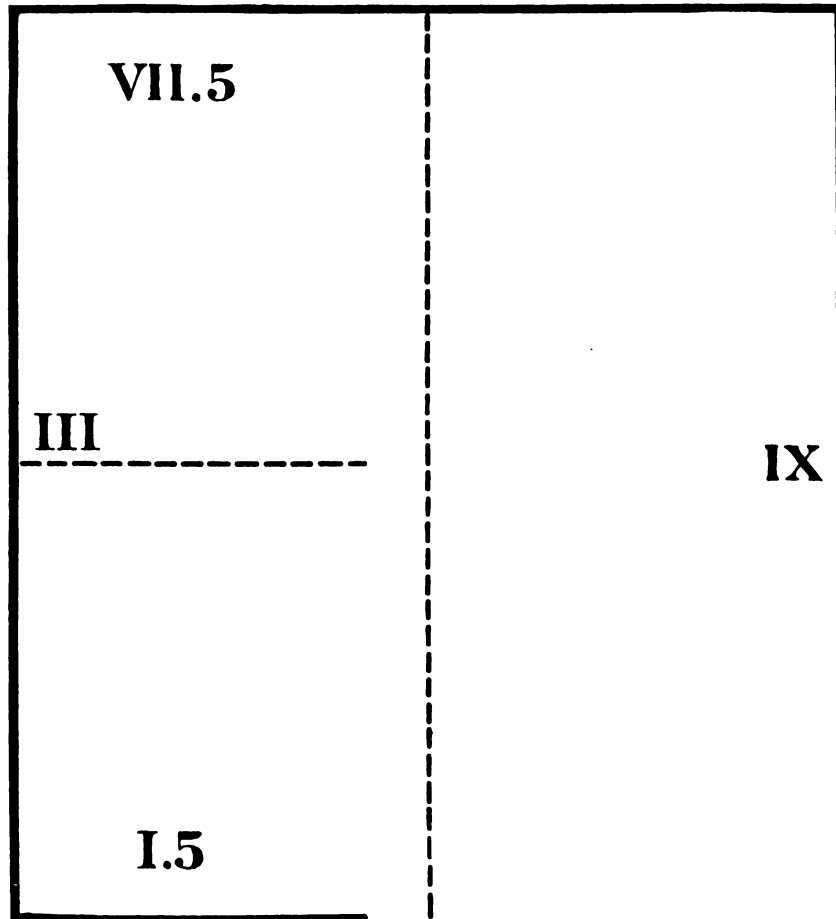


that the cotidal lines are crowded together near the nodal lines of the component oscillations. The crowding will differ from that shown in Fig. 4, if we use another amplitude ratio and phase difference. But this figure will give a general idea of how the cotidal lines should be drawn for a square area. In all cases where the phase difference is not zero or 180° there must be a no-tide point around which the numbers of the cotidal lines progress, but not uniformly or at the rate due to depth. Where the phase difference is zero or 180° there will be a nodal line and no progression.*

* §§ 26, 32, Part IV A and § 4, Part IV B.

Returning now to the Arabian Sea and adjacent waters we note that the conditions are far from having the simplicity of a square. Nevertheless, considering the superposition of two independent systems, it can hardly be doubted that there exists a point near the intersection of the nodal lines (Fig. 1) where the range of tide is very small. It is almost certain that the cotidal lines should approximately radiate from this point being crowded together in the vicinity of Cape Guardafui and spread apart in the

No. 5.



Arabian Sea and toward the Mozambique Channel. Moreover, the progression around this point should be clockwise and apparently irregular.

Instead of the complete square, Fig. 4, imagine a region having nearly the form of a square, but deprived of a portion of its boundary, as shown in Fig. 5. The half-wave area occupies the greater part of the western half of region. The western half of the whole-wave area covers a little more than the northern half of the region. The eastern boundary, although imaginary or latent,* is as good as rigid so far as producing the tide is concerned. Although, as will be noted in § 31, the cotidal hour for the region

* § 29, Part IV A.

lying between the central loop of the North Indian system and that of the South Indian system is not far from VIII, and so the hour for the whole of the eastern end of the hypothetical square differs but little from IX, it is impossible to have an east-and-west oscillation across the southern portion of the square because no whole-wave area is possible immediately south of the North Indian system, and so the latent boundary of the eastern side of the square can not extend beyond the southern limits of this system.

At the India end of the Mozambique-India region the east-and-west or whole-wave oscillation is probably as important as the north-and-south or half-wave, while at the south end the east-and-west oscillation is not directly felt. Hence, at India the tide occurs between the hours III and VII.5. In the Mozambique Channel, on the contrary, it occurs at I.5, or probably a little earlier (say at I), because of the slight influence of the north Indian system already referred to. Around the southern Maldivé Islands, and especially around the Chagos Archipelago, the half-wave oscillation does not exist. From the Arabian Sea to the loop south of Ceylon the tide is small, and there an irregular progression occurs, as is usually the case between two nonsimultaneous regions not too far apart.* There is also an irregular progression from the loop south of Ceylon to northern Madagascar. The direction of these progressions is clockwise, as was that inferred at the close of the preceding paragraph. In the Gulfs of Aden and Oman the tide consists chiefly of stationary oscillations, although there is some progression on account of the inner waters beyond. Each of these two gulfs constitutes a dependent canal-like area whose length is a small fraction of the wave length λ .†

The Persian Gulf being a large, shallow body of water connected with the Gulf of Oman by a large strait, has a tide which progresses at about the rate due to depth. There is some retardation and therefore some crowding up of the cotidal lines in the strait.

27. *The Red Sea.*‡

The tide in this sea is composed of a stationary wave caused by the tidal forces acting on its waters and a progressive wave produced by the outside tidal disturbances acting through the Strait of Bab el Mandeb. The force arrows applied to the nodal line of this sea, regarded as a simple canal-like area whose period is about a half day, give IV.5 and X.5 as the cotidal hours of the north and south ends, respectively.

Because the range of the outside tide diminishes rapidly in passing through the strait, it is difficult to ascertain where a wave progressing at the rate due to depth will be established. But observations made at the north end of the sea indicate that high water of the resultant or combined wave occurs there at about III.75 hours, whereas, according to what has just been said, the high water of the stationary part should occur at IV.5. If now we assume the amplitude of the progressive wave over the Red Sea to be constantly equal to the amplitude of the stationary wave at the loops, and assume the phase of the progressive wave to be 45° in advance of the phase at the loops of the stationary wave, the result of combining the two is as shown by the cotidal lines, the extreme southern end of the sea being excepted (§ 3). On account of the extreme narrowness of the strait, these assumptions concerning the progressive tide seem to be reasonable. In making the computations the depth of the sea was assumed

* Lemma 25.

† Lemma 12.

‡ §§ 80-82, Part IV A.

to have a certain constant value over the region north of the nodal line and a slightly smaller but constant value over the region south of it. The manner in which the cotidal lines should be distributed in and near the strait is somewhat uncertain because the theory underlying such cases has not yet been worked out.*

As indicated by the map the tide in the narrow but deep Gulf of Akabah is chiefly a stationary wave, whose high water is about simultaneous with that at its mouth.

The tide in the much shallower Gulf of Tor is also chiefly a stationary wave with a node at Tor Bank.† However, as the ending of the gulf is not extremely abrupt there must be some small progression upward. Assuming the cotidal hour of both stationary and progressive waves to be III.75 at the mouth and the constant amplitude of the progressive wave to be one-third of the maximum amplitude of the stationary wave, we obtain the cotidal lines shown on the map.

28. No large wave progresses from the South Indian Ocean into the North Indian, producing the tide of the latter.

An harmonic analysis of the tides at Freemantle, Swan River entrance, western Australia, shows that the average range of semidiurnal tide is there only 0.4 foot. A similar analysis at Port Louis, Mauritius Island, gives for the mean range of tide 1.1 feet.

Upon examining any chart of the Indian Ocean which shows the depths approximately, it will be seen, upon making a few measurements, that the region of small range extending from Freemantle to Mauritius Island can not indicate a true nodal line. Again, observations do not indicate that nearly the whole of the Indian Ocean belongs to one system, as such an extended nodal line would imply.

We are therefore led to believe that there is a region of small semidiurnal tides extending from Freemantle to Mauritius Island, and that the absence of a good tide is due to the fact that the distance between western Australia and Madagascar does not approximate to λ or to one-half λ , as an east-and-west stationary wave would require. Some tide, however, exists, and for reasons which will be given in § 31.

By referring to the table given in § 97, Part IV A, it will be seen that for Freemantle (690), Kupang (614), Banjuwangi (561), and Port Louis (870), the ratio $S_2 M_2$ is larger than the corresponding ratio of the tidal forces, i. e., than 0.465. The same is true of Port Darwin (681.5), given in § 19, Part IV B. This indicates that the length of the whole-wave area of the North Indian system, especially of this area when joined with or influenced by the waters between Australia and Madagascar, corresponds better to a solar-wave length than to a lunar. This accords with inferences made from known distances and depths by aid of Table 51.

Again referring to the same tables it will be seen that the ratio $O_1 K_1$ is considerably less than the corresponding ratio of the forces ($= 0.711$), not only for the stations just mentioned but for the Indian Ocean generally. This shows what might have been inferred from Fig. 24, Part IV A, that the Indian diurnal system responds better to the period of K_1 than to the period of O_1 .

There is some inward progression from Rodriguez northwesterly toward the Farquhar Islands and Cape Amber, but it is to some extent mixed up with the

* 113, Part IV A.

† 81, Part IV A.

stationary wave around northern Madagascar. In fact the tide between Rodriguez Island and Cape Amber is due chiefly to the rise and fall at the north end of Mozambique Channel; that is, a species of dependent transverse oscillation, part stationary and part progressive, is maintained by this rise and fall. A somewhat similar effect may exist off Northwest Cape, Australia. The cotidal lines must change from VIII to I in going from the early region east to Reunion Island to the northern end of Mozambique Channel, and from VIII to III in going from this early region to the loop northwest of Australia.*

29. *Miscellaneous remarks.*

Upon referring to Fig. 1 it will be noticed that because of proximity to nodal lines the range of tide around Ceylon and southern Hindustan must be small. For the same reason the tide along the outer or southwestern coast of Sumatra should be small. The tide at the western end of the southern coast of Java should be smaller than is the tide farther east. All these requirements accord well with the observed facts, as can be seen upon referring to the large map, Fig. 7.

The distribution of the cotidal lines through each of the nodal lines respectively located east of Ceylon, off the Malabar coast, and off Somaliland has not been made upon the map in accordance with any assumed mathematical law. The lines are chiefly conjectural, although in accord with the observations.

Upon measuring the dimensions of the oscillating areas, it may appear that the actual lengths are a trifle too short for the existence of large tides, although no definite criterion has been laid down in such matters. There are difficulties in the application of Lagrange's rule to even a canal-like body of variable depth. It seems, however, upon comparing the times required for the eruption of Krakatoa to have been felt at ports in India, Arabia, and Africa, that the areas as laid down on Fig. 1 must have a free period sufficiently long to permit good stationary oscillations.†

The map shows several cases in which a derived wave is produced at a sudden shoaling. For example, the Gulf of Martaban, the vicinity of the mouths of the Ganges, the Gulf of Cambay, and off the Gulf of Kutch. The progression north of Australia is due largely to such sudden shoalings as the Sahul Bank. A dependent stationary wave is apparent in the Gulf of Kutch, and some traces of dependent stationary waves, shown in the acceleration of the times of the tide, can be seen at the 80-mile beach (on the northwestern coast of Australia) and near and in the mouth of the Hugli River.

Sokotra Island and neighboring shoal have some influence upon the distribution of the cotidal lines in that locality. A similar remark may be made concerning the Laccadive and Maldivé Islands.

In passing through Sunda Strait, the range rapidly diminishes, while the cotidal lines bunch up.‡ A somewhat similar statement is true of the narrow straits farther east.**

* Lemma 25.

† Cf. The Eruption of Krakatoa, and Subsequent Phenomena, tabulation opposite page 148 and Plate XXXV.

‡ Lemma 10.

** §§ 104-106, Part I V A.

The tide wave entering Palk Strait proceeds southwesterly up Palk Bay, at about the rate due to depth, nearly to Adams Bridge.

The tide proceeds southeasterly through Malacca Strait, but not at the rate due to depth alone, excepting in the broader portion.

30. *Evidence obtained from tidal streams.*

The directions of the observed tidal streams as given in the Admiralty Pilots afford some clue to the character of the tidal oscillation, especially in localities where the motion is rectilinear. Around the Maldivé Islands the flood stream sets easterly and around the Chagos Archipelago it sets southeasterly. In the northern end of Mozambique Channel the flood sets southerly and in the southern end northerly. Moreover, we find that flood slack occurs soon after high water in the channel, as a stationary oscillation requires. These facts could have been inferred from Figs. 1 and 7.

On the southern coast of Cape Colony, from Table Bay eastward to Port Alfred, no sensible tidal stream exists, although there is a moderately large rise and fall. This is in accordance with the lines of motion terminating at the shore which runs nearly perpendicular to them. See Fig. 1.

The southern shore of Baluchistan lies at the loop of the half-wave area as also of the whole-wave area. Consequently the tidal streams should there be weak, and observation shows this to be the case.

Across the shoals and around the islands east and northeast of Madagascar there are strong tidal streams. According to the map of cotidal lines, this is a region where the tide varies both in time and in range; such conditions always imply large accelerating forces for the water masses and generally large velocities, especially over shoals.

In the short straits which separate from one another the islands east of Java, the currents should be swift. This statement is confirmed by observation. But whether or not the time of maximum flood velocity is an hour or so before the time of high water at the southern ends of the straits has not yet been ascertained.

Strong tidal currents occur among the Nicobar Islands and generally among the Andaman Islands. At Table Island the streams turn at about the times of high and low water.

The streams in the Gulf of Suez accord well with the notion of a stationary wave, the times of slack water very nearly coinciding with the times of the tide at the head of the gulf.*

31. *The South Indian system.*

The South Indian system has been briefly considered in § 77, Part IV A. It extends from the south coast of Australia southwesterly $\frac{1}{2} \lambda$ to where it is supported by the Antarctic Continent; thence northwesterly $\frac{1}{2} \lambda$ to Madagascar and South Africa. By constructing a diagram similar to Fig. 2, it will be found that the theoretical cotidal hour for either end is about III, and for the middle IX. According to Figs. 6 and 40, the cotidal hour for the south coast of Australia is about III.

By referring to Fig. 6, it will be noticed that there exists a region north of Kerguelen Island, from which the numbers of the cotidal lines increase in whatever direction one goes. The approximate time of tide of this region can be seen from theoretical

* § 80, Part IV A.

considerations, viz.: It should be not far from IX, because this is the time for the loop of the systems lying to the north and to the south of it.* On account of the magnitude of the space between the two systems, it is reasonable to expect to find small ranges between Australia and Madagascar. Farther south the range is greater; the mean range at St. Paul Island being 3.0 feet, and at Betsy Cove, Kerguelen Island, 3.2 feet. According to Fig. 6, the hour for this region is VIII. Figs. 6 and 7 show a number of progressions in various directions whose antecedent waves extend to this region † and have the effect of causing its tides to occur a little earlier.

Just westward of this early region, particularly of its northern part, a great crowding up is necessitated because the tide south of Africa and Madagascar occurs at I, Greenwich time, and its range is there considerable.

Spencer Gulf.—According to Fig. 40, the tide is partly progressive and partly stationary. The same is true of the Gulf of St. Vincent. This can be seen by using the depths given in Fig. 34, Part IV A, and Table 50 or 51.

The tides off Kangaroo Island are delayed by openings on either side of the island.‡

Bass Strait.—The Pacific tide joins the Indian tide near the western end of the strait. The range of tide is somewhat greater at the eastern end than at the western.

In Port Phillip the tide is delayed and its range diminished by the narrow entrance. (See Figs. 34, 35, and § 105, Part IV A.)

South of Tasmania there is a large region whose cotidal hour is about X. The tide is largely due to the Pacific acting through the channel between New Zealand and Tasmania. But upon referring to the South Pacific system (Fig. 23, Part IV A) it will be seen that the loop marked XII lies near the Antarctic Continent southeast of New Zealand, while the loop marked IX of the South Indian system rests upon the Antarctic Continent southwest of Australia. Hence the time of tide along the Antarctic Continent changes from IX in this last locality through X south of Tasmania to XII southeast of New Zealand.

From this X region to the loop marked III just south of Australia the time of tide changes rapidly, as is indicated by the crowding up of the cotidal lines just west of Tasmania.

32. *References to Indian Ocean tidal observations and discussions.*

India.—Reports of Survey of India, referred to in § 18.

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Madagascar and Indo-China.—Comptes-Rendus de l'Association Géodésique Internationale, 1900.

Annales Hydrographiques, 1901.

* Lemma 25, last line.

† Lemma 29.

‡ Lemma 31, Case 3.

CHAPTER VI.
THE SEMIDIURNAL TIDES IN THE ATLANTIC OCEAN.

33. *Character of Atlantic tides.*

The tides of the Atlantic Ocean are remarkable for the smallness of the diurnal wave, a circumstance due to the absence of oscillating systems whose free period approaches 24 hours. * As the small tides of the Mediterranean Sea caused the civilized nations of antiquity to pay little attention to the semidaily rising and falling of its surface, so in later times the small diurnal inequality in the Atlantic caused the western nations to regard as remarkable the tides observed in other seas where this inequality becomes prominent. But when Newton calculated its theoretical amount by his equilibrium hypothesis he saw that the forces indicated an inequality in the tides several times greater than any which had been observed on the coasts of Europe. This led him to an erroneous explanation of the phenomenon. The error was pointed out by Laplace. But he in turn makes the erroneous suggestion that the smallness of the diurnal rise and fall in the Atlantic may be analogous to its total absence in case of a globe uniformly covered with water. †

The semidiurnal tides of this ocean are due chiefly to two oscillating systems, which are described in §§ 72-73, Part IV A.

By referring to Fig. 6, it will be seen that there is one conspicuous amphidromic region southeasterly from Newfoundland, the center being placed at 40° N. and 40° W.

It will also be seen that there is, generally speaking, a northerly progression in the time of tide from the Antarctic Continent south of Cape Colony, through the main body of the Atlantic. The progression from South Africa northward led to the once common belief that the tides of the Atlantic are largely, or even chiefly, derived from the waters farther east. It is true that unaccountable irregularities in the rate of this progression and great variations in the range of tide caused many students of the subject to doubt the derived-wave hypothesis.

The reasons for a general northward progression along the western coasts of Africa and Europe are not difficult to find, nor are its irregularities surprising. Given several regions of high water, such as would be produced by two oscillating systems, it is clear that in going from one region to another the times of tide for all intervening points will be forced to assume all intervening values. The change may be made suddenly, causing a crowding together of the cotidal lines, or it may be made in such a way as to distribute them more equably. ‡ Progressive waves in the ocean are generally caused by openings in the shore lines at or near the loops of the oscillations where the rise and

* § 92, and Fig. 24, Part IV A. † See §§ 88, 101, Part I. ‡ Lemma 25. Cf. § 35, Part IV A.

fall is considerable.* A similar cause is the sudden receding of the shore line from a part of it which supports a good rise and fall.† The openings around the northern part of the North Atlantic generally favor northward progression; so do the sudden recessions of the coast and changes of depth near Cape Frio, South Africa, and between Sierra Leone and Cape Verde.

But even in this ocean there are progressions taking other than northerly directions. The tidal hour for the waters around southern Greenland lies between VIII and IX, and for deep water off the eastern coast of the United States it is about XI½. Some kind of southerly progression from the former region to the latter must, in consequence, take place.‡ The tide progresses southwesterly from a region west of the Cape Verde Islands to the northeastern coast of Brazil. This is largely due to the overlapping of the two systems (§ 4).

However, on account of the general tendency of the progressions of the times of tides, we shall begin with the southeastern corner of the ocean and gradually work northerly and westerly. The tides of Patagonia and Argentina will be described in connection with the South Pacific system, from which they are derived.

34. *South Atlantic Ocean.*

The first half wave of the South Atlantic system (Fig. 23, Part IV A), extends from the Antarctic continent to latitude about 27° S. The southern coast of Cape Colony is therefore less than ½ λ distant from the Antarctic continent, and so the intervening region responds to the solar forces more readily than to the lunar.

The east-and-west extent of the Cape Colony coast line is not, however, sufficient for the establishment of an independent solar system of large range of tide.||

Doubtless the South Indian system, taken in connection with the adjoining waters to the north of it, also favors the solar forces. Hence it is not surprising to find that the ratio S_2/M_2 is 0.55 for Durban, Port Natal, and 0.47 for Port Elizabeth, Algoa Bay. Cape Town being less influenced by the South Indian system, the ratio S_2/M_2 becomes somewhat smaller, viz., 0.42.

The considerable progressive wave over this half-wave area is necessitated by the progressive wave generated near Cape Frio, as mentioned in the preceding section, and which is chiefly responsible for the tides in the Gulf of Guinea.

Between the loop marked XII, west of South Africa, and the eastern coast of Brazil is a branch of the next or second area of the South Atlantic system.

The great distance between the lines V½ and VI, east of Brazil, Fig. 9, indicates the stationary character of the oscillation. The decrease of range in going from Pernambuco to Rio de Janeiro indicates approach to the free southern boundary of this branch area.

The Admiralty Tide Tables give 2 feet as the value of the spring range at Ascension Island. This indicates that the nodal line of the area probably passes not far from it.¶

* Lemma 14.

† Lemma 19.

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|| Lemma 3.

‡ Lemma 25. Cf. § 35, Part IV A.

¶ § 79, Part IV A.

35. *From Liberia to the Windward Islands.*

The tides of Liberia and Sierra Leone are not much affected by the North Atlantic system, but belong chiefly to the South Atlantic system. Hence, unless delayed by shoals, the tidal hour should be VI. A nodal line of the North Atlantic system probably passes just south of the Cape Verde Islands. Hence it happens that from a region south of these islands or even from Liberia to the Barbados the time of tide changes by scarcely more than half an hour. There are extensive shoals west of Sierra Leone and around the Bijonga Islands.

The northeastern coast of Brazil forms a portion of the end boundary of the North Atlantic system, and so the theoretical time of tide for the greater part of this coast is about VIII.* The southern lateral boundary of the South Atlantic system is situated some distance out from the coast of Guiana and Brazil, somewhat as indicated by a heavy line in Fig. 23, Part IV A. Hence the theoretical time of tide off this coast lies between VI and VIII (§ 4). The extensive shoals off the Amazon and Para rivers cause a considerable progression, and delay the time of tide for this part of the coast.

In going up the shallow delta of the Amazon, the tidal hour is much increased; but tidal data and even soundings are so meager that cotidal lines can not be drawn far up the river. A bore occurs when the range of tide and stage of the river are favorable.†

The tides around Trinidad Island belong almost entirely to the North Atlantic system. The range is much greater on the south side of the island than on the north side, the oscillation in the former locality being better supported than in the latter. Hence the strong currents experienced by Columbus in the Dragon's Mouth.‡

From Trinidad to Guadeloupe the semidiurnal range of the tide diminishes rapidly. The tides in the southeastern part of the Caribbean Sea are derived from the loop marked VIII of the North Atlantic system influenced, however, by the loop marked VI of the South Atlantic system.||

36. *From Guadeloupe to the United States.*

The range of the semidaily wave on the south coast of St. Thomas Island is a little less than 0.3 foot, and the mean range of tide San Juan, P. R., is 1.1 feet. These indicate that the nodal line lies not far east of St. Thomas. Figs. 11, 14, 16, show how nearly simultaneous are the tides along the greater part of the Atlantic coast of the United States, the Bahamas, the northern coast of eastern Cuba, the northern coast of Haiti, and of Porto Rico. Moreover, the range steadily increases in going from St. Thomas to Savannah River entrance where it is 6.8 feet. All these facts point to a stationary oscillation, and Fig. 23, Part IV A, indicates that it forms part of the South Atlantic system.

Here we have one of those rare cases where a nodal line of a system is not obscured by progressive waves or by the overlapping of another system.

Upon consulting the charts of soundings, Figs. 19, 20, Part IV A, and making use of Table 51, it will be seen that the length of the South Atlantic system is a little more than $3/2 \lambda$. Hence unless neighboring waters alter the mode of oscillation for the solar wave it follows that the ratio $S_1 M_1$ must be less than the ratio of the corresponding

* Lemma 9.

† § 15, Part I.

‡ § 75, Part I.

|| Lemma 14.

forces. The quarter wave length here considered is almost free from such influences. Hence the remarkably small value of this ratio for tides along the Atlantic coast of the United States. See table in § 97, Part IV A, Nos. 32-105.

37. *The Caribbean Sea.*

The tide along the coast of Panama agrees well with the corrected equilibrium theory both as to time and range (§§'2, 3, Part IV A).

The tide in the northern portion of the sea is due chiefly to the tide north of Cuba and Haiti acting through the Windward Passage. Farther west the effect of the tide in the Gulf of Mexico becomes apparent. See Figs. 11 and 17.

As already noted, the tide in the southeastern corner of the sea depends directly upon the tide in the ocean just outside.

The northeastern corner of the sea can have only very small ranges because a nodal line exists in the outside waters, and because the numerous openings between islands which bound the sea on the north and east prevent the formation of an equilibrium tide. The hour of tide changes rapidly, increasing toward the north.

Very little is known of the tides along the southern coast of Haiti or along the opposite coast of South America.

38. *The Gulf of St. Lawrence.*

The tides of the Gulf and River of St. Lawrence are interesting in several respects, but chiefly on account of a large, stationary oscillation, a landlocked wave eddy or amphidromic region, and a wide variation of range at different points of the river. The first of these features has been noted in § 91, Part IV A, and the other two are shown upon Fig. 13, Part IV B. A map of soundings covering this region is given as Fig. 31, Part IV A.

North of the Magdalen Islands the range of tide is small while in the channel off Cape Rosier or Gaspé the range is 4 or 5 feet. The shore line here suddenly turns off from the direction of the main channel, forming the western boundary of a broad, shallow embayment. Hence, by lemma 19, a southward progression takes place following the general direction of this shore line. Section 13 deals with cases of this kind; what is there referred to as the "outside body" is here represented by the channel between the Magdalen Islands and Anticosti Island. The range of this progressive shore wave diminishes as it proceeds. The part entering Northumberland Strait from the west meets the wave from the east at Cape Tormentine.

Upon referring to the cotidal chart of this region as well as to the chart of depths, it will be seen that progression in the river at the rate due to depth does occur below the mouth of Saguenay River. The greatest range occurs off the lower end of the Isle of Orleans, where it is nearly 17 feet; the tide disappears at Lake St. Peter.

As might be expected, the duration of fall in the upper part of the river considerably exceeds the duration of rise; at Quebec the difference is two solar hours.

The tide of Chaleur Bay is chiefly stationary.

A branch of the main channel extends northeasterly between Newfoundland and Anticosti Island. The tide in the center of this branch is nearly simultaneous with that at the point where it joins the main channel; but some delay occurs in reaching

either shore. Moreover, the gradual narrowing and shoaling of this area between Newfoundland and Quebec causes the range on either side to be considerable, viz., 3 or 4 feet. The tidal hour off the eastern end of Anticosti Island being $11\frac{1}{2}$ and off the western end a little more than $V\frac{1}{2}$, the cotidal lines must crowd together in the strait north of this island.

The Strait of Belle Isle has little influence upon the tides of the Gulf.

The tidal hour is probably between XII and $XII\frac{1}{2}$ for the eastern coast of the Magdalen Islands, and probably assumes all values from $\frac{1}{2}$ to XII along the remainder of the coast, but no data are available for verifying this conclusion.*

39. *From Nova Scotia to Rhode Island.*

Along the greater part of the Atlantic coast, extending from Nova Scotia to Florida, the time of tide is but little delayed on account of the strip of shallow water which borders it. (For depths, see Figs. 19 and 31, Part IV A.) This is an application of lemma 9, since the offshore flood and ebb streams are probably nearly normal to the coast line. We can imagine canals leading from deep water to or into the coast line, each having a stationary oscillation. Generally such canals are less than $\frac{1}{4}\lambda$ in length. The Gulf of Maine has a shoaling at Georges Bank which causes a progression to there be formed; that is, the Gulf of Maine can not be considered as forming part of the large body of water to the southeast or even as being a dependent area synchronizing with it. The virtual length of the Gulf of Maine and the Bay of Fundy being nearly $\frac{1}{4}\lambda$, there must result large tides for the inner waters, sustained by the progression, and occurring three hours later than the tides outside. This has been explained in § 34, Part IV A and alluded to under lemma 12. A similar explanation applies to Long Island Sound, but instead of a shoaling at the mouth we there have a contraction.

In case of tidal rivers, the tide wave is generally progressive, so that there is no tendency to synchronization with the tide outside. Off the mouths of large rivers, antecedent waves become noticeable.

Large as is the tide in the Bay of Fundy, it doubtless would have been a little larger, in the bay proper, had there been no Basin of Mines. For reasons similar to those just referred to, the oscillation of the bay is interfered with by any dependency which can not synchronize with it. As it is, this branch implies an antecedent wave, and so there must be some progression up the bay from this cause.

The converging shore line and lessening depths increase the range in either stationary or progressive waves. For this reason the largest tides of all occur near the head of the Basin of Mines and up the Petit Condiac River, the range in the former reaching about 43 feet and in the latter 40 feet.†

At Boston the range of tide is 9.6 feet and at St. John it is 20.8.

The smallest known mean range of tide connected with the Gulf of Maine oscillation occurs at Tom Nevers Head, on the south coast of Nantucket Island, and amounts to only 1.2 feet. (See Figs. 14 and 15.)

* For tides in the Gulf of St. Lawrence, see Dawson's Reports on Survey of Tides and Currents in Canadian Waters for 1898, 1899, and 1903.

† For tides in the Bay of Fundy, see Dawson's Report on Survey of Tides and Currents in Canadian Waters for 1899. For tides in St. John River, see Bulletin XV of the Natural History Society of New Brunswick, pp. 65-82.

The tide north of Nantucket Island comes from the east and its hour is about $IV\frac{1}{2}$. The XII-hour line passes a short way south of Nantucket Island and meets the land at the western end of Marthas Vineyard. Hence the change of four hours in going through Muskeget Channel and the rapid currents. The change is also rapid between Marthas Vineyard and the mainland. The tide wave in Buzzards Bay is almost stationary, the time of occurrence being less than half an hour later at the head than at the mouth.

The tide reaches the southern side of Sable Island about one hour before it reaches the northern side. This shows the delaying effect analogous to that referred to in lemma 28. (See also Fig. 13, Part IV A.)

In going from the Greenland loop of the North Atlantic system to the United States loop of the South Atlantic system, the tidal hour must change from VIII to XII. The manner in which this change takes place depends largely upon the depths encountered and upon the presence of antecedent waves which are necessitated by the progressive waves, the latter being due to openings in the shore line or to off-lying shoals, lemmas 10, 14; e. g., Cabot Strait, the submerged strait south of Nova Scotia, Georges Bank, Browns Bank, and Florida Strait.

40. *From Rhode Island to Florida.*

The tidal hour for this stretch of coast is about XII. The earliest point is Cape Lookout, where the hour is $XI\frac{1}{2}$.

The tides in Long Island Sound are analogous to the tides in the Gulf of Maine and Bay of Fundy, and so at the western part of the Sound are three hours later than the tides outside and upon which they depend. There is, however, some progression. The range varies from 1.9 feet at Montauk Point to $7\frac{1}{4}$ feet at the western end of the Sound.

On the outer or southern coast of Long Island the tide occurs remarkably early, for reasons given in connection with lemma 32 (case 3). This is so notwithstanding the existence of Fire Island Inlet and other smaller straits leading to Great South Bay. For, the flow through the inlet being simply a hydraulic effect, the tide just south of the inlet will be accelerated rather than retarded. (Cf. § 25 and statements under lemma 29.)

The tides in New York Harbor and the lower part of the Hudson River have nearly the same range as the tide at Sandy Hook, where it is 4.6 feet. At Governors Island the tidal hour is XII.72, while at Willets Point it is III.66. This in a general way explains the strong tidal currents in East River, as was pointed out in § 37, Part I, and § 106, Part IV A.

The tides of Peconic, Great South, Jamaica, and Barnegat bays are chiefly due to hydraulic effects, the rising and falling of the waters outside causing a rapid flow through the connecting straits or inlets. For Great South and Barnegat bays the range is much reduced and the time of tide delayed, as shown in Fig. 15. (See §§ 104, 105, 111, Part IV A.) (In the formula there given for κ , the words "amplitude of tide outside" should be stricken out.)

Up the Delaware Bay and River the wave progresses at nearly the rate due to depth. The values of Fig. 14 show that the range increases considerably above its

value at the capes. The difference between the duration of rise and fall at Philadelphia is 2 h. 35 m.

While the funnel-shaped Delaware Bay favors an increasing range of tide, Chesapeake Bay, with its contracted entrance and subsequent expanse and connections with dependent tributaries, tends to reduce the range. The tide progresses up the bay, but in a somewhat irregular manner, as shown in Fig. 14. Near the mouth the high water occurs on the western shore of the bay before it occurs in the western shore of Cape Charles; this is a case coming under lemma 28. Up the numerous tidal rivers the wave progresses regularly. However, by lemma 32 the progression is slower than mean depth would call for.

The tides in Albemarle Sound, Pamlico Sound, and in New River are very small, owing to the narrow openings connecting them with the sea.

For South Carolina, Georgia, and northern Florida the range of tide is larger than for the outer coast farther north. Here the last loop of the South Atlantic system finds a good support; moreover, the distance to the free northeasterly boundary of the system is considerable.

The tide in the St. Johns River is greatly reduced in range at Jacksonville, where there is a constriction in the river.

As the Strait of Florida is approached the range decreases.

41. *The Gulf of Mexico.*

Upon referring to § 3 and Fig. 23, Part IV A, it will be seen that the tidal hour for the eastern part of the Gulf should be III, according to the corrected equilibrium theory, and the hour for the western part should be IX. Upon referring to the chart of cotidal lines for the Gulf, Fig. 17, this will be seen to be the case; but it will also be noticed that, excepting the eastern part, the hours generally progress in going westward, and so the cotidal lines are not amphidromic, as the equilibrium theory requires. Hence arises evidence of a progressive wave from the Strait of Florida. A little west of the center of the Gulf the hour increases rapidly in going westerly, indicating that the amplitude of the progressive wave must there be small; for, if its amplitude were zero, the tidal hour would change suddenly from III to IX in crossing the center of gravity of the equilibrating surface.

Consider the derived wave front extending from Yucatan to the Dry Tortugas. Suppose its center to progress through deep water toward the Mississippi Delta. The eastern wing of this wave crest will approach parallelism to the coast of Florida on account of the rapidly decreasing depth found between the deep water and this coast. (See Fig. 14, Part IV A.) The wave will reach the coast of Louisiana remarkably early or at about the time indicated by the depths of the Gulf. The range in deep water will become smaller as the distance from the source of disturbance increases, because of the ever-increasing length of the crest of the wave, which disperses the motion more and more widely. Hence the range continually diminishes as we go northward parallel to the coast of Florida. The eastern wing of the crest, by moving eastward over continually shoaling water, will cause the range to be much increased by the time the Florida coast is reached. Of course, a somewhat similar east-going progressive wave results from the equilibrium tide, and the range of this derived wave will also increase toward the coast.

The tides between Cape San Blas and the Mississippi Delta can be approximately determined by considering both the progressive wave and equilibrium tide near the edge of deep water, and adding the time of transmission to the shore.

As the eastern wing hinges upon Florida Keys so the western wing hinges upon northeastern Yucatan.* The range diminishes in going from the northern coast of Yucatan northwesterly. Consequently the tides in the southwestern part of the Gulf should be eventually equilibrium tides, and observations indicate that such is the case.

Small as the progressive wave is, its range is greater than that of the equilibrium tide in the deep water south of the coast of Louisiana, thus causing a westerly progression, instead of an easterly one, in this part of the Gulf. From Fig. 23, Part IV A, it is seen that the equilibrium tide off Port Eads, Mississippi Delta, is only 0.1 foot; this, however, increases to 0.4 foot off the mouth of the Rio Grande. The time of tide for the edge of deep water off the coasts of Louisiana and Texas can be estimated by giving greater weight to the derived wave off Port Eads and by gradually increasing the weight given to the equilibrium tide as one goes toward the Rio Grande. If to the times thus determined be added the time of transmission to the shore, the tides of Texas and southern shore of Louisiana will be nearly explained.

As already noted, the equilibrium tide controls the tide in the southwestern corner of the Gulf.

Between Galveston and Tampico no data are available concerning the semidiurnal tides.

The table given under § 97, Part IV A, shows that throughout the Gulf the ratio S_2/M_2 is considerably increased over its value for the Atlantic coast, indicating that the equilibrium tide of the Gulf is generally comparable in magnitude with the tide derived from the ocean. This is also indicated by the fact that the age of the phase inequality, as shown by the value of $S_2^\circ - M_2^\circ$, is much less for the Gulf than for the outside ocean.

The broad simultaneous region in the eastern half of the Gulf is in a general way explained by the fact that the derived and equilibrium tides do for this region approximately agree in phase, while the crowding up of the cotidal lines in the western half results from the two waves being in approximately opposite phases. The time required to transmit a disturbance from the western approach end of Florida Strait to the deep water line off the Rio Grande is about $2\frac{1}{2}$ lunar hours.

Some account of the tides of the Gulf has been given in §§ 89, 90, 92, 96, Part IV A.

42. *Northwestern Africa and Southwestern Europe.*

In passing northeasterly through the Cape Verde Islands, the tidal hour increases rapidly; this indicates proximity to a nodal line of the North Atlantic system, as well as proximity to the free boundary of the South Atlantic system. A tongue-shaped region of early tide extends, as already noted, from south of these islands to the Barbados. South of Cape Verde Islands the hour is VI, while at the loop off Morocco it is II. The general reasons for a northward instead of a southward progression have been given in §§ 8, 33. The wide distances between the cotidal lines north of the Canary Islands and the increased range indicate a loop of the system.

* § 36, Part IV A, and lemma 28.

Upon inspecting Fig. 18 it will be seen that the range of tide decreases from 9 feet near Cape St. Vincent, to $4\frac{1}{2}$ at the eastern Azores and to $2\frac{1}{2}$ feet at Flores, the most western. This shows that the tide is approaching zero range in going westward, and, taken in connection with the fact that all these places have nearly the same tidal hour, throws much light upon the position of the no-tide point. By comparing the times and ranges of Nova Scotia, Sable Island, and the eastern coast of Newfoundland, some evidence of the existence and location of the no-tide point can be obtained.

The early tides at the Strait of Gibraltar and at the head of the Bay of Biscay have been noticed in § 8 and § 9 case 5.

43. *The Mediterranean Sea.*

The tides of this sea have been discussed at some length in §§ 84-88, Part IV A. The conclusions there reached appear to be mainly correct, with the exception of one concerning the Adriatic found near the close of § 87.

The tidal hours for the eastern part of the sea nearly obey the equilibrium theory, as a comparison of Fig. 19, Part IV B, with Fig. 23, Part IV A, will show. The cotidal lines are probably amphidromic, radiating from the Isle of Crete. High water on the coast of Syria occurs at the time of low water off Tripoli. The tide from this part of the sea joins that of the western part through the straits on either side of Sicily. The cotidal lines are crowded closely together in the straits because the time of high water off the southeastern ends of the straits is II and off the northwestern a little more than VII.

The tide in the Adriatic is partly stationary and partly progressive. The Adriatic constitutes a dependent oscillating area whose length is nearly $\frac{1}{2}\lambda$. The crowding together of the cotidal lines indicates proximity to the nodal line. The effect of the earth's rotation is shown by the tendency to radiate from a point on the Italian shore. This body of water is quite analogous to the English Channel, as has been noted in § 12.

The tides of the Ægean Sea being in part derived from those of the Mediterranean near the Isle of Crete are probably small.

The tide in the western part of the Mediterranean is of nearly opposite phase to that of the Atlantic just outside. This accords with the theory of a strait leading from a tided ocean to a near-by but deep tideless sea of moderate dimensions. (§ 103, Part IV A.)

References to Mediterranean Sea tidal observations and discussions.

Malta.—Philosophical Transactions of the Royal Society of London, 1878.

Toulon, Malta, Marsilles.—Proceedings of the Royal Society of London, Vol. 39 (1885).

Genoa, etc.—Annali Idrografici, Vol. 1. Genoa, 1900.

Ragusa, etc.—Mitteilungen des k. u. k. Militärgeographischen Institutes, Vol. 23. Vienna, 1904.

Adriatic Sea.—Admiralty Tide Tables.

44. *The British Islands and the North Sea.*

Tacitus thus refers to the tides of the Isle of Britain:

It is not the business of this work to investigate the nature of the ocean and the tides; a subject which many writers have already undertaken. I shall only add one circumstance: that the dominion

of the sea is nowhere more extensive; that it carries many currents in this direction and in that; and its ebbings and flowings are not confined to the shore, but it penetrates into the heart of the country, and works its way among the hills and mountains, as though it were in its own domain.*

The fourth summer [A. D. 81] was spent in securing the country which had been overrun; and if the valour of the army and the glory of the Roman name had permitted it, our conquests would have found a limit within Britain itself. For the tides of the opposite seas, flowing very far up the estuaries of Clota and Bodotria [Clyde and Forth], almost intersect the country; leaving only a narrow neck of land, which was then defended by a chain of forts. Thus all the territory on this side was held in subjection, and the remaining enemies were removed, as it were, into another island.†

The tides around Great Britain and in the North Sea are remarkable in many respects. They are best described by constantly referring to the cotidal maps of these regions (Figs. 20-22).

The cotidal line IV extends from the northwest corner of France to the southwest capes of Ireland, and thence to Rockall Island. The ranges for these three places are 14, 7, and $4\frac{1}{2}$ feet, respectively. This line shows how the tide approaches the land. The diminution of range gives some indications of the nodal line of the northern half of the North Atlantic system, although it is covered up by the northeasterly progressing wave.

Depending upon the incoming wave are the tides of the English and Irish channels. In these bodies of water the tide is partly stationary and partly progressive, much influenced by the deflecting force of the earth's rotation. (See §§ 3 and 12.)

The oscillation in North Channel is stationary, because high water at the south end of the channel occurs at very nearly the time of low water north of Ireland § 35, Part IV A. Instead of a nodal line extending across the channel, the deflecting force of the earth's rotation causes the cotidal lines to be amphidromic about a no-tide point, off the Mull of Cantire, § 12.

Along the northern coast of Scotland to the Orkney and Shetland islands is an easterly progression. Between these two groups of islands and through Pentland Firth, the tidal hour changes rapidly and the flood and ebb streams are strong. The motion thus transmitted, together with that of a wave rounding the Shetland Islands, is the chief cause of the tides along the eastern shore of Scotland and England. However, a part of the wave passing north of the Shetland Islands proceeds northeasterly along the coast of Norway. Evidently the coast line at the south end of Norway is not touched by these waves, unless the progression up the coast of Norway entails a sensible antecedent wave, which may reach southward toward the Naze.

Down the eastern coast of Scotland and England the wave proceeds at about the rate due to depth.

All accounts agree that there is very little tide between the Naze and Stavanger. Now imagine lines drawn from this region to the Shetland Islands, Orkney Islands, and northern Scotland, and we have reasons to believe that the large wave going down the coast of Scotland and England as far as Norfolk will control the tide over a sector of the North Sea, and that the cotidal lines will radiate from a point a little way off the southwest coast of Norway. (See cotidal maps of the North Sea, Figs. 21, 22, and § 13.)

The main progression from the northwest turns to the east off eastern Norfolk and proceeds along the north coast of Holland and Germany to Holstein and Schleswig. From this locality northward the range of tide rapidly diminishes.

*Life of Agricola, Ch. 10 (Oxford Trans.).

†Ibid., Ch. 23.

The tidal streams in the arm of the sea between Belgium, Holland, and England indicate that it is essentially a stationary wave about $\frac{1}{3} \lambda$ long with a maximum northeasterly stream occurring at II, as one would expect from the fact that the tidal hour in the strait is XI and the range in the North Sea is comparatively small and of nearly opposite phase. (§§ 102, 103, Part IV A.)*

The earth's rotation should therefore cause the tidal hour at the Holland end of the hypothetical nodal line to occur at II and at the England end at VIII. This agrees fairly well with the observed facts. The stationary oscillation in this arm of the sea is sustained chiefly by the rise and fall at Dover Strait, and so the effect of the main body of the North Sea is little felt excepting where the arm joins it. The theoretical time of high water being II on the Holland coast, and the wave on this coast progressing northeasterly as the amphidromic requires, it will be seen that much crowding up of the cotidal lines must occur just south of Texel. Moreover, the influence of the amphidromic wave causes the tides off Texel to occur earlier than the time indicated by the North Sea tides alone. An opposite effect is produced on the English side. Hence the crowding up of the cotidal lines in the vicinity of Yarmouth and the broad space in the North Sea between the cotidal lines VI $\frac{1}{2}$ and VII.

The cotidal lines shown in Figs. 21, 22 have a striking resemblance to the scheme proposed by Whewell (Phil. Trans., 1833, 1836), and which Airy discredits in Arts. 525-528 of his Tide and Waves.

As stated elsewhere, the point at which Captain Hewett observed tides and found no rise and fall has been taken as the no-tide point for the amphidromic region between England and Holland.

In passing up the Zuider Zee the tide is much retarded at the narrows near the middle of that body. South of the narrows the range of tide is diminished to about 1 foot.

The form assumed by the cotidal lines at the mouth of the estuaries of the Forth, Humber, and Thames has been explained in § 31, case 4.

In going up the estuaries and funnel-shaped bays of England and Scotland the range of tide generally increases.

Much might have been said concerning the peculiarities of the tides of Europe, especially the want of simplicity of the wave manifested in double high waters, long stands, short stands, etc.; also the want of equality between the times of rise and fall. But no explanation will now be attempted, for such phenomenon can better be treated in Part V.†

One curious feature of the tides of Europe, and probably of those along the Atlantic coast of Africa as well, is the great variation in the ratio of O_1/K_1 , and the large value, positive or negative, of $K_1^\circ - O_1^\circ$. (See Nos. 900 to 995 of the table given under § 97, Part IV A and Nos. 910 to 1011, § 19, Part IV B.)

Upon consulting the same table it will be noticed that $S_2^\circ - M_2^\circ$, and so the age of the phase inequality, is negative for Arendal, Oscarsborg, Christiania, and Copenhagen.

* See charts entitled "Tidal Streams in the North Sea," by the Admiralty.

† See, for example, *Étude Pratique sur les Marées Fluviales et Notamment sur le Mascaret*, by Comoy.

References to tidal observations and discussions for the British islands and the North Sea.

General references.—Tide Tables for the British and Irish Ports, by the Admiralty.

Philosophical Transactions, 1831, 1833, 1836, 1845.

Proceedings of the Royal Society of London, Vols. 39, 45.

Norway.—Norske Gradmaalings-Kommission, 1882, 1904.

Holland.—Algemeene Dienst van den Waterstaat. Verzamelingstabel der Waterhoogten, for the month of April, 1894.

Comptes-Rendus de l'Association Géodésique Internationale, 1900.

45. *From the Färöe Islands to Newfoundland.**

By consulting the chart of soundings, Fig. 19, Part IV A, it will be seen that a broad strait exists on either side of Iceland. Now, if a wave of uniform range were progressing northerly up the northern Atlantic Ocean it would be reasonable to expect to find a wave transmitted northward through both of these straits. But upon referring to Fig. 23, Part IV A, it will be seen that the strait between Greenland and Iceland is at the loop of the stationary oscillation while the strait between Iceland and the Färöe Islands is much nearer to the nodal line. Hence, the wave transmitted through Denmark Strait being of much greater amplitude than that which would be transmitted through the other strait, it is easily seen that it might proceed along the northern coast and govern the tides to the northeast of Iceland. As a matter of fact the tides in this last-mentioned region are almost opposite in phase to those just south of the strait. Consequently, § 35, Part IV A, the oscillation in this strait must be chiefly stationary. However, there is some southward progression down the eastern coast of Iceland and some northward progression up the western coast of the Färöe Islands, the direction in the first case being governed by the preceding motion along the northern coast of Iceland and in the latter by the fact that the range north of the Färöe Islands is smaller than the range south of them, so that these islands form a part of a boundary, along the southern side of which is a good rise and fall. Again referring to the chart of soundings and to the chart of systems, it becomes evident that the range of tide for the western coast of the Färöe Islands must considerably exceed that of the eastern. Observations indicate about $7\frac{1}{2}$ feet for the former and $4\frac{1}{2}$ for the latter. The motion thus set up causes the hours to progress southerly down the eastern coast of these islands; that is, the wave is turned or refracted, as it were, around the northern end of the islands because of the shoals and deeper off-lying waters.

The oscillation in the strait between the Färöe and Shetland islands is, in deep water, nearly stationary, since the waters to the southwest of it are nearly opposite in phase to the waters to the north. However, along the northern coast of Scotland and the western coasts of the Orkney and Shetland islands the tide consists chiefly of a wave progressing northeasterly at about the rate due to depth.

The short length of the walls or sides of these two broad straits makes the effect of the earth's rotation, if sensible at all, of secondary importance.

Between Rockall and Labrador there is a considerable progression, due chiefly to the opening between southern Greenland and Labrador.

* See Figs. 12, 18, 20, 23.

The range of tide at Nennortalik, near Cape Farewell, is 6.3 feet and at St. Johns, 2.6; for the coast of Labrador it is 4 or 5 feet. These values indicate something about the position of the no-tide point.

The tide in Hudson Strait is unexpectedly large. The range at Port Burwell near the eastern end being 15.1 feet; at Ashe Inlet, Big Island, 23.5 feet, and at Port Laperrière, Digges Island, at the western end of the strait, 6.6 feet. The lateness of the tide on the southeastern shore of Ungava Bay has been explained in § 7.

In the northern branch of Hudson Strait, constituting the southern portion of Fox Channel, the range is large, being 9.5 feet at Winter Island.

The cotidal lines of Hudson Bay, as shown in Fig. 12, are little more than conjecture. The lack of reliable observations for the southeastern portion of the bay, and the uncertainties connected with the geography of Southampton and neighboring islands, cause inferences to be of little value. The range of tide is probably about 3 feet on the eastern side of the bay and more than twice that amount on the western.

It may be well to here point out some evidence which goes to prove that the north-western corner of the North Atlantic Ocean, together with its tidal dependencies, belongs to the North Atlantic system.

Upon referring to Fig. 23, Part IV A, it will be seen that the loop off Gibraltar is not overlapped by any other system, and that the same is true of the loop between Labrador and Iceland. Hence the magnitude of the phase inequality, as well as its age, should be comparable at the two ends of this area. That such is the case can be seen by comparing the values S_2/M_2 , $S_2^\circ - M_2^\circ$ for Nos. 6-28 of the table given under § 97, Part IV A, with similar values for Nos. 932-990 of the same table.

As has been already noted, the ratio S_2/M_2 is much smaller for the coast extending from Nova Scotia to Florida Strait, showing that the tides of this coast belong to a different system.

CHAPTER VII.

THE SEMIDIURNAL TIDES IN THE ARCTIC OCEAN.

46. *Indications of land near the pole.*

Before attempting to draw cotidal lines for the Arctic regions, some assumptions have to be made as to the distribution of land and water; but in making such assumptions observations upon the tides and surface drifts form the principal guides, as the matter which follows will show. By considering this question in detail, an estimate can be made of the relative merits of the cotidal lines as drawn for different parts of the Arctic Ocean. Except for slight alterations, the remainder of this section is as it recently appeared in the *National Geographic Magazine*.*

It is a well-established fact that there are two important surface currents (or drifts) in the Arctic Ocean. One of these flows easterly along the northern coast of Alaska, through the Arctic Archipelago, finally reaching the Atlantic Ocean through Davis and Hudson straits. The other starts in the neighborhood of Herald Island, northwesterly from Bering Strait, and thence flows northwesterly, passing to the north of New Siberia; thence to the north of Franz Josef Land and the Spitzbergen Islands, and through Denmark Strait to and around Cape Farewell. Therefore these currents are near together when north of Bering Strait and again when in the vicinity of southern Greenland.

Some evidence of the American current may be cited. The ships *Advance* and *Rescue*, of the first Grinnell expedition, were for a while carried northerly in Wellington Channel by the drifting ice; but when near the northern end of the channel the current reversed, and thereafter they were carried southerly and easterly through Barrow Strait, Lancaster Sound, Baffin Bay, and Davis Strait, to latitude $65^{\circ} 30' N.$, where they got themselves free from the ice. The amount of southeasterly drifting measures about 1 000 nautical miles, and required a little more than six months, extending from November, 1850, to June, 1851. This gives an average rate of 5 miles per day.

In May, 1854, the British ships *Intrepid* and *Resolute* were abandoned off the western end of Barrow Strait. The *Resolute* was picked up off Cape Mercy, in the south end of Davis Strait, in September, 1855. During these 16 months 1 100 miles were covered, making an average rate of $2\frac{1}{3}$ miles per day.

Strong easterly currents are encountered in Fury and Hecla Strait and in Bellot Strait.

Northeasterly currents off the northwestern coast of Alaska have been noted by Captain Collinson,† and easterly currents along the northern coast by Captain McClure.‡

* Vol. XV (1904), pp. 255-261.

† Collinson: *Journal of H. M. S. Enterprise*, edited by his brother, pp. 137-142.

‡ McClure: *The Discovery of the Northwest Passage*, edited by Osborn, p. 71.

Collinson noted an eastern set in Dease Strait far to the east,* and McClure found a large quantity of American pine, almost certainly from the Mackenzie River, drifted into Prince of Wales Strait.†

McClure Strait is constantly filled with ice, probably coming in chiefly from the west.

The existence of the current far to the north of Russia is pretty well established by the drifting of the steamship *Jeannette* from Herald Island to a point northeast of New Siberia, where she was crushed in the ice, and by the subsequent drifting of some papers and clothing from the sunken vessel across the polar sea to Julianehaab, near Cape Farewell. The *Jeannette* was frozen in the ice September 6, 1879, and was crushed June 12, 1881, having made good a distance of 600 miles. During the last five of these 21 months much more than half of all the distance made good was covered, and during the last 26 days almost one-sixth. The relics were picked up in 1884, or three years after the sinking of the boat, having gone a distance of at least 2 900 miles. (See Fig. 24.)

Before undertaking his famous voyage in the *Fram*, Nansen adduced, as further evidence of this current, the finding on the coast of Greenland of an implement which almost certainly came from the Alaskan Eskimos in the vicinity of Bering Strait; also the prevalence of driftwood on the Greenland coasts and the north coast of the Spitzbergen Islands, the species indicating that a large portion of this wood came from northern Siberia.

The voyage of the *Fram* verified his previous calculations in a remarkable manner. That vessel became fast in the ice at a point northwesterly from New Siberia, September 22, 1893. It thence drifted to a point north of the Spitzbergen Islands, having passed about midway between Franz Josef Land and the North Pole. It was released from the ice June 14, 1896, thus having drifted for 33 months, the distance made good being 900 miles. At the beginning of the drifting the rate of the current was a little more than half a mile per day, and increased to 1 mile near the end.

Having established the existence of these two prevailing surface currents, and noting that both eventually flow to southern Greenland, the question arises as to why the *Jeannette* did not drift almost due north instead of bearing off to the west. The *Fram* went almost directly toward the eastern coast of Greenland. It is true that after the loss of the *Jeannette* Commander De Long and his party found themselves on ice drifting rapidly northward. As already noted, the last 26 days' drifting of the boat covered about one-sixth of the entire distance. These facts suggest a broad strait north of Bennett Island, beyond which is the corner of a large tract of land dividing the deep Arctic channel traversed by the *Fram* from the shallow sea through which the *Jeannette* drifted. The final accelerated rate and northward direction of De Long's drift seem to indicate proximity to this strait.

This sea extends from Bennett Island to Banks Land. It is about 30 or 40 fathoms deep along the track of the *Jeannette*, and perhaps from 100 to 200 fathoms west of Banks Land, where it is known as Beaufort Sea.

That land probably extends to the north of Beaufort Sea can be inferred from the fact that the ice found here is very old, the sea seeming to have no broad outlet through which the ice can escape, as it does north of Siberia. The openings to the east are long

* Collinson: L. c., p. 291.

† Richardson: The Polar Regions, p. 232.

and rather narrow channels. This does not argue against a tolerably broad expanse of water extending westward, for the currents setting eastward prevent the ice from escaping to the west. It seems probable that land, continuous or nearly so, must extend far westward from off Banks Land, for this supposed land and the eastward currents might well explain why it is that the ice never recedes far northward from the northern coast of Alaska nor westward from Banks Land.

Osborn thus speaks of the ice encountered by McClure in Beaufort Sea: "Ice of stupendous thickness and in extensive floes, some 7 or 8 miles in extent, was seen on either hand; the surface of it not flat, such as we see in Baffins Strait and the adjacent seas, but rugged with the accumulated snow, frost, and thaws of centuries."*

Such are the arguments for the existence of a tract of land extending from near the northwest corner of Banks Land, or from Prince Patrick Island, to a point north of New Siberia, based upon the drifting of the ice on the one hand and upon its age and comparatively slight movement on the other hand.

Let us next consider what are the indications from the tides. In the first place, the tide at Point Barrow is semidiurnal in character, with a mean range of 0.4 foot, the flood coming from the west. This can not come through Bering Strait, because the tide immediately south of the strait has scarcely one foot range, with a large diurnal inequality, and at a short distance north of the strait, at Pitlekaj, where the *Vega* wintered in 1878-79, the range of the semidiurnal tide was carefully measured and found to be only 0.2 foot. Whence comes the Point Barrow tide? It can not come from the north or east, because all observers agree that the flood comes from the west, and that it is high water on the western side of the point considerably earlier than on the eastern.† De Long's party made careful observations upon the tide at Bennett Island, and these show a range of 2 feet. Such a range, diminished by the broadening of the shallow sea to the east of this island, might well be reduced to that found at Point Barrow, provided one considers that the range generally diminishes off headlands and capes. On the other hand, if no land exists north of Point Barrow, how can the tide there be much less than that found at Bennett Island, and how can the flood come from the west? For practically all of the Arctic Ocean tide is derived from the Atlantic, chiefly through the Greenland Sea, and without land near the Pole one of these stations would be reached about as well as the other.

The reasons for not drawing the boundary straight from the Bennett Island corner to the Banks Land corner, but deflecting it to the south, are, first, the apparent necessity for such a bend in order that the direction of the flood may better accord with observation, and that the times of the tides of northern Alaska may be consistent with those at Bennett Island, and, second, the small north-and-south movement of the ice north of Alaska indicating that the sea is here probably narrower than it is farther west, or north of Siberia.

In the extreme north this land can not extend much beyond the Pole toward Franz Josef Land, because this would undoubtedly have there caused a bend in the track of

*McClure: L. c., p. 83.

†Thomas Simpson: Discoveries on the North Coast of America, 1836-1839, pp. 161, 162, 167. Accounts and Papers, Navy, vol. 42 (1854), p. 162. Lieut. P. H. Ray: Report of the International Polar Expedition to Point Barrow, Alaska, p. 678.

the *Fram's* drift. Furthermore, the undiminished range of tide at Bennett Island perhaps indicates that the Nansen channel does not greatly broaden at the Pole.

Between this supposed land and the islands recently discovered by Sverdrup may be other islands, forming a continuation of the Arctic Archipelago and separated from one another by channels of moderate depths, or perhaps this land approaches the Garfield Coast and Grant Land. At any rate, the range of tide diminishes from 2 feet at Cape Sheridan to $1\frac{1}{2}$ feet at Northumberland Sound, Penny Strait, and Lockwood and Brainard judged the tide to be small at Greely Fiord. These indicate that the access of the tide from the north is not altogether unrestricted; in fact, part of the tide at Northumberland Sound comes from the east through Belcher Channel.

We come now to another question. A few tides have been observed along the northern coast of Alaska by the explorer Thomas Simpson.* They show that the tide on the outer coast occurs nearly simultaneously from Point Barrow to Camden Bay and Simpson Cove. But as the international boundary line is approached a great change takes place; the tide at Demarcation Point, not 100 miles farther east, is about seven hours later in its time of occurrence. Observations are not sufficient for showing how this change takes place, but it certainly occurs. A few tides in Mackenzie Bay and eastward were observed by Captain Richardson† and Commander Pullen.‡ The set of the flood along the outer coast is given as easterly for all points where it has been observed from Point Barrow to and beyond Cape Bathurst; but such observations are very meager, probably on account of the smallness of the tide. This would seem to preclude the possibility of the principal part of the tide coming from the north or east; hence the probable approach of the polar land to Banks Land, or to Prince Patrick Island, or to Grant Land.

Suppose an island about 100 miles in diameter to be separated from the coast by a shallow strait about 75 miles wide in its narrowest part. By assuming that deeper water exists to the west of the strait and island, and that the tide comes from the west, it seems possible to account for the sudden change in the time of tide; for the main wave, going north of the island, would control the time of the tide to the northeast of it, and deep water west of the island and shallow strait would cause the tide at Camden Bay and westward to occur remarkably early, just as if this coast were at the head of a deep, suddenly terminated canal extending northwesterly.

Immediately eastward from this supposed strait both Simpson§ and McClure|| found that the waves became more like those upon a sea of some magnitude, and the latter, sailing a little north of east, found the depths to rapidly increase from 9 to 32 fathoms, and soon to 195 with no bottom.

Now the question is, Why this more sea-like appearance unless some huge obstruction lies immediately to the west? It may, of course, be partly due to the open water caused by the influx of the Mackenzie.

* Simpson: Discoveries on the North Coast of America, 1836-1839, pp. 115, 117, 121-123, 132, 138, 161-162, 167, 178, 183.

† Richardson: Arctic Searching Expedition, pp. 144, 154, 157-160, 169, 175.

‡ Pullen in Reports on Arctic Expeditions, 1852, pp. 35, 38, 40, 51.

§ Simpson: L. c., p. 176.

|| McClure: L. c., p. 82.

It will be of interest to note that several Arctic authorities have at various times suspected or inferred the existence of land near the Pole. Richardson says:

The Eskimos of Point Barrow have a tradition, reported by Mr. Simpson, surgeon of the *Plover* [in 1832], of some of their tribe having been carried to the north on ice broken up in a southerly gale, and arriving, after many nights, at a hilly country inhabited by people like themselves, speaking the Eskimo language, by whom they were well received. After a long stay, one spring in which the ice remained without movement they returned without mishap to their own country and reported their adventures. Other Eskimos have since then been carried away on the ice, and are supposed to have reached the northern land, from whence they have not as yet returned. An obscure indication of land to the north was actually perceived from the masthead of the *Plover* when off Point Barrow.*

On August 15, 1850, Captain McClure, anchored off Yarrowborough Inlet, about half way from Point Barrow to Demarcation Point, writes:

The packed ice to-day, as far as the eye can reach, appears solid and heavy, without a drop of water discernible. The refraction has been considerable, giving to the edge of the pack the appearance of a continuous line of chalk cliffs, from 40 to 50 feet in height. From the light shady tint, which, in different parts of the pack, is distinctly visible, I should be inclined to think that there may be many of the same kind of islands as those we have met with, extending to the northward, and impeding the progress of the ice, thereby keeping this sea eternally frozen.†

Captain Collinson, who wintered at Simpson Cove, 1853-54, actually undertook a sledge journey, in the spring, northward, one object of which was to see if land would not be reached. The roughness of the ice caused him soon to abandon the project. He writes:

I therefore returned, and with sorrow gave up an attempt which * * * I had looked forward to with much interest; thinking that, with anything like a favorable road, I should reach 73° north latitude, and settle the question with regard to the open sea, which certainly does not appear to exist here in the same manner as it does to the north of the Asiatic continent.‡

In 1873 Admiral Sherard Osborn read a paper before the Royal Geographical Society, in which he predicted the existence of an archipelago or land extending from near Prince Patrick Island up very near to the Pole and thence to Wrangell Island, thus forming the northern boundary of a nearly inclosed sea.§

A probably less happy prediction was made by Petermann, who contemplated land extending northeasterly from Greenland, thence across the Pole to Wrangell Island.

Sir Clements Markham is quoted as having said, in November, 1896:

Personally, as I do not believe in any land near the Pole, or on this side of it beyond Franz Josef Land, I trust an attempt will be made to explore another portion of the Arctic regions. I believe there is land, probably in the form of large islands, between Prince Patrick Land and the New Siberia Islands.||

Prentiss discredits there being much land north of Bering Strait, but his reasons for so doing can hardly be regarded as convincing.

The following quotation is from a paper by Marcus Baker, in volume 5 of the *National Geographic Magazine*, entitled "An Undiscovered Island off the Northern

* The Polar Regions, p. 240.

† McClure: L. c., p. 81.

‡ Collinson: L. c., p. 312.

§ Clements R. Markham: *The Threshold of the Unknown*, pp. 216-224.

|| Prentiss: *The Great Polar Current*, p. 105; see also p. 19.

Coast of Alaska." He suggests that the supposed land be called Keenan Island. The following statements are there furnished by Capt. Edward P. Herendeen, who for many years was engaged in whaling:

It is often told that natives wintering between Harrison and Camden bays have seen land to the north in the bright, clear days of spring.

In the winter of 1886-87, Uzharlu, an enterprising Eskimo of Ootkeavie, was very anxious for me to get some captain to take him the following summer, with his family, canoe, and outfit, to the northeast as far as the ship went, and then he would try to find this mysterious land of which he had heard so much; but no one cared to bother with this venturesome Eskimo explorer. So confident was this man of the truth of these reports that he was eager to sail away into the unknown, like another Columbus, in search of an Eskimo paradise.

The only report of land having been seen by civilized man in this vicinity was made by Capt. John Keenan, of Troy, N. Y., in the seventies. He was at that time in command of the whaling bark *Stamboul*, of New Bedford. Captain Keenan said that after taking several whales the weather became thick, and he stood to the north under easy sail, and was busily engaged in trying out and stowing down the oil taken. When the fog cleared off, land was distinctly seen to the north by him and all the men of his crew; but, as he was not on a voyage of discovery and there were no whales in sight, he was obliged to give the order to keep away to the south in search of them. The success of his voyage depended on keeping among whales.

The fact was often discussed among the whalers on the return of the fleet to San Francisco in the fall. The position of Captain Keenan's ship at the time land was seen has passed from my mind, except that it was between Harrison and Camden bays.

It will be noticed that these statements would place the island a little to west of the position shown in Fig. 24.*

47. *Character of Arctic tides.*

An inspection of the tidal forces (Fig. 1, Part IV A) shows that the semidiurnal equilibrium tide, even if possible, would be very small, because the forces vanish at the pole. Again, the dimensions are probably such that no free period of the tide is approached. For these reasons the semidiurnal tide observed in the Arctics must be almost entirely derived from other waters. The cotidal lines indicate that nearly all of the disturbance producing the Arctic tides acts through the waters of Greenland Sea. The small mean range of tide (0.2 foot at Pitlekaj) and the smallness of the diurnal wave at Point Barrow show that the influence of the Pacific extends but a very short distance northward from Bering Strait. The shallowness of Barents Sea prevents its influence as a channel of transmission from being of much consequence to the east of Franz Josef Land and Nova Zembla. In fact, it is probable that the tides around northern Nova Zembla are derived from the wave north of Franz Josef Land. (See Figs. 23, 25.)

The northern half of the North Atlantic system (e. g., western Europe and southern Greenland and Iceland) possesses a good phase inequality which has an "age" generally of about 30 hours along the outer coasts. In the Arctic the phase inequality is of a similar character, with an age varying from 30 to 60 hours.† The tropic range of the diurnal wave is about 2 feet for the Arctic Archipelago, diminishing toward the west, and about 0.3 foot for Franz Josef Land and Bennett Island, and scarcely 0.2 foot for Point Barrow.

* Since this appendix was prepared the writer has noted in a paper read before a section of the Eighth International Geographic Congress that the small diurnal tides at Pitlekaj and Point Barrow furnish additional evidence of a large tract of land near the North pole.

† Part IV A, § 97, Nos. 6-25; also Part IV B, § 19, Nos. 1011, 1030.

48. *Greenland Sea branch.*

From the north end of the North Atlantic system where the range is large (the mean range at Reikiavik being 10 feet) the disturbance acts through Denmark Strait. A smaller disturbance comes chiefly from the loop off Portugal, passing up the outer coast of Ireland, Scotland, and Norway. The result is that the tide is about simultaneous over the deep Greenland Sea, having a range at Jan Mayen of 3 feet. The tide proceeds from this sea toward the Lena Delta, probably at a rate due to depth. The mean range is 2.0 feet at Cape Sheridan and at the southern part of Bennett Island. Northeastern Franz Josef Land is a turning point of the tide and beyond which the sea suddenly widens; the range of tide at Teplitz Bay is only 1.0 foot. The tide is of about this magnitude on the southern coast (at Cape Flora) probably because the wave passing between Norway and the Spitzbergen Islands is reduced in amplitude in the northern part of the somewhat expanded Barents Sea. From Bennett Island toward Alaska the wave proceeds at about the rate due to depth, the range diminishing because of the breadth and shallowness of the intervening waters. Judging from the few tidal observations made along the northern coast of Alaska and eastward, and which have already been referred to, the tide appears to progress in an easterly direction somewhat as shown in Fig. 26. If the assumption of a hypothetical island separated from Manning Point by a shallow strait is correct, it is probable that nearly all of the tide between Cape Bathurst and Dolphin and Union Strait, is due to the wave passing north of the island. At Coronation Gulf the semidiurnal tide practically disappears. At Cambridge Bay, Dease Strait, the tide is felt and doubtless comes from the east.* The flood in Prince of Wales Strait has the appearance of setting northeasterly.†

Having traced the tide from Greenland Sea to where it disappears at Coronation Gulf, it will now be well to note that one branch of probably considerable width passes north of Greenland, causing a tide of 2.0 feet at Cape Sheridan, the same wave going southwesterly produces the rather small tides at Greely Fiord, the western coast of Ellesmere Land, and among the islands to the west. The range of tide at Northumberland Sound is 1.5 feet, a part of which doubtless comes from Jones Sound through Belcher Channel.‡ Another branch from the main channel passes around the northeastern corner of Franz Josef Land, producing tides in the Yenisei Gulf, Gulf of Ob and the Kara Sea, the tide being in general small, probably about 1 foot on the outer coasts, but less up the gulfs and rivers.

The tides of the White Sea are worthy of note. At the southern end of the entrance the time of tide changes rapidly—from IX to XII—in a very short distance. The north side of the sea is much deeper than are the gulfs of Archangel and Onega on the south side; the tide in the former locality is nearly simultaneous, but there is progression in the gulfs. The range of tide is probably about 3 or 4 feet.

A great crowding together of the cotidal lines appears to occur in the strait between Kolgoniev Island and Kanin Peninsula.

* Thomas Simpson: *Discoveries on the North Coast of America*, pp. 267, 288, 289, 357, 363. Capt. Richard Collinson: *Journal of H. M. S. Enterprise*, p. 291, also map opposite p. 153.

† Capt. R. McClure: *The Discovery of the Northwest Passage*, edited by Osborn, p. 197.

‡ Capt. Sir Edward Belcher: *The Last of the Arctic Voyages*, Vol. 1, p. 105.

Collinson: *L. c.*, map opposite p. 255.

Accounts and Papers, Arctic Expeditions, Vol. 25 (1855), pp. 118–120.

It is probable that the small tide in the Gulf of Ob progresses at the rate due to depth.

No tidal data for the coast of Siberia, extending from the Yenisei Gulf to Pitlekaj, is available, and so the cotidal lines for this coast are only conjectural.

49. *Baffin Bay branch.*

We come now to the Baffin Bay branch of the Arctic tides. This covers much less area than does the Greenland Sea branch, but the amount of the rise and fall is in general much greater. The waters south of Davis Strait, in Davis Strait, and in Baffin Bay are characterized by a stationary wave, but everywhere there is a considerable rise and fall. The cotidal lines are crowded together in the northern portion of the strait and spread apart in the northern portion of Baffin Bay.

A stationary strait wave exists in Robeson and Kennedy channels, Kane Basin being the body of greater range and Lincoln Sea the body of smaller range. The mean range for Kane Basin is nearly 8 feet, for Fort Conger 4.3 feet, and for Cape Sheridan 2.0 feet. The fact that the tide at Cape Sheridan occurs about $1\frac{1}{2}$ hours earlier than at Kane Basin proves that the tide of Lincoln Sea comes from the north. The known tides of this locality gave strong evidence in favor of Greenland being an island some years before that fact was practically established by Lieutenant Peary.*

The Greenwich time of tide for Kane Basin is a little more than III, the cotidal hours increasing toward this body both from the south and from the north.

The range of tide at Cumberland Sound is unexpectedly large, the mean value being 15.9 feet for Kingua Ford and 14.7 feet for Ananito Harbor.

The tide progressing up Lancaster Sound soon divides, one part going westerly through Barrow Strait, the other going southerly through Prince Regent Inlet. The former produces the tide in Wellington Channel, Melville Sound, Franklin Strait, McClintock Channel, James Ross Strait, Victoria Strait, Dease Strait, and probably Coronation Gulf; the latter, the tide in the Gulf of Boothia.

The tides of Fox Channel are produced by the Hudson Strait tides. They appear to be amphidromic, so that the flood goes northward on the east side and southward on the west. The cause is probably the sudden eastward trend of the coast of Fox Land at Capes Dorchester and Willoughby, which turns a wave of large amplitude toward the northeast. The geography is but imperfectly known on the east side of this channel, and no tides or depths have there been measured.

In the straits of Fury and Hecla,† Bellot,‡ and Simpson§ the tidal streams are strong, because the tides at either end generally differ in range and in phase.

* E. Bessels: Scientific Results of the U. S. Arctic Expedition, C. F. Hall commanding, Vol. I, pp. 85, 86.

Capt. Sir G. S. Nares: Voyage to the Polar Sea, 1875-6, Vol. II, p. 356, Appendix by Rev. S. Haughton.

Lieut. A. W. Greely: Report U. S. Expedition to Lady Franklin Bay and Grinnell Land, p. 699, Appendix by A. S. Christie.

†Journal of Parry's Second Voyage for the Discovery of the N. W. Passage, pp. 319, 336-7.

‡McClintock: Franklin and his Discoveries, pp. 181-3.

§Simpson, l. c., p. 365.

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CHAPTER VIII.

THE SEMIDIURNAL TIDES IN THE PACIFIC OCEAN.

51. *Character of Pacific tides.*

Upon referring to the chart of diurnal systems (Fig. 24, Part IV A) it will be seen that large diurnal tides can not originate in the South Pacific, and observations show that the diurnal inequality is comparatively small in these waters.

The two systems for the semidiurnal tide have been described in §§ 74, 75, Part IV A.

By referring to Fig. 6 it will be seen that there are three principal amphidromic regions, the positions of the no-tide points being taken as $30^{\circ} 24' N.$, $141^{\circ} 25' W.$; $14^{\circ} 7\frac{1}{2}' S.$, $153^{\circ} 13' W.$; and $51^{\circ} 30' S.$, $172^{\circ} 10' W.$ The first lies between California and Hawaii, at the intersection of two nodal lines; the second, northwest of the Society Islands, near two nodal lines and a free boundary; the third, southeast of New Zealand, near a free boundary.

Some progression is inherent in the overlapping of the systems, as has been pointed out in § 15. This applies to the first amphidromic region mentioned, especially to the coast of Lower and Upper California, and also to the second.

But openings in the coast line have a widely spread influence. Important openings are between Cape Horn and the South Shetland Islands; between the Peninsula of Alaska and Kamchatka; through the Kuril Islands; between Japan and Formosa; between Formosa and Luzon; through the Sulu Archipelago; between New Zealand and Australia.

The shoaling found to the north and northwest of New Zealand, together with the Fiji and neighboring islands and North Island, New Zealand, form an imperfect boundary which gives rise to westerly and southwesterly progressions.

The central portion of the American coast line is farthest removed from openings of this description. Hence it is not surprising to find no progression toward the waters off Panama and Colombia, and very little up the Bay of Panama. We find what would be expected, viz., small progressions outward from the vicinity of the Galapagos Islands.

The following are localities of rapid change of time of tide, due to the fact that regions having good ranges but different times of tide lie near each other: Coast of Peru, connecting the Panama loop to the loop at Chile; western coast of southern Chile, connecting the loop at Chile with the Cape Horn loop; straits through the Aleutian Islands, connecting the northern Pacific and Bering Sea. (See lemma 25.)

Owing to the great extent of the Pacific Ocean, the large number of islands, and the considerable variations in depth, it is not difficult to believe that forces of slightly different periods may be responded to by modes of oscillation quite different in some cases. The land boundaries here are not sufficient for necessitating one particular mode

of oscillation for both the lunar and solar forces; there are many chances for the portions of the sea best responding to the lunar forces to annex or discard neighboring masses of water which islands and shoals may partially define. (See § 47, Part IV A.) Hence we can not generally infer by the equality of S_2/M_2 or $S_2^\circ - M_2^\circ$ at two distant loops that the loops form or do not form parts of the same system. Of course between neighboring places either of these quantities will generally vary but little. The ratio S_2/M_2 for the Pacific is, as a rule, less than the corresponding ratio of the forces. The cases where it is greater, like Tahiti* and Mazatlan, are so near the nodal lines that they little affect the generality of this rule; they simply indicate that the modes of the solar oscillation do not exactly agree with those of the lunar, and so at the nodal line of the lunar tide there might exist a sensible solar tide. Even Japan is too near to a nodal line for having the fact that S_2/M_2 there approaches its theoretical value seriously affect the generality of the rule.

On the other hand, the ratio S_2/M_2 is unusually small at Wellington, Port Russell, and Port Chalmers, New Zealand, and even at Apia, Samoan Islands.

52. *From the Galapagos Islands to Cape Horn.*

The change of tidal hour from the Panama loop to the Chile loop, and thence to the Cape Horn loop, has been noted in the preceding section. But the law of the change is influenced by the antecedent wave made necessary by the progression through the strait south of Cape Horn. (See Figs. 6, 27, 28, 30, 41.)

In the deep water surrounding the Galapagos Islands the tidal hour is a trifle less than VIII, but amongst these islands it is a trifle more than VIII. The tidal hour changes rapidly along the coast of Peru.

In latitude about 28° , on the Chilean coast, the time of tide appears to be about an hour earlier than in latitude 23° or 24° . This anomaly is probably due to the directions taken by the tidal streams of the oscillation and the progressive wave, causing a dependent stationary wave at the first-mentioned locality; lemma 32, case 4. In latitude about 41° there is a crowding up of the cotidal lines at the coast. This is a species of turning point of the tide and is to be regarded as analogous to a cape guarding the entrance to a broad bay; lemma 28. The direction of the tidal streams—say 100 or more miles offshore—are, at this latitude, probably nearly parallel to the shore instead of being normal to it, as an early tide requires. The crowding up of the cotidal lines near the western end of Magellan Strait is explained in a similar way.

In Corcovado Gulf is a dependent stationary wave having a 12 or 13 foot range at the head. This produces strong currents (4 to 5 knots) in Chacao Narrows. (Cf. § 106, Part IV A.)

The range of tide is 3.0 feet at Valparaiso and 4.3 feet at Cape Horn.

The axis of the southeastern half-wave length of the South Pacific system may be supposed to terminate in longitude 69° W. The theoretical tidal hour is VI at this Cape Horn loop. Now, superimpose upon this stationary wave an easterly progressing wave of equal amplitude and whose tidal hour is VIII† at longitude 69° W. (§ 3); the result will be the irregular progression shown in Fig. 6 along the axis of the area. Here λ is taken as 84° of a great circle.

* See Ferrel's Tidal Researches; also this manual, Part I, Fig. 17.

† Lemma 21.

53. *From Cape Horn to Uruguay.*

Upon referring to the chart of semidiurnal systems (Fig. 23, Part IV A), it will be seen that a loop of the South Pacific system has southern Chile and Graham Land for its eastern boundary. The intervening strait, constituting an opening in this boundary, causes a large wave to progress into the Atlantic. Fig. 29 shows that the disturbance thus transmitted produces the tide on the eastern coast of South America up to and including the tide in the Rio de la Plata. North of this river the tide becomes very small, especially at Rio Grande do Sul, and a crowding together of the cotidal lines is there necessitated in order to coalesce with the Brazilian tide due to the South Atlantic system.

The tide occurs earlier on the southeastern coast of the Falkland Islands than elsewhere in this group. One branch of the tide swings around to the north, going rapidly in the deep water northeast of the islands. The other branch appears to progress slowly along the south coast. In reality this rapid change in the tidal hour is due to the fact that the times are here governed by the large tides between Staten Island and Port Santa Cruz. (See § 14.) The two branches are united into one off the western extremity of the group. The tide enters Falkland Sound from both ends and meets near the center. For most of the inland water between West Falkland Island and Pebble Island the tide progresses easterly, the larger opening being to the west.

Three stationary waves, existing because the resultant flood is in certain localities largely directed shoreward, occur along the eastern coast of Patagonia. The head of the first extends from Magellan Strait to Port Santa Cruz. The length (to the 100-fathom line) may be considered as being close to $\frac{1}{4} \lambda$, and therefore favorable to the production of large tides. The second includes the Gulf of St. George, with off-lying shallow water; its length lies between $\frac{1}{4} \lambda$ and $\frac{1}{2} \lambda$. The third includes the Gulf of San Matias and immediate approach; its length lies between 0 and $\frac{1}{4} \lambda$, but nearer to $\frac{1}{4} \lambda$.

The tide in the first is caused by the tide between the Falkland Islands and Tierra del Fuego. The shore line receding at Port Santa Cruz a progressive wave moves northward (lemma 19). This combines with a wave coming from the east, causing the tide which sustains the stationary oscillation in the Gulf of St. George. At Cabo del Sur, marking the northern limit of this gulf, the receding shore line gives a northward progressive wave. On account of the early arrival of the wave coming from the east, the resultant tide must change its tidal hour rapidly between this cape and Delagada Point, Valdes Peninsula. Similarly, the tide east of this peninsula sustains a large oscillation in the Gulf of San Matias and Port San José. Hence the greater range of tide on the north side of the peninsula and its connecting isthmus. At Point Raza the shore line suddenly recedes, giving rise to a northerly progression, which follows the shore line around El Rincon until it meets the wave coming from the southeast.

Thus we see the necessity for two wave eddies such as have been referred to in the latter part of § 13. It is probable that § 5 may be of some assistance in explaining them.

The positions assigned to the no-tide points are $45^{\circ} 18' \text{ S.}$, $63^{\circ} 37' \text{ W.}$, and $40^{\circ} 55' \text{ S.}$, $60^{\circ} 45' \text{ W.}$

The tide in the Rio de la Plata is a progressive wave, advancing at about the rate due to depth. Although this estuary is funnel-shaped and of decreasing depth, the

irregular shoals so impede the progress of the wave that the range of tide remains about constant. It is 2 feet at Montevideo and 1.8 feet at Buenos Ayres. Here the tide occurs eleven hours later than in the deep water off the mouth of the river.

The range of tide in the eastern portion of Magellan Strait is about 30 feet; after passing the first narrows it is about 16 feet, and in Broad Beach, beyond the second narrows, 4 feet. The tidal streams in these narrows are strong and due to hydraulic effects, §§ 9, 105, Part IV A. The tidal hour increases southward and westward. The hour increases from Froward Reach to Cockburn Channel along either end of Clarence Island and coalesces with that of the tide which progresses eastward through the strait between Cape Horn and Graham Land, thus completing a cycle of values. The branch of the tide which follows the strait proper from Froward Reach westward meets the tide from the west near the eastern end of Desolation Island. One branch of the tide from Froward Reach produces the tide in Otway Water.

54. *From the Galapagos Islands to British Columbia.*

The Panama loop of the North Pacific system, marked IX in Fig. 23, Part IV A, is made possible by the fact that the trapezoidal branch of this system has a loop not far away also marked IX. Hence by the latter part of lemma 25 the tidal hour for the intervening waters should be IX. The Panama loop may, therefore, be described as having a large portion of its southern boundary latent (§ 29, Part IV A). Only at the extreme eastern end is there a rigid southern boundary. Observations indicate VIII instead of IX for the tidal hour of this loop. (See Fig. 30.) This discrepancy is probably due to the progressions (antecedent waves) setting out from this loop both southerly and northerly.

Some inward progression occurs in the gulfs along this coast. How the tide is delayed at the mouth of a bay and how the far side of the mouth of a bay is sometimes reached as early as the near side have been explained under lemma 31, cases 3, 4.

The nodal line terminating just above Acapulco is remarkably well defined both by the small range and remarkably sudden change in the tidal hour.

The tide in the Gulf of California is chiefly stationary, although there is enough inward progression to prevent there being a no-tide point from which the cotidal lines would radiate.

On account of the low latitude and the obstructions due to islands, the effect of the earth's rotation in crowding together the lines on the Lower California side and spreading them apart on the opposite side is probably less marked than in the case of the English or Irish Channel.

The range of tide at Mazatlan is 2.6 feet and at the head of the Gulf about 20 feet. Strong tidal currents due to hydraulic effects (§ 106, Part IV A) occur between Tiburon Island and the mainland.

The progression along both Lower and Upper California is due to the overlapping of the two Pacific systems (§ 15).

The curved nodal line of the North Pacific system terminating at Point Arguello is obscured by a loop of the South Pacific system. From this cape northward to Alaska, the range of tide increases. It is 3.6 feet at Point Arguello and 7.7 feet at Sitka. Ranges varying from 10 to 15 feet are usual for the inner waters of Alaska. (See Figs. 32, 33.)

The Gulf of Georgia, including the Juan de Fuca Strait, has a large stationary wave whose nodal line is near Discovery Island Light. There is, however, some progression, so that the influence of the earth's rotation is to cause the cotidal lines to crowd together at the Vancouver end and to spread apart on the Washington side, where, as is usual in such cases, the range of tide is considerable.

The tide entering Queen Charlotte Sound reaches the vicinity of Discovery Passage several hours before the time of tide in the upper end of the Gulf of Georgia. The range of tide in the first locality is somewhat larger than the range in the second. The hydraulic effect is a current in Seymour Narrows of from 4 to 8 knots (§ 106 and Fig. 32, Part IV A).

Upon referring to the table given under § 97, Part IV A, Nos. 215-219, it will be seen that the ratio S_2/M_2 for Mazatlan and near-by stations in Lower California is larger than the ratio of the corresponding forces. For all other parts of the Pacific coast of America from Cape Horn to Bering Strait the tidal ratio is less than the ratio of the forces.

55. *Alaska and Bering Sea.*

The north angle of the large triangle forming the greater part of the North Pacific system is bounded for a considerable distance by the coast of Alaska. In § 74, Part IV A, it was pointed out that the range of tide should here be large and that its theoretical hour should be not far from IX. Upon referring to the charts of cotidal lines, Figs. 31, 33, we see this to be the case. The chart covering lower Alaska, Fig. 33, shows that the tides in the numerous canals seldom occur more than half an hour later than do the tides outside. The range, however, is considerably increased. These facts show that the tide in these canals consists largely of a stationary wave a moderately small fraction of a wave length long. (See also § 91 and Fig. 32, Part IV A.) The small difference in both time and range of tide throughout these canals and straits prevents the tidal streams from becoming as violent as the large range of tide and the peculiar formation of the coast might indicate. But hydraulic effects are not altogether wanting; in Clarence Strait, northeast of Prince of Wales Island (Fig. 33, Part IV A) the tidal current is 5 knots. Strong currents are likewise found in Sergius Narrows, Peril Strait. Throughout most of the canals and straits tolerably strong currents are necessitated for carrying in and out the large tidal volumes.

The large tides in Cook Inlet probably indicate that the virtual length of this body is about $\frac{1}{4} \lambda$.

The diminution of range of the Pacific tides experienced in going west from the Gulf of Alaska indicates the existence of a nodal line farther on. But the Bering and Okhotsk seas are bodies of water which permit extensive progressions, and so it is not strange to find an antecedent wave as far east as the Gulf of Alaska, and to find the nodal line off Rat Islands much obscured in consequence.

The tides of Bering Sea are derived from those of the Pacific. (See chart of soundings, Fig. 19, Part IV A, and chart of cotidal lines, Fig. 34, Part IV B.)

The Gulf of Anadir, although shallow, is characterized by a stationary wave, and so by nearly simultaneous tides.

Between St. Lawrence Island and Alaska the tidal hour changes rapidly and the range diminishes in going northeasterly through this strait.

At the mouth of the Yukon River the range of tide is 1.4 feet; 60 nautical miles up the river, the range is 0.4 foot.

Norton Sound is an amphidromic region, the no-tide point being about $\frac{1}{4}\lambda$ from the head of the sound. Without the deflecting force of the earth's rotation, this body of water would constitute a dependent fractional area with a nodal line running nearly north and south. But this deflecting force, on account of the narrowness of the sound, causes the water to pile up on the south side and to recede from the north side when the eastward motion across the nodal line prevails. The reverse is true for the west-going stream. On account of the relatively large diurnal wave in Norton Sound there is generally but one high water per day when the moon is far from the equator.

Around St. Lawrence Island the time of the tide probably assumes all values, as shown on the cotidal chart, but no observations for establishing this supposition are available. The range of tide is probably about 1 foot at the east end and somewhat greater at the west end.

The range of tide in the deep water south of the Pribilof Islands is about 2 feet; but the extensive shoaling and favorable configuration of the shore line cause the range to become 20 feet near the heads of the estuaries terminating Bristol Bay. There is undoubtedly considerable tide in Kuskokwim Bay.

From the Peninsula of Alaska westward to Umnak Island are numerous passages leading from the Pacific to Bering Sea. Through these the time of tide changes rapidly. The tidal streams are swift, particularly in the narrow passes, and are, in the main, hydraulic effects (§ 106, Part IV A). The range of tide is much greater on the southern side of these islands than on the northern side or in the passes.

56. *Islands of the North Pacific.*

The cotidal charts covering this region are Nos. 31, 35, 36.

The progression for waters lying easterly and northerly from the no-tide point situated between California and Hawaii has just been explained as being due to the overlapping of systems in the one case and the broken northwestern boundary of the Pacific in the other. The question now arises as to what may produce a southerly progression for the waters lying to the west of this point. By referring to the chart of systems it will be seen that north of this point is a region marked IX and south-southwest of it one marked XII. But XII or 0 combined with III, which number covers a very wide region, gives an intermediate value, say $I\frac{1}{2}$. We should therefore expect to find a change from IX to $I\frac{1}{2}$ for the western side of the amphidromic region, especially if there exists a favorable progressive wave in the vicinity of the nodal line west of the no-tide point. Such a wave is the antecedent wave moving toward the Fiji and New Hebrides Islands and involved in the progression caused by the shoaling and openings north and northwest of New Zealand, where the rise and fall of the South Pacific system is considerable (Fig. 20, Part IV A). It extends northeasterly to and even beyond the Hawaiian Islands, giving to the tides of all intervening waters a tendency to progress southwesterly.

South of the no-tide point is a region over which the time of tide changes but little, that time being between $I\frac{1}{2}$ and II, which is, of course, intermediate between XII and III. In this region progressive waves, if felt at all, are small in comparison with the resultant stationary wave.

West of the Hawaiian Islands a general westerly progression is apparent because of the small range of the stationary wave over the extensive region marked III on the chart of systems. This is caused in part by the antecedent wave just referred to, but chiefly by similar waves entrained by the tides of the Sea of Okhotsk and the Yellow and China seas. In the western angle of the North Pacific triangle (between the Philippines and Guam) this progression is no longer apparent, partly because of the directions and limitations of the antecedent waves and partly because the range of the stationary tide becomes great as the angle of the triangle is approached (§ 27, Part IV A).

57. *From Kamchatka to Japan.*

The tides at Petropauloysk, although not different in character from the tides of British Columbia and Alaska, have some historic interest because they were observed as long ago as 1828 and found to possess a large diurnal inequality, especially in the low waters.

Very little is known of the tides in the Sea of Okhotsk and comparatively few soundings have been made.

The tide is probably large in gulfs of Jijiginsk and Penjinsk; the range for the western and northern shores of the sea is probably 7 or 8 feet.

Upon referring to Fig. 23, Part IV A, it will be seen that a nodal line passes through the Caroline Islands just east of the Ladrone Islands and terminates off the eastern coast of Japan. On account of the progression up the Yellow Sea, and into the China Sea, it is probable that the northern end of this nodal line will be somewhat obscured by the antecedent wave. However, Fig. 36 shows a sufficient crowding up of cotidal lines to prove the existence of a nodal line. The fact that the tide reaches the eastern coast of the Philippine Islands within less than an hour after the time it reaches Guam, and that the range in the former locality is several times greater than in the latter, shows clearly the existence of a nodal line and loop.

Along the eastern coast of Japan to about the latitude of Tokyo the range of tide is about 2 feet. Going thence westerly along the coast the range continually increases, being 6.2 feet at Nagasaki.

The tide, entering from the south, progresses up the Inland Sea as far as the most northern point of Sikok Island, where it is met by the tide from the northern entrance.

The tidal hour changes rapidly from IX½ to I½ and from X to I½ in the southern and northern straits, which connect this sea with the northern entrance, or Kii Channel.

The tide reaches a point on the eastern coast of Kiusiu Island remarkably early for reasons given under lemma 32.

The tide which progresses westerly through Bungo Channel coalesces with that of Korea Strait a short distance west of Simonoseki.

The tides in the Sea of Japan are caused mainly by the disturbance acting through the Strait of Korea, where the cotidal lines are amphidromic. It is high water nearly simultaneously over the southern half of the sea, and this happens at about the time of low water for the southern end of the strait. The case is one of a tidal body producing tides in a nearly tideless sea. Hence § 35, Part IV A, the greatest inward velocity occurs three hours after high water at the south end of the strait. The extreme south end of the sea being narrow, the deflecting force of the earth's rotation causes it

to then be piled up along the Japanese coast, while it recedes from the Korean coast. The reverse occurs on the outgoing stream.

The table given under § 97, Part IV A, and continued under § 19, Part IV B, shows great activity on the part of the Japanese in observing and analyzing tides. The ratio S_2/M_2 has here generally about its theoretical value; but as the nodal line is not far away, the value of the ratio here can not be expected to compare closely with its value in other parts of the system.

58. *Philippine Islands and China.*

The cotidal chart of the Philippine Islands, Fig. 37, shows that the tidal hour for the Pacific coast of these islands is about IX $\frac{1}{2}$.

The tide of the Celebes Sea is almost simultaneous with the tide east of the islands. The tide is delayed and its character somewhat altered in passing through the Sulu Archipelago into the Sulu Sea.

In passing through the channel north of Luzon the tide is much retarded. Over the deeper portion of the China Sea the tide is nearly simultaneous and the range small. Somewhat similar instances to this are Denmark Channel and the basin extending thence to Spitzbergen, also the strait south of Cape Horn and the deep waters towards South Georgia and the Sandwich Group. As the tide enters the China Sea it is delayed at the coast of Luzon, lemma 28, and the cotidal line becomes nearly parallel to the western coast of Luzon and Palawan.

Which part of the tide comes directly from the Pacific and the Celebes Sea and which part from the China Sea can be seen upon the map. It will be noticed that the tide ranges of Sulu Sea comes mainly from the south. By observing where bracketed occur, localities which at times have but one high water daily can be inferred.

In the southern part of San Jacinto Strait the time of tide changes suddenly from X $\frac{1}{2}$ to II, and in San Bernardino Strait from IX $\frac{1}{2}$ to II.

The tide passes up the Yellow Sea at nearly the rate due to depth. Between Shantung Promontory and Korea there is some crowding together of the cotidal lines. By lemma 28 the northern shore of Shantung Promontory should have very late tides, and observations at Wei-hai-wei show that such is the case. (See also § 14.) The wave advances northwesterly through the waters connecting the gulfs of Pechili and Liaotung, to the northwestern shore near the terminus of the Great Wall. One branch passes northeasterly up the Liaotung Gulf; the other follows the western and southern shores of the Gulf of Pechili, finally coalescing with the incoming waves, thus forming a wave eddy (§ 13).

The chart of cotidal lines (Fig. 36) shows the range to be generally large for the eastern coast of China, and especially for the western coast of Korea. The range at Wei-hai-wei is, however, only 4.5 feet.

Some observations made upon the bore of the Tsien-tang Kiang are shown in Fig. 19, Part I.*

Lemma 28 applies to the northern and southern ends of the island of Formosa, and the cotidal lines finally become parallel to the sides of Formosa Strait.

* This bore is described by Prof. G. H. Darwin on pp. 59-71 of his book entitled "The Tides and Kindred Phenomenon in the Solar System."

In passing up the Gulf of Tonkin the small semidiurnal tide from the China Sea becomes very small toward the head of the gulf, while the diurnal tide suffers no such diminution. The tide here is of historic interest, having been observed by Francis Davenport and discussed by Dr. Edmund Halley and Sir Isaac Newton.*

From the deep portion of the China Sea an extensive progression extends to Singapore and to the Gulf of Siam. This gulf causes an increase in range as Bangkok is approached.† The progression to Singapore coalesces with that of the Strait of Malacca and continues southeasterly for a short distance along the coast of Sumatra. At Banca Island it turns east, and reaching Borneo turns northerly, finally coalescing with the incoming tide. This is a species of wave eddy, mentioned in § 13, whose period requires 24 instead of 12 hours. It resembles the one occupying the greater part of the North Sea. It is probable that for considerable distance around the no-tide point the range of tide is very small.

The tides of Gillolo Passage, Molucca Passage, and Macassar Strait are shown in Figs. 7, 36. They coalesce with the Pacific tides at the northern ends of the passes. In case of the first two some crowding up of the lines is required, while the northern portion of Macassar Strait is characterized by simultaneous tides. (See §§ 35, 102, Part IV A.)

By means of a large-scale chart of soundings the apparently irregular arrangement of the cotidal lines in the Banda and Java seas are easily accounted for, at least in a general way. The tide progresses westerly in the Java Sea because the range of the ocean tide upon which it depends is large near the loop and small near the node of the North Indian system.

59. *Islands of the South Pacific.*

The sequence of tidal hours around the no-tide point near the Society Islands can be obtained from the following considerations: According to the loops of the oscillating systems (Fig. 23, Part IV A), east of the point the hour should be IX; north of the point it should be I and II; west of the point it should be VI; south of the point there should be little or no tide. The antecedent wave involved in the progression resulting from the shoals and openings north and northwest of New Zealand indicates that northwest of the no-tide point there is a progressive wave moving southwesterly. Southwest of the point is a sector over which there is little progression, as the tidal hours for the Samoan and the Hervey or Cook Islands go to show. South of this point the range of tide is small, but an eastward progression is necessitated because the tidal hour for the Cook Islands is VI while for Rapa or Oparo Island it is $IX\frac{1}{2}$ and for Tahiti between X and $X\frac{1}{2}$. East of the point there is a northward progression, as observation made in the Low Archipelago and on the Marquesas Islands prove. Observations made on the Marquesas, Caroline, and Penrhyn islands prove a westerly progression to exist north of the no-tide point. Northeast of the point, the change in tidal hour is very slow, as might have been inferred from the chart of systems. All these considerations agree in

* See §§ 85, 88, Part I.

† Since Fig. 36 was drawn, new constants for several places in Cochin China have become available through the French colonial tide tables for the year 1904. They indicate that the tidal hours for Nha-trang and vicinity should be increased by an hour or more; also, that between Cambodia Point and Hatien the tide is much delayed in accordance with § 7.

giving an amphidromic region similar to the one shown in Fig. 38. The position of the no-tide point is quite accurately fixed by the known tides at Tahiti, and Tonga-reva or Penrhyn islands; at these two places the range is small, and when it is high water at the one it is almost low water at the other.

It may be well to note that no observations appear to have been made for the central portion of the loop marked IX.

The tides at Pitcairn, Henderson, and Ducie islands would probably be little influenced by those not belonging directly to the system.

Between Easter Island and Sala-y-Gomez a rapid change takes place in the tidal hour. This crowding up of the cotidal lines is due to the proximity of a nodal line modified by the wave antecedent to the Cape Horn eastward progression.

The Kermadec, New Hebrides, Fiji, and Ellice islands all lie at or near the western loop of the South Pacific system. Their theoretical tidal hour is VI. However, the southwesterly progression started by the imperfect character of the outer boundary of this loop has the effect of making the number of the hours increase southwesterly.

The tide on the southwestern coast of New Caledonia occurs 2 hours later than on the northeastern. (Cf. lemma 28.)

In going westward from the Gilbert and Ellice islands to the northern coast of New Guinea the range of tide continually decreases, indicating approach to the nodal line which passes just east of the Ladrone Islands.

The progression from the Marshall and Gilbert islands through the Solomon Islands, thence across the Coral Sea to the Australian coast, seems to be the result of two neighboring regions having tides at different hours. In the first region the tidal hour is about V and in the latter IX. Thus the tide in the Coral Sea comes from both the north and the east, the tide from the east being the controlling factor.

The extreme shallowness of Torres Strait prevents the Pacific tide from exerting much influence on the tides in the Gulf of Carpentaria.

60. *New Zealand and eastern coast of Australia.*

The westward and southwestward progression resulting from the imperfect support of the western loop of the South Pacific system constitutes the tides along the eastern coast of Australia and in the adjacent waters. One portion going southwesterly through the broad passage between New Zealand and Australia has the effect of producing, with the aid of the South Indian system and the XII loop of the South Pacific system, a region south of this passage over which the tides are nearly simultaneous. The progression extends completely around New Zealand, the portion found east of North Island being an antecedent wave. The tidal hours thus form a complete cycle of values (§ 14).

The tidal hour at the south end of Cook Strait is $IV\frac{1}{2}$. The hour increases in going northward to X on the southern shore of North Island and to $IX\frac{1}{2}$ near Cape Farewell, where it meets the tide progressing down the western coast of the islands. The range at the south end of the strait is considerably less than at the north end. The motion in the strait is largely a stationary oscillation. The maximum velocity occurs about 3 hours after the maximum surface slope through the strait. (Cf. §§ 11, 35, 102, Part IV A.) In many parts of this strait there are strong tidal currents.

A no-tide point southeast of New Zealand appears to be required for the following reasons, although its exact location is, of course, doubtful: Observations show a sequence of hours, I to VI, to the west and northwest of the assumed point. To the north there must be a crowding together of cotidal lines because of the proximity to a nodal line. For the Chatham Islands the tidal hour is $IV\frac{1}{2}$ and for Rapa or Oparo, near the loop marked IX on the chart of semidiurnal systems, it is $IX\frac{1}{2}$, indicating that the tidal hour increases in going northeasterly from Chatham Islands. East of the point the theoretical considerations indicate that the tidal hour must be XII (Fig. 23, Part IV A). Hence the southeasterly increase from $IX\frac{1}{2}$ to XII. This change is probably helped along by the antecedent wave of the Cape Horn progression. To the south must be a wide sector of nearly simultaneous tide, because observations made at Auckland and Campbell islands, west of the point, give a tidal hour but little different from the theoretical tidal hour southeast of the point.

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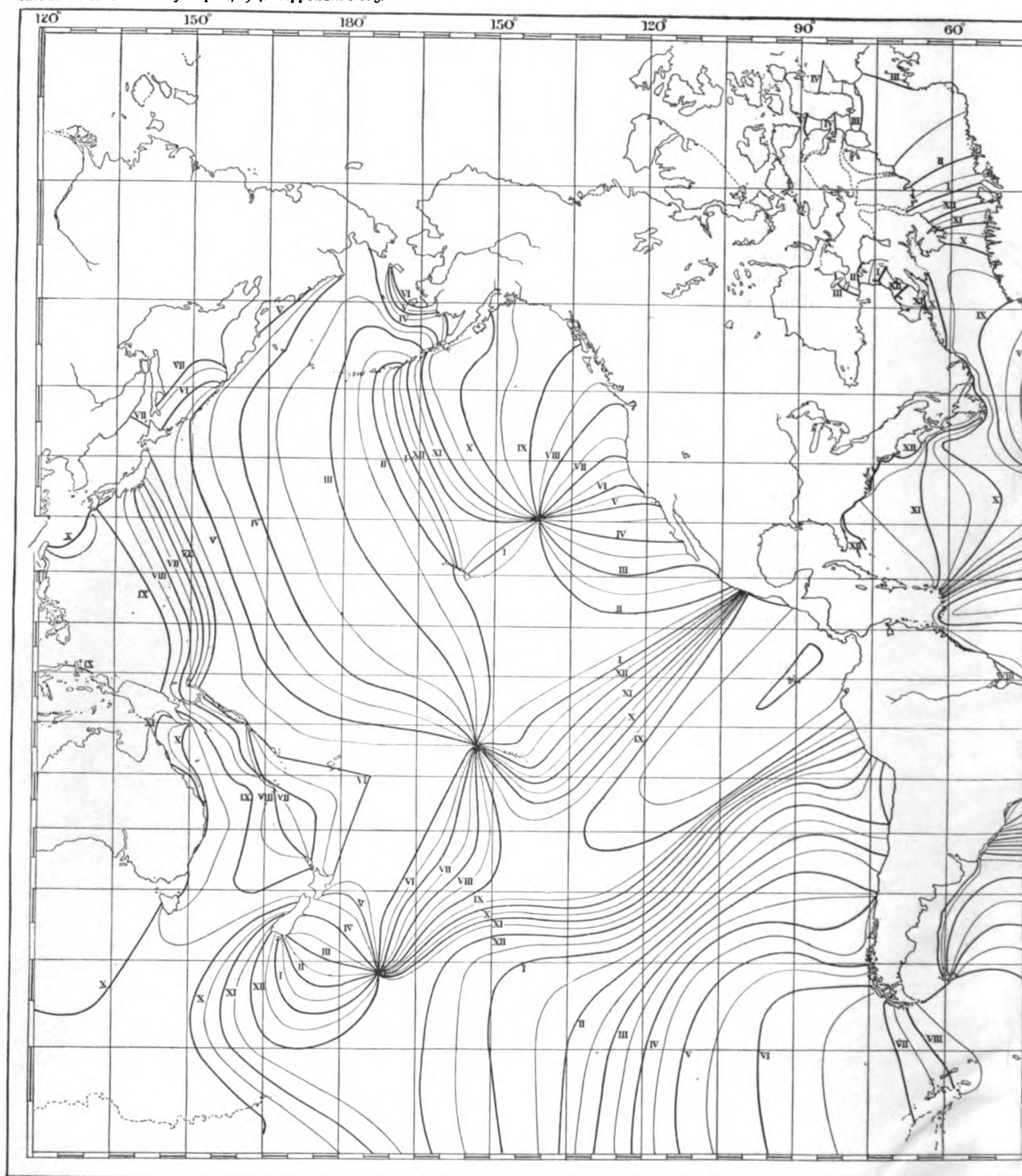
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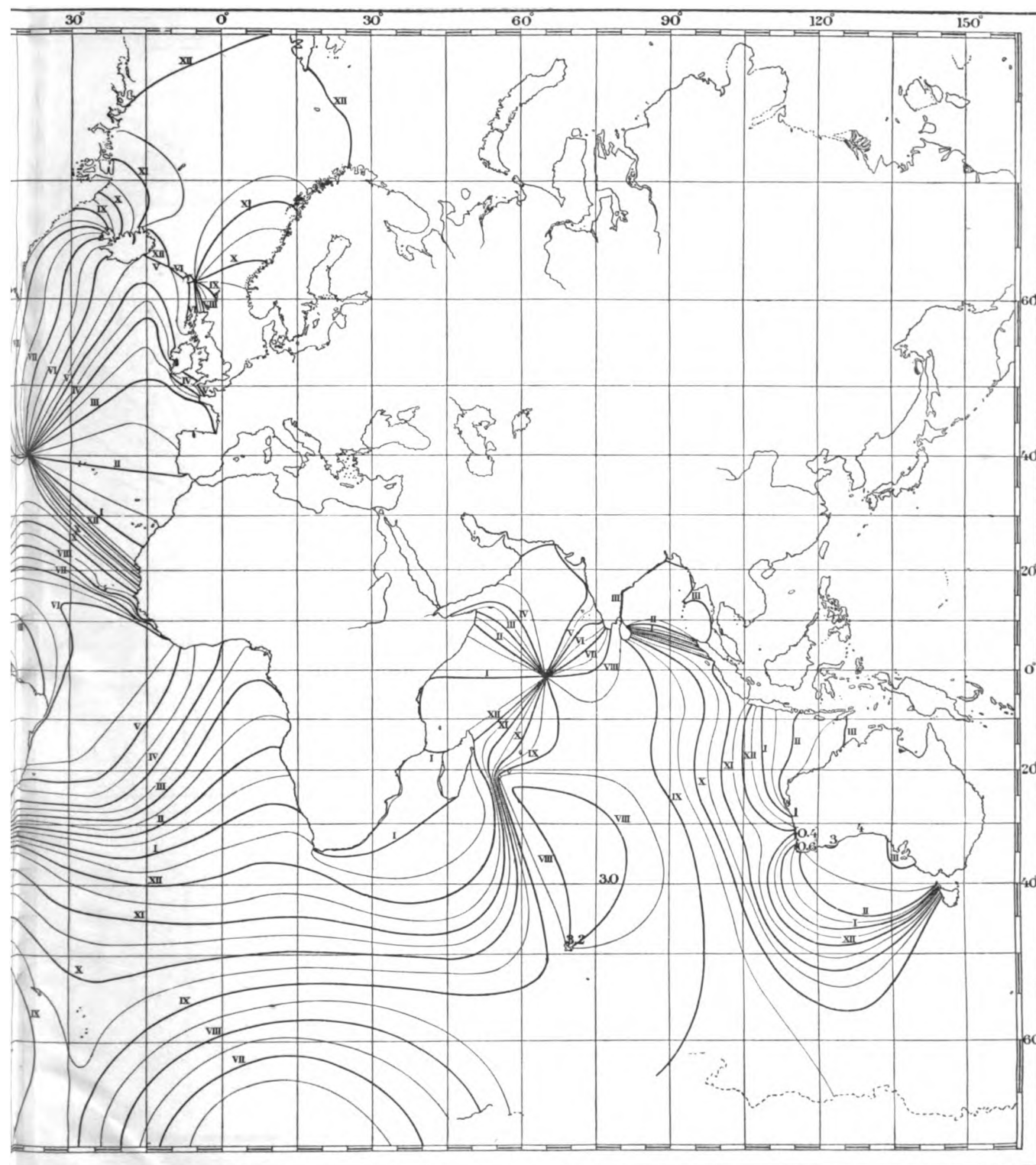
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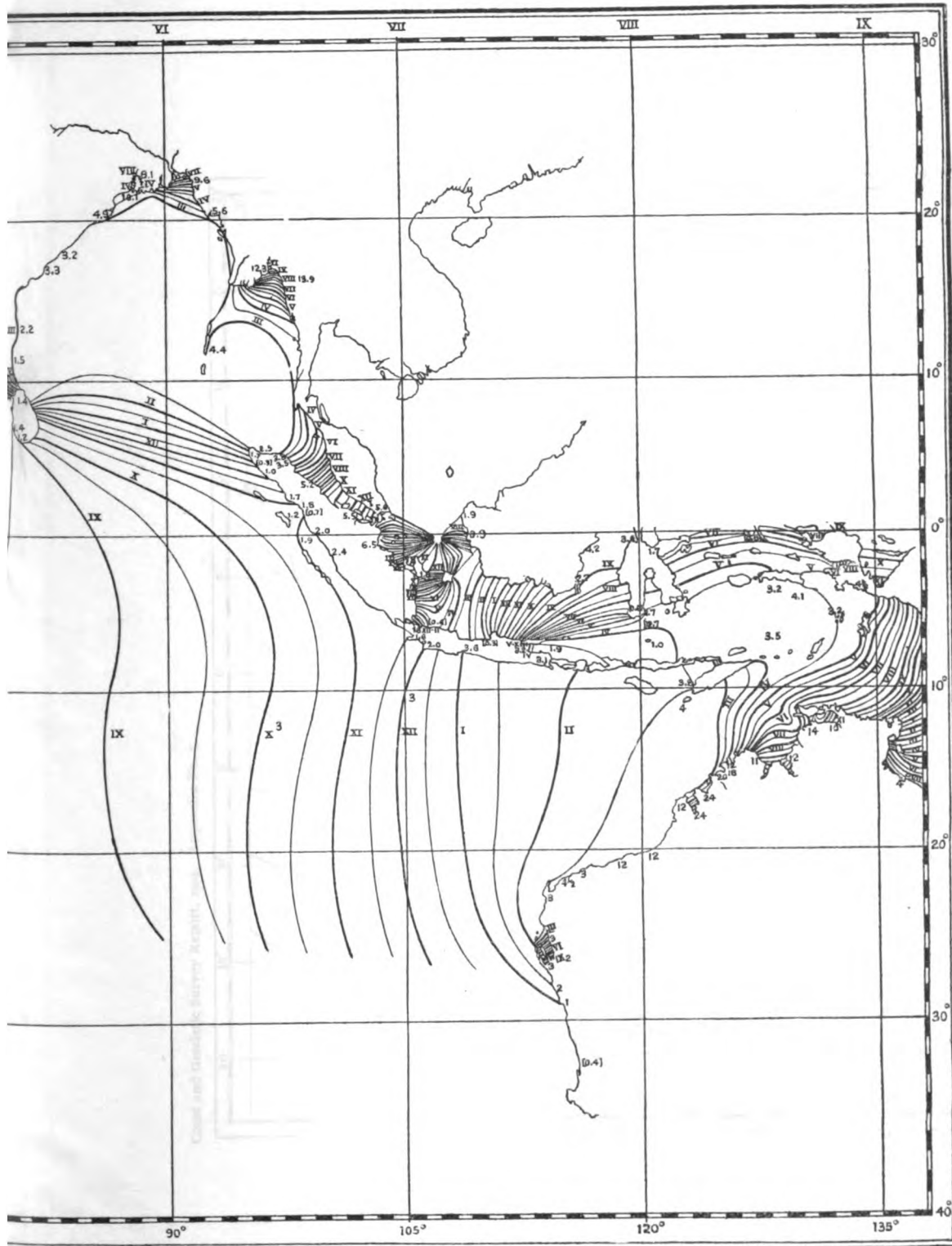


Cotidal line

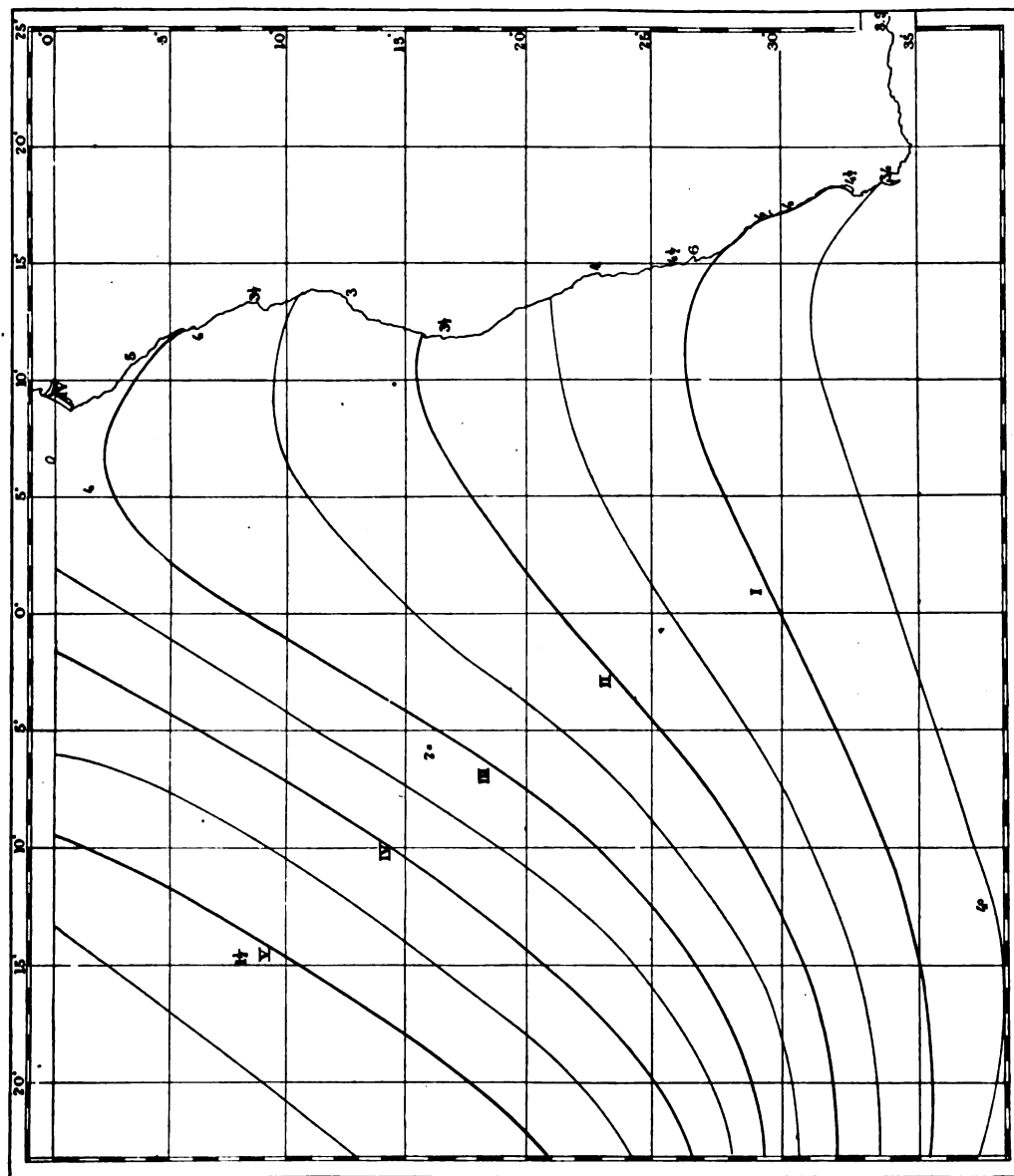


for the World.

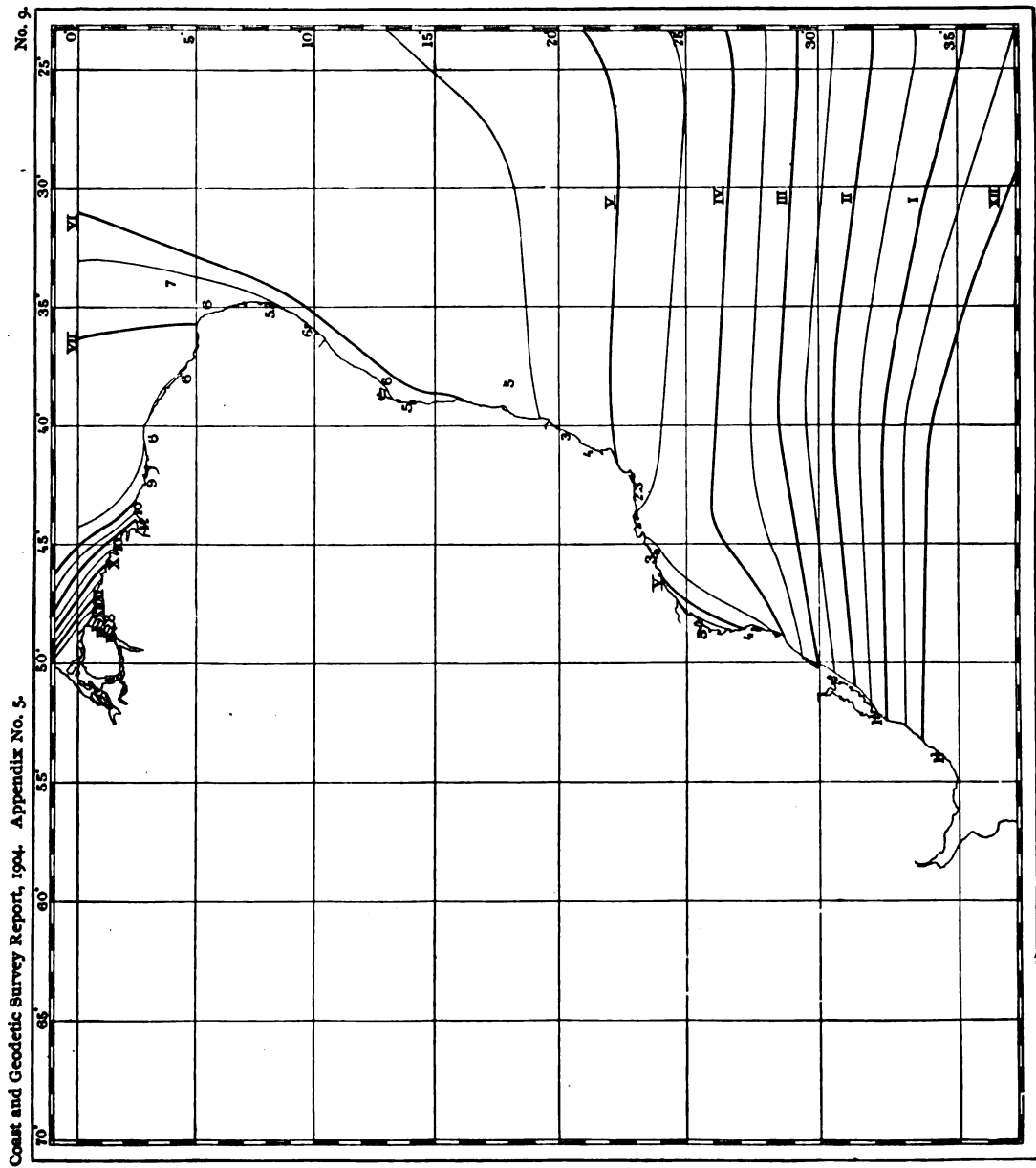


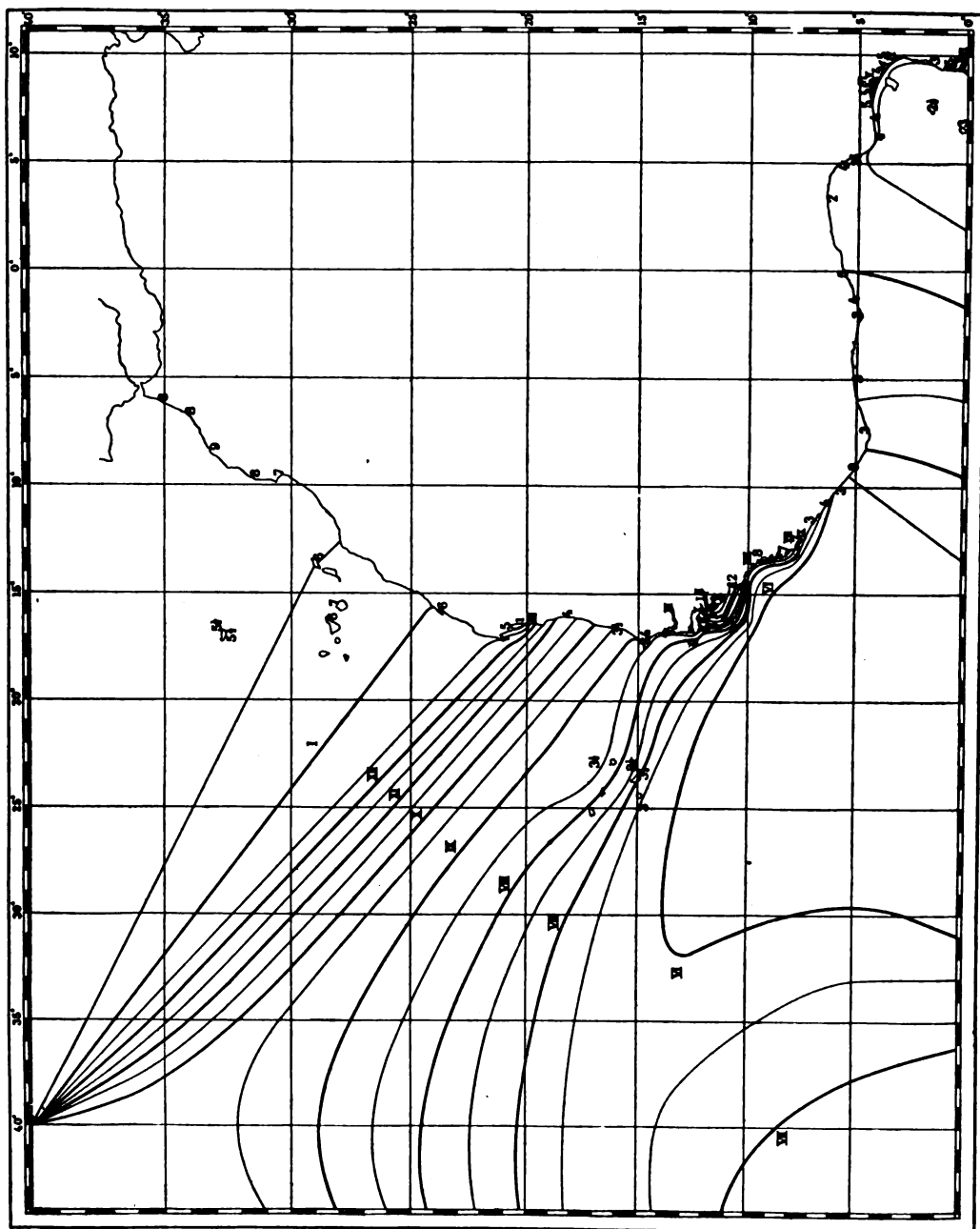


part of the Indian Ocean.

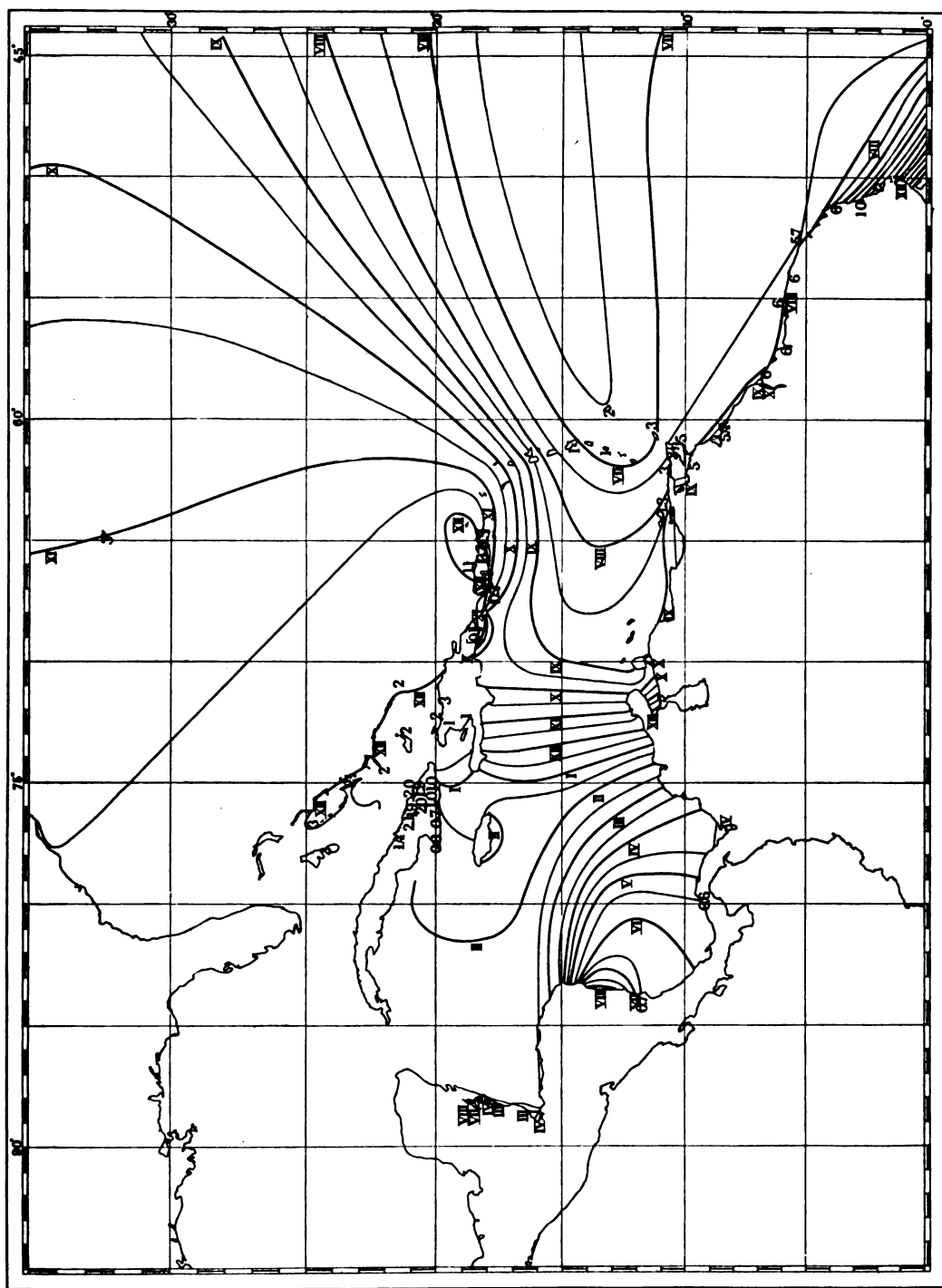


Cotidal lines west of Southern Africa.

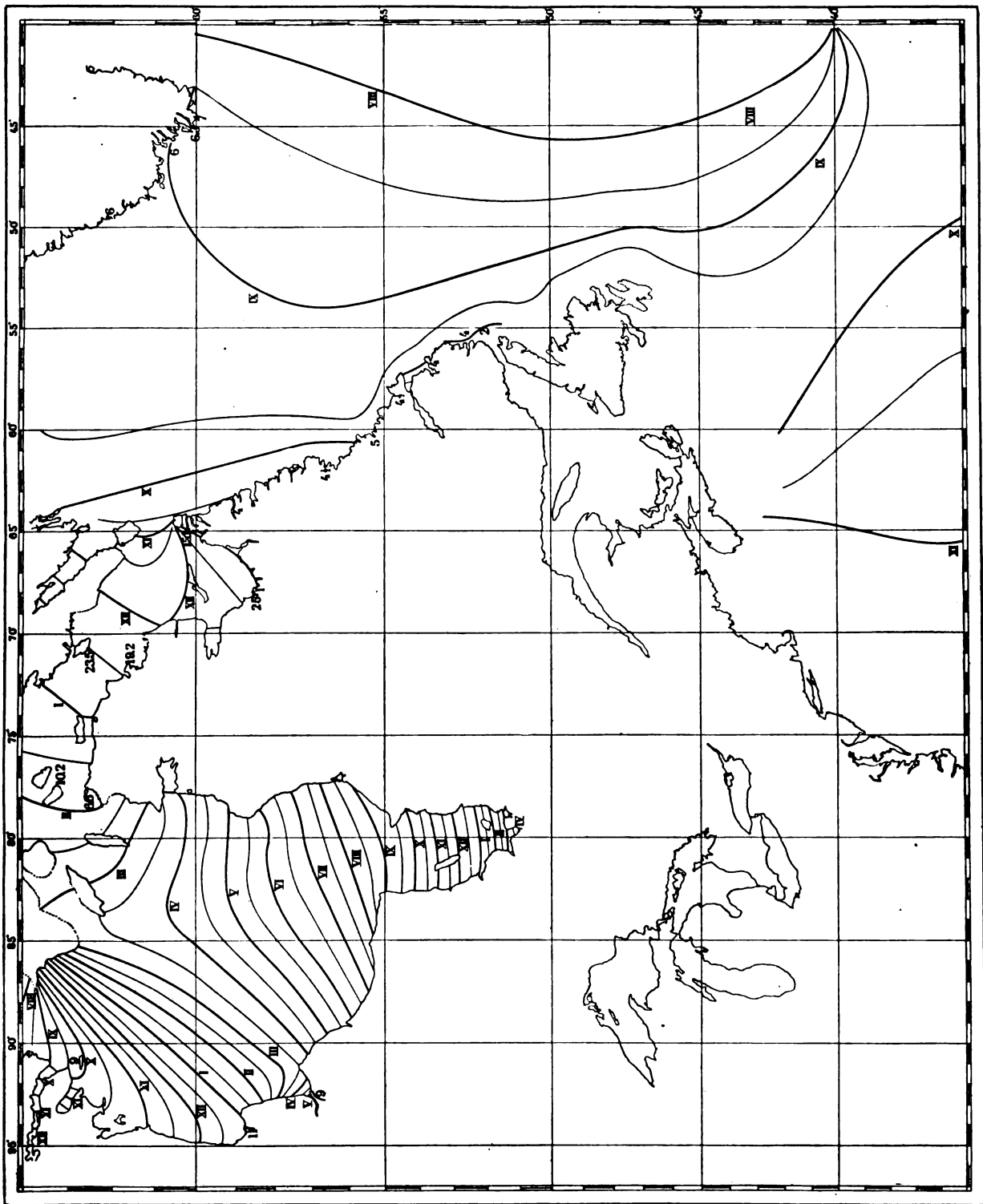




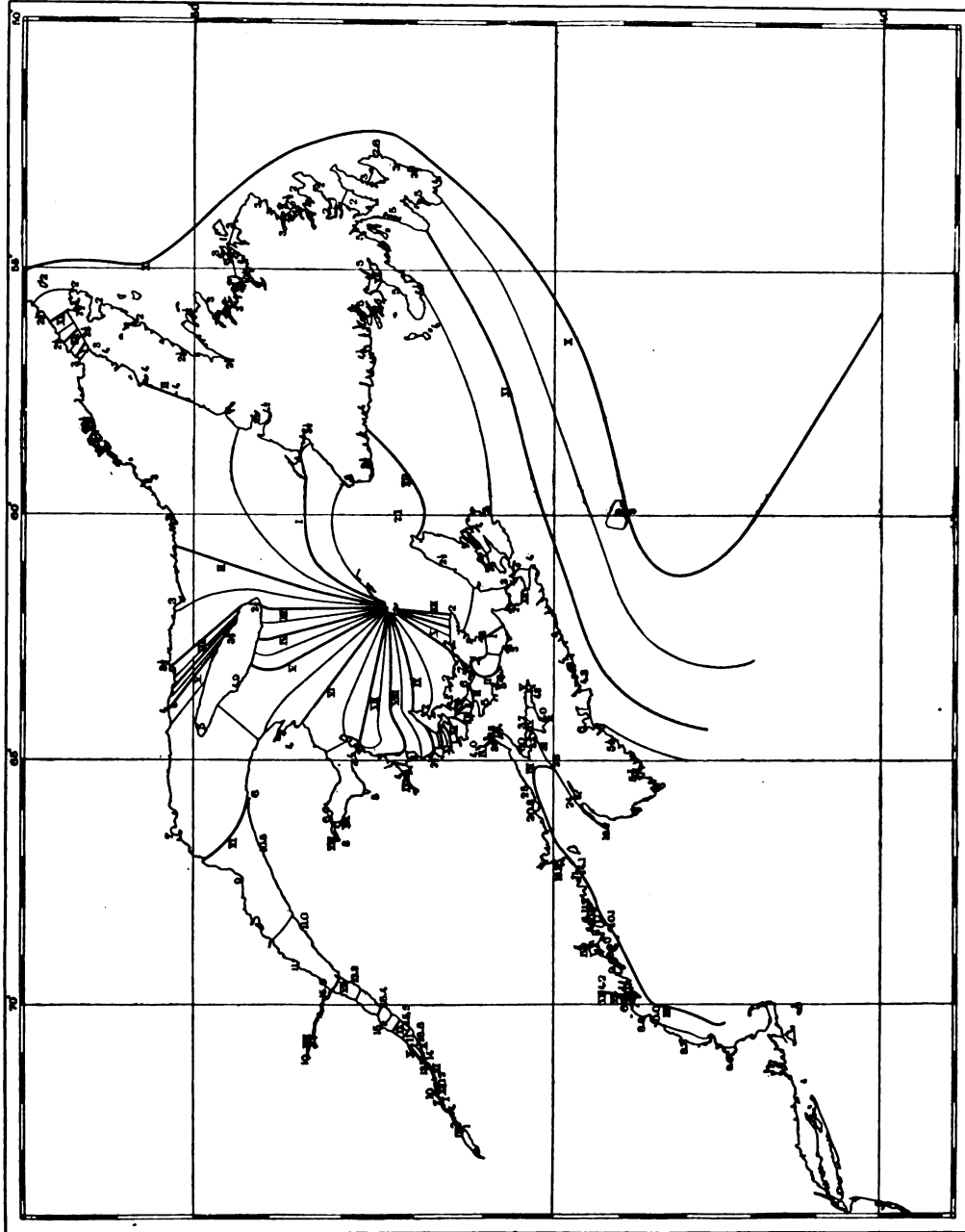
Tidal lines west of Northern Africa.



Cotidal lines north of South America.

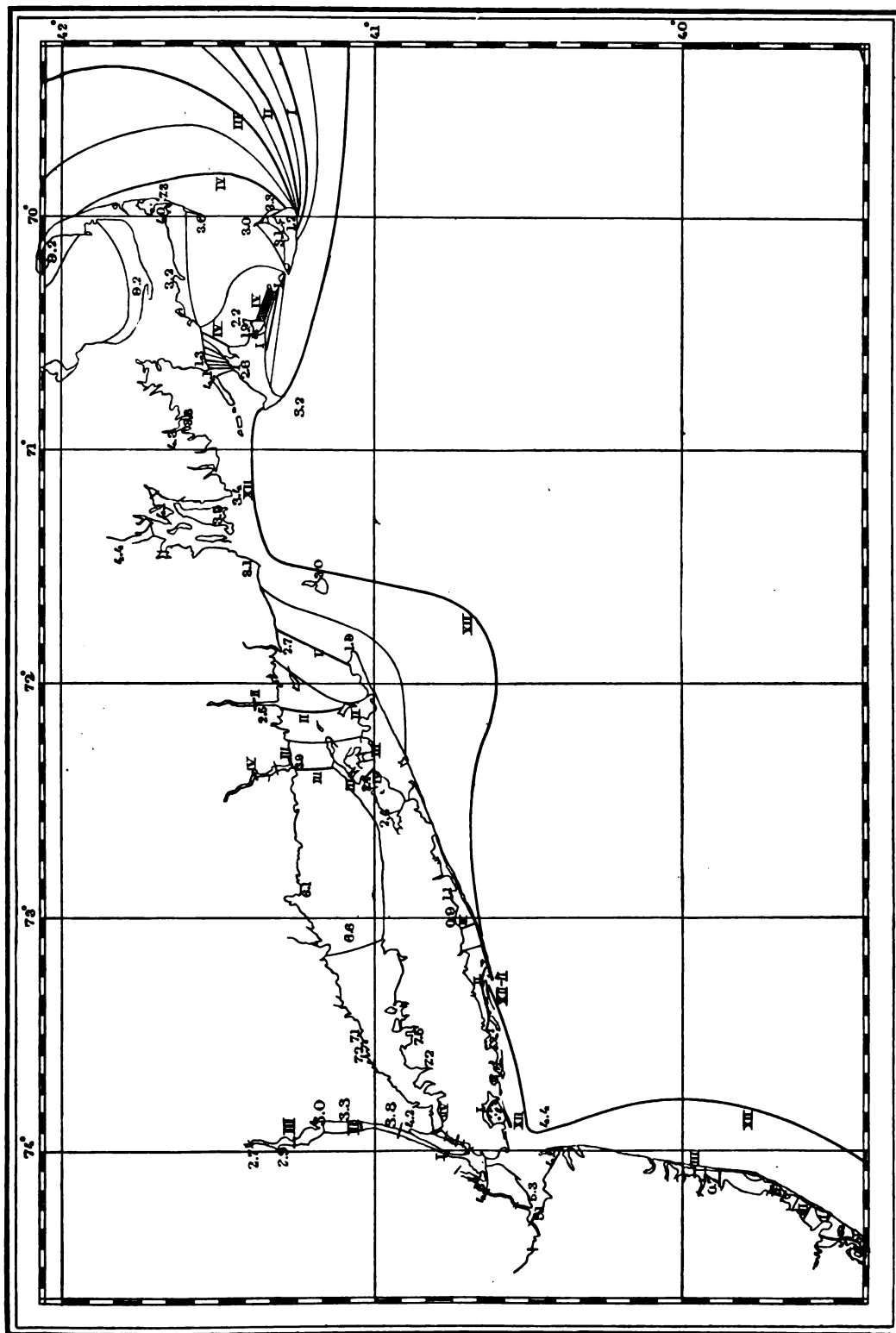


Cotidal lines east of Canada.

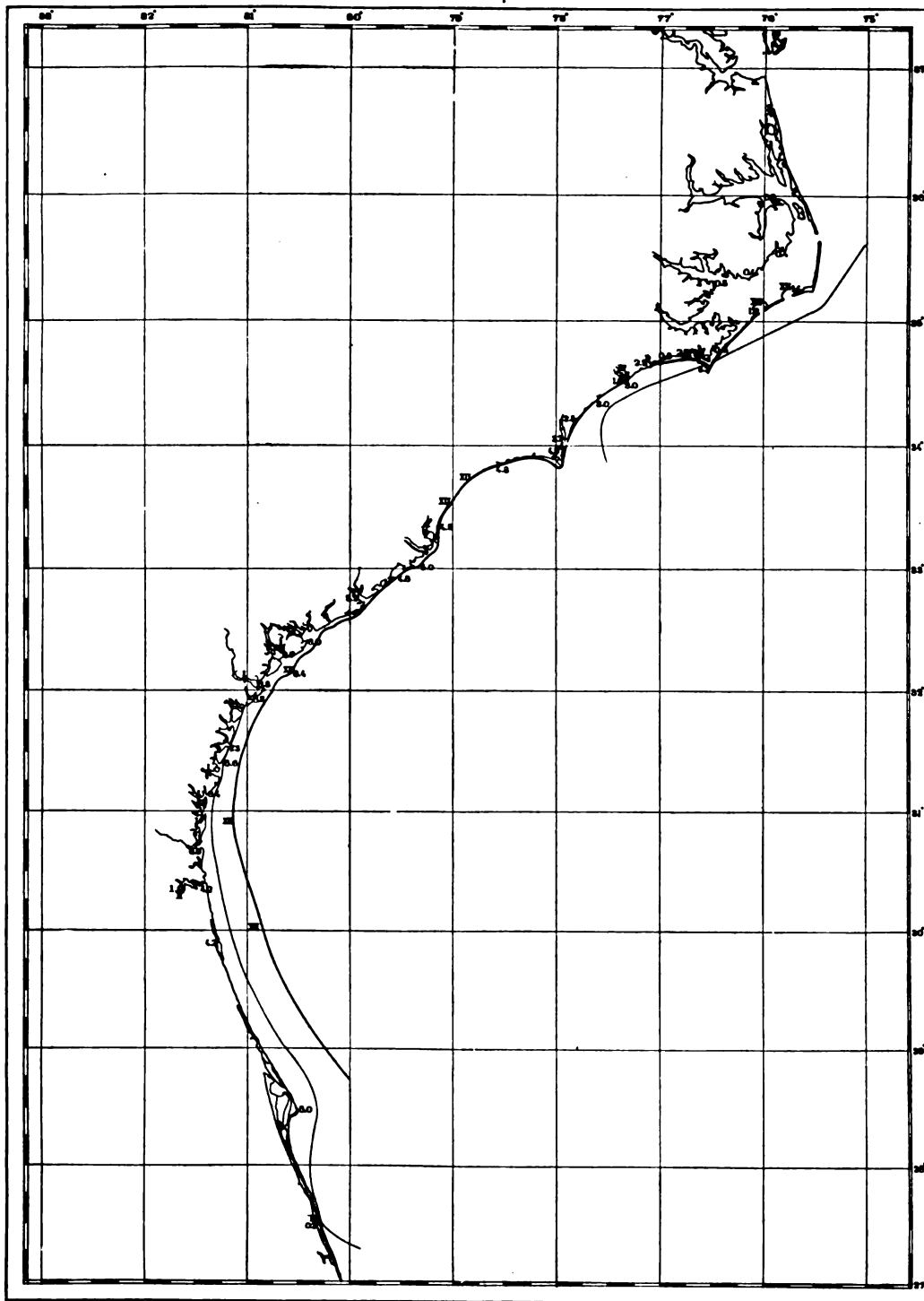


Cotidal lines for the Gulf of St. Lawrence.

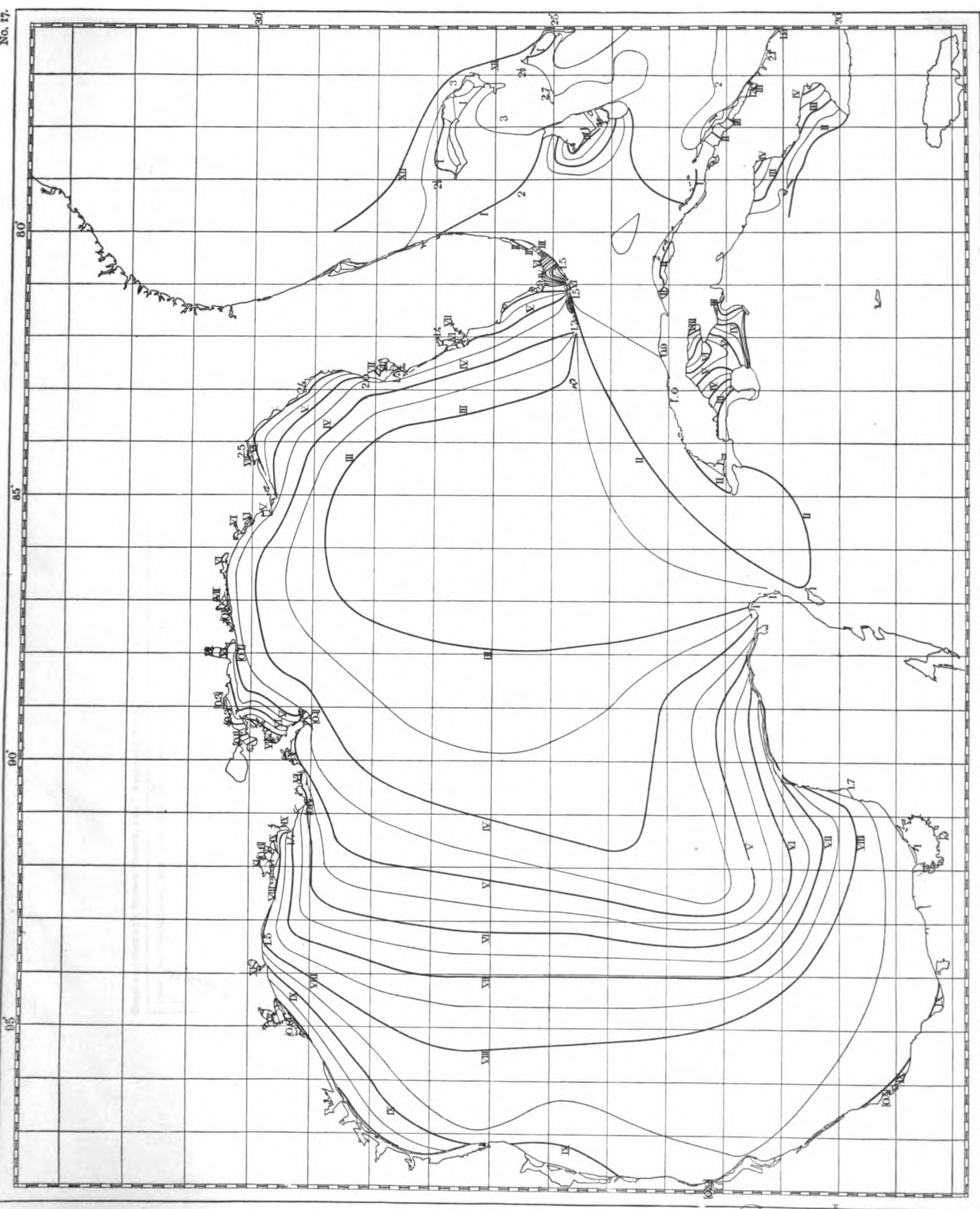




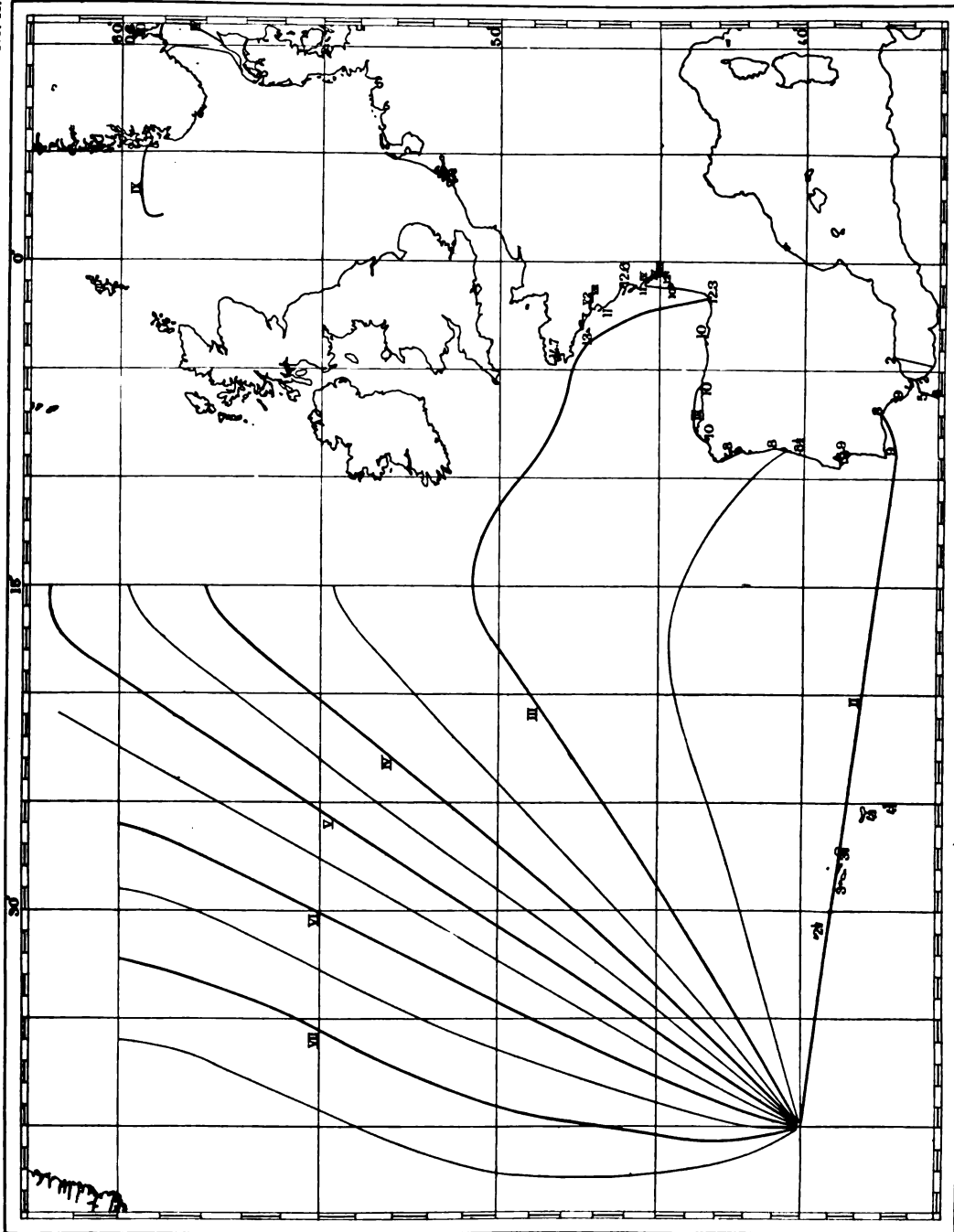
Cotidal lines for New York Harbor and approaches.



Cotidal lines for the Southern Atlantic States.

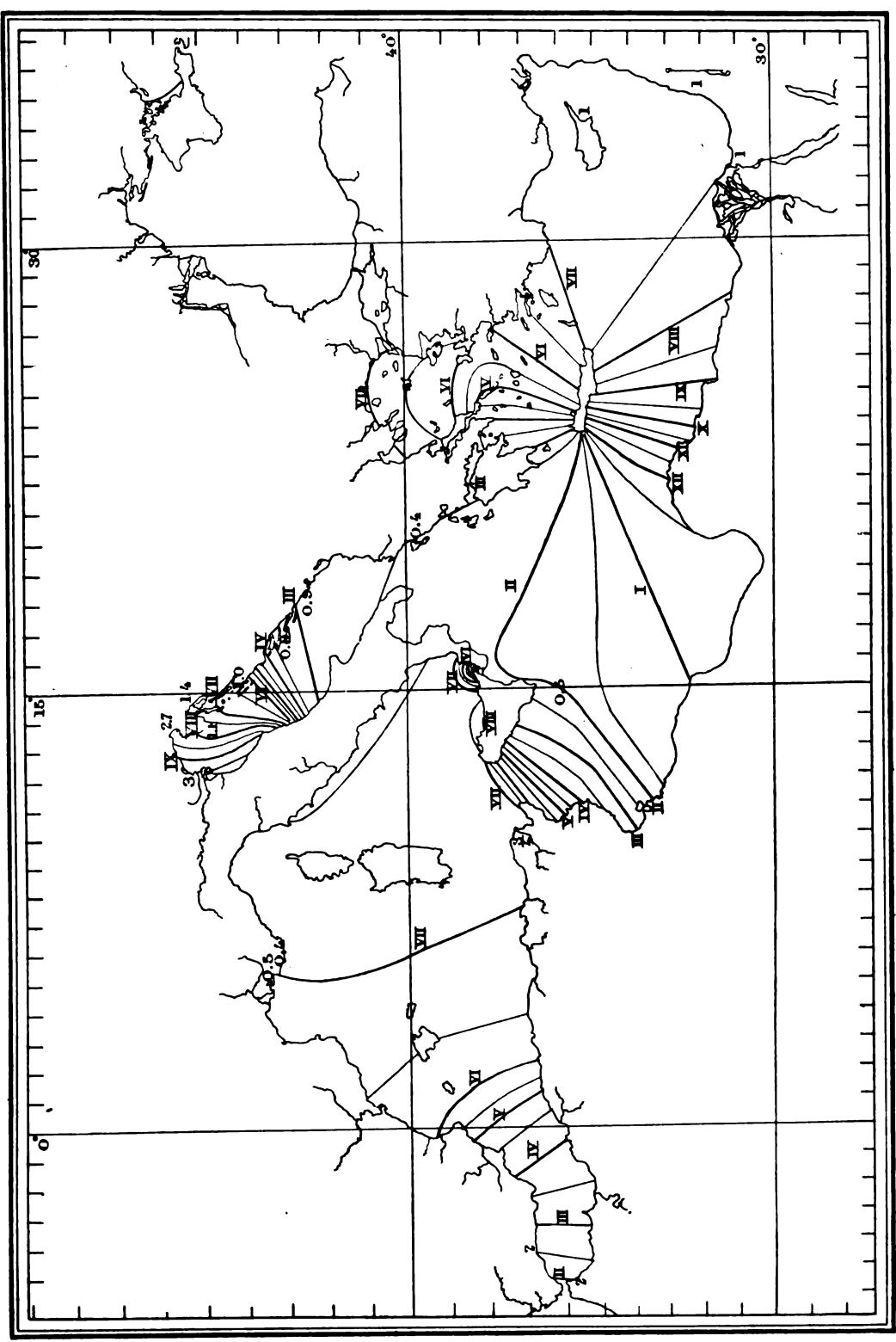


Cotidal lines for the Gulf of Mexico.

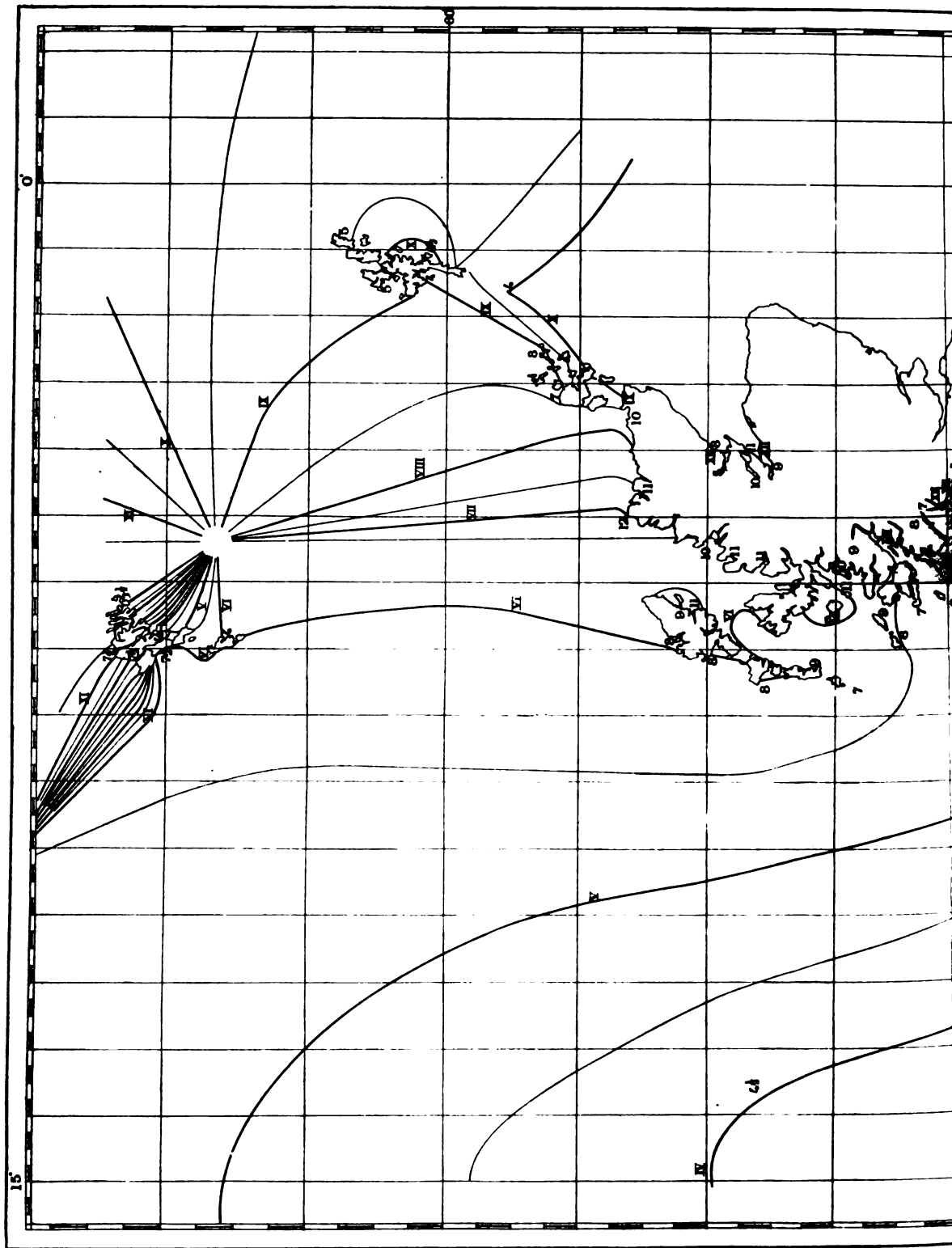


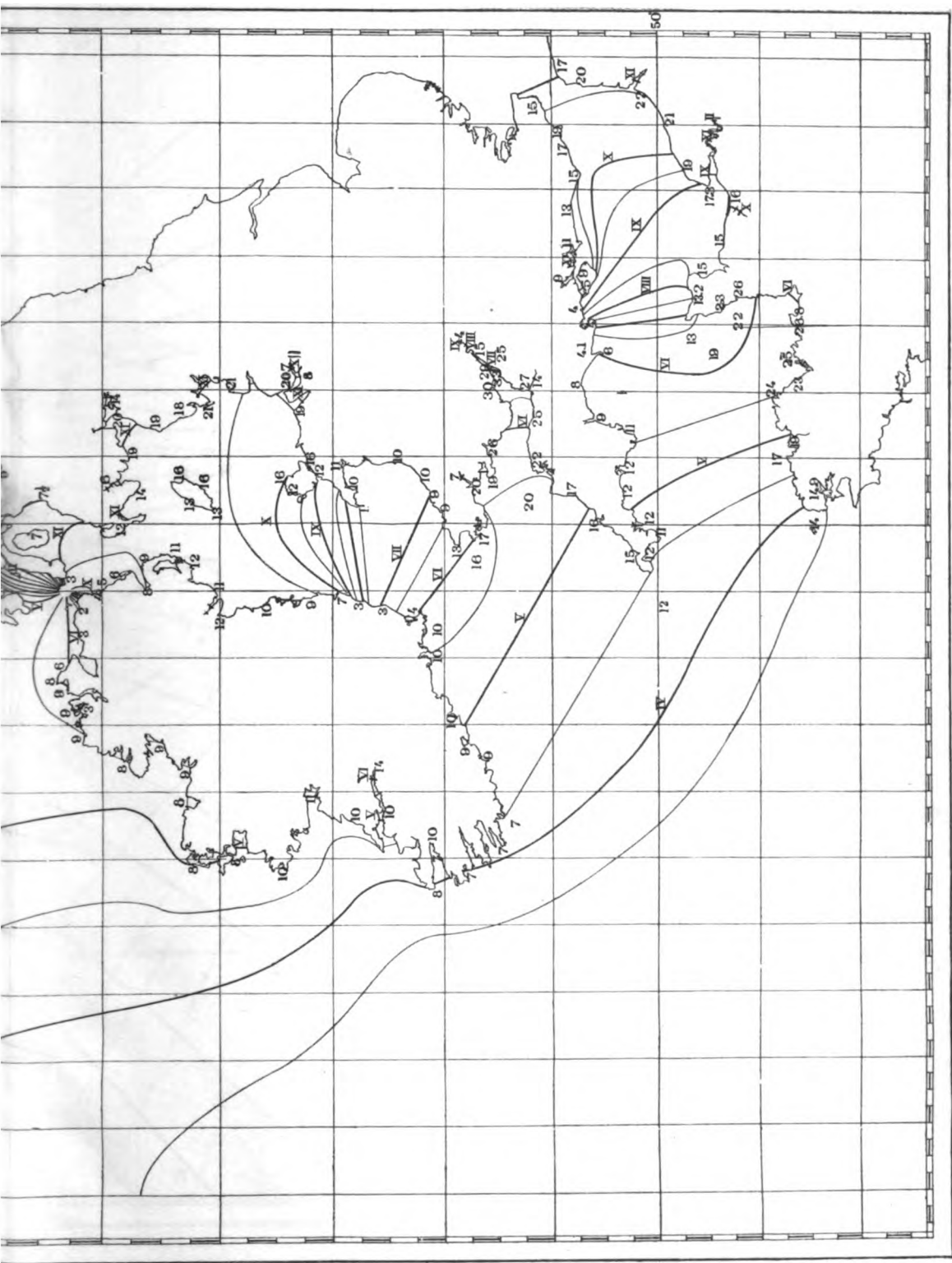
Cotidal lines west of Europe.

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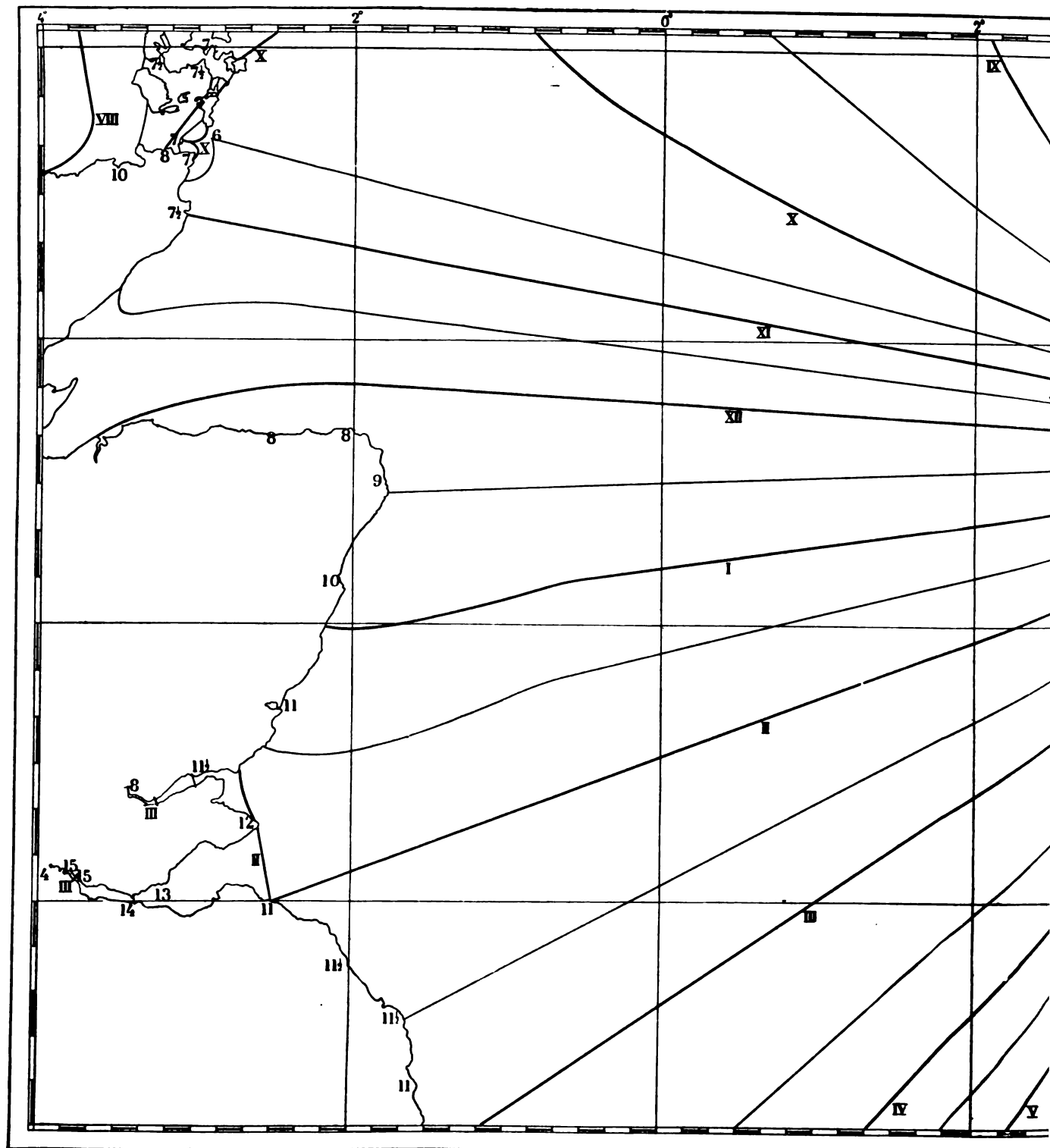


Cotidal lines for the Mediterranean Sea.

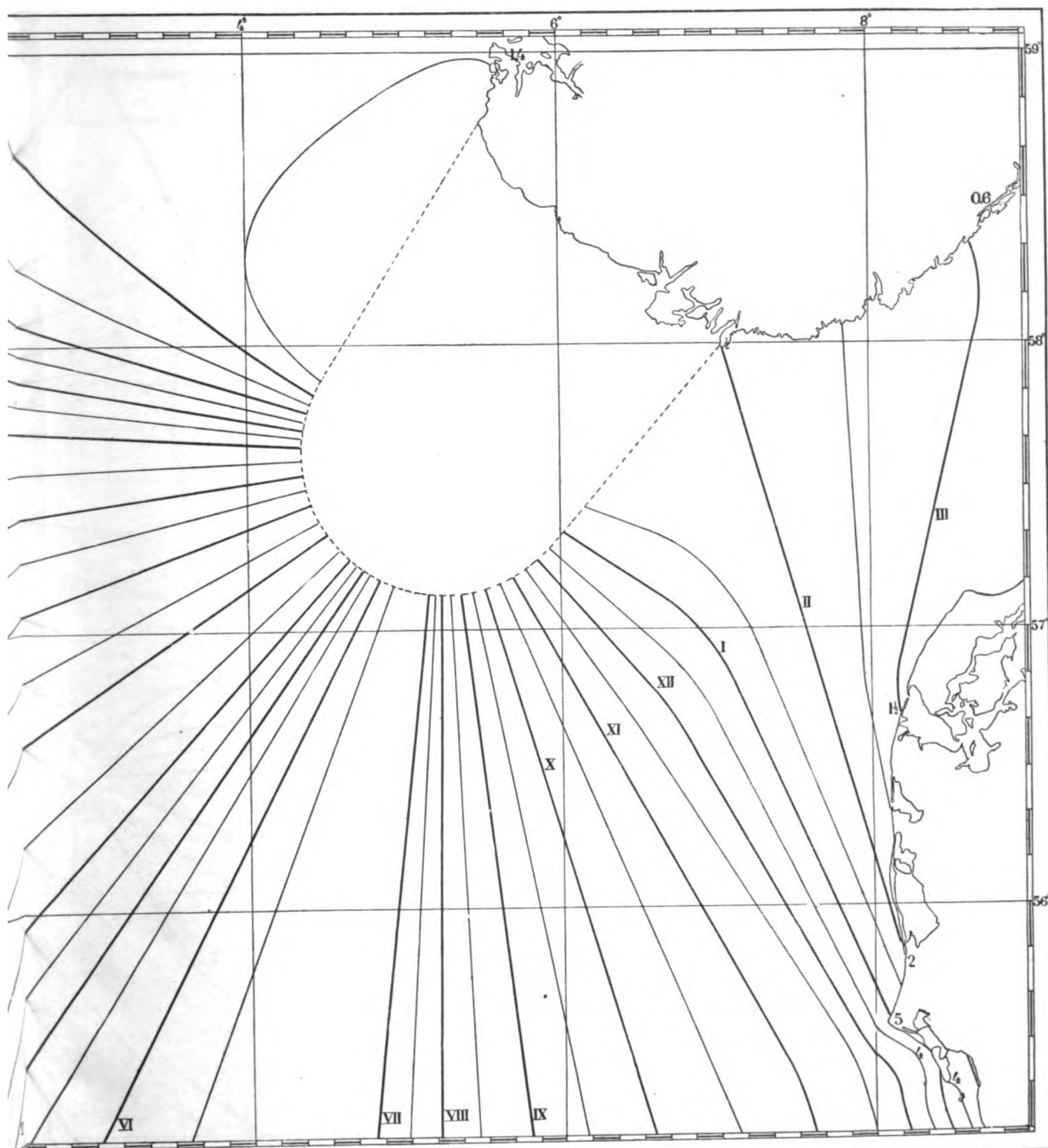




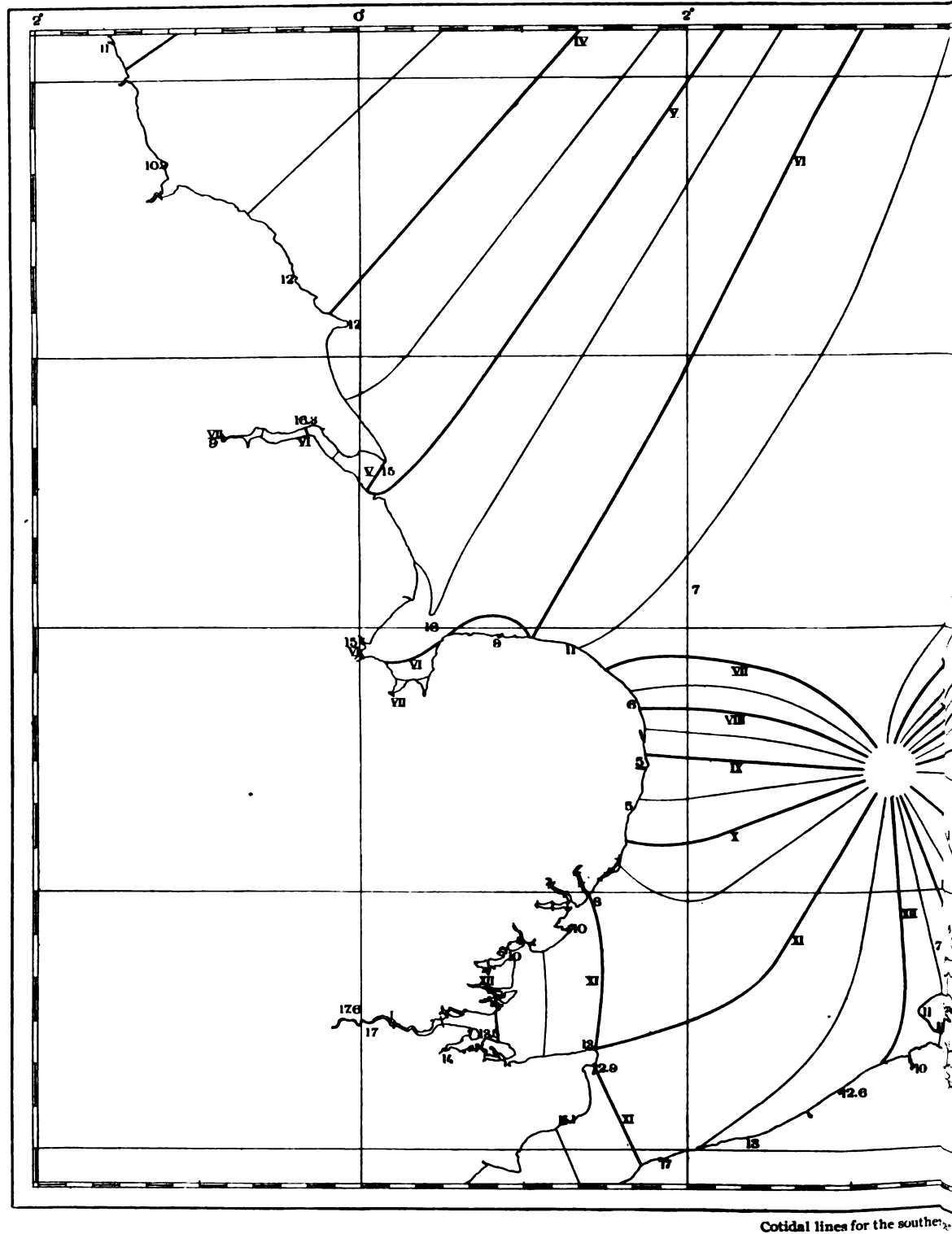
Cotidal lines for the British Islands.



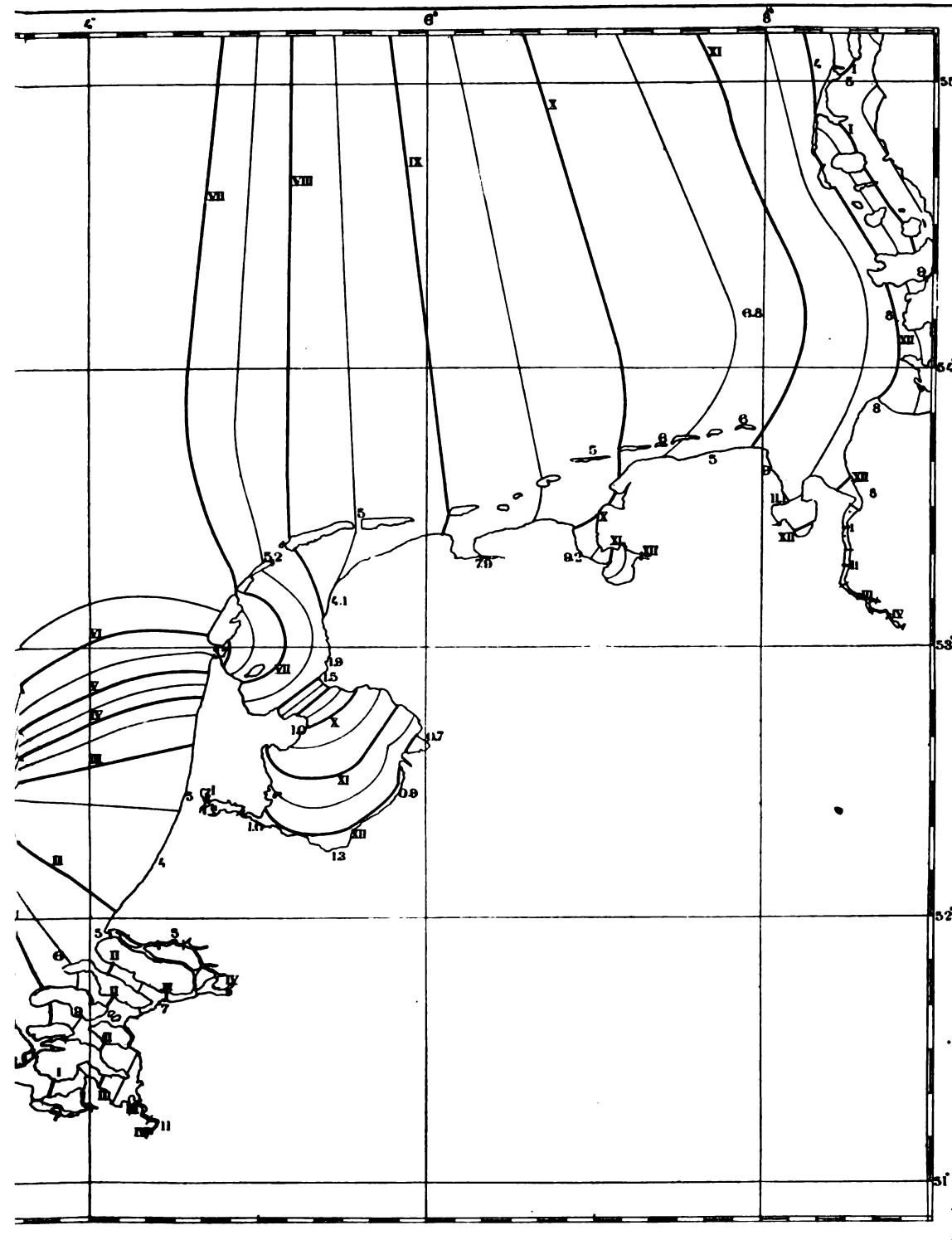
Cotidal lines for the north

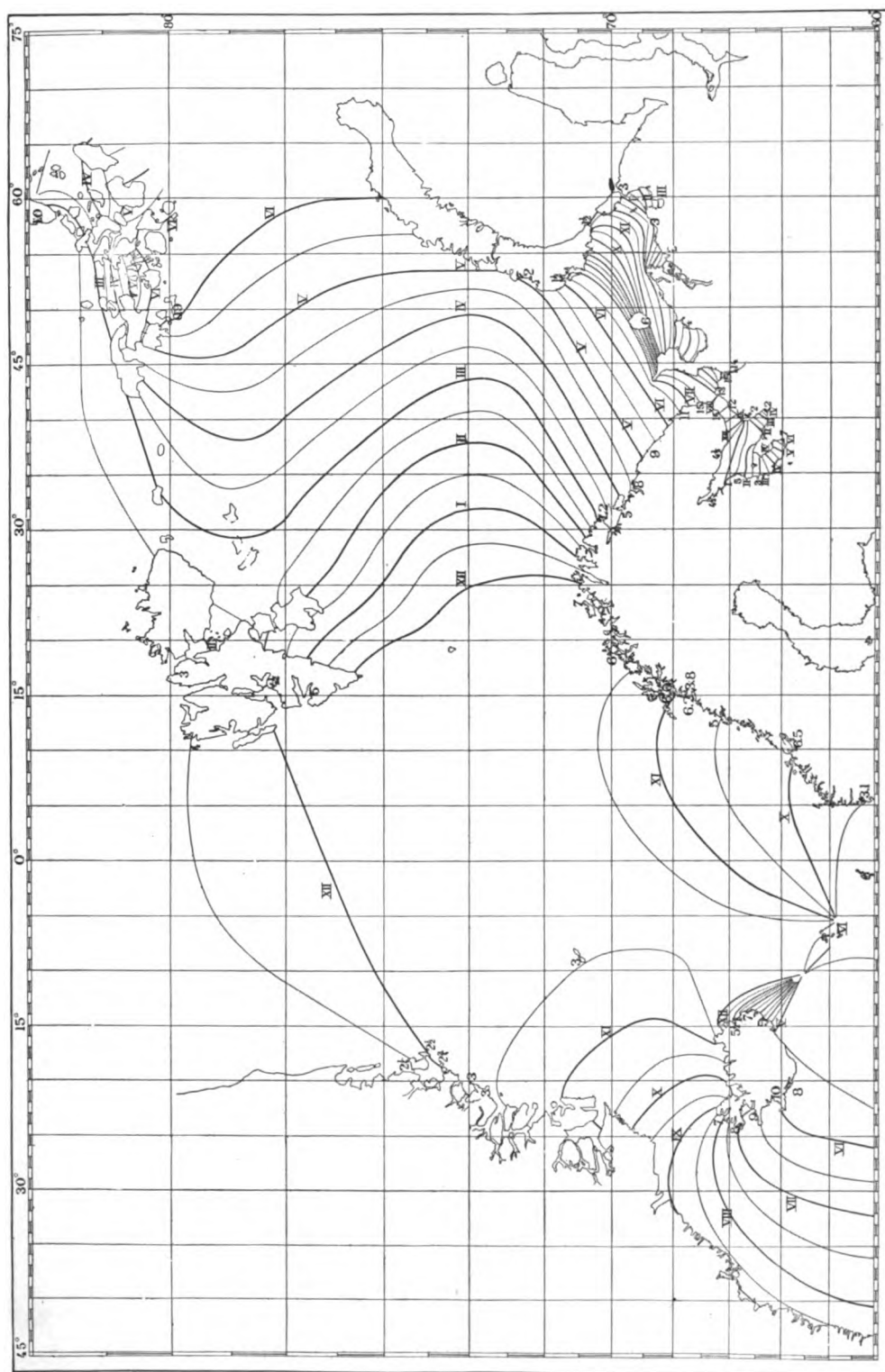


part of the North Sea.

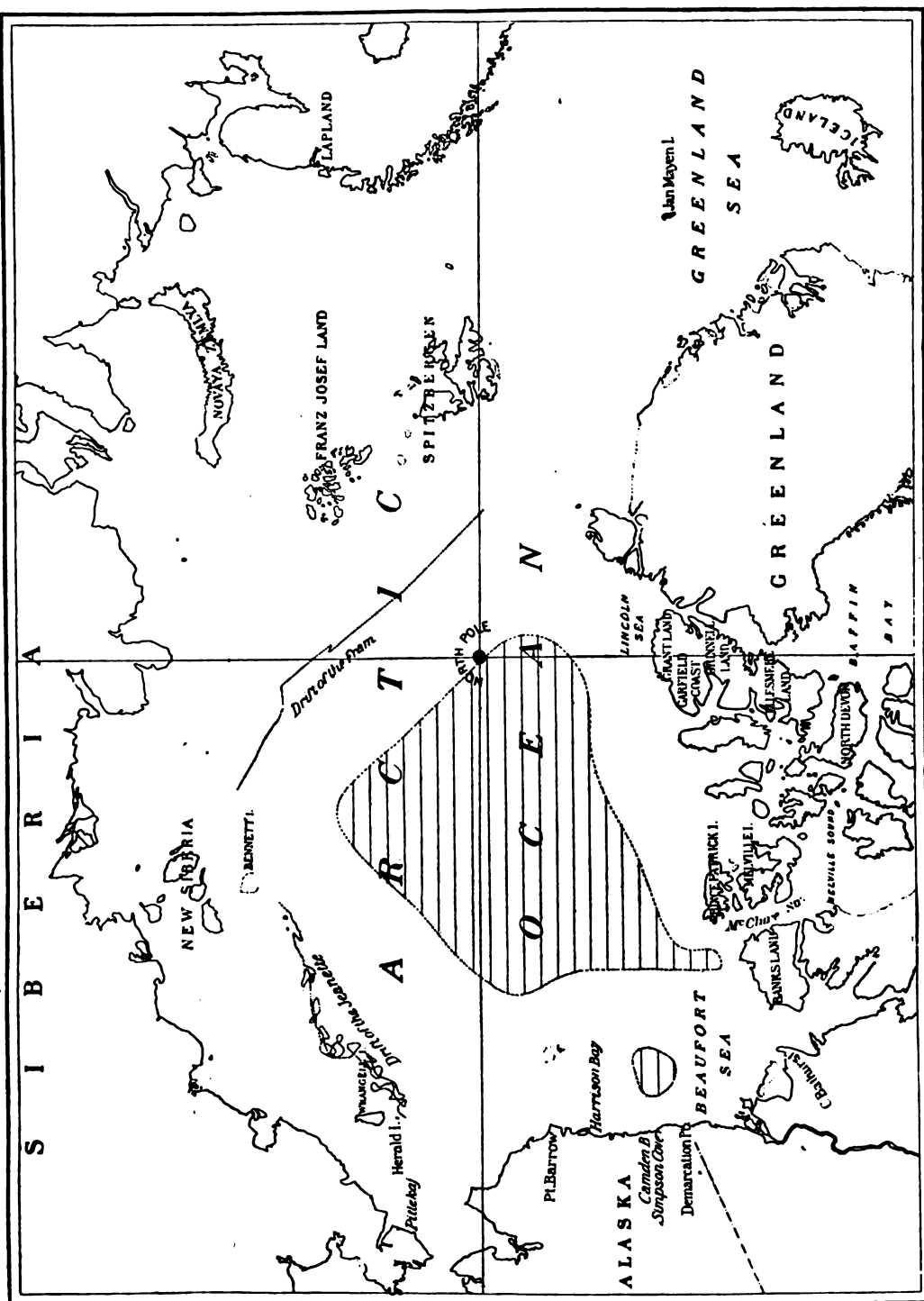


Cotidal lines for the southern

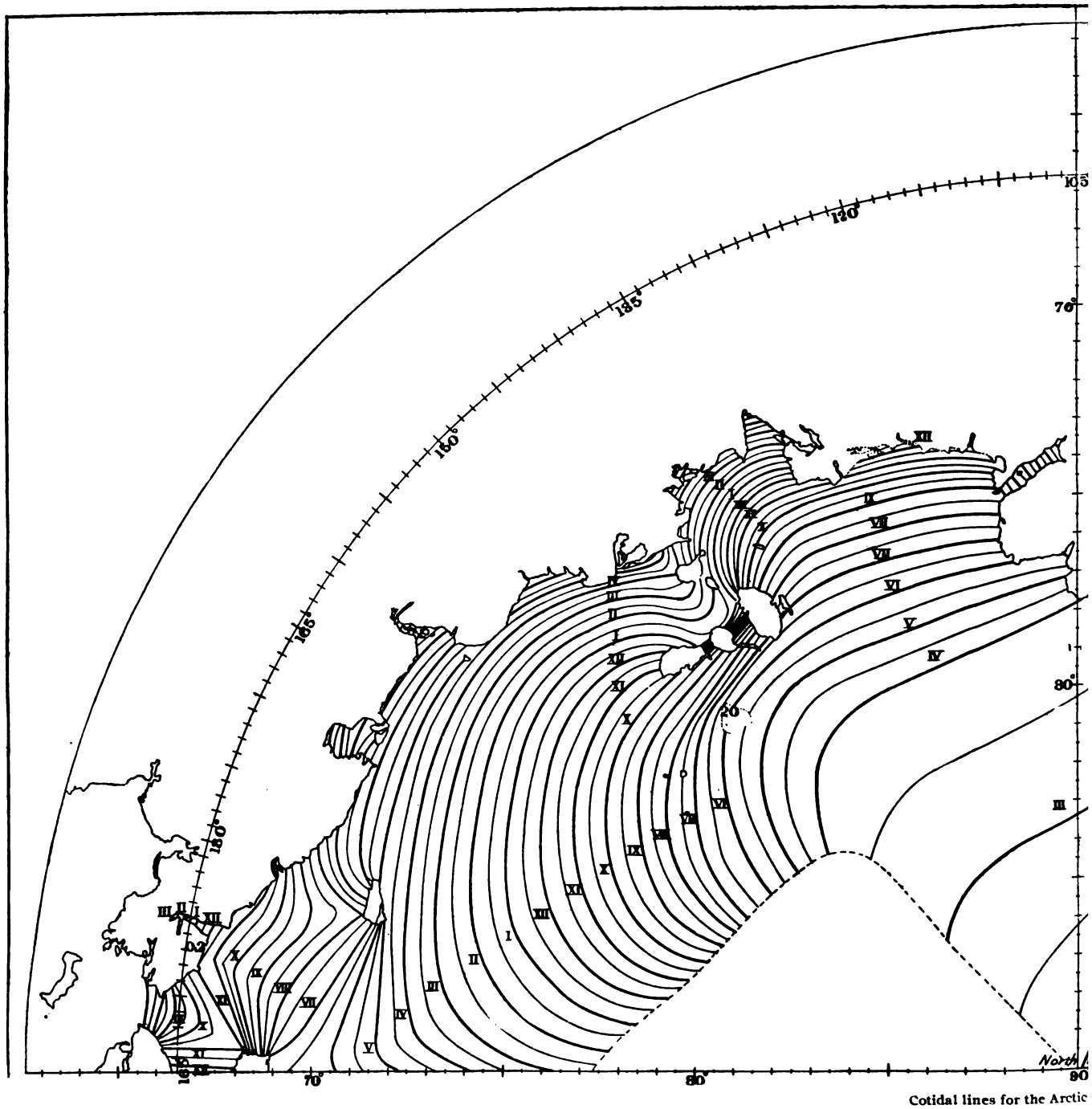


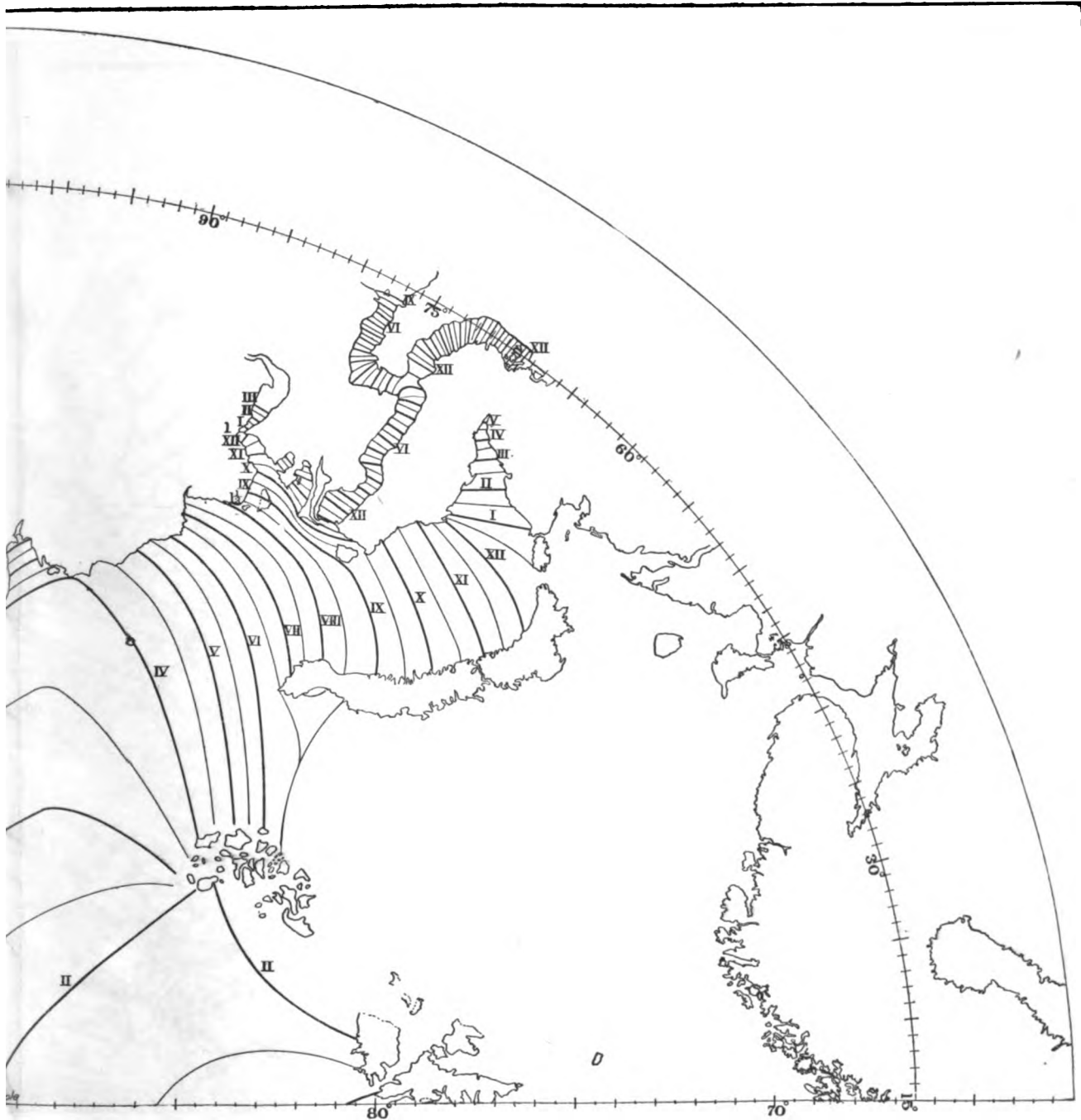


Cotidal lines for Greenland and Barents seas.

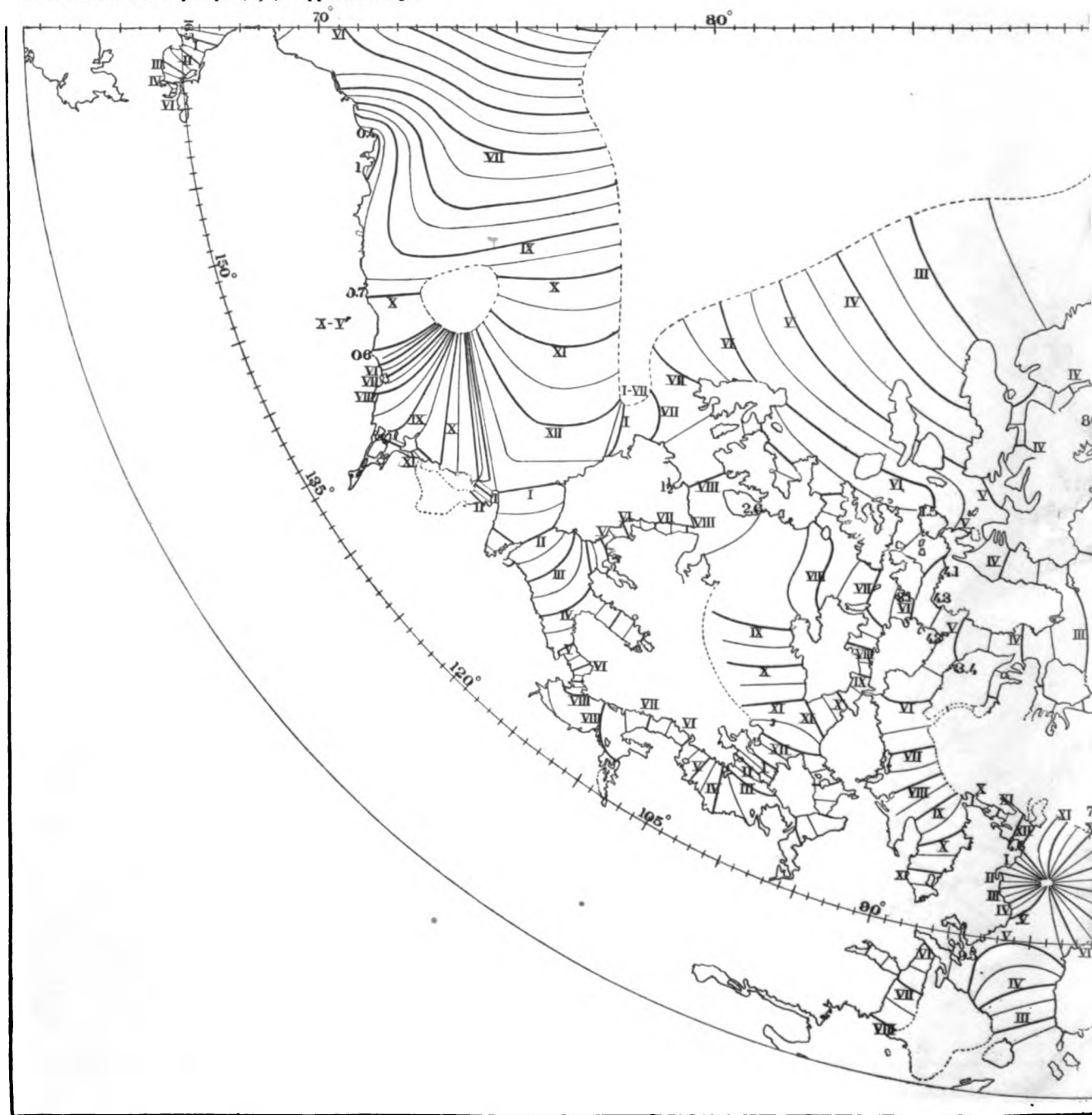


Sketch of Polar Regions.

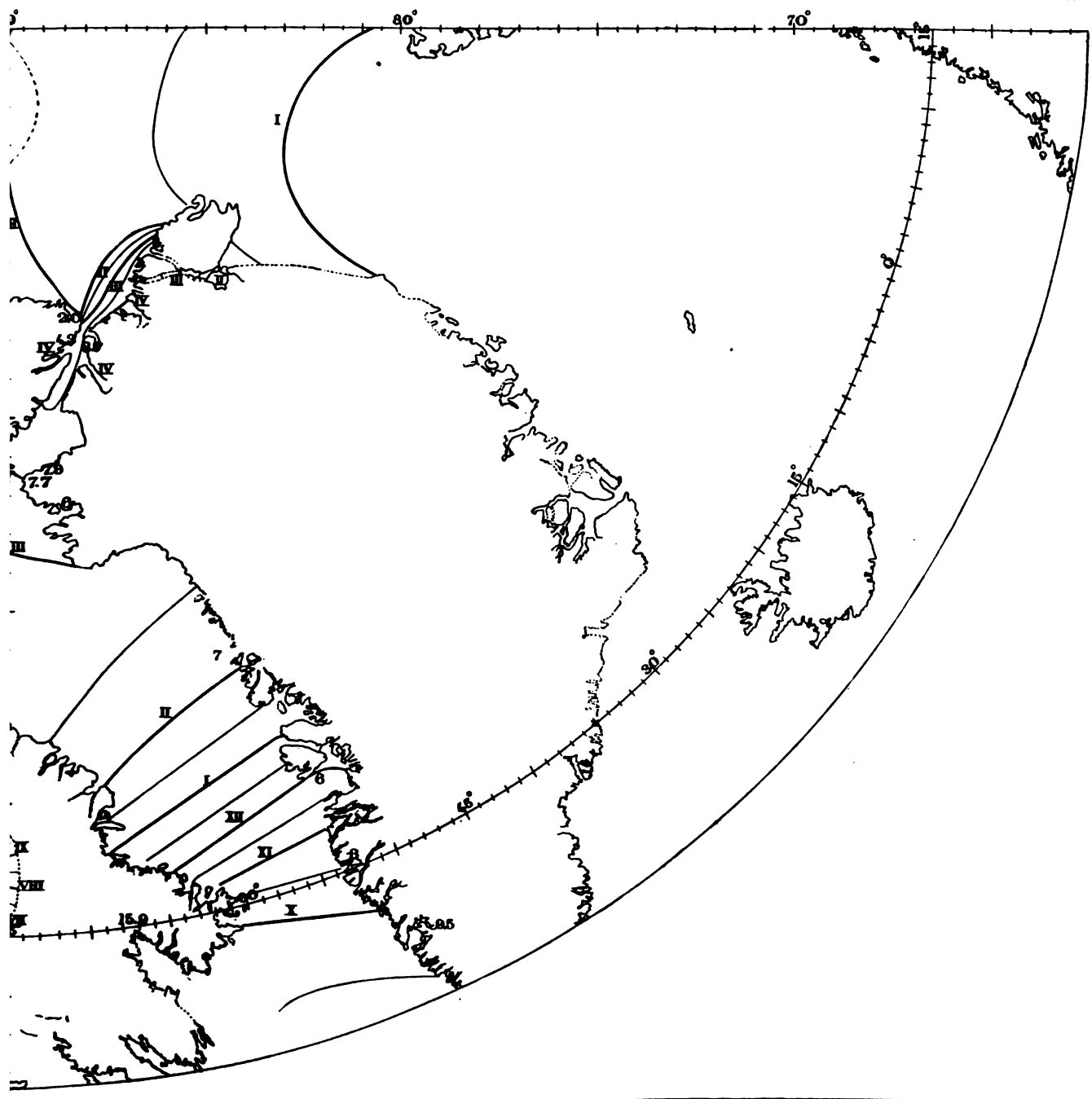




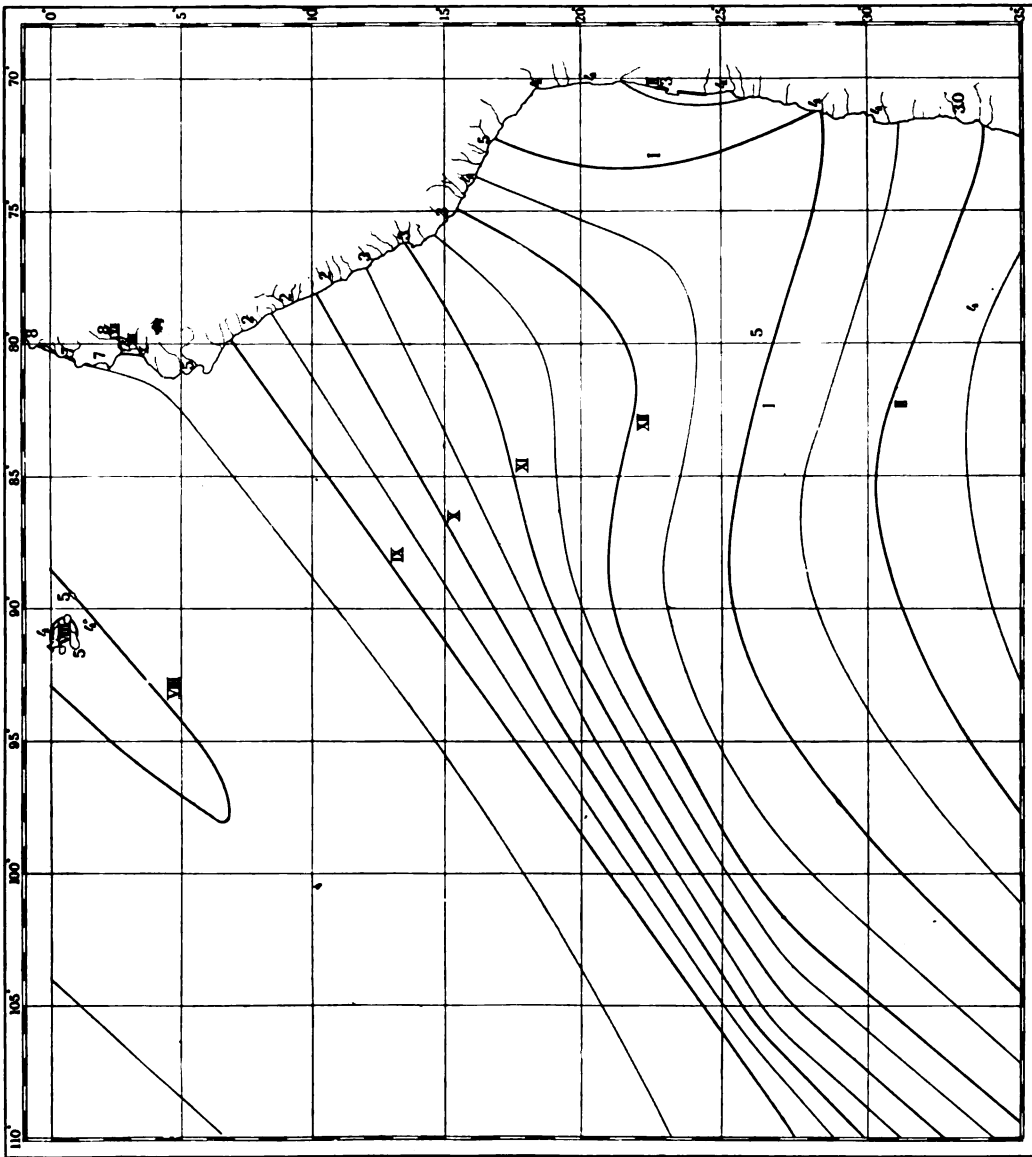
regions. Siberian side.



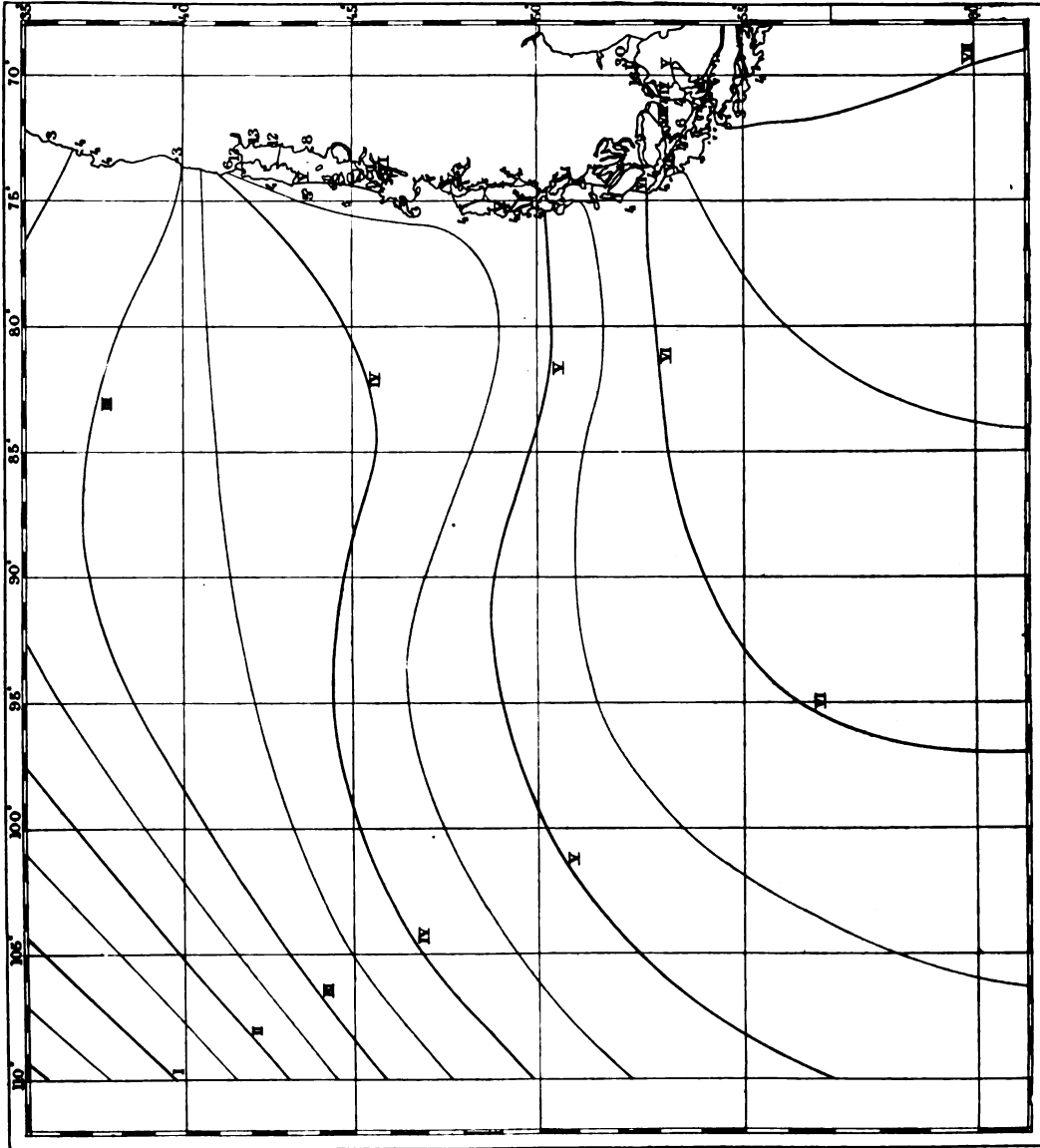
Tidal lines for the Arctic Ocean



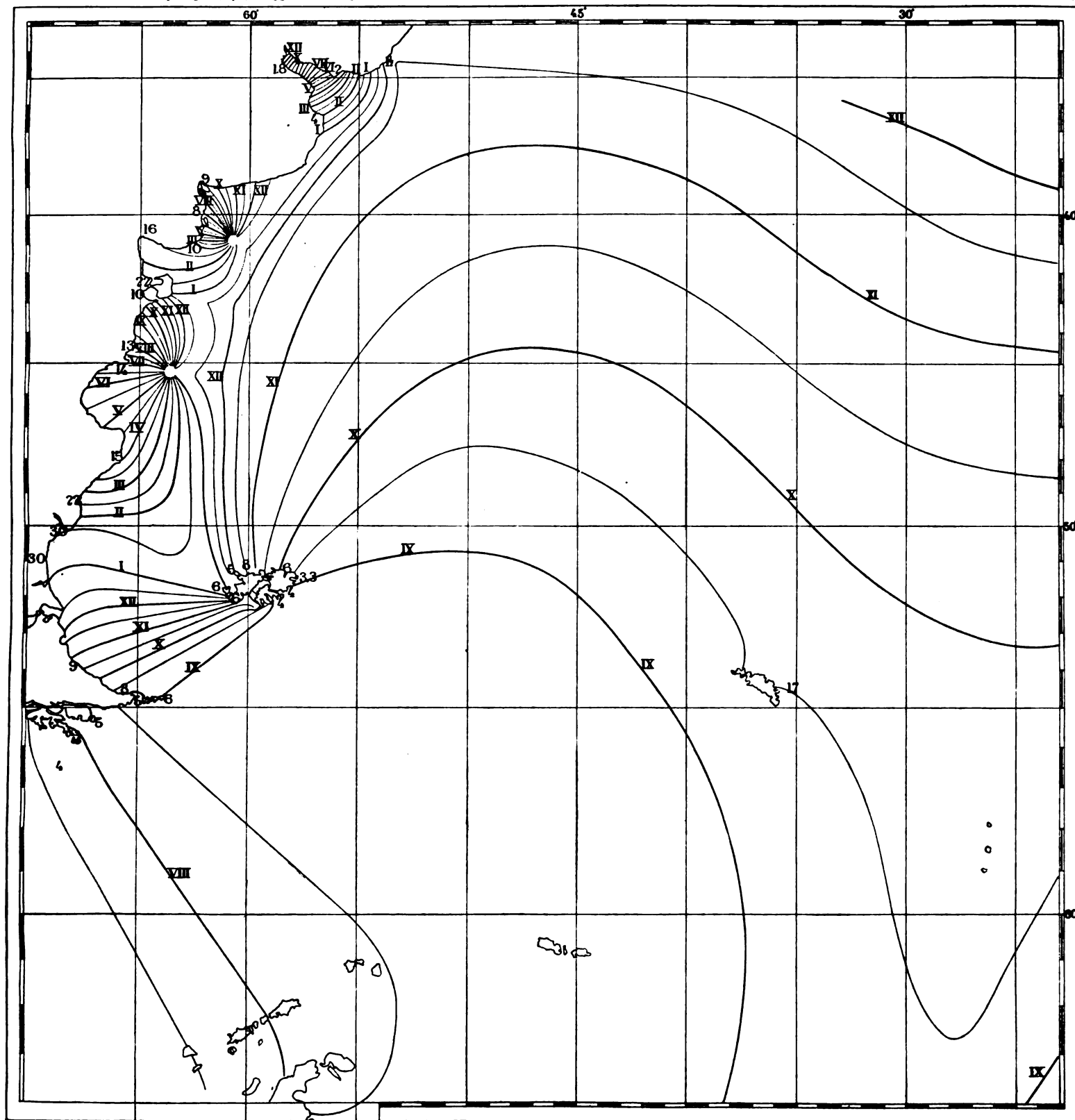
: Regional. American side.



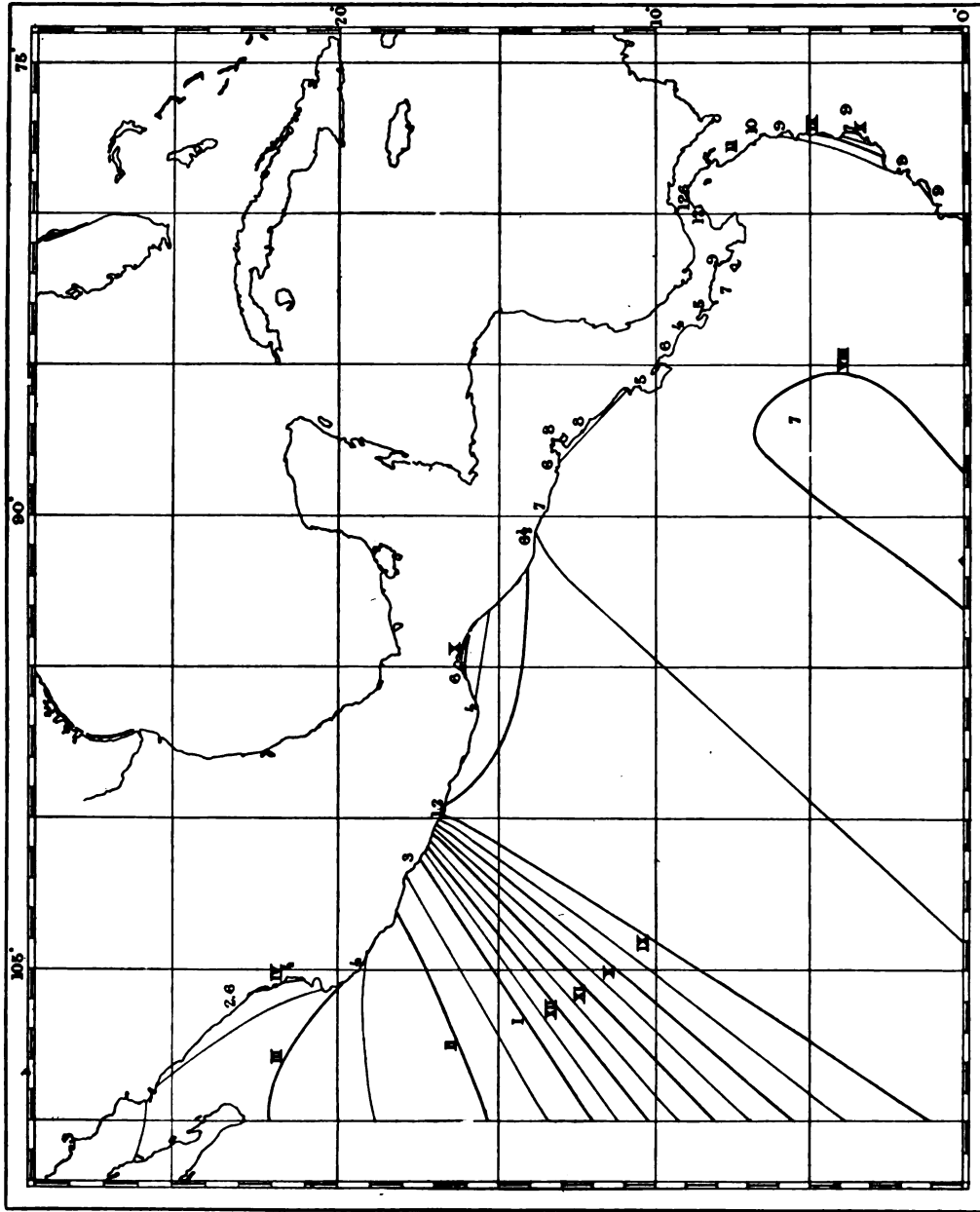
Cotidal lines west of Peru.



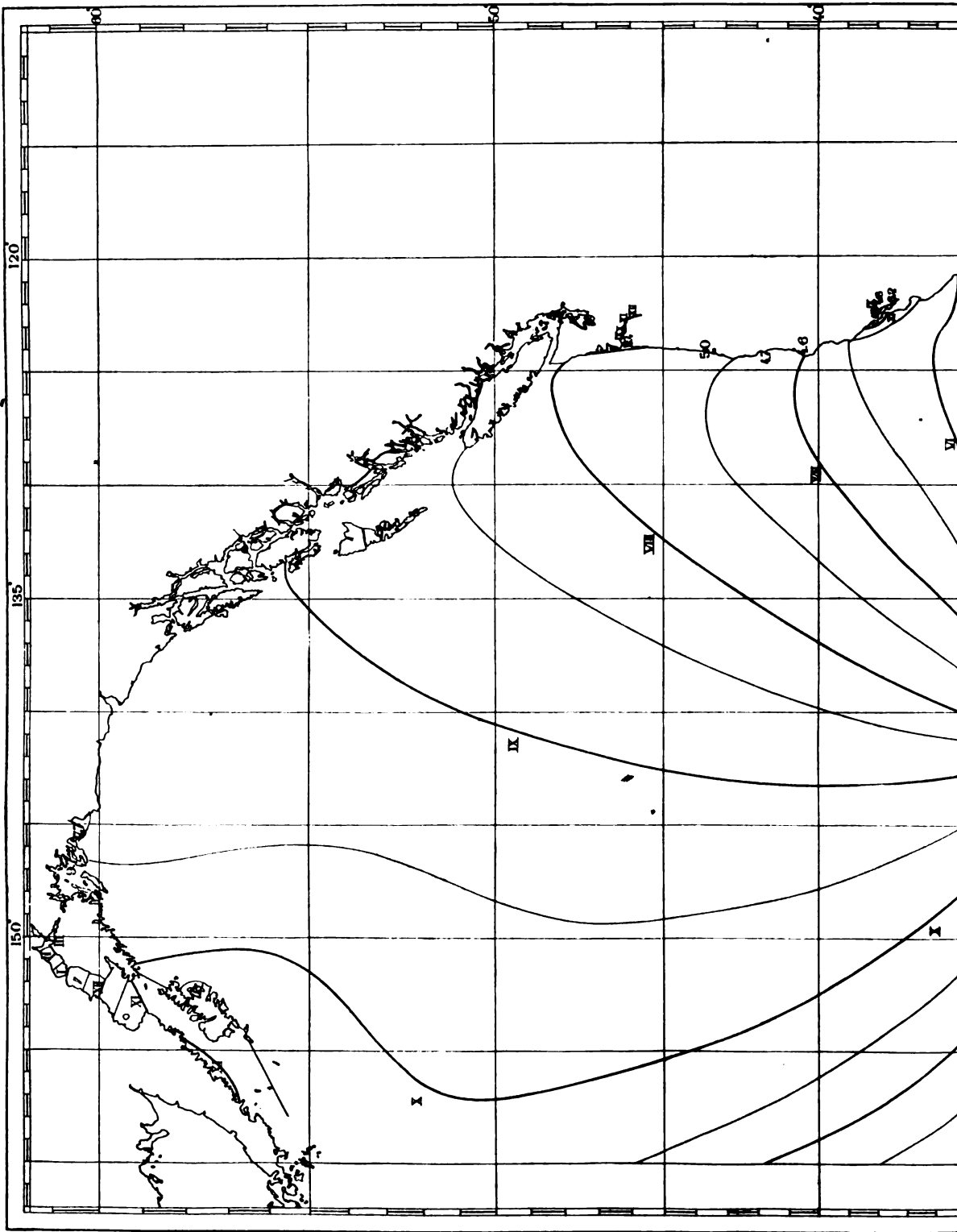
Cotidal lines west of Chile.

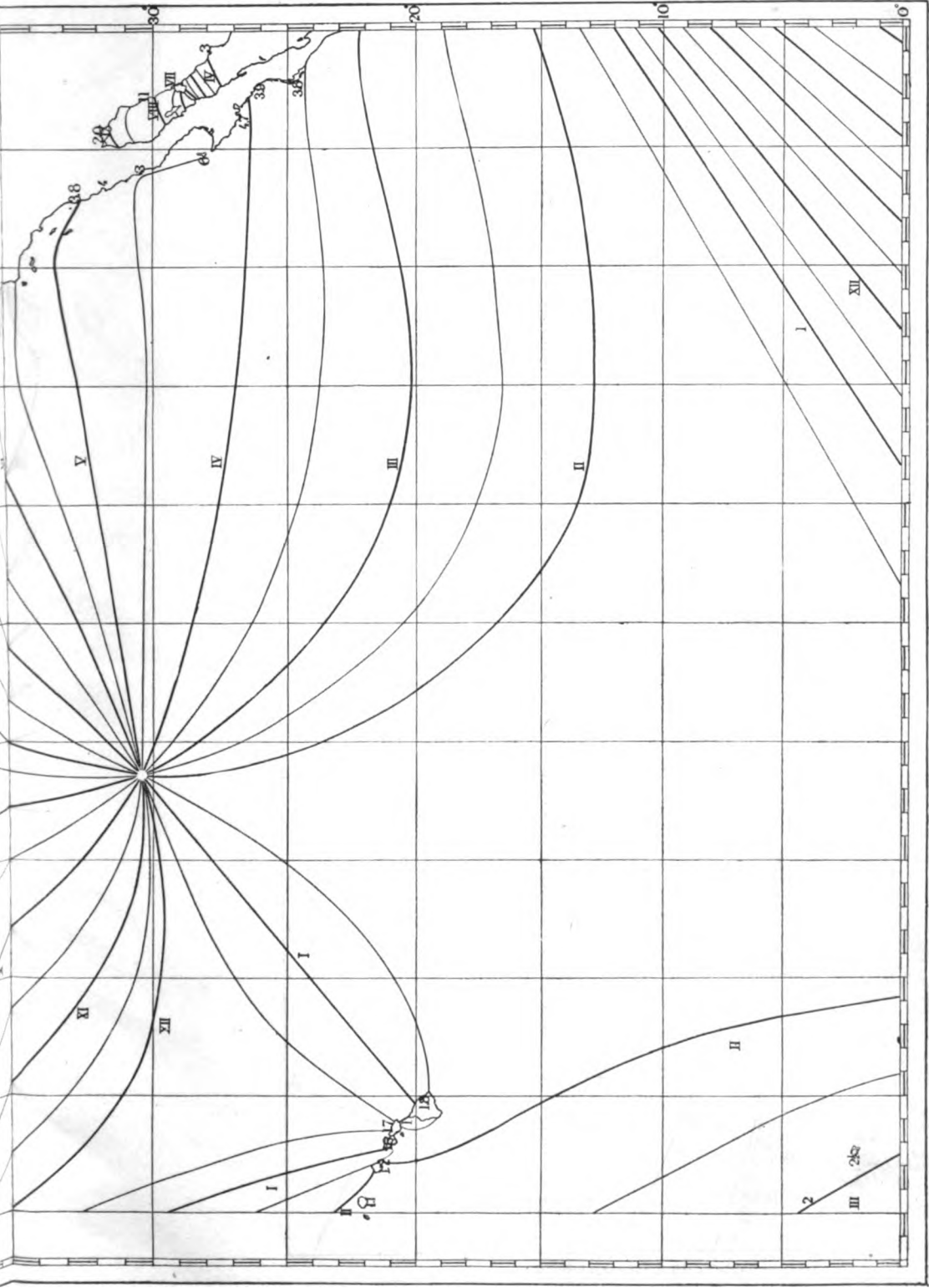


Cotidal lines east of Patagonia.

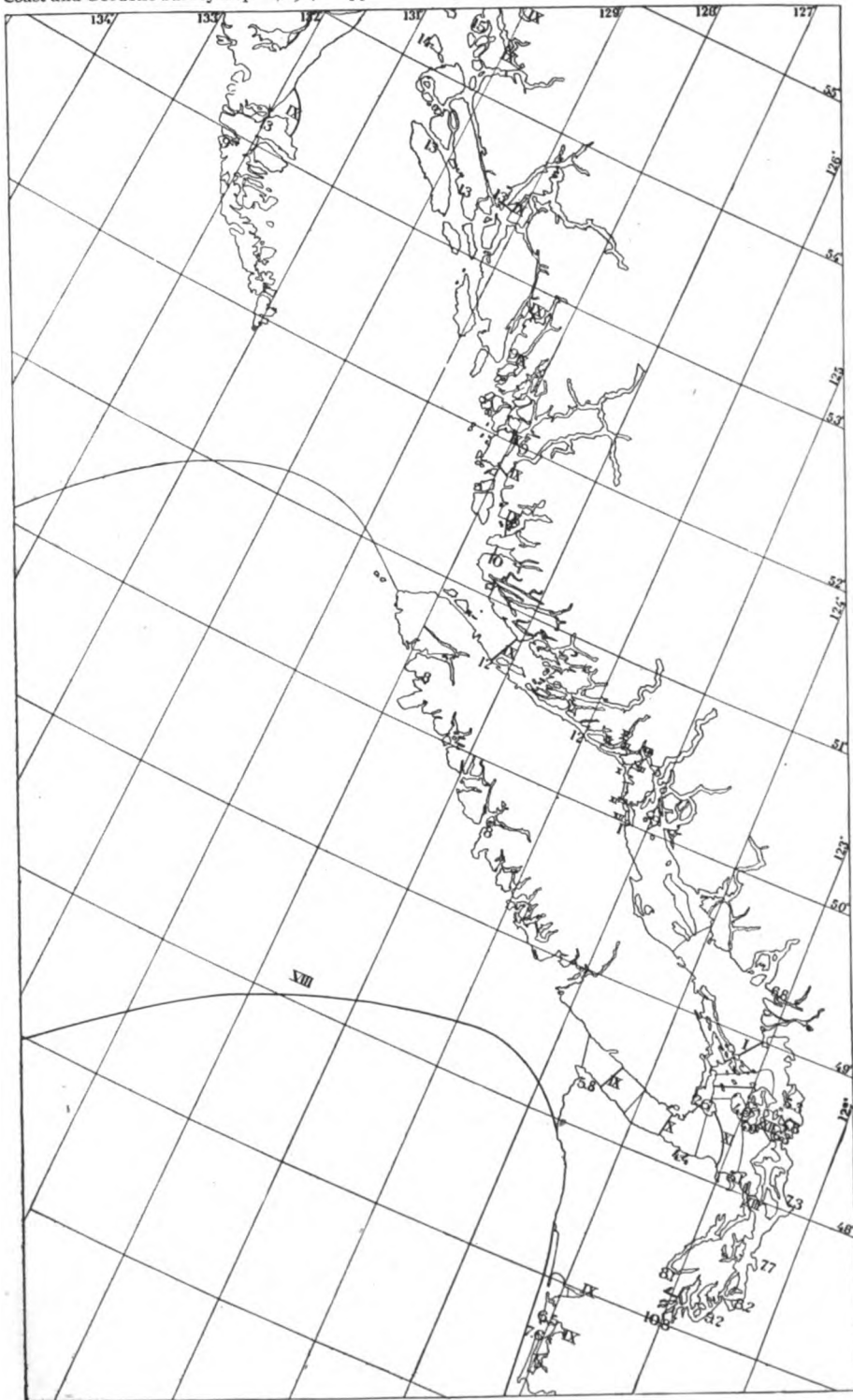


Cotidal lines south of Mexico.

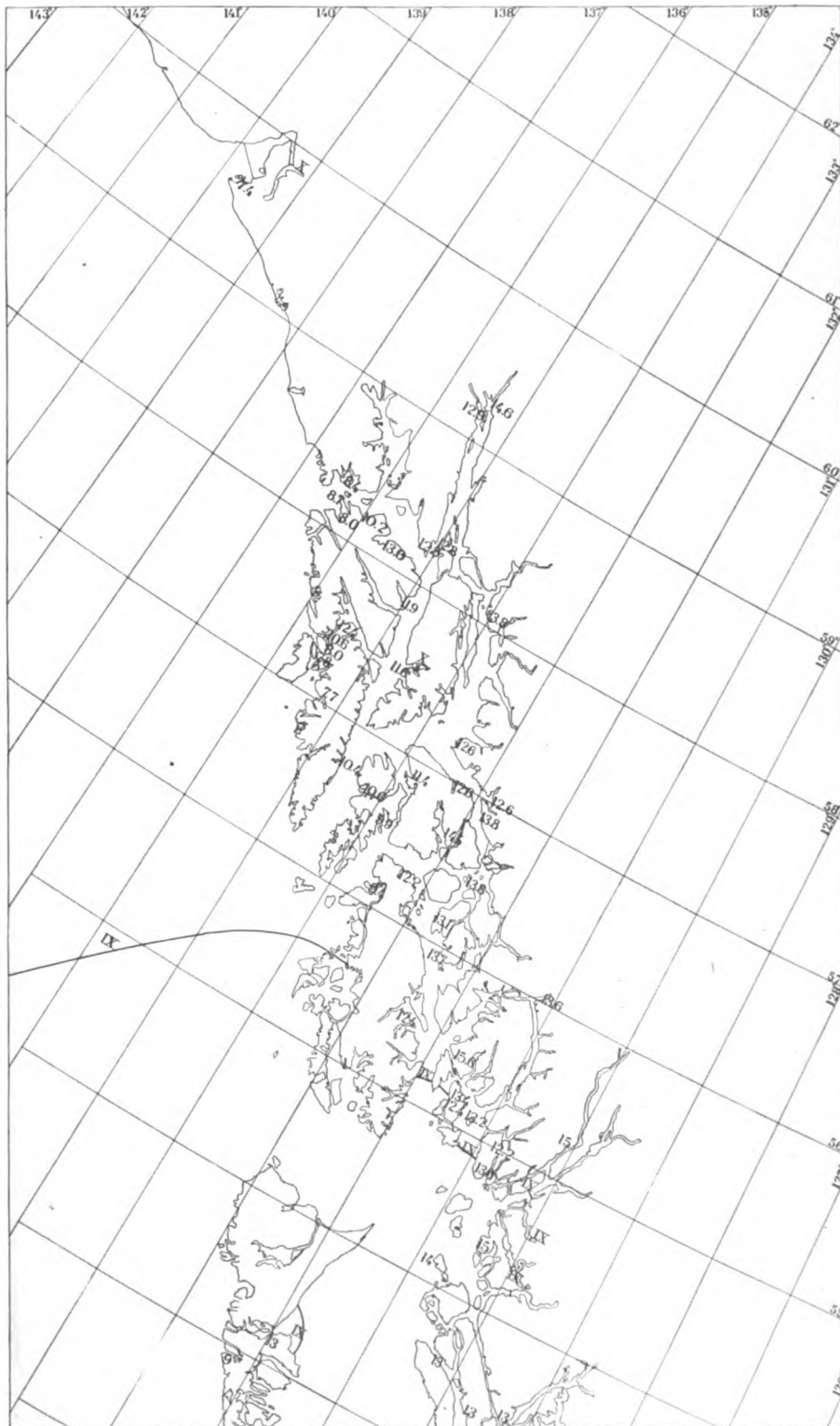




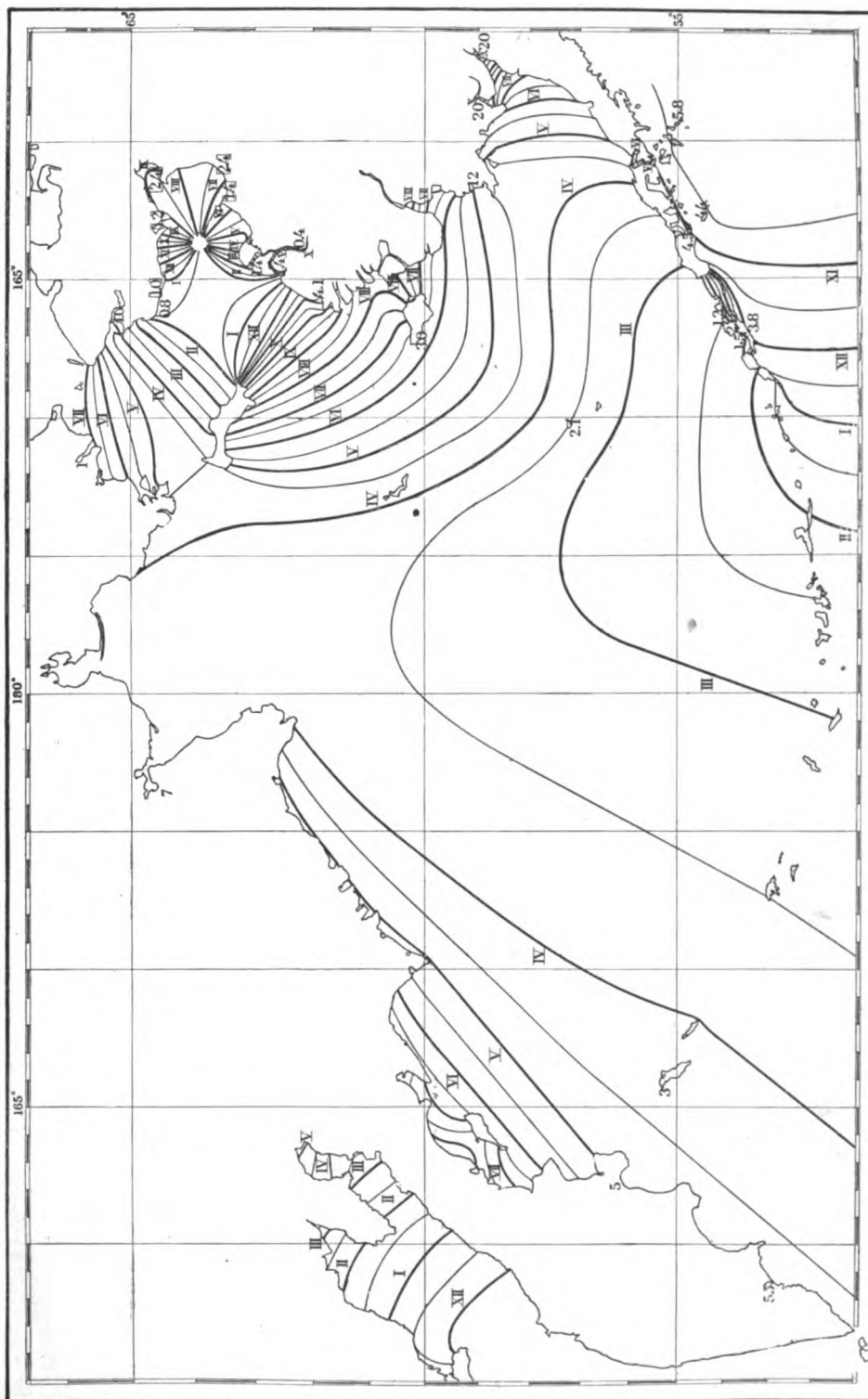
Cotidal lines west of the United States.



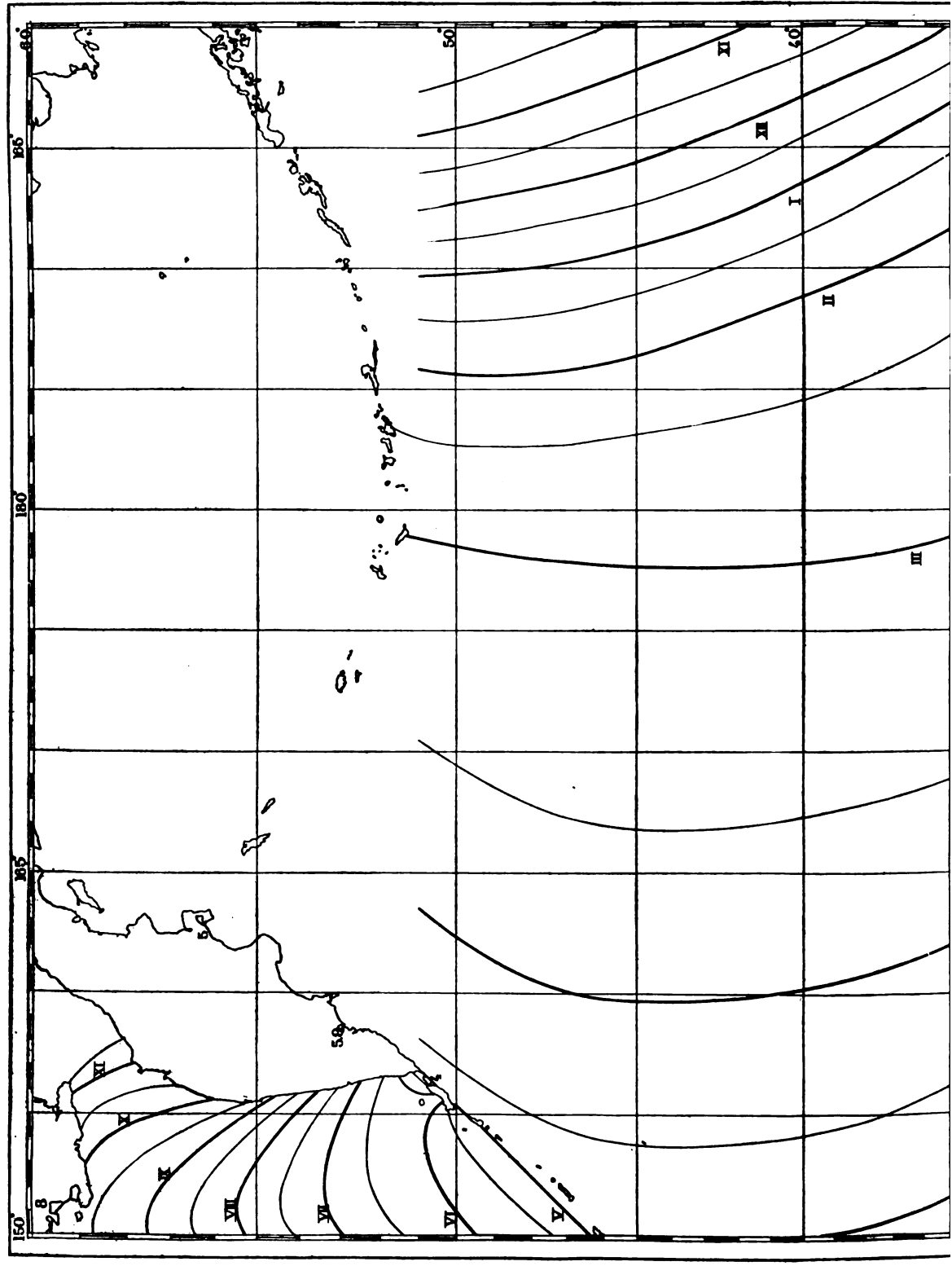
Cotidal lines for British Columbia.

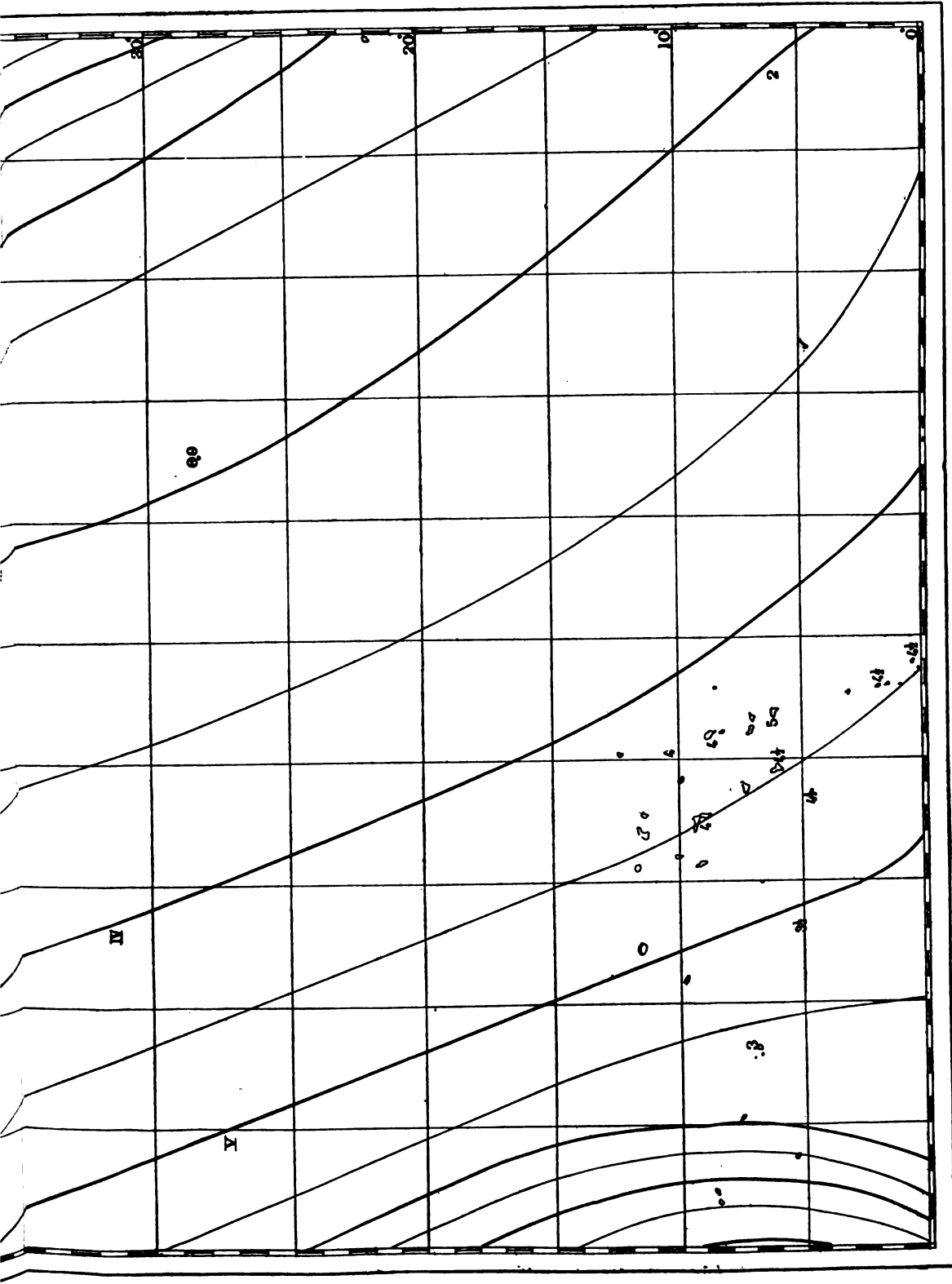


Cotidal lines for Southeastern Alaska.

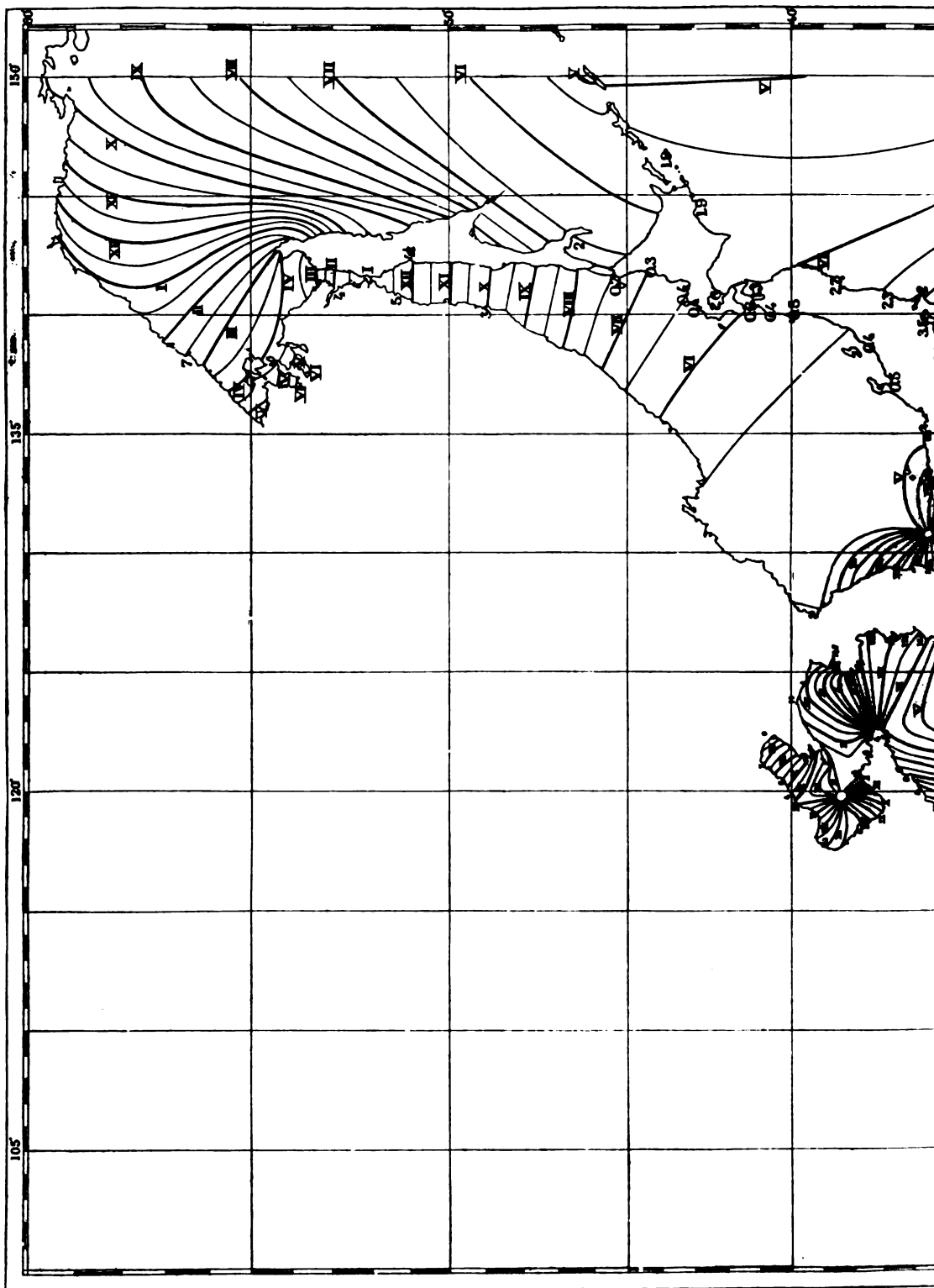


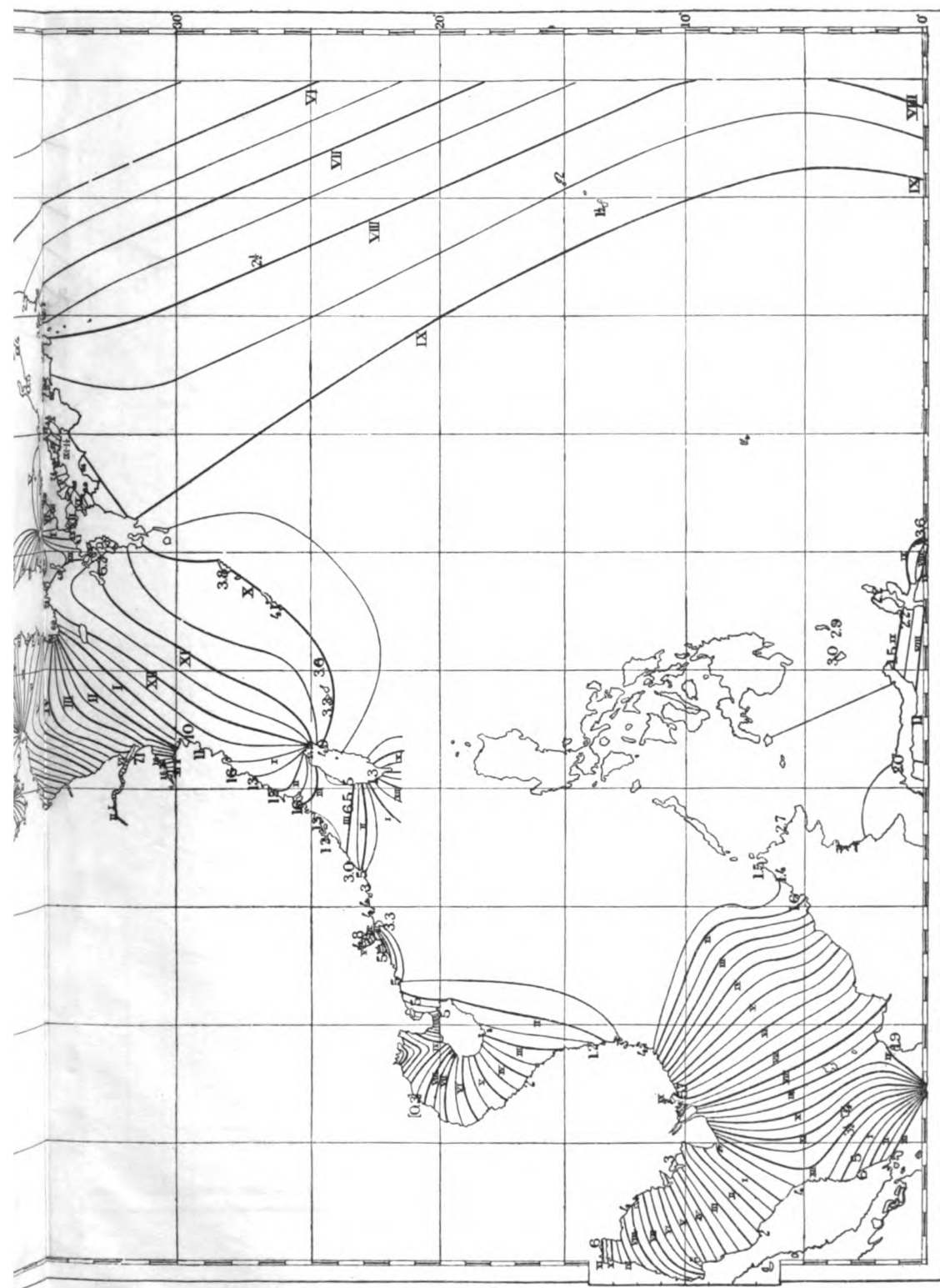
Cotidal lines for Bering Sea.

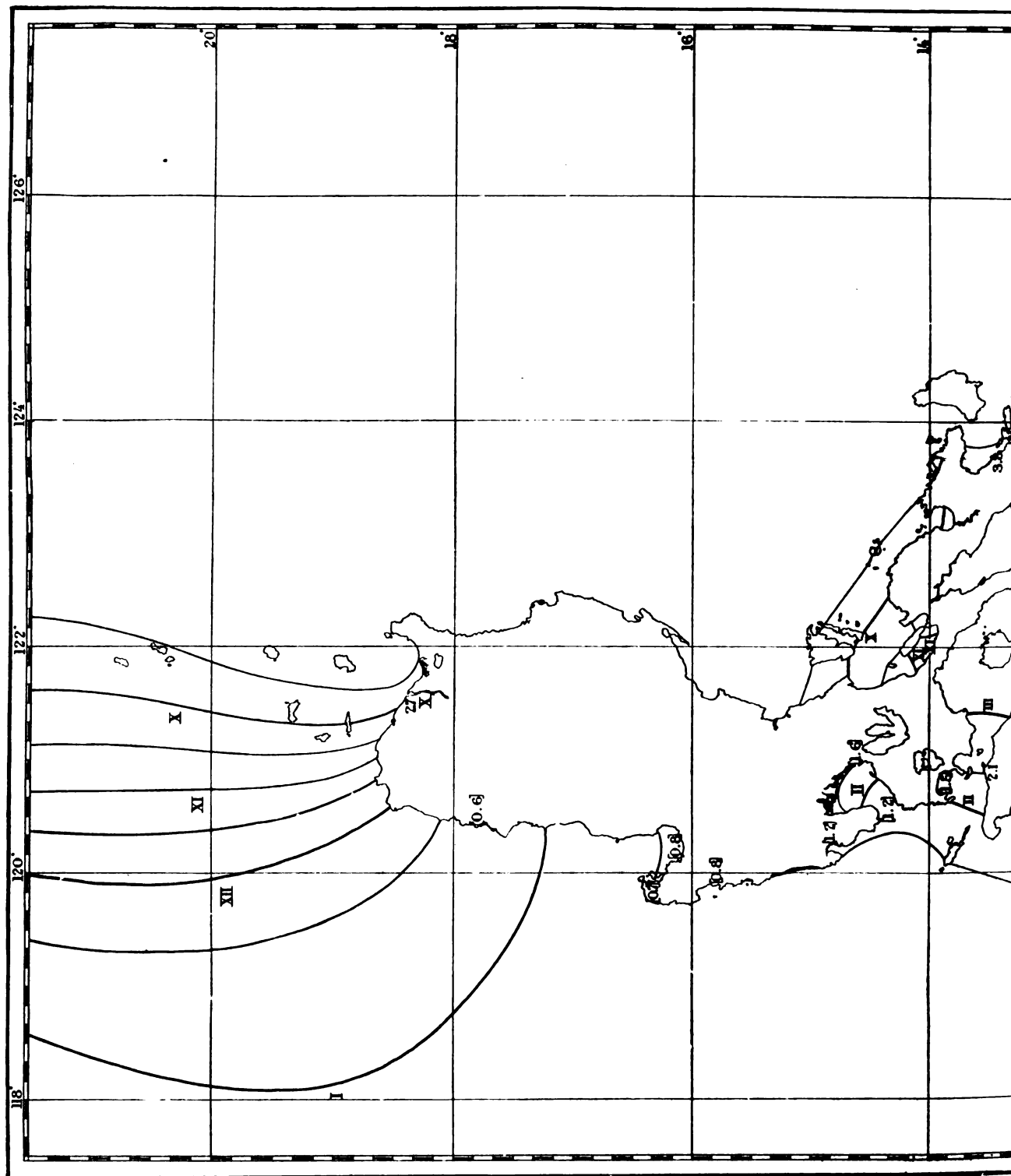


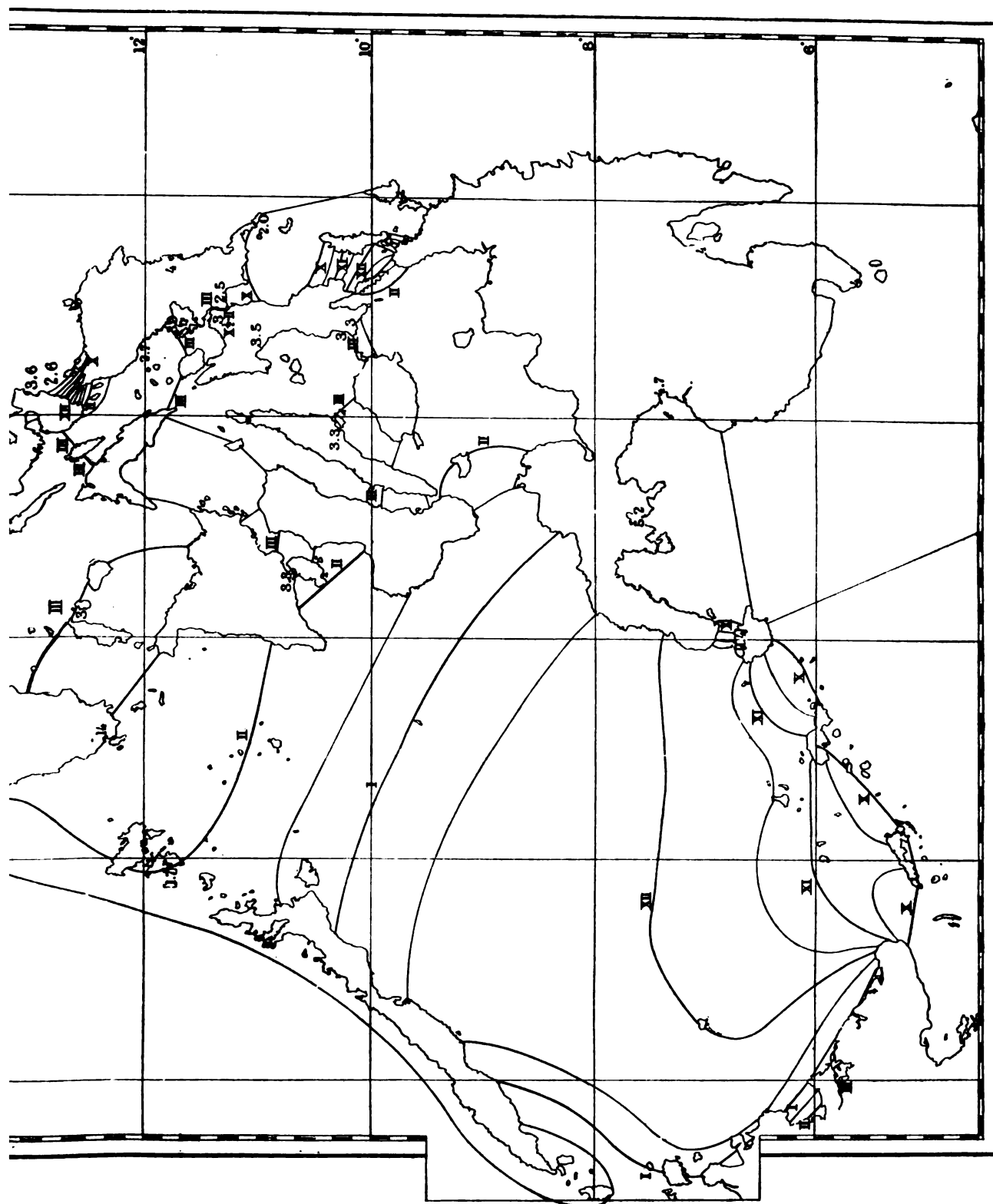


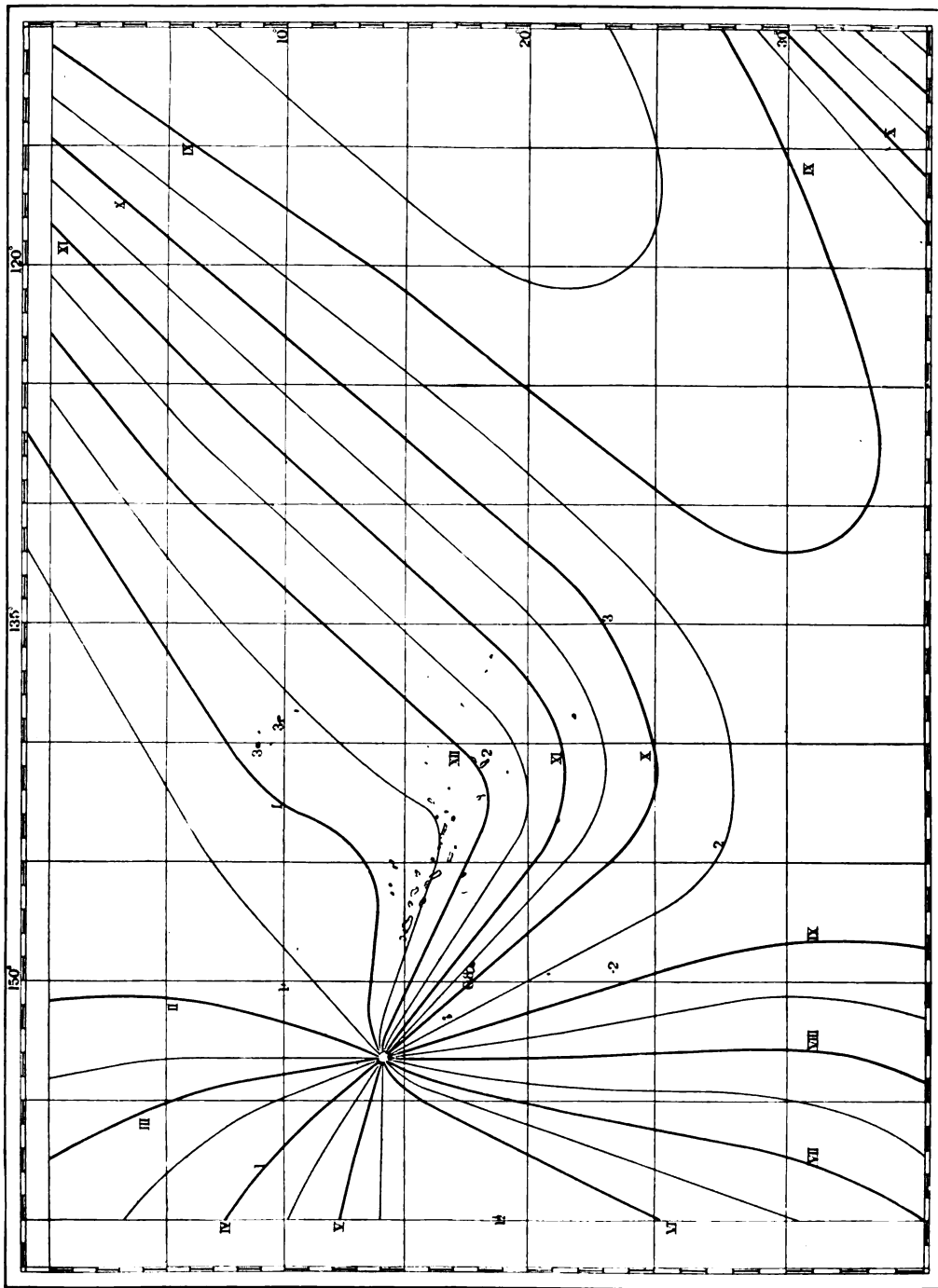
Cotidal lines west of Hawaii.



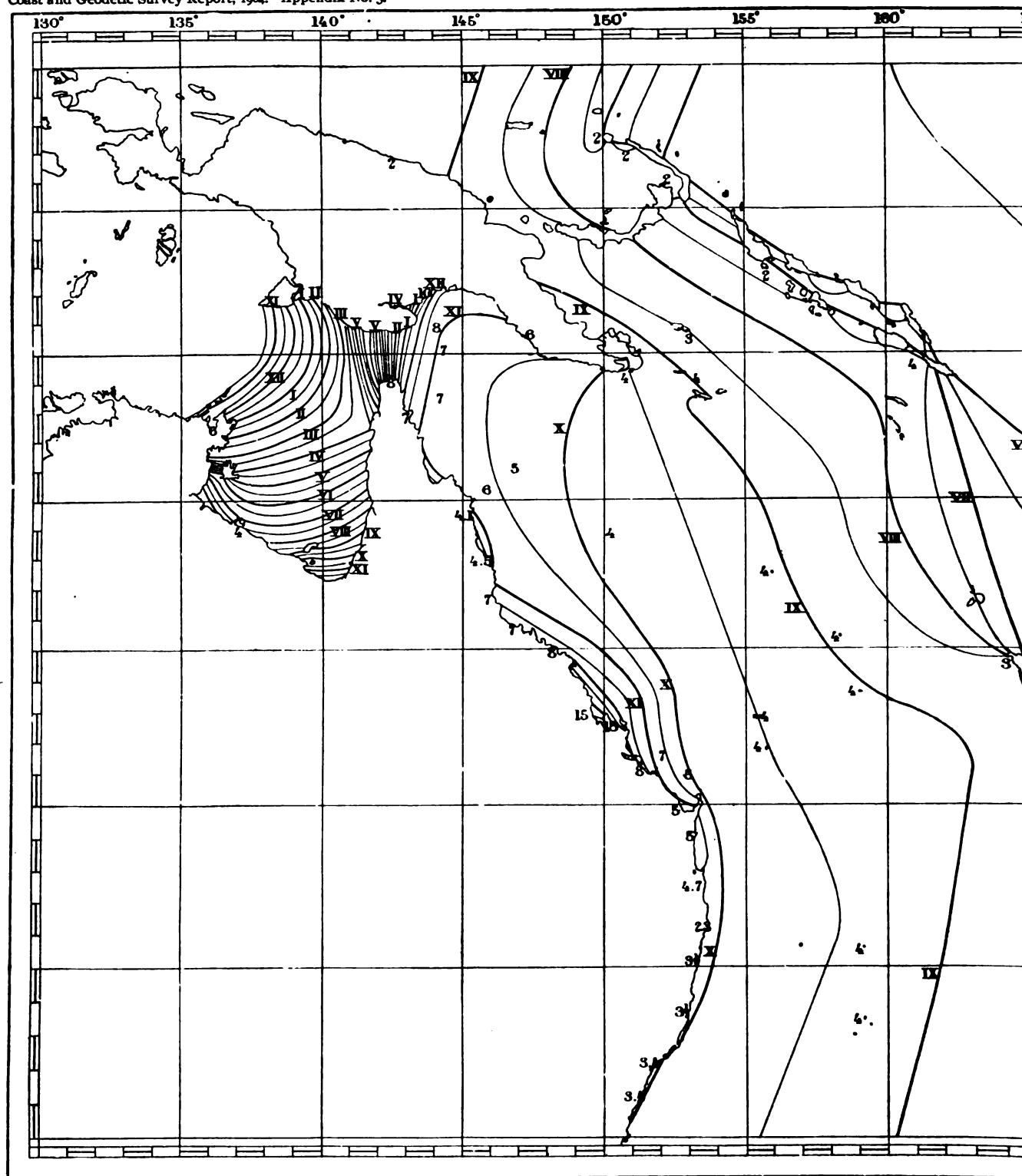




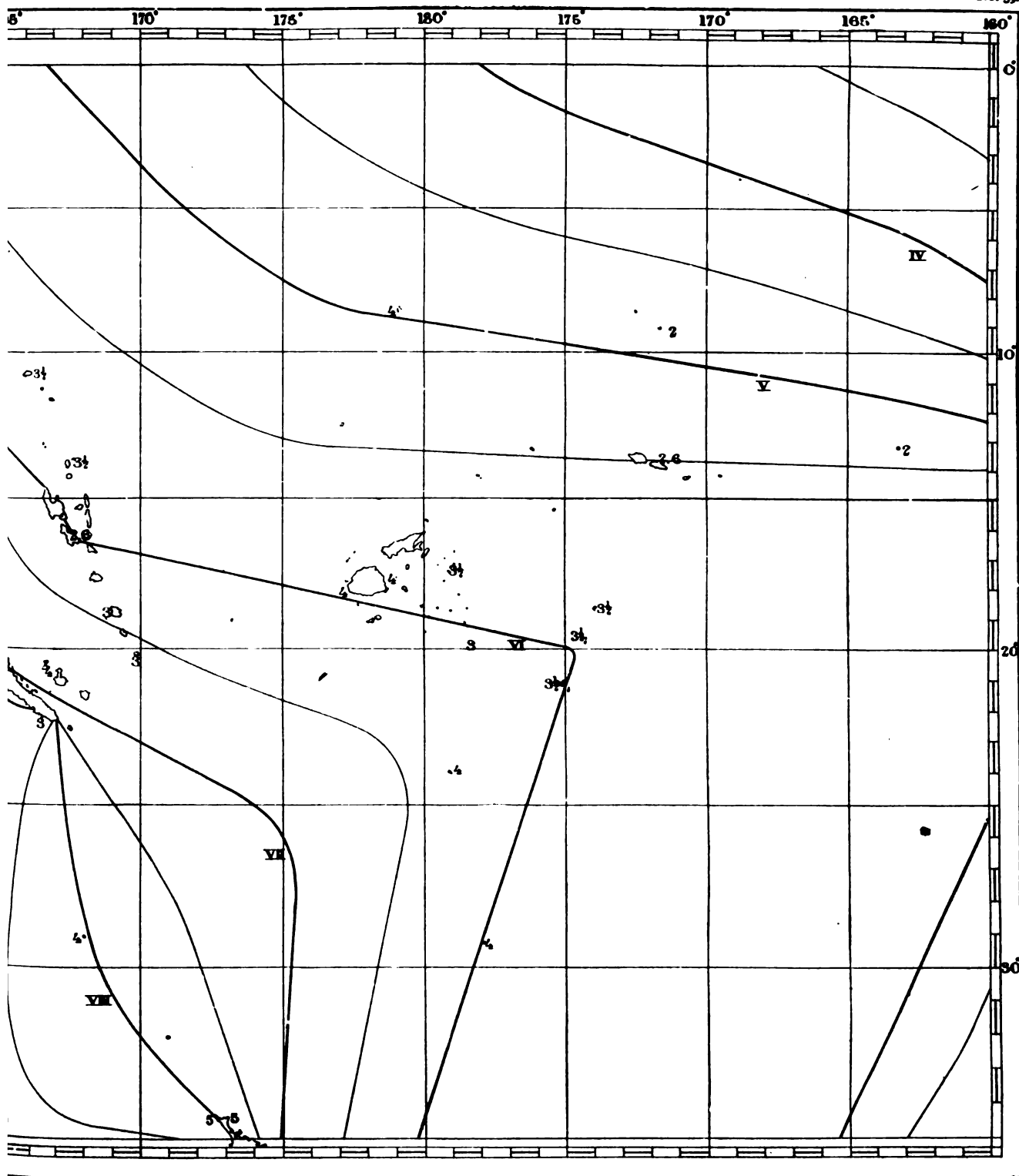




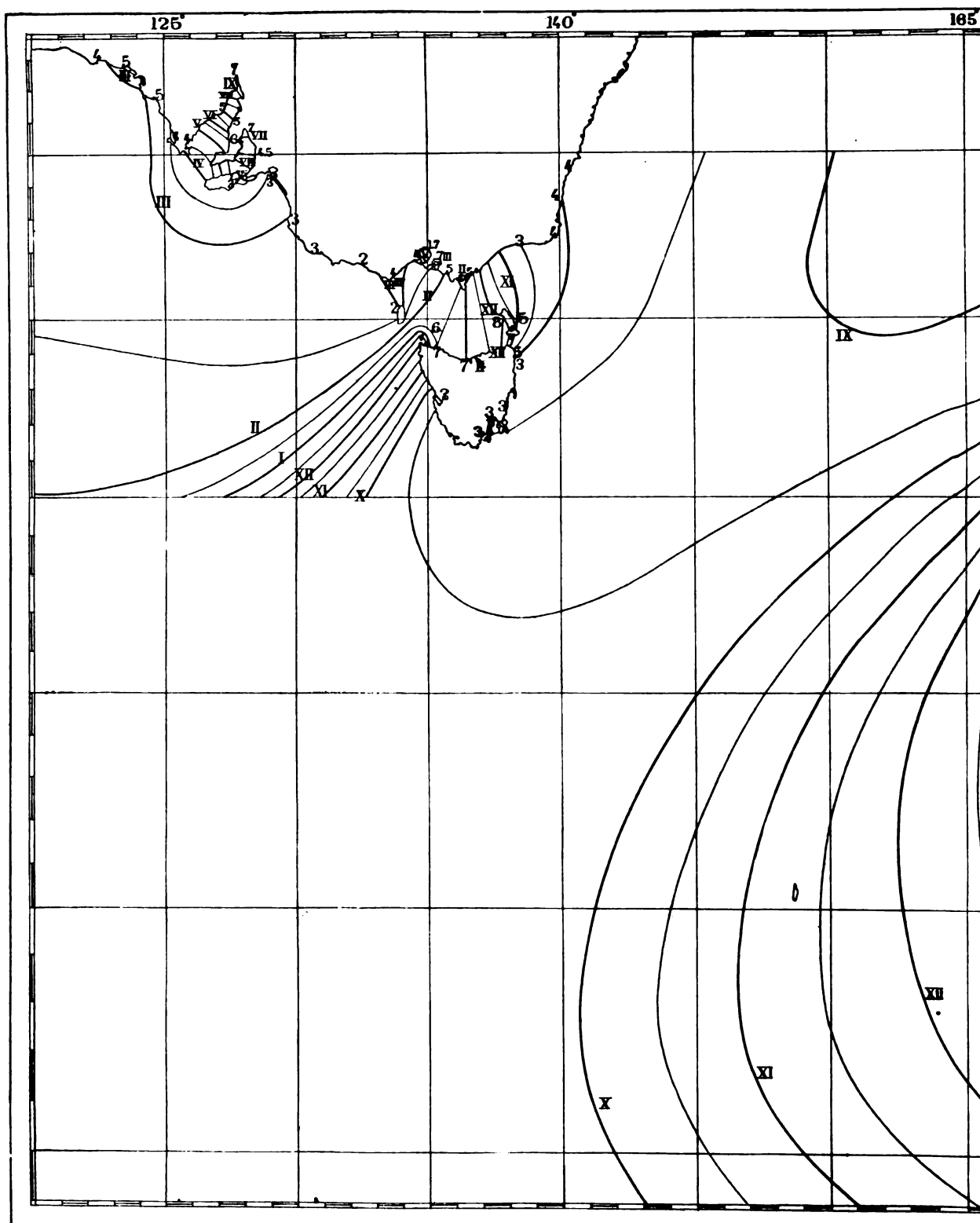
Cotidal lines for Paumotu Group.



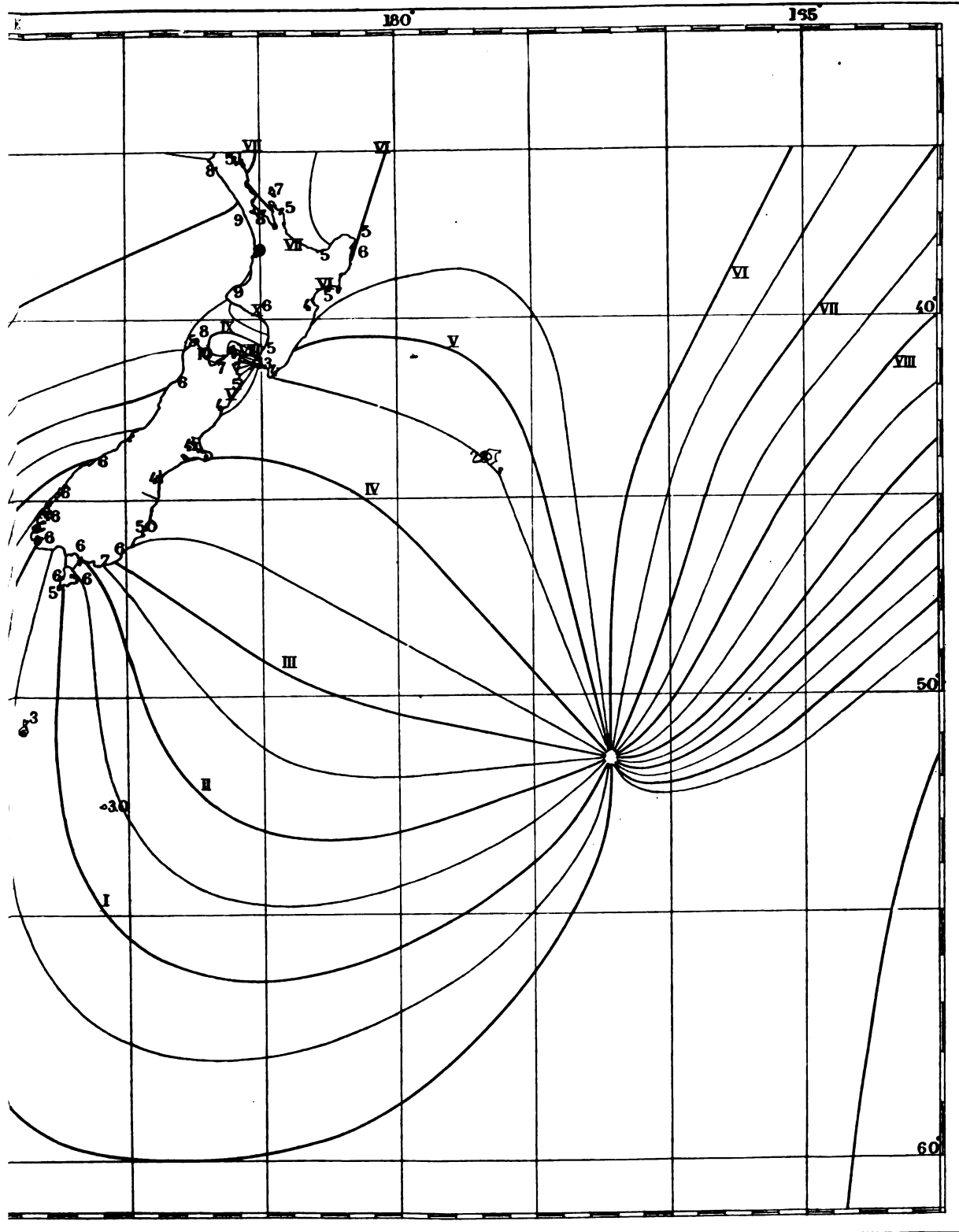
Tidal lines fo



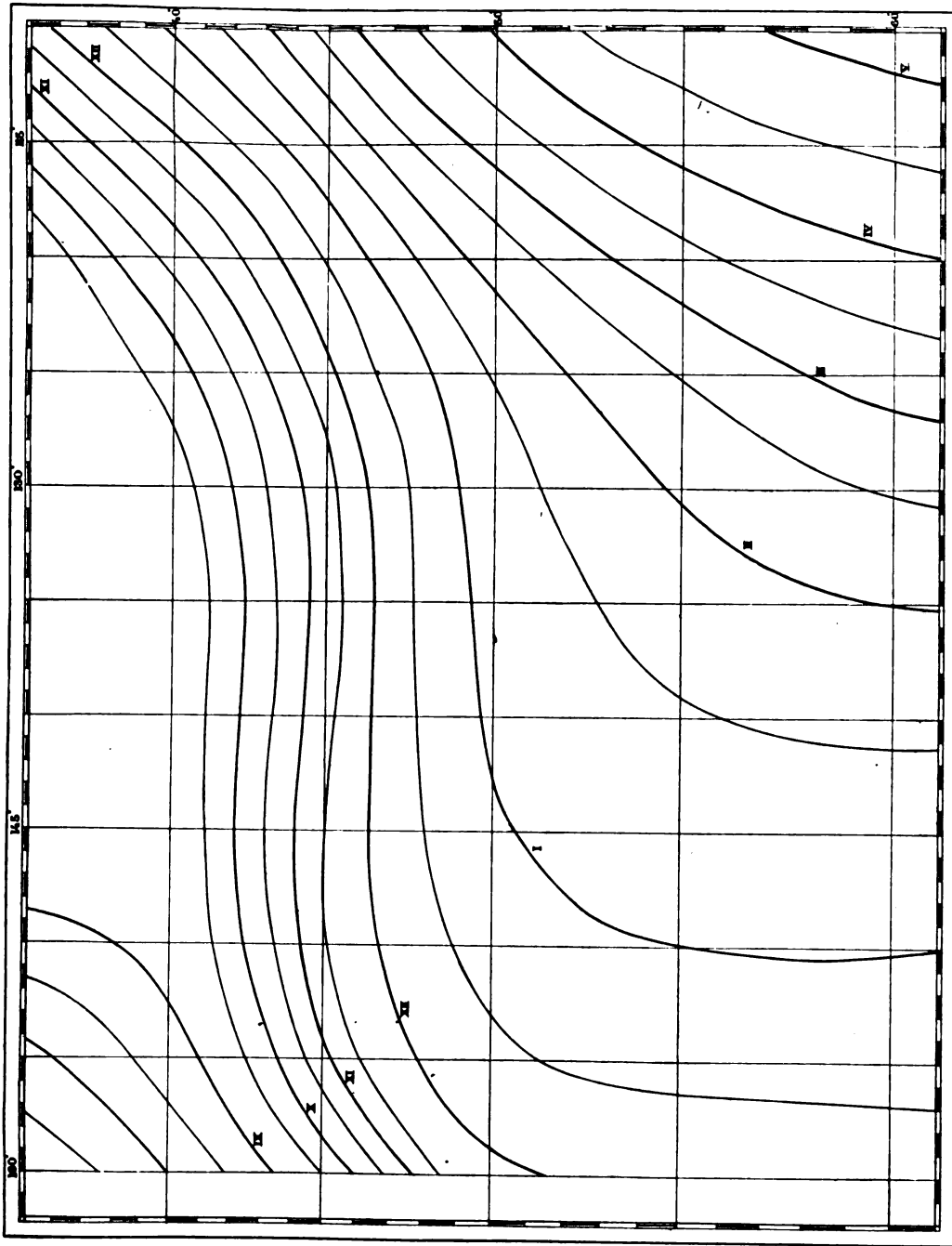
Western Polynesia.



Cotidal lines for



New Zealand.



Cotidal lines for a portion of the South Pacific.

APPENDIX 6

REPORT 1907

MANUAL OF TIDES—PART V
CURRENTS, SHALLOW-WATER TIDES,
METEOROLOGICAL TIDES, AND
MISCELLANEOUS MATTERS

By ROLLIN A. HARRIS

P R E F A C E .

This paper, constituting the concluding chapters of a manual of tides, treats of a variety of matters more or less connected with the main subject. The other parts appeared in the reports of the Survey for the years 1894, 1897, 1900, and 1904.

Chapter I treats of the motion of liquids, with special reference to the possible modes of flow and the nature of the resistance experienced.

Chapter II considers in some detail the kind of resistance (dissipation) which practically controls the principal ocean tides.

Chapter III is devoted to the discussion of shallow-water or river tides. Airy's treatment of this subject is slightly extended and is compared with more recent work. The peculiarities of many tide curves are accounted for in this chapter, but others defy explanation. A conclusion is drawn as to the forms of estuaries from the known laws of friction.

Chapter IV treats of the combinations of motions such as occur in connection with currents and tides. Special points styled "circular points," or points where the tidal current never slackens, are described.

Chapter V treats of the observation of currents and modes of reducing the observed data.

Chapter VI with the included maps, tables, and quotations contains most of the available information concerning tidal currents which seems likely to throw light upon the oscillations in oceanic basins. Observational data for the coasts of the United States are given in considerable detail. The greater portion of this matter has been worked up during the past two years. The connection between the observed tides and currents is, in many cases, pointed out and briefly explained.

A few matters relating to marine engineering comprise Chapter VII.

One of the principal aims of Chapter VIII is to point out the causes which produce the annual meteorological tides, and to briefly consider the causes of ocean currents.

In Chapter IX the question of seiche oscillations is considered for both open and closed bodies. It appears from this discussion that regular oscillations observed even in small partially-inclosed harbors depend, as a rule, directly upon the dimensions of such harbors rather than upon the dimensions of the sea with which they are connected.

Chapter X treats of lake tides, and shows from observations that Lake Superior very nearly obeys the equilibrium theory.

Chapter XI is to some extent supplementary to the other parts of the manual, particularly to Part III.

As usual, the author has received assistance from other members of the Tidal Division of the Survey in the preparation of the tables contained in and following the text.

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MANUAL OF TIDES—PART V. CURRENTS, SHALLOW-WATER TIDES, METEOROLOGICAL TIDES, AND MISCELLANEOUS MATTERS.

By ROLLIN A. HARRIS.

CHAPTER I.

FLOW AND RESISTANCE.

1. *The flow of water in uniform canals and pipes.*

Before about the middle of the last century it was commonly assumed that the greatest velocity of a stream should, except for the effect of winds, be found at the surface. Various assumptions were made as to the law of velocity decrease in going downward. Among the curves taken to represent the velocities were: A parabola with vertical axis, the vertex being placed beneath the bed of the stream; a sloping straight line; an ellipse; and a logarithmic curve.

Raucourt (b. 1779, d. 1841) from observations upon the Neva believed that the velocities could be represented by the ordinates of a vertical ellipse whose minor axis lies a little below the surface and the lower extremity of whose major axis lies a little below the bottom of the river. When the river was frozen over, the maximum velocity was found to be a little below middle depth.

Boileau, about 1850, obtained from his measurements of the Mosel, at Metz, a parabola with horizontal axis along the surface, whose equation could be written

$$u = A - Bz^2,$$

z denoting depth below the surface, the measurements being taken in the main current. However, this equation does not apply to the part of the velocity curve above the line of maximum velocity where the velocities are assumed to follow another curve. For a small canal in which he experimented, he found the maximum velocity to occur below the surface from one-fourth to one-fifth of the water's depth.

From observations made on the Irrawaddy, Gordon found the line of greatest velocity to lie as a rule at one-tenth of the depth, and in a few cases at from two-tenths to four-tenths.

The velocities change rapidly in going from the shore to the center of a stream. Humphreys and Abbot found that at Columbus, Ky., the velocities of the Mississippi River 5 feet below the surface vary according to the law of a parabola; also that—

The parameter of the curve of velocities 5 feet below the surface at any stage is proportional to the square root of the corresponding mean velocity of the river.

From their measurements of the velocities of the lower Mississippi, Humphreys and Abbot drew the following conclusions:

The velocities at different depths below the surface, in a vertical plane, vary as the abscissæ of a parabola whose axis is the axis of N and parallel to the water surface.

The position of the axis in calm weather is about three-tenths of the depth below the surface, whatever be the mean velocity of the river.

The effect of the wind, whether blowing up or down stream, is directly proportional to its force, in the former case lowering and in the latter raising the axis. Also, the amount of such lowering or raising is independent of the mean velocity of the river.*

From their work it is evident that even if the wind had the velocity and direction of the stream the maximum velocity would still be found below the surface and at nearly three-tenths of the entire depth. This fact seems to oppose the theory that the resistance of the atmosphere is necessary for causing the position of greatest velocity to lie below the surface.

Bazin assumes the velocity curve for a regular canal to be an ordinary parabola with its axis horizontal; he finds this to lie from 0 to 0.2 of the total depth below the surface for natural channels, and as deep as 0.35 for some artificial channels.†

The relative velocities for different parts of the cross section of rectangular conduits and open canals, also of open canals of trapezoidal, triangular, and semicircular section, are given by Darcy and Bazin on Plates XVIII–XXIII of their *Recherches Hydrauliques*. The velocity curves for a canal of rectangular cross section are roughly parabolic in both horizontal and vertical planes. When the canal is open, the parabolic velocity curve of the vertical plane has its vertex somewhat below the surface. Their experiments show that the air-resistance is not the most important factor in depressing the thread of maximum velocity.

That considerable transverse motion occurs in a stream flowing along an open channel is obvious from the most casual observations made upon floating objects. That vertical motion also occurs is not so evident because of the difficulty in following water particles below the surface.

In 1867 Mr. James B. Francis, at Lowell, Mass., experimented upon the cross flow of water currents in uniform canals by discharging whitewash at points near the bottom and noting when and how far downstream it would appear at the surface.‡ He says:

From these experiments it appears that the water at the bottom of the streams came to the surface at distances varying from about ten to thirty times the depth, the shorter distances being in the canal of the least depth and most uneven bed.

But he also states that the whitewash continued to come to the surface for a considerable portion of the time which the part first appearing required in ascending from the bottom to the surface.

Observation curves showing the manner in which the velocity diminishes from the center to the sides of canals with vertical sides are given by James B. Francis, on Plate XVII of his *Lowell Hydraulic Experiments*, and described in sections 207 and 208 of that

* Report upon the Physics and Hydraulics of the Mississippi River, Philadelphia, 1861, pp. 234, 243, 257, 262, Pl. XI. Velocity curves are given on Pl. XI.

† *Encyclopædia Britannica*, Vol. XII, p. 498.

‡ Transactions of the American Society of Civil Engineers, Vol. VII (1878), pp. 109–113. See also remarks upon this paper, pp. 122–130.

work. He notes that the velocity at any given point, or rather at any given part of the section, varies continually, although the mean velocity for the whole section remains sensibly constant. This he attributes to an interchange of the currents. In a general way the velocity curves are parabolic.

According to measurements made by W. E. Spear, the tidal currents in Boston Harbor have maximum velocities at considerable depths below the surface.* The velocity curves resemble parabolas.

If a stream be covered with ice, the maximum velocity lies one-third or more of the way down from the surface. For broken and tilted ice, this fraction may be increased to one-half or more.†

Dr. E. C. Murphy draws the following conclusions from observations made at the hydraulic laboratory at Cornell University. The canal is 16 feet wide, 10 feet deep, and 415 feet in length.

The thread of the maximum velocities is at the surface for depths less than 2 feet and unobstructed flow at the lower end of the canal. For depths of 5 feet or more and discharge checked at the lower end of the canal this thread is from 0.2 to 0.4 depth below the surface, the mean for 21 experiments being 0.31 depth.

The position of the thread of mean velocity varies from 0.5 depth for small depths to 0.73 depth for larger depths. For the 31 experiments by the ordinary method of series C and D it is 0.64 depth below the surface.

In ordinary streams where the depth varies from about 1 to 6 feet, the thread of mean velocity is about 0.6 [of the entire depth] below the surface.‡

For historical notices of the movement of water in canals, see Rühlmann's *Hydro-mechanik* (2d ed., 1879), section 121, and Chapter III of Humphrey's and Abbot's Report.

2. *Cross-sectional velocity variation connected with the constant of resistance.*

Observation shows that the velocity is greatest along the axis of a canal and least near the sides and bottom.

For simplicity, first suppose the canal so deep that the effect of the bottom may be neglected. Every elementary mass may be supposed to cross the canal while traveling downstream or onward a distance equal to q , on the average; the mass may not actually cross the stream while traveling this distance, but its transverse motions are such that so far as affecting the onward flow is concerned they may be supposed to give way to this hypothetical uniform crossing. For, the cross-sectional variation in velocity being a continuous and uniform function of the distance from the axis of the stream, all transverse motions of the particles will imply determinate losses of energy. As an element leaves the axis of the stream it loses a portion of its energy. In this way each element takes away (periodically on the average) energy from the energy of the

* Report of the Committee on Charles River Dam (1903), pp. 387-466, Appendix by J. R. Freeman.

† See Water Supply and Irrigation Paper No. 76, U. S. Geological Survey, "Observations on the Flow of Rivers in the Vicinity of New York City," by H. A. Pressey, Washington, 1903; also Paper No. 187, "Determination of Stream Flow during the Frozen Season" (1907), by H. K. Barrows and Robt. E. Horton.

‡ Water Supply and Irrigation Paper No. 95, U. S. Geological Survey, "Accuracy of Stream Measurements," by E. C. Murphy, Washington, 1904.

stream. The impacts of the elements set up rotations and other cross motions. These rotations and cross motions are diffused throughout the liquid, and their maintenance against viscosity requires the expenditure of considerable energy, which is eventually converted into heat. They become apparent when lime water or other colored liquid is mixed with the water.

The diminution in velocity shows how much energy per element is lost to the stream during an excursion of an element from the axis to one side and back again. If v_c denotes the velocity at the center and v_s at the side or bank, the energy which must be supplied to each unit mass while the latter travels a distance q in order to maintain the flow is

$$\frac{1}{2} (v_c^2 - v_s^2), \quad (1)$$

and so the average resisting force per unit mass is

$$\frac{1}{2q} (v_c^2 - v_s^2). \quad (2)$$

Now, it is found from observation that for moderately smooth walls q is about $60b$ where b denotes the breadth of the stream. The resisting force per unit mass is therefore about

$$\frac{1}{120} b (v_c^2 - v_s^2). \quad (3)$$

The side resistances upon a slice of water one unit long, one unit deep, and of a width b , may (§ 7) be written

$$2\zeta' v_m^2 \frac{\gamma}{2g},$$

v_m denoting the mean velocity over the section.

This slice contains b cubic units and the mass is $b \frac{\gamma}{g}$; \therefore the resistance per unit mass

is $\zeta' \frac{v_m^2}{b}$, and so

$$\zeta' = \frac{1}{120} \frac{v_c^2 - v_s^2}{v_m^2}. \quad (4)$$

Assuming, for the moment that the velocity of a stream is greatest at the surface, and that the resistance due to the bottom is similar to that due to one side in the case just considered, it follows that b can be replaced by $2h$ and that $2h/q$ is the average upward slope of an ascending element, $\frac{1}{2} q$ being the x -distance over which a particle placed at the bottom travels before reaching the surface.

In Francis's experiments $2h/q$ varies from one-tenth to one-thirtieth or less. This rapid ascension is in part due to transverse or upward motions of the particles in addition to the motion of the element as a whole. Moreover, for some of the particles $2h/q$ is considerable less than one-thirtieth.

3. *Wave motion and flow in a vertical plane, the depth being uniform or nearly so.*

Suppose the horizontal displacement be assumed to be of the form

$$\mathbf{x} = \left[A + B \frac{h-z}{h} + C \left(\frac{h-z}{h} \right)^2 + \dots \right] F(x, t) + \phi(z, t). \quad (5)$$

Then because of the equation of continuity,

$$\frac{\partial \mathbf{x}}{\partial x} + \frac{\partial \mathbf{z}}{\partial z} = 0,$$

the vertical displacement will be

$$\mathbf{z} = \left[-Az + B \frac{h}{2} \left(\frac{h-z}{h} \right)^2 + C \frac{h}{3} \left(\frac{h-z}{h} \right)^3 + \dots \right] \frac{\partial F(x, t)}{\partial x} + \psi(x, t), \quad (6)$$

A similar expression could have been assumed for the horizontal velocity and by making use of the equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,$$

an expression similar to (6) would have been obtained for the vertical velocity.

Now, it seems reasonable, and in accordance with common observation, to assume that the horizontal displacement or velocity has a factor independent of time and distance. This amounts to assuming that the displacement or velocity has a factor depending upon the depths alone; . . . $\phi(z, t)$ should be zero. In a somewhat similar manner it can be shown that $\psi(x, t)$ should be zero. Along the bottom of the sheet of water $z=0$. \mathbf{z} (or w) will there be zero, provided that either

$$B=0, \text{ or } \frac{B}{2} + \frac{C}{3} = 0 \text{ or } \frac{B}{2} + \frac{C}{3} + \frac{D}{4} = 0 \quad (7)$$

etc., according as 2, 3, 4, etc., terms are assumed to occur in the brackets.

In case u does not contain x —that is, in case the velocity of a particular thread or depth is the same at all cross sections of a uniform canal—then w (the x -axis coinciding with the nearly horizontal bottom) will be zero at all depths regardless of the nature of the z -function in the brackets. Hence it is kinematically possible to have at a given time and depth any constant velocity for all points situated at the given depth. But the constant velocities belonging to the other depths may differ in any manner whatever amongst themselves, or from the velocity belonging to the depth to which attention is directed. In reality the manner in which uniform flow can take place is limited or defined by the nature of the external and internal resistances. According to statements made in section 1 concerning the motions of the particles, the velocity in a broad canal should be a parabola whose vertex lies somewhat below the surface, unless the bottom is rough; in which case the maximum velocity may be at the surface.

When F contains x , the simplest mode of flow is that where $B=0$, and where no further terms occur in the coefficients. This means that water moves in vertical slices, all of whose particles travel the same horizontal distance. This is evidently an impossible case on account of the resistance at the bottom. The next mode in order of simplicity is that where $\frac{B}{2} + \frac{C}{3} = 0$, and where no terms beyond the C -term occur in the coefficient. This may be styled the parabolic flow for reasons which will soon appear.

It is the simplest possible mode of flow consistent with a bottom resistance, and independently of the considerations of section 1, it would be the natural assumption to take. The case where $\frac{B}{2} + \frac{C}{3} + \frac{D}{4} = 0$, and where no terms beyond the D -term occur in the coefficient, may be styled parabolic flow of order three.

Let $\frac{h-z}{h} = r$, and omit the terms beyond C ; then

$$A + Br + Cr^2 \quad (8)$$

is the coefficient of the displacement. This may be written in either of the two forms

$$A + Br - \frac{3}{2}Br^2 \quad (9)$$

or

$$A - \frac{2}{3}Cr + Cr^2 \quad (10)$$

From $\frac{\partial x}{\partial r} = 0$, it follows that the maximum value of x corresponds to $r = \frac{1}{3}$. The values of the coefficient of the displacement at the surface, one-third of the way down, and at the bottom are, respectively,

$$\begin{aligned} A, \\ A + \frac{B}{3} + \frac{C}{9} = A + \frac{B}{6} = A - \frac{C}{9}, \\ A + B + C = A - \frac{B}{2} = A + \frac{C}{3}. \end{aligned} \quad (11)$$

Consequently the maximum displacement or velocity exceeds the displacement or velocity at the bottom by four times the amount it exceeds that at the surface.

$$\text{For } r = \frac{1}{2}, \text{ the coefficient} = A + \frac{B}{2} + \frac{C}{4} = A + \frac{B}{8} = A - \frac{C}{12} \quad (12)$$

$$\text{For } r = \frac{2}{3}, \text{ the coefficient} = A + \frac{2}{3}B + \frac{4}{9}C = A. \quad (13)$$

The average value of the coefficient $= A + \frac{B}{2} + \frac{C}{3} = A$, the surface value, or the value two-thirds of the way down.

In case of the tides

$$F(x, t) = \sin(at - lx + \alpha),$$

$$\frac{\partial F(x, t)}{\partial x} = -l \cos(at - lx + \alpha),$$

for a progressive wave: and

$$F(x, t) = \sin[l(L - x)] \cos(at + \alpha),$$

$$\frac{\partial F(x, t)}{\partial x} = -l \cos[l(L - x)] \cos(at + \alpha),$$

for a stationary wave in a canal whose length is L , the origin being taken at its mouth.

If u contains x , because the depth is subject to small gradual changes, it can be shown that the simplest possible type of steady motion is that in which the velocity curve is a parabola with vertex one-third of the depth of the stream below its surface.

Let u and w be written thus,

$$u = \left[a + b \frac{h-z}{h} + c \left(\frac{h-z}{h} \right)^2 \right] f(x), \quad (14)$$

$$w = \left[-az + \frac{bh}{2} \left(\frac{h-z}{h} \right)^2 + \frac{ch}{3} \left(\frac{h-z}{h} \right)^3 \right] \frac{\partial f(x)}{\partial x}. \quad (15)$$

These satisfy the equation of continuity; w will vanish at the bottom if $\frac{b}{2} + \frac{c}{3} = 0$.

Let h denote the average depth for a distance over which the depth changes by only a small fraction of itself. z is supposed to be reckoned from the nearly horizontal bottom. The expressions in the brackets have nearly constant values at each cross-section of the reach considered. For another reach, h may have a sensibly different value. But in each case the simplest possible mode of flow is the parabolic kind referred to above. This will apply to reach after reach whose depth variations are everywhere gradual. Even if a stream in an earlier part of its course flowed over a bed not of the kind here supposed, the effects upon its flow will, later on, practically disappear and the parabolic mode of flow be finally established if the length of the stream over tolerably even bed be sufficiently great. As the observed velocity curve approaches in form a parabola of the second order, the depth of maximum velocity will approach $\frac{1}{3}h$.

As already noted, roughness of the bed, or a shallowing of the stream, throws the line of maximum velocity nearer to the surface. A channel broader at the top than at the bottom has its line of maximum velocity raised, and vice versa for one narrow at the top and broad at the bottom.

See Darcy and Bazin: *Recherches Hydrauliques*, Plates 18-23.

4. *Kinds of resistance.*

If a solid be held in a fixed position while the adjacent liquid has a steady flow, or if the solid move while the liquid remains at rest, three kinds of resistance will generally be experienced: skin friction, resistance due to impact, implying discontinuous motion, and generally some wave resistance.

Viscosity proper—i. e., the force required to overcome the shear of the elements as one lamina moves over a neighboring lamina—will not be considered here, as it is probably of little consequence. But its indirect effect is of great importance, as will presently be seen. A resistance (dissipation) described in the next chapter is a species of wave resistance.

5. *Skin friction as varying on account of the velocity.*

If an elementary mass of water move through or into a larger mass of water having less velocity than the given element has, the latter experiences a retarding force; some of the original energy will be retained in the element, some will be utilized in directly helping along the neighboring particles, and some will be consumed, i. e., turned into

heat. The change of motion experienced is quite analogous to that resulting from the impact of inelastic bodies.

For, imagine a long pipe or canal filled with very small elastic balls. Let the whole collection of them move forward because of gravity or pressure. If they impinge against an object, or if the size of the pipe suddenly change, the chance is very small that important rebounds, like those belonging to a single ball, will take place for a considerable portion of the mass of balls, because the forces resulting from the impacts are largely consumed in moving and rotating the particles among themselves and in urging forward any balls which may lag behind in the rebound. Thus the aggregate of these assumed elastic balls behaves nearly like an aggregate of inelastic balls. With greater reason a liquid will so behave because the particles are indefinitely small.

For perfectly inelastic bodies, M' , M'' , moving with velocities v' , v'' in the same direction, the velocity after their impact is

$$\frac{M' v' + M'' v''}{M' + M''},$$

because the quantity of motion remains unaltered. Now, if v' and v'' before impact be each increased n -fold, so will be the velocity after impact. Since the diminutions in the velocities of a large portion of the liquid, especially near a solid or solid boundary, may be considered as depending upon impacts, it follows that such diminutions in velocities probably bear nearly fixed ratios to the general velocity of the stream, whatever be the actual value of the latter, within tolerably wide limits.

The average force imparted to the boundary walls when the velocity of an element of unit mass is reduced from its maximum value, v_1 , to its minimum value, v_2 , is $\frac{1}{2}(v_1^2 - v_2^2) \div s$, where s denotes the distance along the stream between successive points of minimum velocity of the element as it pursues its slightly undulating or sinuous path. Assume that all motions are geometrically similar. Because of this assumption, when v_1 becomes nv_1 , v_2 will become nv_2 , while s remains the same as before. The energy in either case divided by the length of the path s is the average value of the resisting force acting upon the element. These forces are therefore to each other as 1 is to n^2 . Hence the resisting force when two scales of velocities are employed is proportional to the square of the velocity, i. e. to the square of the initial velocity of the element, or velocity at a certain point in the cross section.

Consider now a rigid body immersed in a uniformly flowing stream. Observations indicate that there is a film of water adhering to the body, otherwise there would be a slipping along the surface of the body. The influence of this film extends outward to a considerable distance in accordance with some law not well understood. A moving element enters the zone of retarded flow which forms a sort of cushion around the rigid body. For any symmetrical surface the impinging particles produce a dragging force upon the body, whose resultant effect acts in approximately the direction of flow. As has just been seen, this portion of the force of impact is proportional to the square of the velocity of the stream. But nothing here stated contemplates a comparison between bodies of different shapes or sizes. All that it is intended to show is, assuming the flows are geometrically alike for different scales of velocity, that what is commonly known as the skin friction upon a body is proportional to the square of the velocity.

This resistance arises from changes in the velocities of the moving liquid elements as they enter the field of influence of the rigid body.

For the resistance due to impact and which does not imply changes in the magnitudes of the velocities, see section 12.

6. *Skin friction as varying on account of the areas of the rigid boundaries.*

Suppose that we compare the resistance found in a given long pipe or canal with the resistance experienced in a pipe or canal whose cross section is similar to that of the former, but whose length and diameter or width are not the same as before.

As already intimated, the resistance to a moving liquid involves what may be termed a misdirection of the particles, and that this in turn is primarily due to the adhesion of a liquid film to the rigid boundary, to viscosity, and to sensible irregularities of the boundary. That is, because of the adhesion and viscosity, however small they may be, a deflecting force must exist (particularly near the boundaries), which sooner or later will cause the rapidly moving particles to enter regions of less onward motion and even to arrive so near the boundary that the motion becomes much diminished.

It may be laid down as the fundamental property of liquid flow in long pipes or canals that each particle will in time occupy all cross-sectional positions of the stream from the axis to the boundary. For short pipes or canals the same tendency to exchange positions is going on, but particles may leave these conduits before experiencing great transverse displacements.

The question as to how viscosity and the presence of rigid boundaries can cause deflections of the particles need not be discussed. It may, however, be noted that since the side of a particle toward the axis of the stream is urged forward by the adjacent liquid while the side toward the rigid boundary is retarded, there must be a couple tending to produce rotation, such that the forward part of the particle is crowded toward the boundary. But the result when all particles are considered is sinuous flow.

In a uniform pipe or canal the impacts of the forward-directed elements against those moving slower produce a traction which is ultimately exerted along the boundary. The continuity of the liquid in a pipe or canal does, in a sense, prescribe the paths of the particles, and these paths may, in the long run, all be considered as alike in all respects. Now, whatever element is followed, it attains the maximum velocity near the axis of the pipe and later on loses this velocity and attains a minimum velocity near the bounding walls.

If two long pipes have the same diameter but differ only in length, then, by what has just been said, the geometrical character of the flow in each is identical. For, the number of minimum velocities experienced by the elements in the two cases will be directly proportional to the lengths, and the forces required to impart a given motion to the particles, which at intervals, regular in the long run, lose a certain part of their maximum velocity, must be proportional to the number of elements concerned.

If the pipes have the same length but differ only in diameter, it is reasonable to assume that the number of minima experienced by the particles in the two cases will be inversely proportional to the diameter. This will evidently be so if the motions are geometrically similar.

The amount of matter to which the forces must impart velocities is directly proportional to the area of the cross section of the pipe. But the number of minimum

velocities experienced by the moving particle is inversely proportional to the diameter. Consequently the required force is directly proportional to the diameter.

Hence it is reasonable to conclude that the resisting force or drag upon the pipe (or bed of the canal) is directly proportional to the length of the pipe (or canal) and directly proportional to the diameter (or wetted perimeter); but the average resistance per unit of cross-sectional area is consequently inversely proportional to the diameter (or hydraulic mean depth). From what was shown in section 5 the resisting force is directly proportional to the square of the velocity (i. e., scale of the velocities).

Since the direct effect of viscosity is assumed to be negligible, and since the flow in the same body under different conditions is assumed to be geometrically similar, it follows that the diminution in velocity, or the loss of head, is independent of the pressure under which all motions take place. For, the differential pressure required to produce a certain scale of velocities is the same, whatever the general or atmospheric pressure, because it is consumed in the same way (i. e., to the same advantage) in accelerating the water particles. Experiments made with water pipes buried at various depths, and Coulomb's experiments with an immersed pendulum disc, the apparatus being under the receiver of an air pump, all indicate that the general pressure has no effect upon the resistance. From what has just been said, this indicates that the motions of the water particles are geometrically similar.

It is here convenient to regard as a fundamental type of skin-frictional resistance that which occurs when a thin board of considerable length is placed lengthwise in a large, uniformly-flowing stream. Moreover, for a given fluid, only velocities falling within certain limits will be considered.

It may be postulated as a law derived from observation that in some unknown manner a very thin fluid film clings to the solid with such tenacity that moving adjacent particles can not impart motion to it, and that this stationary film influences the motion of the fluid to considerable distances from the solid. This conception in part explains the fact that for most rigid and tolerably rigid surfaces the amount of resistance for like velocities and dimensions is approximately the same per square unit. But it increases somewhat with the roughness of the boundary surface.

The amount of this resistance (force) can be approximately represented by

$$\zeta' v^2 \frac{\gamma}{2g} A,$$

where A = resisting area, γ the heaviness of a cubic foot of the fluid, and so γ/g = the density. ζ' is an abstract number, supposed to be constant for a body of given shape, and to remain nearly the same where the size of the body varies.

7. Numerical values for skin-frictional resistance.

Let F = resistance per square unit; then

$$F = \zeta' v^2 \frac{\gamma}{2g} \text{ or } \zeta' v^n \frac{\gamma}{2g}. \quad (16)$$

Using the foot as unit, this becomes

$$F = \zeta' v^2 \frac{\gamma}{64.3444} \text{ or } \zeta' v^n \frac{\gamma}{64.3444}. \quad (17)$$

For pure water, sea water, and air, γ may be taken as 62.4 pounds, 64 pounds, and 0.08 pounds, respectively; then

$$\begin{aligned} F &= 0.9698 \zeta' v^2, 0.9946 \zeta' v^2, 0.00124 \zeta' v^2 \\ \zeta' &= 1.0312 F v^2, 1.0054 F v^2, 804.305 F v^2 \end{aligned} \quad (18)$$

The value of ζ' for a smooth surface is approximately 0.004 in all three fluids. Since F is a force per unit area, it follows that the dimensions of ζ' are zero in time, length, and mass, provided the exponent equals 2; but if the exponent 2 be replaced by n , then the dimensions of ζ' become

$$M^0 T^{n-2} L^{2-n}.$$

It will be noted that the dimensions of F must be

$$ML^{-1} T^{-2}$$

for all the values of n .

If we are dealing with only one liquid or fluid, as water, the resisting force per unit area may, for convenience, be written

$$F = \zeta'' v^n, \quad (19)$$

where $\zeta'' = \zeta' \frac{\gamma}{2g}$. For water, $\gamma/(2g) = 0.9698$; or $\zeta'' = 0.9698 \zeta'$.

The values of ζ' and ζ'' , from various authorities, for several kinds of resisting surface, are given below, together with values of F for a velocity of 10 feet per second according to Froude:*

Varnished surface	$\zeta' = 0.00258$	$\zeta'' = 0.00250$	$F = 0.25$ lb.
Painted or planed plank	$\zeta' = 0.00350$	$\zeta'' = 0.00339$	
Surface of iron ships	$\zeta' = 0.00362$	$\zeta'' = 0.00351$	
Fine sand surface	$\zeta' = 0.00418$	$\zeta'' = 0.00405$	$F = 0.40$ lb.
New well-painted iron plate	$\zeta' = 0.00489$	$\zeta'' = 0.00473$	
Coarse sand surface	$\zeta' = 0.00503$	$\zeta'' = 0.00488$	
Beds of streams	$\zeta' = 0.007565$	Depends on size and form of cross section.	
Iron pipes	$\zeta' = 0.0075$	Depends on diameter of pipe and velocity of flow.	

For air $\zeta'' = 0.00124 \zeta'$.

According to experiments made by Prof. A. F. Zahm,

$$F = 0.00000671 v^{1.85}.$$

Since the dimensions of γ are $M L^{-2} T^{-2}$ and of g , $M^0 L T^{-2}$, it follows that the dimensions of ζ'' are $ML^{-1-n} T^{n-2}$.

* Weisbach: Mechanics of Engineering, Vol. I, pp. 867, 965. Unwin: Encyclopædia Britannica, Vol. XII, pp. 482, 483. Bovey: Hydraulics (1901), p. 124.

8. *Uniform flow impeded by skin-frictional resistance.*

To find the relation between velocity and slope of an indefinitely long pipe or channel of uniform cross section.

If the slope is uniform, the pressure at the two ends of a moving elementary slice will be equal; i. e., $d p = 0$. A resisting surface of unit area exerts a retarding force equal to F . If m units of volume stand upon this area, or rather have this area as lateral boundaries, the force of gravity will drive the slice of water forward with the force $\gamma m \frac{h}{l}$, h here denoting fall and l length. \therefore for uniform motion

$$F = \gamma m \frac{h}{l} = \zeta' \frac{v^n}{2g}, \quad (20)$$

or
$$\zeta' \frac{v^n}{2gm} = \frac{h}{l} = \text{slope} = i, \quad (21)$$

and
$$v^n = \frac{2gm}{\zeta'} \frac{h}{l}. \quad (22)$$

In a pipe, $m = \frac{1}{2}r$, (23)

$$\text{slope} = \zeta' \frac{v^n}{gr}, \quad (24)$$

$$= v^n \frac{gr}{\zeta'} \frac{h}{l}. \quad (25)$$

In a broad channel, m = the depth; also

$$\text{slope} = \zeta' \frac{v^n}{2gm}, \quad (26)$$

$$v^n = \frac{2gm}{\zeta'} \frac{h}{l}. \quad (27)$$

In general m denotes the hydraulic mean depth.

As ζ' is known to depend in a measure upon the velocity, it has sometimes been replaced by $g(a + \frac{b}{v})$. Hence the dimensions of a are $M^0 L^{-1} T^2$ and of b , $M^0 L^0 T$. Consequently, to turn a , expressed in meters, into values expressed in feet, we must divide by 3.28083; b remains the same in both systems.

$$F = (a + \frac{b}{v}) v^2 \frac{\gamma}{2} = \gamma m \frac{h}{l}. \quad (28)$$

the height corresponding to the loss through friction may be written,

$$h = (\alpha' v^3 + \beta' v) \frac{l}{m}, \quad (29)$$

if ζ' be replaced by $2g(\alpha' + \frac{\beta'}{v})$, where $\alpha' = \frac{a}{2}$, $\beta' = \frac{b}{2}$.

In case of a pipe, d = diameter = $4m$.

The formulas of Prony, Eytelwein, and d'Aubuisson, for long tubes are

$$\begin{aligned} h &= (0.0013932v^2 + 0.0000693v) \frac{l}{d}, \\ h &= (0.0011213v^2 + 0.0000894v) \frac{l}{d}, \\ h &= (0.001370v^2 + 0.0000753v) \frac{l}{d}, \end{aligned} \quad (30)$$

if the meter is the unit of length, or

$$\begin{aligned} h &= (0.00042465v^2 + 0.0000693v) \frac{l}{d}, \\ h &= (0.00034177v^2 + 0.0000894v) \frac{l}{d}, \\ h &= (0.00041758v^2 + 0.0000753v) \frac{l}{d}, \end{aligned} \quad (31)$$

the mean being $(0.00039467v^2 + 0.0000780v) \frac{l}{d}$,

if the foot is the unit. These coefficients divided by 4 give α' , β' , and by 2, a , b . ζ' is of the form $g \left(a + \frac{b}{v}\right)$.

It is to be especially noted that the ζ used by Weisbach in pipe formulæ is by definition four times ζ' used throughout this paper, and so the numerical values in Vol. I, § 429, of his Mechanics must be divided by 4 in order to make them comparable with the above ζ 's or with his own ζ when used in connection with streams.

The relations

$$F = \zeta' v^2 \frac{\gamma}{2g} = \left(a + \frac{b}{v}\right) v^2 \frac{\gamma}{2g} = \gamma m \frac{h}{l}, \quad (32)$$

when applied to long tubes, give

$$\begin{aligned} F &= \frac{\gamma m}{l} (0.00039467v^2 + 0.0000780v) \frac{l}{d} \\ &= \gamma (0.00009867v^2 + 0.0000195v), \\ &= 0.006157v^2 + 0.001217v, \end{aligned} \quad (33)$$

if $\gamma = 62.4$, and

$$F = 0.006315v^2 + 0.001248v, \quad (34)$$

if $\gamma = 64$.

Weisbach proposes as the value of ζ , $\alpha + \frac{\beta}{\sqrt{v}}$, where h is of the form

$$\left(\alpha + \frac{\beta}{\sqrt{v}}\right) \frac{l}{d} \frac{v^2}{2g},$$

which implies that ζ' takes the value $\frac{1}{4} \left(\alpha + \frac{\beta}{\sqrt{v}} \right)$ if h has the form

$$\left(\alpha + \frac{\beta}{\sqrt{v}} \right) \frac{l}{m} \frac{v^3}{2g}.$$

Here $\alpha = 0.0143$ and $\beta = \begin{cases} 0.010 \\ 0.018 \end{cases}$ according as the unit is the meter or the foot.

For an open channel Prony and Eytelwein respectively find

$$h = (0.000\ 094\ 277\ v^2 + 0.000\ 044\ 50\ v) \frac{l}{m}, \quad (35)$$

$$h = (0.000\ 111\ 415\ v^2 + 0.000\ 024\ 265\ v) \frac{l}{m}, \quad (36)$$

the foot being the unit of length. These coefficients are α' and β' . They are somewhat greater than the α' and β' connected with pipes because of the considerable irregularities in stream beds.

These values substituted in $F = \gamma m h / l$ give

$$F = 0.005\ 883\ v^2 + 0.002\ 777\ v, \quad (37)$$

$$F = 0.006\ 952\ v^2 + 0.001\ 514\ v,$$

if $\gamma = 62.4$, and

$$F = 0.006\ 034\ v^2 + 0.002\ 848\ v,$$

$$F = 0.007\ 131\ v^2 + 0.001\ 553\ v, \quad (38)$$

if $\gamma = 64$.

Omitting the term in v to the first power in (35), (36),

$$\sqrt{mi} = 0.009\ 710\ v,$$

$$\sqrt{mi} = 0.010\ 56\ v;$$

$$v = 102.99 \sqrt{mi}, \quad (39)$$

$$v = 94.74 \sqrt{mi}. \quad (40)$$

If ζ' is assumed to so depend upon the hydraulic mean depth, m , that $\zeta' = \frac{\gamma}{2g} \left(\alpha + \frac{\beta}{m} \right)$, the equation $\gamma mi = F = \zeta' v^2 \frac{\gamma}{2g}$

gives

$$v^2 = \frac{mi}{\alpha + \frac{\beta}{m}}, \quad (41)$$

the formula deduced by Darcy and Bazin for a channel of rectangular or trapezoidal section. For a channel through earth, $\alpha = 0.000\ 085$ when the foot unit is used and $0.000\ 28$ when the meter; in either case $\beta = 0.000\ 35$.

Bazin's formula (1897) is

$$v = \frac{157.6}{1 + \frac{\gamma'}{\sqrt{m}}} \sqrt{mi} \quad (42)$$

for feet, and

$$v = \frac{87}{1 + \frac{\gamma'}{\sqrt{m}}} \sqrt{mi} \quad (43)$$

for meters, γ' is a coefficient, being about 0.1 for a smooth artificial surface, about 1.5 for earth, and 3 or more for exceptionally rough channels. The corresponding values when the meter is used are 0.06, 0.85, and 1.7.

In Ganguillet and Kutter's formula ζ' is taken to be a function of the roughness of the channel, of the slope, and of the mean depth.

Tadini's formula is

$$v = 91\sqrt{mi} \quad (44)$$

when the foot is the unit and

$$v = 50\sqrt{mi} \quad (45)$$

when the meter is the unit.

The following are a few references to formulæ relating to the flow of water in pipes and canals:

Julius Weisbach: *Mechanics of Engineering*, Vol. I, secs. 420-479.

Humphreys and Abbot: *Report on the Physics and Hydraulics of the Mississippi River* (1861), Chap. V.

Darcy and Bazin: *Recherches Hydrauliques* (1865).

Greenhill and Unwin: *Encyclopædia Britannica*, Article, *Hydromechanics* (1881).

James B. Francis: *Lowell Hydraulic Experiments*, 4th ed. (1883), secs. 177-246.

Hamilton Smith, jr.: *Hydraulics* (1886), pp. 17-24, 195-198, 271-276.

Ganguillet and Kutter: *A General Formula for the Uniform Flow of Water in Rivers and Other Channels*, translation (1889), pp. 1-76.

Bovey: *Hydraulics*, 2d ed. (1901), pp. 123-144, 246-253.

9. *Bernoulli's theorem when friction is taken into account.*

Equation (52), Part IV A, expresses that the energy of all kinds possessed by an element which passes a cross section in unit time is constant for any cross section. If to the second member be added the energy which this unit-time mass loses in going from the first cross section to the second, the equation becomes

$$\rho_1 v_1 \Omega_1 + \frac{\gamma}{g} v_1 \Omega_1 \left(\frac{1}{2} v_1^2 + V_1 \right) = \rho_2 v_2 \Omega_2 + \frac{\gamma}{g} v_2 \Omega_2 \left(\frac{1}{2} v_2^2 + V_2 \right) + \int_{s=s_1}^{s=s_2} F P v ds, \quad (46)$$

where ds means $\frac{\partial s}{\partial t} dt$, $V = gh$, and P denotes the wetted perimeter. This equation

reduces to

$$\frac{\rho_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{\rho_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + \int \frac{F P v ds}{\gamma \Omega v}. \quad (47)$$

The last terms may be written $\zeta' v^n \frac{P}{\Omega} \frac{s}{2g}$ if F be put equal to $\frac{\zeta' v^n}{2} \frac{\gamma}{g}$, and v denote average velocity.

The same result can be established in connection with section 8, Part IV A.

Effective force = acceleration \times mass of element = impressed force

$$\begin{aligned}
 &= -\Omega dp - \Omega \gamma dz - \zeta' P ds \frac{v''}{2g} \gamma. \\
 \therefore v dv &= -g \frac{dp}{\gamma} - g dz - \zeta' \frac{P v''}{2\Omega} ds; \\
 \frac{v_1^2 - v_2^2}{2} &= -\frac{g}{\gamma} [p_1 - p_2 + \gamma(z_1 - z_2)] + \zeta' \int_{s=s_1}^{s=s_2} \frac{P v''}{\Omega} ds. \quad (48)
 \end{aligned}$$

The last term becomes $\zeta' \frac{P v''}{\Omega} s$ if v is the constant average value.

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + \zeta' \frac{P v''}{\Omega} s. \quad (49)$$

If n equal 2, and the initial velocity be zero, as when a pipe leads from a large tank, then

$$v^2 = \frac{2gh}{(1 + \zeta' \frac{P}{\Omega} l)}, \quad (50)$$

This equation is of fundamental importance both for the flow of water through pipes and in open channels.

In neither of the foregoing modes of establishing Bernoulli's theorem with friction added is it supposed that the velocity is alike from axis to boundary. It may vary according to any law. The velocity used in the formula may be the mean cross-sectional velocity, the axial velocity, or any other specified with reference to the cross section; for each case ζ' will have a different value. Unless otherwise indicated, the mean cross-sectional velocity is understood to be the one associated with ζ' .

If the pipe or channel consist of several parts which have different cross-sectional areas, a resistance term can be added in the denominator of (50) for each length. Moreover, for each bend or sudden change of cross section, a resistance term, depending in a measure upon the slope and size of the stream, will occur.

10. *Nonuniform flow impeded by skin-frictional resistance.*

If the cross section changes slowly, and if we assume the flow to be without frictional resistance, the case comes under Bernoulli's theorem without resistance terms.

If resistance exists, a modification of the work just given will become necessary, because there the pipe or channel was assumed to have a uniform cross section for a considerable distance. This can be readily accomplished for a pipe of continually varying diameter. For it would only be necessary to divide the pipe into elements so short that the resistance for each could be easily ascertained, the relative velocities being known through the condition of continuity. If water flows in an open channel, it would still generally be possible to make an estimate of the velocities in a similar manner, provided we somewhere know the depth and velocity, and especially if we know the depth at two points a considerable distance apart and the velocity at one of them, or know the velocities at the two points and the depth at one of them.

If any filament of water in an open channel be considered, it is readily seen that the forces which drive along an elementary mass, gravity directly and pressure (gravity indirectly), are the same as the force along the surface, directly above the element and which is direct gravity alone, viz., $g \cos \phi$ per unit mass, where ϕ is the angle between the surface and the nadir. Consequently, in the motion of an element, the pressure at each end of the element may be regarded as one and the same; and so the pressure terms may be omitted from Bernoulli's theorem.

The differential equation thus becomes

$$v dv = -g dz - \zeta' \frac{P v^n}{2\Omega} ds, \quad (51)$$

and the same integrated between the limits $s=s_1, s=s_2$, becomes

$$\frac{v_1^2 - v_2^2}{2} = -g(z_1 - z_2) + \zeta' \frac{P}{2\Omega} v^n s. \quad (52)$$

Or by equation (87) Part IV A

$$\begin{aligned} v \frac{\partial v}{\partial s} &= -g \frac{\partial z}{\partial s} - k v^n, \\ \frac{1}{2}(v_1^2 - v_2^2) &= -g(z_1 - z_2) + k \int v^n ds \end{aligned} \quad (53)$$

where v is a function of s .

For any cross section the quantity of water passing in unit time must be constant.

$$\begin{aligned} \therefore \Omega v &= \text{constant} \\ \therefore \frac{d}{ds}(\Omega v) &= v \frac{d\Omega}{ds} + \Omega \frac{dv}{ds} = 0. \end{aligned} \quad (54)$$

If b denotes the constant breadth of the surface of the stream and h its variable depth, then

$$\frac{d\Omega}{ds} = b \frac{dh}{ds}, \quad (55)$$

and so

$$\Omega dv + v b dh = 0.$$

Let $-dz$ denote the fall of the surface in the distance ds , dh the increase in depth in the same distance, i the inclination of the bed, radian measure, taken as positive if the bed slope downward in the direction of the motion; then

$$ids = dh - dz,$$

and so the differential equation becomes

$$v dv = g(ids - dh) - \zeta' \frac{P v^n}{2\Omega} ds,$$

or

$$-v \frac{v b dh}{\Omega} = g(ids - dh) - \zeta' \frac{P v^n}{2\Omega} ds; \quad (56)$$

$$\frac{dh}{ds} = i \frac{1 - \zeta' \frac{P v^n}{2\Omega g i}}{1 - \frac{v^2 b}{\Omega g}}. \quad (57)$$

Putting $n=2$,

$$v^2 = \frac{i - \frac{dh}{ds}}{\zeta' \frac{P}{2\Omega g} - \frac{b}{\Omega g} \frac{dh}{ds}} \quad (58)$$

If the channel be of uniform depth as well as breadth

$$\frac{dh}{ds} = 0,$$

and so

$$i = \zeta' \frac{P v^n}{2\Omega g} \quad (59)$$

is the equation for steady flow along a uniform channel.

Let H and V refer to the principal portion of the stream where the course is uniform. Suppose the stream to be very wide in comparison with its depth. Then

$$v^2 = \frac{H^2}{h^2} V^2$$

and from (59), putting $n=2$,

$$V^2 = \frac{2\Omega i g}{\zeta' P} = \frac{2H i g}{\zeta'}; \quad (60)$$

$$\therefore v^2 = \frac{H^2}{h^2} \frac{2g i}{\zeta'};$$

$$\therefore \frac{dh}{ds} = i \frac{1 - \left(\frac{H}{h}\right)^3}{1 - \frac{2i}{\zeta'} \left(\frac{H}{h}\right)^3}.$$

By integration s can be expressed in terms of h and $\frac{h}{H}$. The expression involves what has been styled the backwater function.

When i is assumed to be very small, the sign of $\frac{dh}{ds}$ will depend upon that of the numerator. Consequently, if $h > H$ the depth increases in going downstream from where the depth approximates H to where it equals h , and according to the law involved in the above equation. Similarly for $h < H$, the depth decreases in going downstream.*

11. Resistance of impact.

That portion of the force of impact which depends on the component of the centrifugal force of the moving particles can be easily seen to vary as the square of the velocity if two different scales of velocity are compared.

*The following are a few references to the backwater or *rémou*; Bennett's translation of D'Aubuisson de Voisius' *Treatise on Hydraulics* (Boston, 1852), secs. 161-174; Rühlmann, *Hydro-mechanik*, 2d ed., under "Stauweite," secs. 155-159; Merriman, *A Treatise on Hydraulics*, under "Backwater;" Bovey, *A Treatise on Hydraulics*, 2d ed., secs. 13-17.

In computing this force, the velocity of a particle is supposed to be unaltered by the presence of the submerged object, although eventually it will be reduced in value, because after leaving the front surface of the object the particle moves obliquely to the general direction of the stream.

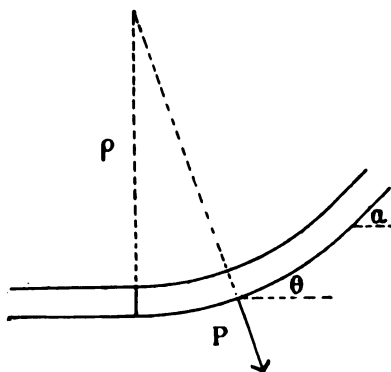
12. To find the dynamical pressure in the original direction of motion upon a curved pipe.

The centrifugal force of a body of mass M moving with velocity v is

$$\frac{Mv^2}{\rho},$$

where ρ denotes the radius of curvature of the path.

No. 1.



At a point P the centrifugal force of an elementary mass is $dM \frac{v^2}{\rho}$; and the component directed along the x -axis is

$$dM \frac{v^2}{\rho} \sin \theta.$$

$$\text{Total force} = \int_{s=s_1}^{s=s_2} \frac{v^2}{\rho} \sin \theta dM, \quad (61)$$

$$s_2 - s_1 = l, \quad dM = \frac{M}{l} ds, \quad ds = \rho d\theta.$$

$$\begin{aligned} \therefore \text{total force} &= \int_{\theta=0}^{\theta=\alpha} \frac{v^2}{\rho} \sin \theta \frac{M}{l} \rho d\theta, \\ &= \frac{v^2 M}{l} (1 - \cos \alpha). \end{aligned} \quad (62)$$

If the pipe had the velocity v' , the total force would become

$$\frac{(v-v')^2 M}{l} (1 - \cos \alpha).$$

because the water during its time of exerting pressure has an x -velocity of $v-v'$ relatively to the pipe.

$$M = l \cdot \text{area} \cdot \text{density} = l \cdot \text{area} \cdot \frac{\gamma}{g};$$

$$\therefore \text{total force} = (v-v')^2 \cdot \text{area} \cdot \frac{\gamma}{g} (1 - \cos \alpha). \quad (63)$$

Hence the total longitudinal pressure is independent of the length or law of curvature of the curved pipe.

This evidently applies to a jet impinging upon a surface of revolution, either stationary or moving in the direction of the line of motion of the jet. For, the jet in the region where curvature takes place can be divided into tubes of flow.

Applying equation (63) to a stationary plate whose direction is perpendicular to the direction of the stream, the force of resistance is

$$v^2 \frac{\gamma}{g} \cdot \text{area of jet}.$$

If the area of the jet be the same as that of the plate, this formula becomes

$$v^2 \frac{\gamma}{g} \Omega,$$

a value much in excess of the true resistance, because only a small portion of the approaching column of water whose cross-sectional area is Ω is turned or deflected through the angle of 90° . If the average angle of deflection were 60° , the factor $(1 - \cos \alpha)$ of (63) would equal $\frac{1}{2}$.

If a column of water of cross-section Ω moving with velocity v could have this velocity entirely destroyed, and without interfering with neighboring stream lines, the energy consumed per unit time would be

$$\Omega v \frac{\gamma}{g} \frac{v^2}{2}.$$

If the pressure Ωp could act through the distance traveled in one time unit it would do, per unit time, work represented by $\Omega p v$;

$$\therefore \Omega p v = \Omega v \frac{\gamma}{g} \frac{v^2}{2},$$

$$p = \frac{\gamma}{g} \frac{v^2}{2}.$$

This being the resistance per unit area the total resistance will, upon the above hypothesis, be

$$\Omega \frac{\gamma}{g} \frac{v^2}{2}. \quad (64)$$

Or, if h denote the height a body must fall to acquire the velocity v , this expression is the statical pressure of a column of water whose height is h .

Kirchoff has calculated this resistance upon an immersed plate for stream-line two-dimensional flow, and found it to be

$$\frac{\pi}{4+\pi} \Omega \frac{\gamma}{g} v^2. \quad (65)$$

If the plate is inclined to the stream at an angle α , $\frac{\pi}{4+\pi}$ is replaced by $\frac{\pi \sin \alpha}{4+\pi \sin \alpha}$.*

The dynamical pressure of a jet striking against a plane perpendicular to its direction is by (63)

$$v^2 \cdot \text{area of jet} \cdot \frac{\gamma}{g}, \quad (66)$$

and this must be in excess of the true value when the plane is no larger than the cross section of the jet, because the threads of the liquid are deflected, as a rule, much less than 90° . In case of a plane immersed in a stream of indefinite extent, the normal impulse must be less than the above expression, for reasons just stated. The experiments of du Buat and Thibault gave $\zeta = 1.86$ in the formula

$$\zeta \frac{v^2}{2} \frac{\gamma}{g} \Omega. \quad (67)$$

Comparatively recent determinations give to ζ values between 1.25 and 1.75.

It is generally assumed that the resistance or force of impact is the same whether the body moves through the water or the water impinges upon the body held stationary. In case of a plate wholly submerged and held perpendicular to the lines of motion, it is evident that the greater resistance will occur in the case where the plate is stationary. For, in this case, on account of the discontinuous motion and the inertia of the impinging water, a considerable amount of dead water will lie behind the plate; the stream lines will be prevented from so closing in behind as to somewhat resemble their appearance in front of the plate; that is, the impinging force due to the curvature of an elementary stream in front of the plate may not to any considerable extent be offset by a similar force in the rear. But if the plate moves through the water, the streams around its edges continually turn inward, thus preventing the occurrence of a void, and later give rise to a force urging the plate forward.

Wherever discontinuous motion occurs, it is not probable that the resistance determined by moving the body will be equivalent to that obtained by assuming it fixed and the water in motion.

13. *Resistance for cylindrical bodies.*

Experiments made by myself for the force of impact upon steel wire and rods varying in diameter from 0.036 to 0.5 inch, and with the velocity of the water ranging from 1 to $1\frac{1}{2}$ feet per second, showed that the force is well represented by the expression

$$\zeta \frac{v^2}{2} \frac{\gamma}{g} l d \text{ or } \zeta \frac{v^2}{2} \frac{\gamma}{g} \Omega \quad (68)$$

where the abstract number $\zeta = 0.95$.

* Rayleigh: Collected Works, Vol. I, pp. 287-296; or Phil. Mag., Vol. II (1876), pp. 430-441. Lamb: Hydrodynamics, 2d ed., article 77.

If

$$Z = cz + a^2 \frac{c}{z},$$

then

$$\begin{aligned} X &= c \left(r + \frac{a^2}{r} \right) \cos \theta, \\ Y &= c \left(r - \frac{a^2}{r} \right) \sin \theta. \end{aligned} \quad (69)$$

$Y = \text{constant}$, denotes the stream lines for steady two-dimensional flow past a circular cylinder placed transversely to the stream. For, if $Y = 0$, either $\theta = 0$ or $r = a$. This is the case where the stream line breaks up into a straight line and circle. If $Y = a$ large constant, r becomes large in comparison with $\frac{a^2}{r}$, and we eventually have

$$r \sin \theta = \text{the large constant}$$

which gives the equation of a straight line parallel to x -axis.*

By assuming only statical pressure on approximately the rear half of the cylinder, it is possible to make some estimate of the pressure due to motion on the remaining surface. For this purpose consider the greatest deflection experienced by each tube of flow.

The central one is deflected 90° ; the one originally in front of the extreme edge of the cylinder is deflected about $21\frac{1}{2}^\circ$.

The longitudinal force of each tube of flow is

$$v^2 \cdot \text{cross section of tube} \cdot \frac{\gamma}{g} (1 - \cos \alpha) \quad (70)$$

The longitudinal thrust of all tubes of flow thus computed divided by the area of the diametrical plane of the cylinder gives $\zeta \frac{v^2}{2} \frac{\gamma}{g}$, from which ζ can be determined, v now denoting the general velocity of the stream before being influenced by the presence of the cylinder.

Since the resistance of impact concerns the front half of the cylinder, it is of interest to note that the resistance upon planes whose traces form a half hexagon inscribed in a semicircle is, when computed by Kirchhoff's formula (65) nearly equal to the observed resistance of the cylinder.

A similar method is roughly applicable to a sphere. The observed value of ζ for a sphere lies, according to Weisbach, between 0.5 and 0.6, the resistance being

$$\zeta \frac{v^2}{2} \frac{\gamma}{g} \Omega, \quad (71)$$

Ω denoting the area of a great-circle section.

* See Lamb: Hydrodynamics, 2d ed., sec. 68.

14. *The transporting power of water.*

If water impinges upon two solids of like densities and geometrically similar, the forces just sufficient to move them along the horizontal bed of a stream are proportional to their weights, and so to the cubes of their homologous linear dimensions, l_1, l_2 .

$$\frac{(\text{Force required})_1}{(\text{Force required})_2} = \frac{l_1^3}{l_2^3}$$

By sections 12 and 13 the ratio of the forces of impact is

$$\frac{(\text{Force of impact})_1}{(\text{Force of impact})_2} = \frac{v_1^2 l_1^2}{v_2^2 l_2^2} = \frac{v_1^2}{v_2^2} \frac{(\text{Force required})_1}{(\text{Force required})_2}. \quad (72)$$

If the solids just move, the force of impact must equal the required force, and so

$$\frac{(\text{Force})_1}{(\text{Force})_2} = \frac{v_1^6}{v_2^6}. \quad (73)$$

15. *Obstruction caused by sudden enlargement.*

The momentum possessed by a mass of water which passes a fixed point near the mouth of a small pipe in unit time is $\Omega_1 v_1 \frac{\gamma}{g} \cdot v_1$; the momentum of a mass passing a fixed point of the large pipe in the same time is $\Omega_2 v_2 \frac{\gamma}{g} \cdot v_2$. Since the motion is steady, the momentum between the two elementary masses considered remains constant in time.

The loss of momentum per unit-time mass occasioned by the sudden increase in the size of the pipe is therefore

$$\Omega_1 v_1^2 \frac{\gamma}{g} - \Omega_2 v_2^2 \frac{\gamma}{g} = \Omega_1 \frac{\gamma}{g} v_1 (v_1 - v_2) = \Omega_2 v_2 \frac{\gamma}{g} (v_1 - v_2), \quad (74)$$

since by the condition of continuity $\Omega_1 v_1 = \Omega_2 v_2$. The pressure in the small pipe is p_1 , that in the large one, not too near the mouth of the small pipe, is p_2 . Since increase of pressure does not appear until the velocity of the stream becomes diminished (because $\frac{p}{\gamma} + \frac{v^2}{2g} = \text{constant}$), and since the pressure in dead water can not differ much from that in the neighboring stream, it may be assumed that for the upper end of the large pipe the pressure equals p_1 rather than p_2 . The accelerating resultant pressure is

$$p_1 \Omega_1 + p' (\Omega_2 - \Omega_1) - p_2 \Omega_2,$$

or

$$(p_1 - p_2) \Omega_2,$$

where p' is put equal to p_1 .

This resultant force acting upon the water of this region does, during each unit of time, increase the momentum of a unit-time mass from what it possessed upon entering to what it possesses upon leaving.

$$\therefore (p_1 - p_2) \Omega_2 = -\Omega_2 v_2 \frac{\gamma}{g} (v_1 - v_2). \quad (75)$$

The energy after the shock is therefore, per unit mass,

$$\frac{p_2 - p_1}{\gamma} = \frac{v_1(v_1 - v_2)}{g}.$$

For motion without loss of energy

$$\frac{p_2'' - p_1}{\gamma} = \frac{v_1^2 - v_2^2}{2g}$$

since the pressure head (p_1/γ) is assumed to be the same in both cases. The second member of this equation exceeds that of the preceding by

$$\frac{(v_1 - v_2)^2}{2g},$$

or since $\Omega_2 v_2 = \Omega_1 v_1$,

$$\frac{v_1^2}{2g} \left(1 - \frac{\Omega_1}{\Omega_2}\right)^2$$

is the loss of energy per unit mass, and this is equal to the loss of pressure head at the lower cross section (i. e. $\frac{p_2 - p_2''}{\gamma}$) because of shock. The factor $\left(1 - \frac{\Omega_1}{\Omega_2}\right)^2$ is Borda's coefficient of resistance.

16. Hydraulic coefficients.

Through orifices the amount of the theoretical discharge is diminished by a factor styled the coefficient of discharge. This is the product of the coefficient of velocity times the coefficient of contraction or *vena contracta*. For orifices, the coefficient of velocity is nearly equal to unity, especially for high velocities, and the coefficient of discharge about 0.6. The value diminishes slightly as the size of the orifice is increased and shows that the motion is nearly but not exactly geometrically similar for various velocity scales.

As noted in section 20, Part IV A, the coefficient of contraction for a two-dimensional stream is $\pi/(2 + \pi) = 0.611$.

Bovey considers the coefficient of contraction to be approximately 0.64 for sharp-edged orifices of any form and nearly unity for those perfectly rounded.*

At the mouth of a projecting tube or mouthpiece the velocity is theoretically that due to height. Consequently if the mouthpiece be divergent the velocity at a narrow section will exceed that due to height.

This has been shown by Bernoulli, Venturi, Eytelwein, and Francis. The increased velocity will not be realized in vacuo, nor if an open channel take the place of the tube or mouthpiece. Hence the flow along an open short channel connecting two bodies of water can never from this cause exceed the velocity due to difference between the surface levels even at the most contracted part of the channel.

17. A compound vessel.

A vessel is supposed to be divided into several compartments by means of vertical partitions through each of which is a small opening. At one end of the vessel and near the bottom is an opening through which the liquid escapes into the air. The sur-

* Bovey; A Treatise on Hydraulics, 2d ed., rewritten, sec. 12.

face of the first compartment is maintained at a fixed height by continually supplying the liquid, and all other surfaces will eventually assume certain heights. Let these heights be $h_1, h_2, h_3, \dots, h_l$ reckoned from the center of the lowest orifice. Let $A_1, A_2, A_3, \dots, A_l$ denote areas of the free liquid surfaces in the several compartments and $\Omega_1, \Omega_2, \Omega_3, \dots, \Omega_l$ the effective areas of the several small openings in the partitions.

Given the $n+1$ quantities $h_1, \Omega_1, \Omega_2, \Omega_3, \dots, \Omega_l$ to find the $2n-1$ quantities $v_1, v_2, v_3, \dots, v_l; h_2, h_3, \dots, h_l$. The relations between the v 's and h 's of the form

$$v_1^2 = 2g(h_1 - h_2), v_2^2 = 2g(h_2 - h_3), v_3^2 = 2g(h_3 - h_4), \dots, v_l^2 = 2gh_l \quad (76)$$

are n in number; and between the v 's and Ω 's

$$v_1^2 \Omega_1^2 = v_2^2 \Omega_2^2 = v_3^2 \Omega_3^2 = \dots = v_{l-1}^2 \Omega_{l-1}^2 = v_l^2 \Omega_l^2$$

are $n-1$: all relations thus number $2n-1$.

From the first set we have

$$v_1^2 + v_2^2 + v_3^2 + \dots + v_l^2 = 2gh_1;$$

and from the second set

$$\frac{v_1^2}{v_l^2} = \frac{\Omega_l^2}{\Omega_1^2}, \frac{v_2^2}{v_l^2} = \frac{\Omega_l^2}{\Omega_2^2}, \frac{v_3^2}{v_l^2} = \frac{\Omega_l^2}{\Omega_3^2}, \dots, \frac{v_{l-1}^2}{v_l^2} = \frac{\Omega_l^2}{\Omega_{l-1}^2}.$$

Substituting these values of $v_1, v_2, v_3, \dots, v_{l-1}$, we have

$$v_l = \frac{\sqrt{2gh_1}}{\sqrt{1 + \frac{\Omega_l^2}{\Omega_1^2} + \frac{\Omega_l^2}{\Omega_2^2} + \frac{\Omega_l^2}{\Omega_3^2} + \dots + \frac{\Omega_l^2}{\Omega_{l-1}^2}}} \quad (77)$$

From v_l any other v follows from the equations just written, and two adjacent v 's will by aid of (76) determine any required h .

The coefficient

$$\frac{1}{\sqrt{1 + \frac{\Omega_l^2}{\Omega_1^2} + \dots}} \quad (78)$$

depends only on the ratios Ω_l/Ω_1 , etc., and is independent of the velocity, agreeing in this respect with the other coefficients of resistance. The energy lost is converted into heat in the several compartments and the body into which the final discharge takes place.

If the constrictions consist of well-rounded capes, so that no energy is lost, we have

$$v_1^2 - v_0^2 = 2g(h_1 - h_2), v_2^2 - v_1^2 = 2g(h_2 - h_3), v_3^2 - v_2^2 = 2g(h_3 - h_4), \dots, \\ v_l^2 - v_{l-1}^2 = 2gh_l; \\ \therefore v_l^2 - v_0^2 = 2gh_1;$$

or

$$v_l = \sqrt{2gh_1},$$

if

$$v_0 = 0.$$

18. *The principle of similitude.**

It is assumed that two configurations of matter, or two mechanisms, are to be geometrically similar at certain times; also that the masses of the corresponding moving parts have a constant ratio to one another; also the motion is sustained or governed by forces which have a constant ratio to one another in the two cases.

Let the size of the second configuration be l times that of the first; the times in passing from one position to a similar position be τ times as great in the second as in the first; the corresponding masses μ times as great; and the corresponding impressed forces F times as great.

In the indeterminate equation of motion

$$\sum \left\{ \left(X - m \frac{d^2 x}{dt^2} \right) \delta x + \left(Y - m \frac{d^2 y}{dt^2} \right) \delta y + \left(Z - m \frac{d^2 z}{dt^2} \right) \delta z \right\} = 0 \quad (79)$$

we can suppose that X, Y, Z are the components of the impressed moving forces on the element or particle because that portion of the impressed forces which is balanced by the reactions can be associated with no displacements other than zero which are compatible with the connections of the system.

Substitute for X, Y, Z, x, y, z, m , and t , the quantities $FX, FY, FZ, lx, ly, lz, \mu m, \tau t$ in the indeterminate equation.

The equation will be identical with (79), provided

$$\mu l = F \tau^2. \quad (80)$$

Kepler's third law.—For the motions of two planets the equation

$$F \tau^2 = \mu l$$

becomes, because $F = \mu l^2$, or $F = \frac{\mu M}{l^2}$ if the planets have different central bodies whose mass ratio is M ,

$$\tau^2 \frac{\mu M}{l^2} = \mu l. \quad (81)$$

Resistances of model and ship.—Assuming that the resistance is proportional to the area and the square of the velocity; also, that the density of both bodies is the same. The first question is, what velocity will satisfy the requirement

$$F \tau^2 = \mu l?$$

Let v denote velocity ratio; then, by hypothesis,

$$F = v^2 l^2, \quad \mu = l^3;$$

$$\therefore v^2 l^2 \tau^2 = l^4.$$

This gives

$$v \tau = l.$$

Now assume

$$v = \sqrt{l}.$$

$$\therefore F = \frac{\mu l}{\tau^2} = \frac{l^4}{l^2} v^2 = l^2 v^2 = l^3. \quad (82)$$

* See Routh; *Elementary Rigid Dynamics*, sec. 367.

That is, if we use a velocity for the model such that the ship's shall be \sqrt{l} times as great, the total resistance of the ship will be l^3 times that of the model.

This is known as Froude's theorem.

Torricelli's theorem.—Imagine two vessels of like form (orifice included) to contain a like portion of their total volumes of liquid. Then

$$F\tau^2 = \mu l$$

becomes, since the ratio of the impressed force is $g' l^3$, g' denoting the ratio of gravity in the two cases,

$$\begin{aligned} g' l^3 \tau^2 &= l^3 l, \\ \tau &= \sqrt{\frac{l}{g'}}; \end{aligned} \quad (83)$$

\therefore the times of discharging the n th part of the vessels varies directly as the square root of their linear dimensions. Cf. § 9, Part IV A. Under ordinary circumstances g' is very nearly unity. If one liquid were more dense than the other, τ would not be altered, because F and μ would both be altered by the same factor for density.

Long wave motion.—Consider the case of the free oscillation of two shallow sheets of water having the horizontal dimensions proportional, but with any uniform depths whose ratio is d .

Assume amplitudes proportional to depths, which we may do on the principle that the periods of small oscillations are independent of amplitudes. This makes it easy to compare (accelerating) impressed forces.

By section 11, Part IV A, the force of restitution is $g \times \text{slope} \times \text{density} \times \text{volume}$; the mass is density \times volume.

$$\begin{aligned} F\tau^2 &= \mu l \\ \text{becomes} \quad g' \frac{d}{l} \cdot d l^3 \tau^2 &= l^3 d l; \\ \tau^2 &= \frac{l^2}{g' d}, \\ \tau &= \frac{l}{\sqrt{g' d}}. \end{aligned} \quad (84)$$

\therefore The periodic times of the motions are directly as the ratio of the horizontal dimensions of the bodies and inversely as the square root of the ratio of the depths. g' , the ratio of the forces of gravity, may be taken as unity.

Supposing that for small motions, equation 94, Part IV A,

$$\frac{\partial^2 \zeta}{\partial t^2} = g h \left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right),$$

represent the motion in one body of water; similar motion in the second body will take place provided

$$\frac{d'}{\tau^2} \frac{\partial^2 \zeta}{\partial t^2} = g' d' g h \left(\frac{d'}{l^2} \frac{\partial^2 \zeta}{\partial x^2} + \frac{d'}{l^2} \frac{\partial^2 \zeta}{\partial y^2} \right), \quad (85)$$

where d' denotes the amplitude ratio, reduces to the above equation. This it will do, provided

$$\tau = \frac{l}{\sqrt{g' d}}. \quad (86)$$

If the depths of the two bodies be not uniform, but are characterized by a fixed ratio d , the equations

$$\frac{\partial^2 \zeta}{\partial t^2} = g \left\{ \frac{\partial}{\partial x} \left(h \frac{\partial \zeta}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial \zeta}{\partial y} \right) \right\}, \quad * (87)$$

$$\frac{d'}{r^2} \frac{\partial^2 \zeta}{\partial t'^2} = g g' \left\{ \frac{dd'}{l^2} \frac{\partial}{\partial x} \left(h \frac{\partial \zeta}{\partial x} \right) + \frac{dd'}{l^2} \frac{\partial}{\partial y} \left(h \frac{\partial \zeta}{\partial y} \right) \right\}, \quad (88)$$

become identical if

$$\tau = \frac{l}{\sqrt{g'd}}.$$

In long-wave motion the vertical and horizontal scales may be anything whatever and the character motion will not be altered. The periods will be inversely as the square roots of the depths.

The only requirement is that the motion at all points be sensibly horizontal; that is, if considerable slopes anywhere exist, the motion must there be small.

19. *To change a formula expressed in certain units into one expressed in other units.*

Ascertain the dimensions of every numerical term or coefficient or factor by aid of the formula itself. Multiply these numerical quantities by the unit ratios raised to powers indicated by the dimensions of the factors of the several terms. By unit ratios are meant quotients obtained by dividing the magnitudes of the units used in the original formula by the magnitudes of the units to be used in the transformed formula.

For example, Torricelli's theorem might be written

$$v^2 = 19.62h,$$

where the meter is the unit of length and g is assumed to be 9.81. Now the numerical coefficient is of dimension +1 in length. \therefore assuming a meter = 3.28 feet, 19.62 must be multiplied by this quantity, thus giving

$$v^2 = 64.3h.$$

But if we write

$$v^2 = 2gh,$$

then the numerical coefficient 2 is of dimension zero, and so to pass from meters to feet, 2 must be multiplied by $(3.28)^0 = 1$, and the formula remains

$$v^2 = 2gh$$

for all units. Here the dimensions in time do not have to be considered because the second is the unit in both cases.

If c of Chézy's formula,

$$v = c\sqrt{mi}, \quad (89)$$

be taken as 90 when the foot is the unit of length, it becomes $\left(\frac{1}{3.28}\right)^{\frac{1}{2}} \times 90 = 50$ when the meter is the unit.

* Lamb: Hydrodynamics, sec. 189.

CHAPTER II.

CONSIDERATIONS OF DIMENSION AND RESISTANCE IN TIDAL WAVES.

20. *Tides in a canal encircling the earth along a parallel of latitude, friction being taken into account.*

Since the tide in a uniform canal coinciding with a parallel of latitude must be periodic in time and distance, we may write

$$\xi = A \cos (a't - l'x + \alpha), \quad (90)$$

where x and ξ are reckoned westward.

By equation (10), Part IV A, the westward tidal force may be written

$$H \sin (a't - l'x), \quad (91)$$

where the origin of x is the initial meridian for time as well as distance.

The dynamical equation to be satisfied is

$$\frac{\partial^2 \xi}{\partial t^2} + \mu \frac{\partial \xi}{\partial t} - gh \frac{\partial^2 \xi}{\partial x^2} = H \sin (a't - l'x). \quad (92)$$

$$\text{while } \zeta = -h \frac{\partial \xi}{\partial x}.$$

Upon substituting for ξ in (92) its value (90) and equating the entire coefficient of $\sin (a't - l'x)$ and of $\cos (a't - l'x)$ to zero, we have

$$\tan \alpha = \frac{a'\mu}{a'^2 - gh l'^2}, \quad (93)$$

$$A = \mp \frac{H}{[(a'^2 - gh l'^2)^2 + a'^2 \mu^2]^{1/2}}, \quad (94)$$

where the upper or lower sign is to be used according as $a'^2 - gh l'^2$ is positive or negative.

$$\therefore \xi = \mp \frac{H}{[(a'^2 - gh l'^2)^2 + a'^2 \mu^2]^{1/2}} \sin \left(a't - l'x + \tan^{-1} \frac{a'\mu}{a'^2 - gh l'^2} \right), \quad (95)$$

$$\zeta = \mp \frac{H h l'}{[(a'^2 - gh l'^2)^2 + a'^2 \mu^2]^{1/2}} \cos \left(a't - l'x + \tan^{-1} \frac{a'\mu}{a'^2 - gh l'^2} \right). \quad (96)$$

By section 9, Part I, these may be written in the form

$$\xi = - \frac{H}{(a'^2 - gh l'^2)^2 + \mu^2 a'^2} [(a'^2 - gh l'^2) \sin (a't - l'x) + \mu a' \cos (a't - l'x)], \quad (97)$$

$$\zeta = - \frac{H h l'}{(a'^2 - gh l'^2)^2 + \mu^2 a'^2} [(a'^2 - gh l'^2) \cos (a't - l'x) - \mu a' \sin (a't - l'x)]. \quad (98)$$

When $a't - l'x = 0$, the tidal body is on the meridian of the place. For moderately deep water $a'^2 - gh l'^2$ is positive, and for very deep water, negative. Upon referring to (96) and (98) it will be seen that in the case of moderately deep water the argument or angle of ζ at the time of transit of the tidal body is greater than zero. Hence, low water occurs a little before the time of transit. In case of very deep water the angle of ζ is less than zero, and so high water will occur a little after the time of transit.

For an equatorial canal, $l' = a' \sqrt{gh}$, and for one following a parallel of latitude,

$$l' = \frac{a'}{\cos \lambda} \sqrt{gh}.$$

The more general case, where the canal follows any circle great or small drawn upon the earth, has been discussed by Levy in sections 96-101 of his treatise on tides.

21. *Tides in a canal closed at both ends whose waters are acted upon by a periodic force.*

From equation (320), Part IV A,

$$\zeta = \frac{hf}{a'\sqrt{gh} \cos \frac{a'L}{\sqrt{gh}}} \sin \frac{a'(x-L)}{\sqrt{gh}} \cos (a't + \alpha) \quad (99)$$

where the impressed periodic force is $f \cos (a't + \alpha)$.

Let l' be written for $a' \sqrt{gh}$. This equation becomes

$$\zeta = \frac{f}{g l' \cos l' L} \sin (l' x - l' L) \cos (a't + \alpha) \quad (100)$$

Consider the tide at the far end of the canal (where $x=2L$) at the time when the force has its maximum value in the direction $+x$; then $\sin (l' x - l' L)$ becomes $\sin l' L$ and $\cos (a't + \alpha)$, unity.

$$\therefore \zeta = \frac{f}{g l'} \tan l' L = \frac{f}{g} \frac{\sqrt{gh}}{a'} \tan l' L. \quad (101)$$

In case of M_2 , $f=0.000\ 000\ 076\ 5\ g$, $a'=0.000\ 140\ 519$ radian per second.

$$\therefore \zeta = 0.003\ 088 \sqrt{h} \tan l' L \quad (102)$$

The equilibrium tide is

$$0.000\ 000\ 076\ 5\ L \quad (103)$$

Suppose λ denote the length of wave due to depth h and whose period is τ , i. e., $\lambda = \tau \sqrt{gh}$. Suppose the length of the canal to vary from nothing to λ , the intermediate length being fractions of λ . The following table shows the value of ζ from (102). For a depth of 10 000 feet, $\sqrt{h} = 100$ feet:

Height of tide.

Length = $2L$											
$0 \frac{0}{360} \lambda$	$20 \frac{20}{360} \lambda$	$40 \frac{40}{360} \lambda$	$60 \frac{60}{360} \lambda$	$80 \frac{80}{360} \lambda$	$100 \frac{100}{360} \lambda$	$120 \frac{120}{360} \lambda$	$140 \frac{140}{360} \lambda$	$160 \frac{160}{360} \lambda$	$170 \frac{170}{360} \lambda$	$175 \frac{175}{360} \lambda$	$179 \frac{179}{360} \lambda$
$1 \frac{1}{h}$ 0.000 0000	$1 \frac{1}{h}$ 0.000 5 445	$1 \frac{1}{h}$ 0.00 1 124	$1 \frac{1}{h}$ 0.00 1 783	$1 \frac{1}{h}$ 0.00 2 591	$1 \frac{1}{h}$ 0.00 3 680	$1 \frac{1}{h}$ 0.00 5 349	$1 \frac{1}{h}$ 0.00 8 484	$1 \frac{1}{h}$ 0.0 1 751	$1 \frac{1}{h}$ 0.0 3 530	$1 \frac{1}{h}$ 0.0 7 073	$1 \frac{1}{h}$ 0. 3 539
$180 \frac{180}{360} \lambda$	$181 \frac{181}{360} \lambda$	$185 \frac{185}{360} \lambda$	$190 \frac{190}{360} \lambda$	$200 \frac{200}{360} \lambda$	$220 \frac{220}{360} \lambda$	$240 \frac{240}{360} \lambda$	$260 \frac{260}{360} \lambda$	$280 \frac{280}{360} \lambda$	$300 \frac{300}{360} \lambda$	$320 \frac{320}{360} \lambda$	$340 \frac{340}{360} \lambda$
$\pm 1 \frac{1}{h}$ α	$-1 \frac{1}{h}$ 0. 3 539	$-1 \frac{1}{h}$ 0.0 7 073	$-1 \frac{1}{h}$ 0.0 3 530	$-1 \frac{1}{h}$ 0.0 1 751	$-1 \frac{1}{h}$ 0.00 8 484	$-1 \frac{1}{h}$ 0.00 5 349	$-1 \frac{1}{h}$ 0.00 3 680	$-1 \frac{1}{h}$ 0.00 2 591	$-1 \frac{1}{h}$ 0.00 1 783	$-1 \frac{1}{h}$ 0.00 1 124	$-1 \frac{1}{h}$ 0.000 5 445

These results should apply fairly well to hypothetical terrestrial canals whose lengths are not greater than, say, 45° of a great circle, if all resistance could be left out of consideration. The value of the sustaining forces taken in the direction of the canal, can be ascertained from sections 1, 2, and Fig. 1, Part IV A, and as a rule only the value at the middle of the canal need be computed.

When the force is variable over the length of the canal, a progressive wave will generally accompany the stationary wave unless the canal lies along a meridian.

22. *Tides in a canal which extends along a meridian from the Equator to either pole.*

By equation (12), Part IV A, the tidal forces for a semidaily tide and acting in a northerly direction may be written

$$-f \cos \lambda \sin \lambda \cos (a't + \alpha) \quad (104)$$

where λ denotes north latitude. The dynamical equation becomes

$$\frac{\partial^2 \xi}{\partial t^2} = gh \frac{\partial^2 \xi}{\partial x^2} + f \cos \frac{x}{r} \sin \frac{x}{r} \cos (a't + \alpha) \quad (105)$$

r denoting the radius of the earth, and x the distance north of the Equator. The equation of continuity is, of course,

$$\zeta = -h \frac{\partial \xi}{\partial x}.$$

The values of ξ and ζ satisfying these equations are

$$\xi = -\frac{1}{2} \frac{r^2 f}{4gh - a'^2 r^2} \sin 2 \frac{x}{r} \cos (a't + \alpha), \quad (106)$$

$$\zeta = \frac{fhr}{4gh - a'^2 r^2} \cos 2 \frac{x}{r} \cos (a't + \alpha), \quad (107)$$

as is easily seen upon substituting the value of ξ in (105).

Hence, the wave is stationary. If the canal have great depth, equation (107) shows that when the tidal body is on meridian, it is high water in latitudes below 45° and low water in higher latitudes. If the canal have ordinary oceanic depth, this rule is reversed.

The general problem of finding the tides in canals, whether circular, closed at one end, or at both ends, friction being taken into account, has been treated by Airy in subsection 6 of his *Tides and Waves*. Similar matters are treated by Ferrel in Chapter IV of his *Tidal Researches*, and by Lévy in Chapter VIII of his treatise on tides.

The problem of finding the tide in a canal of any length bounded at both ends and whose waters are acted upon by a force periodic in time and varying or not varying over the canal, although a special case of the more general problem, is nevertheless one of considerable difficulty. For a solution, see Airy's *Tides and Waves*, article 337, and for numerical examples of east-and-west canals, see Ferrel's *Tidal Researches*, sections 144-149.

On account of the difficulties connected with problems of the class just mentioned and the questionability of the results obtained being applicable to the existing ocean tides, it seems best to here rest content with giving the above references.

23. Concerning the "ages" and coefficients of tidal inequalities.

In case of a compound pendulum oscillating in a resisting medium, it is easy to see that if the sustaining force is not exactly isochronous with its natural period, the departures in phase from the phase obtained upon the assumption of exact isochronism are proportional to the departure of speed from the critical speed.

For, in equation (297), Part IV A,

$$\tan \alpha_1 = \frac{C'' a' \lambda^2}{M(a^2 - a'^2) \lambda^2} \quad (108)$$

Here α_1 denotes the phase of the sustaining force (intensity) when the phase of the oscillation (displacement) is 180° , a' is the speed of the sustaining force, a that of the free pendulum. Suppose $a' = a + \varepsilon$; then

$$\tan \alpha_1 = \frac{C'' \lambda^2}{M(-2a\varepsilon)} \frac{(a + \varepsilon)}{\lambda^2} \doteq - \frac{C'' \lambda^2}{2M\varepsilon \lambda^2} \quad (109)$$

Now since the difference between the phase of the force and phase of the oscillation is about 90° , we may write

$$\tan \alpha_1 = \tan (90^\circ + E)$$

when E is a small angle. But

$$\tan (90^\circ + E) \doteq -\frac{1}{E}; \quad (110)$$

or

$$\varepsilon E = \text{constant};$$

and so the departure of the phase of a pendulum from 90° , or its critical value, is proportional to the departure of the natural "speed" of the pendulum from the "speed" of the sustaining force.

Equation (296), Part IV A,

$$A' = - \frac{F_1 \lambda \lambda_1 \sin \alpha_1}{C' a' \lambda^2} \quad (111)$$

shows that the amplitude of the oscillation varies but slowly on account of the variation in phase, provided α_1 lies near to $\pm 90^\circ$. Other things being equal, the amplitude of the oscillation is then very nearly proportional to the amplitude of the impressed force. The above expression may be written

$$\frac{A'}{F_1} = K \sin \alpha_1 = K \cos E \doteq K \left(1 - \frac{E^2}{2} \right), \quad (112)$$

showing that the defect in this proportionality is proportional to the square of small angle E or to the square $\varepsilon = a' - a$.

Suppose the amplitude of the vibration with resistance to be reduced thereby to μ' times the amplitude of the vibration without resistance.

From equation (298), Part IV A, the amplitude of a vibration without resistance is

$$\pm \frac{F_1 \lambda_1}{M(a'^2 - a^2) \lambda'} \quad (113)$$

Now, by hypothesis the amplitude with friction is to be μ' times this expression. For a pendulum resembling a simple pendulum as most pendulums do $\lambda = \lambda_1 = \bar{\lambda}$. The amplitude with resistance then becomes

$$A' = -\frac{F_1 \sin \alpha_1}{C' a'} = \pm \mu' \frac{F_1}{M(a'^2 - a^2)}; \quad (114)$$

also, from equation (294),

$$\cos \alpha_1 = \frac{A' M(a'^2 - a^2)}{F_1}; \quad (115)$$

$$\therefore \cos \alpha_1 = \pm \mu', \quad (116)$$

a result independent of a'/a ; i. e., α_1 depends only upon the ratio of the amplitude of the actual vibration to the amplitude of the theoretical one without resistance.

Suppose $\mu' = \frac{1}{2}$. Then, $\alpha_1 = 60^\circ$, or 120° ; that is, the force phase relatively to that of the displacement of the pendulum is within 30° of the value which it would assume were the length exactly critical and so resistance in absolute control.

24. Considering, first, only areas in which fairly large tides are produced; it will be seen that the values of S_1/M_1 , N_1/M_1 , $S_2^0 - M_2^0$, $M_2^0 - N_2^0$ (sec. 97, Part IV A; sec. 19, Part IV B) do not differ very greatly in going from place to place. Now, if the departure from the critical dimensions suited to the several semidiurnal components were a matter of prime importance, we would find places where one or more of the ratios of the amplitudes would be widely different from the ratios of the amplitudes of the forces, and where it would be reasonable to suppose one or more of the epoch differences would be approximately 180° instead of zero degrees. For, if the free period of an "area" were greater than the period of one of the components and less than the period of another component, the epochs of the tides with reference to the forces should differ by 180° . At any rate, this condition could be realized in comparing different areas.

From the table in section 21 it can be seen that in the case of no resistance, great variations in amplitude ratios should, in an area of nearly critical dimensions, result from slight variations in the speeds of the components. But, as already stated, such great variations do not occur in areas having fairly large tides. It follows that the resistance, including dissipation, must be paramount.

As already noted, the amplitude of the forced oscillations of a pendulum in a period closely approximating its natural period is nearly proportional to the sustaining force.

On the other hand, the existence of positive ages for most areas having good tides indicates that the difference in speed of the semidiurnal components is felt to a limited extent. That is, the epochs of two components are not equal to each other, nor do they differ by 180° ; but the epoch belonging to the faster component is generally a few degrees greater than the epoch belonging to the other, the amount in degrees, when two differences are compared, being roughly proportional to the speed differences.

This statement accords with what has already been shown with reference to a pendulum sustained by forces whose periods are nearly equal to the free period of the pendulum.

Now if the length (i. e., variation from the critical length) had no sensible effect upon the phase or epoch, it is doubtful if the phase would be altered in this manner by resistance.

To convince one that ages, if due to only resistance and variation in force intensity, must be smaller than those commonly found in nature, suppose, if possible, that the tide owes its existence to $2n$ successive impulses (n in either direction), each contributing equally to its formation. The amplitude of the tide at any given time will then be proportional, not to the forces acting at that time, but to the average value of the forces during the n preceding periods, and so approximately proportional to the force $\frac{n}{2}$ periods before the time of tide. But it is reasonable to suppose that the effect of the impulse nearest the time of tide is much greater than the effect of any one of the earlier impulses. Consequently the greatest tides must follow the greatest forces by a time or age not exceeding a small fraction of $\frac{n}{2}$ periods in length. For the semidaily tide it is probable that n periods do not exceed three or four days.

We are thus led to believe that the small departures of the periods of the components from the free period of a body are necessary in order that the resistances may cause the "ages" of the inequalities; but that such departures, unless considerable, generally have but a moderate influence upon the amplitude ratios.

If a component nearly fits an area, the phase of the tide will not be sufficiently altered to depart far from the phase determined upon the assumption of exact agreement between period of the component and free period of the body and where resistance controls.

In some cases, even where the range of the ocean tide is not remarkably small, the ratios S_2/M_2 , N_2/M_2 may differ greatly from the theoretical values. Similarly the values of $S_2^0 - M_2^0$, $M_2^0 - N_2^0$ may in some instances be almost anything.

These conditions may be brought about through the close correspondence in period between two or more modes of free oscillation and the components involved. This may occur because the outlines of the body are generally irregular and the dimensions not very definite. In fact, it is easy to imagine a rectangular area where the lines of motion in an oscillation suited to S_2 shall lie at right angles to the lines of motion suited to M_2 .

Along the southern coasts of Australia and around Lower California the ratio S_2/M_2 is unusually large. In such localities "dodging tides" may occur in extreme cases.

The ratio S_2/M_2 may be considerably too small in a well-defined area and where but one mode of oscillation could be expected, because the free period of the area differs considerably from the period of S_2 . An example of this occurs along the Atlantic coast of the United States. The coast of New Zealand and the Pacific coast of southern Chile are probably examples of this, but on account of the expanse of the Pacific Ocean there is a possibility of the S_2 -oscillation differing in mode from the M_2 -oscillation.

Where the ocean tide is small the ratios S_2/M_2 , N_2/M_2 may depart from their theoretical values or values where the tide is large; and the values of $S_2^0 - M_2^0$, $M_2^0 - N_2^0$ may

be almost anything, because the nodal line of a stationary oscillation for M_2 may not be the nodal line of a stationary oscillation for S_2 or N_2 . Near the nodal lines the above ratios would naturally vary, especially in a dependent fractional area. Examples of this are the tides at the mouth of the Bay of Bengal and near Portland, England.

If the cause of the small ocean tides can not be seen, it is reasonable to suppose that different modes of oscillation for different components account for many of these apparent irregularities. It may be that in many places of small tides the forced oscillations of bodies whose periods do not closely approach the semidaily period are of sufficient size to be comparable with the other tides there existing. But it is probable that the ratios in such oscillations if considered alone would approach the theoretical values and that the ages would be very short, thus approaching the conditions of equilibrium tides. In fact, there are doubtless basins in all oceans in which approximate equilibrium tides arise (particularly diurnal tides), but these are generally obscured by the derived waves from other sources.

Generally speaking, the bodies of water in which fairly large tides are generated approach more nearly to M_2 in the matter of free periods than to S_2 . Hence the theoretical hours laid down in Fig. 23, Part IV A, belong, as a rule, to the lunar rather than the solar tide. The dimensions of the "areas" generally indicate this, and the smallness of S_2/M_2 gives further confirmation. The values of N_2/M_2 being generally greater than the force ratio or 0.1936 for the Atlantic and Pacific coasts of the United States indicates, perhaps, that N_2 is fitted by the "areas" better than is M_2 . The large value of N_2/M_2 for Suez is probably due to the fact that the Gulf of Suez approaches nearer to a critical length ($\frac{1}{4}\lambda$) for the N_2 -wave than for the M_2 - or S_2 -wave. Owing to the rapidity with which the tidal impulses are destroyed through dissipation and friction, and also to the possibility of somewhat different modes of oscillation for the various semidaily tidal constituents, it is not reasonable to suppose that their amplitudes will be very nearly proportional to the forces, nor that their epochs will follow any simple law.

25. *On the dissipation or want of motion.*

Observation shows that ocean tides generally are too great to be produced by a single impulse of the tidal forces (as in the case of equilibrium tides); it shows the existence of fairly well-defined nodal lines and loops, thus establishing beyond all doubt the existence of stationary waves in certain oceanic areas. Moreover, the computed free periods for such areas generally approximate to the periods of the tidal forces; i. e., the areas have approximately critical dimensions.

Regarding this hypothesis as sufficiently well established, the questions now to be considered relate to the resistances which check or reduce the motion and affect the phase of the oscillation relatively to that of the forces.

In determining the time of elongation of the water particles, according to sections 62-67, Part IV A, it is assumed that the dimensions of the oscillating body are so nearly critical that the resistance is the important factor. It should have been noted earlier that equation (305), Part IV A, can be written in a more general form, thus adapting it to less restricted hypotheses. For example, the assumed mode of division of the body of water into elements might be quite different in the resistance term from the mode of division used in the other terms. That is, it is of no consequence how the

body is divided for resistance provided the resistance from all parts be finally added together. The resistance coefficient may be great in one locality and small in another; that is, C should take a subscript (ν or κ), and so, from what has been said, the last term of (305), Part IV A, may be written in the more general form

$$-Aa \sum_{\kappa} C_{\kappa} m_{\kappa} \sin^2 l_{\kappa} \sin at. \quad (117)$$

Evidently the rules laid down in the sections referred to for connecting the forces with the time of tide will not be altered by this generalization, but it becomes important when one attempts to look more closely into the nature of the resistance; in fact, it is obvious that the distribution of the resisting causes may vary greatly in different cases.

The term resistance will be used to denote the cause of all absence, diminution, or loss of motion, judged from what would have resulted under given sustaining forces had all boundaries been regular and complete and had the liquid been a perfect fluid.

The part of the resistance due to imperfect and incomplete boundaries may be called either "dissipation" or "virtual resistance;" while the part due to motion in the area only, such as friction and viscosity, is real or actual resistance.

For a body of water a little less than $\frac{1}{2}\lambda$ long having a complete regular boundary and being devoid of all resistance, the high water in either half of the body occurs when the horizontal force is greatest in the direction of the half considered. The wave, however, is the result of several or many impulses of these forces, and so the one most nearly coinciding with the time of a particular high water is responsible for only a small part of the motion. Each impulse acting in this way has very little power to sustain the oscillation. Now, if resistance occurs the observed oscillation is the result of forces acting in a manner best suited to sustain it; for if not, some other arrangement would arise which would cause a greater amplitude to the motion.

It is obvious that in order to overcome friction the sustaining forces must act more with the moving particles than against them.

If the area is very nearly $\frac{1}{2}\lambda$ in length and the amplitude is kept down by friction proportional to the velocity, it is easily seen that the forces must be of the same phase as the velocities, in order to sustain the greatest actual oscillation; in other words, the phase of the displacements must be $\frac{1}{4}\pi$ or 90° behind the phase of the forces.

Dissipation because of incomplete boundaries generally takes place through either stationary or progressive wave motion.

Imagine a rectangular body or area of approximately critical dimensions to have solid barriers at the ends only. Experiment shows that the oscillation in such a body has a much smaller amplitude than would have been the case had all boundaries been solid; also that across the imaginary sides, particularly near the loops, a transverse stationary motion simultaneous with the longitudinal motion is going on. The reason for this transverse motion is obvious. For, an elevation at one loop causes an outward slope and so an outward acceleration. This gives a maximum outward velocity when the water at the loop has fallen to mean-water level. Similarly the greatest velocity toward this loop will occur at the time of mean level when the water is there rising.

The outgoing water crowds up the water situated well outside of the area and so tends to cause it there to rise. If a rigid barrier were placed at a suitable distance beyond this free or imaginary side of the area, a considerable rise would actually take place. As no such barrier exists, the outward-going motion, well outside of the area, is not converted into height; it is communicated to waters not susceptible of taking up good oscillations. Consequently the succeeding inward motion does not have the benefit of any considerable height energy in the outer part of the transversely moving water. The same result as to the loss of transverse motion will be obtained by considering in the first place a low water at the loop.

The lateral boundaries of the main area being only imaginary, it is clear that every increment of energy given to the area by the forces is offset by a loss of motion going on across the missing or imaginary boundaries. The shorter the solid ends in comparison with the missing sides, the smaller will be the oscillation set up and maintained. In fact the length of the end walls must be at least about $\frac{1}{4}\lambda$ in length if any considerable oscillation is to occur (see sec. 53, Part IV A).

It will be noticed that energy is leaving the system most rapidly at the time of mean level rising or falling, and that none escapes at the times of high and low water; at intermediate times the escape is proportional to the velocity which could have, under other conditions, been converted into height. Therefore, whether much motion exists or not, it follows that the amplitude of the oscillation is kept down by destructive agencies most active at the times of mean level and which can be approximately represented by the term (117) which includes frictional resistance and is based upon the assumption that all resistance varies with the first power of the velocity. The dissipation term implied in the equation of virtual work [(302), Part IV A] may be written $-\sum_v K_v m_v \xi_v \frac{\partial \xi_v}{\partial t}$. The corresponding term implied in (305), Part IV A, is

$-A_a \sum_v K_v m_v \sin^2 l x_v \sin at$, and so may be considered as being covered by the given resistance term, (117).

If an area were entirely surrounded by solid walls, excepting one narrow opening into a tideless sea, the above law of dissipation, so far as dissipation occurs, holds here because according to sections 35, 103, Part IV A, the motion in the strait is a stationary oscillation simultaneous with the motion in the oscillating area sustaining it.

If one of the four walls surrounding an area consists of a shore line made somewhat irregular by headlands and bays, then, provided the bays are nonpropagative arms of water, it is obvious that the motion in these is greatest at the time of mean level outside. Hence whatever resistance occurs in these arms, whether from friction proper or from the fact that the motion in them sustained by the rise and fall of the area is in part dissipated in the area, must follow the above law, at least very approximately.

In the next place, imagine one of the four rigid walls surrounding an area to contain an opening through which a wave is propagated; for example, the mouth of a tidal river or other propagative arm of the ocean.

The mass of water occupying the immediate approaches to the mouth of the river or bay rises and falls with the area upon which it depends. From the rise and fall of this mass, the energy expended in the river or bay is immediately derived. The size of this intermediary mass as compared with that of the oscillating system indicates, in a

measure, how much the motion in the latter will be reduced. At the time of mean level, energy is being transferred from the system proper to the intermediary mass at a maximum rate; at the time of high or low water none is being thus transferred. The question here simply relates to the transference of energy from the main body to the intermediary mass and has little to do with what goes on between this mass and the river proper.

The resistance in the river tends to diminish the rise and fall at its mouth, hence the necessity of supplying energy to the mass in maintaining the rise and fall at the river's mouth.

If one of the end boundaries of an oscillating area contain an opening of considerable size and through which a progression occurs, a progressive character is given to the tides of the area a considerable distance from the opening (antecedent wave). This progressive wave may result from two causes: (1) that just given, viz., the necessity of agreement in height between the water's surface off the mouth of the river or strait and that of the principal oscillation; (2) the fact that particles in and near the opening are not turned back at the time of elongation of principal oscillation, but somewhat later. In either case it is necessary to go into the area some distance from the opening in order to observe the true time of rise and fall for the principal oscillation. In either case, also, there must be an intermediary mass of water whereby energy is transferred from the principal oscillation to the progressive wave. In deep water this wave may be felt nearly across the area, and is often sustained in part by the horizontal motion across the nodal line.

In the case of a stationary-dissipation movement, the greatest crowding of the waters into which the motion is dissipated takes place on the falling tide of the neighboring loop.

In case of a progressive-dissipation movement, the greatest crowding of the waters immediately adjacent to the area takes place on the rising tide of the neighboring loop.

More generally, the oscillation in an area having incomplete boundaries is kept down because the impulses of the tidal forces in their efforts to sustain or augment the oscillation tend to cause a crowding together or drawing asunder of the water particles near by. If the boundary were complete, an increased oscillation would result. Being incomplete, only a moderate rise and fall results. The failure to make the crowding effective in increasing the height at a loop of the area is most rapid at the time of half-tide level. This is also the time when the height of the surface is changing most rapidly.

Incomplete boundaries are in fact the chief reason why the tide in a given area always falls much short of its theoretical dynamical value. Whether the reduction of motion be due to irregularities in the shore line or to incompleteness of boundary, the forces will select a critical strip of water, if such exist; but on account of these defects of boundary the forces will act to comparatively poor advantage, especially in the case of the terrestrial oceans.

It must not be inferred that energy is always expended most rapidly on account of a break in a boundary of the oscillating area or system at the time of half-tide level. Where hydraulic effects occur it may be otherwise. If the principal resistance of the area were of this character, the forces would not precede the displacements by $\frac{1}{4}\tau$, or 90° , as usual; for example, a lake or bay communicating with the sea through a

narrow and short strait. In this case the energy of the oscillation is being most rapidly reduced at about the time of high or low water outside, for then the surfaces of near-by waters have their greatest difference in height. It does not seem probable that dissipations of this kind are great enough to have a sensible influence upon ocean tides. And even in the case just supposed, as the opening becomes larger there will be less tendency to depart from the general rule.

In section 23 it is shown that if the amplitude of vibration of a pendulum in a resisting medium be μ' times the value when free from resistance, the impressed forces remaining the same in both cases, then the phase of the forces relatively to that of the displacement of the pendulum is given by the equation

$$\cos \alpha_1 = \pm \mu'.$$

Now, assuming that a tidal oscillation follows this law for all forms of resistance, as is probably very nearly the case, it follows that if the observed amplitude has $\frac{1}{4}$ of its theoretical value with no resistance, then the phase of maximum velocity of the particles differs from the phase of maximum forces by $14^\circ 29'$, or about half an hour for a semidaily tide.

The nearer to critical dimensions are the dimensions of the oscillating areas, the greater will be their tides and the more will these tides overshadow all other tides which may arise from less suitable areas, and the more will the resultant tides be controlled by them; hence their great liability to chiefly constitute the observed tide. This state of affairs is, in fact, necessary wherever much incompleteness and irregularity of boundary exist; that is, μ' must be small on account of these defects in boundary and the dimensions of an area must be nearly critical in order that a sensible tide may arise in an area thus imperfectly surrounded.

26. *To find the frictional resistance upon a canal-like sheet of water, the length approximating to $\frac{1}{2}\lambda$.*

Let the body be divided longitudinally into strips one unit in width. Taking the left end of the canal as space origin and the time origin at the time when the particles are at elongation to the left, the equations of the motion are

$$\xi = -A \sin lx \cos at,$$

$$\zeta = A/h \cos lx \cos at, = A' \cos lx \cos at; \quad (118)$$

$$\therefore \text{velocity} = \frac{\partial \xi}{\partial t} = Aa \sin lx \sin at = A' \sqrt{\frac{g}{h}} \sin lx \sin at. \quad (119)$$

If the amplitude of the tide is one foot, the values of the maximum velocity at the nodal line are as follows for various depths, h :

$h = 600, \quad 1200, \quad 3000, \quad 6000, \quad 9000, \quad 12000, \quad 15000, \quad 18000, \text{ feet.}$
 $\sqrt{\frac{g}{h}} = 0.23156, \quad 0.16374, \quad 0.10356, \quad 0.07323, \quad 0.05979, \quad 0.05178, \quad 0.04631, \quad 0.04228 \text{ feet}$
 per second.

It thus appears that for anything like oceanic depths, the maximum velocity of the water particles is less than 1/10 foot per second for each foot of semi-range of tide. The resistance per square foot of bottom area is F (secs. 7, 8). The total maximum resistance of a strip of water $\frac{1}{2}\lambda$ long and one foot wide is

$$\frac{1}{2}\lambda\frac{2}{\pi}F$$

pounds where the maximum velocity at the nodal line is used in F . The weight of water composing this strip is $\frac{1}{2}\lambda h\gamma$. The amplitude of the M_2 tidal force acting upon this strip is, for a canal so short that we may disregard the variations of forces over it,

$$\frac{1}{2}\lambda h\gamma \times 0.000\ 000\ 076\ 5 \quad (120)$$

pounds (sec. 2, Part IV A). The tidal force divided by the resisting force is

$$\frac{0.000\ 000\ 076\ 5h\gamma}{\frac{2}{\pi}F} = 0.003\ 20\ \frac{h}{v} \quad (121)$$

where $\gamma=64$ pounds and $F=0.002\ 4v$. This shows that for ocean depths frictional resistance is only a small fraction of the tidal force.

27. *Tidal retardation of the earth's axial rotation:*

If the earth were a homogeneous ellipsoid of revolution, its moment of inertia about its axis of revolution would be

$$\frac{2}{5} \text{Mass (equatorial radius)}^2. \quad (122)$$

where

$$\text{Mass} = \frac{4}{3}\pi r'^3\mu$$

where equatorial radius = 20 925 000; r' = the average radius = 20 902 000 feet; μ = mass per unit volume = $62.4 \times 5.5 = 343.2$ pounds. These values give for the moment of inertia I , $2\ 299 \times 10^{36}$ foot poundals or $7\ 396 \times 10^{34}$ foot pounds.

The energy of rotation is $\frac{1}{2}I\omega^2$ where ω denotes the angular velocity of rotation = 0.000 072 72 radian per sidereal second.

$$\therefore \frac{1}{2}I\omega^2 = 608 \times 10^{38} \text{ foot poundals.} \quad (123)$$

But the density of the earth increases from the surface toward the center according to some law not fully known. Among the hypotheses which have been used in questions connected with the earth's figure are Legendre's (or Laplace's), Roche's, Lipschitz's, Maurice Levy's, and Wiechert's.

Without going into the computation in accordance with any of these hypotheses, we may use the value

$$504 \times 10^{38} \text{ foot poundals}$$

given in the Smithsonian Tables, and which depends directly upon Harkness' values of the principal moments of inertia.

Let ϵ denote the constant lengthening of the sidereal day per sidereal day; then after t days the original length of the day will be increased by $t \epsilon$. The total time lost during a period of t days will be

$$\epsilon \int_{t=0}^{t=t} t dt = \frac{t^2}{2} \epsilon = s, \quad (124)$$

$$\therefore \epsilon = 2 \frac{s}{t^2}.$$

The energies at $t=0$ and $t=t$ are connected by the equation

$$(\text{Energy})_0 : (\text{energy})_t = 1^2 : \left(\frac{1}{1 + \epsilon t} \right)^2;$$

$$\therefore \frac{\text{energy}_t}{\text{energy}_0} = 1 - 2\epsilon t. \quad (125)$$

The energy lost in t days is therefore very nearly equal to the original (or final) energy multiplied by $2 \epsilon t$, where ϵ is expressed as a small fraction of a day.

If in 100 years, the earth's meridian is 22 (sidereal) seconds behind the position it would have assumed had the rotation not been retarded, we have $s = 22 \text{ seconds} = 0.000255 \text{ day}$.

$$\therefore \epsilon = 2 \frac{0.000255}{366^2 \cdot 100^2} = 3.81 \times 10^{-13} \quad (126)$$

and

$$504 \times 10^{28} \times 2\epsilon = 384 \times 10^{16}$$

for the number of poundals of work lost in a sidereal day.

According to Krümmel the area of the oceans and inland seas is

$$374\,058\,000 \text{ sq. k.} = 144\,424\,168 \text{ sq. st. miles} = 4\,026\,314\,780\,000\,000 \text{ sq. feet.}$$

The average depth exceeds 9 000 feet, and the average range of tide is about 2 feet. These values give for the average maximum velocity in feet per second of the water particles

$$A' \sqrt{\frac{g}{9\,000}} = 0.0598 \doteq 0.06$$

where $A' = 1$ and $g = 32.1722$; the average mean velocity will be $0.06 \times \frac{2}{\pi} = 0.038$.

$$\text{Total resistance} = F \times \text{area} \doteq 0.0024 \times \text{velocity} \times \text{area} = 3.65 \times 10^{11} \text{ lbs.} \quad (127)$$

Average distance traveled by a particle in a sidereal day = 3 274 feet. The energy expended in a sidereal day is

$$3.65 \times 10^{11} \times 3\,274 = 1\,195 \times 10^{12} \text{ foot pounds,} = 384 \times 10^{14} \text{ foot pounds.} \quad (128)$$

According to this estimate, the tidal resistance of the ocean and inland seas causes the earth's meridian to fall behind 0.22 second per century instead of 22 seconds.

This value should be considerably increased on account of the fact that in shallow water the frictional resistance is many times greater per square mile or foot of area than in deep water.

The following are a few references to the question of retardation of the earth's axial rotation due to tidal friction:

Thomson and Tait: *Natural Philosophy*, 2d ed., secs. 276, 830. Ap. G (a) and (b) (by Darwin).

Lord Kelvin: *Popular Lectures and Addresses*, Vol. II (1894), pp. 20-24, 65-72, 90-96, 270-272.

Sir Robt. S. Ball: *Time and Tide*.

This manual, Part I, secs. 140, 142, and references there given.

CHAPTER III.

SHALLOW-WATER AND RIVER TIDES.

28. To Airy belongs the credit of having first obtained the exact equation of long-wave motion (No. 129 below). His approximate solution of this equation enabled him to partially explain the change in form of a wave where the depth is but a moderate multiple of the amplitude of the tide.*

In 1871 M. de Saint-Venant gave as the rate of advance of a free wave $3\sqrt{gz}-2\sqrt{gh}$ where z denotes the depth and h the undisturbed depth.†

In 1892 J. McCowan published an important paper entitled "On the theory of long waves and its application to the tidal phenomena of rivers and estuaries."‡

Prof. Maurice Lévy gives a very complete discussion of river tides in Chapter IX of his *Théorie des Marées*.§

The aim of the present chapter is to give some of the more essential parts of Airy's, McCowan's, and Lévy's developments, together with some modifications and additional matters.

As noted in section 17, Part I, the equation of motion for a canal in which the rise and fall amounts to a considerable fraction of its depths is

$$\frac{\partial^2 \xi}{\partial t^2} = gh \frac{\frac{\partial^2 \xi}{\partial x^2}}{\left(1 + \frac{\partial \xi}{\partial x}\right)^3} = gh \frac{\partial^2 \xi}{\partial x^2} \left[1 - 3 \frac{\partial \xi}{\partial x} + 6 \left(\frac{\partial \xi}{\partial x} \right)^2 - \dots \right] \quad (129)$$

while the equation of continuity is

$$\left(1 + \frac{\partial \xi}{\partial x}\right) \left(1 + \frac{\xi}{h}\right) = 1; \text{ or, } \frac{\xi}{h} = -\frac{\partial \xi}{\partial x} + \left(\frac{\partial \xi}{\partial x}\right)^2 - \dots \quad (130)$$

The approximate solution of (129) where $\frac{\partial \xi}{\partial x}$ is small in comparison with unity is

$$\psi(\kappa t + x) \pm \psi(\kappa t - x) \quad (131)$$

where $\kappa^2 = gh$. Therefore assume as an approximate solution of (129)

$$\xi = A \sin(at - lx + \alpha) \quad (132)$$

* Tides and Waves, secs. 195 et seq. See also Stokes, "Report on recent researches in hydrodynamics," and "Notes on hydrodynamics," Mathematical and Physical Papers, Vol. I, pp. 157-176, and Vol. II, pp. 222-229; also this manual, Part I, secs. 113, 117.

† Comptes rendus de l'Académie des Sciences, Vol. 73 (1871), pp. 147-154. See also Vol. 71 (1870), pp. 186-195.

‡ Philosophical Magazine, Vol. III, pp. 250-265.

§ Première Partie, Paris, 1898.

where $a/l = \sqrt{gh}$. Equation (129) may now be written

$$\frac{\partial^2 \xi}{\partial t^2} - \kappa^2 \frac{\partial^2 \xi}{\partial x^2} = -\frac{3}{2}(Al)^2 l \kappa^2 \sin 2(at - lx + \alpha) + \text{higher powers in } Al \quad (133)$$

where Alh = the original amplitude of the semidaily tide = A' .

If ξ were of the form (132), the left member of (133) would be zero; consequently besides containing terms of the form (132), ξ must contain a part such that when

$$\frac{\partial^2}{\partial t^2} - \kappa^2 \frac{\partial^2}{\partial x^2} \text{ is applied to it the result shall be } -\frac{3}{2}A'^2 l^2 \kappa^2 \sin 2(at - lx + \alpha).$$

The part answering to this description is

$$\frac{3}{8}A'^2 l^2 x \cos 2(at - lx + \alpha) + \text{any function satisfying } \frac{\partial^2}{\partial t^2} - \kappa^2 \frac{\partial^2}{\partial x^2} = 0.$$

Assume this function to be

$$A_2 \sin 2(at - lx + \alpha_2) + Pt + Qx + R$$

and suppose A_2 to be small in comparison with A . Upon substituting the entire value of ξ in the equation of continuity (130), and carrying the result to the second power of quantities of the magnitude Al , there results

$$\begin{aligned} \frac{\zeta}{h} = & Al \cos (at - lx + \alpha) - \frac{3}{4}A'^2 l^2 x \sin 2(at - lx + \alpha) - \frac{3}{8}A'^2 l^2 \cos 2(at - lx + \alpha) \\ & + 2A_2 l \cos 2(at - lx + \alpha_2) + A'^2 l^2 \frac{1 + \cos 2(at - lx + \alpha)}{2} - Q. \end{aligned} \quad (134)$$

If the term Qx were not included in the expression for ξ , it might be inferred from (134) that the mean value of ζ must be

$$\frac{1}{2}A'^2 l^2 h, \text{ or } \frac{1}{2}A'^2 / h \text{ where } A' = Alh$$

is the amplitude of the principal term in the expression for ζ . This would seem to indicate that the mean of all ordinates of the tide curve at any given point of a river or canal lies a little above the level of the ocean or of a tideless canal.* This conclusion is wrong, because, from the nature of the case, the mean level of the river near the mouth can not be sensibly raised above mean sea level just outside.

At the mouth of the river where $x=0$ the tide is assumed to be simply harmonic and so the terms in $2at-2lx$ and whose coefficients do not contain x must vanish for all values of x ; also the mean half-tide level must be that of the sea. Equation (134) then gives, since an amplitude is essentially positive,

$$A_2 = \frac{1}{16}A'^2 l, \alpha_2 = \alpha + \frac{\pi}{2}, Q = \frac{A'^2 l^2}{2}. \quad (135)$$

* Cf. Ferrel, Tidal Researches, Equation 273 and secs. 248-253; and Airy, Tides and Waves, arts. 515, 531.

If $x' = x + \xi$, it will be the coördinate of the particle in its disturbed position. Equation (134) then becomes approximately

$$\begin{aligned} \frac{\zeta}{h} &= Al \cos (at - lx' + \alpha) - \frac{3}{4} A^2 l^3 x' \sin 2(at - lx' + \alpha) \\ &\quad - A^2 l^2 \frac{1 - \cos 2(at - lx' + \alpha)}{2} - \frac{3}{8} A^2 l^2 \cos 2(at - lx' + \alpha) \\ &\quad + 2A_2 l \cos 2(at - lx' + \alpha_2) + A^2 l^2 \frac{1 + \cos 2(at - lx' + \alpha)}{2} - Q \\ &= Al \cos (at - lx' + \alpha) - \frac{3}{4} A^2 l^3 x' \sin 2(at - lx' + \alpha) \\ &\quad + \frac{5}{8} A^2 l^2 \cos 2(at - lx' + \alpha) + 2A_2 l \cos 2(at - lx' + \alpha_2) - Q. \end{aligned} \quad (136)$$

The conditions that the height of water in the mouth of the canal must agree with the height outside gives

$$A_2 = \frac{5}{16} A^2 l, \quad \alpha_2 = \alpha + \frac{\pi}{2}, \quad Q = 0. \quad (137)$$

The value of ξ may now be written

$$\begin{aligned} \xi &= A \sin (at - lx + \alpha) + \frac{3}{8} A^2 l^2 x \cos 2(at - lx + \alpha) - \frac{1}{16} A^2 l \sin 2(at - lx + \alpha) + Pt \\ &\quad + \frac{A^2 l^2}{2} x + R. \end{aligned} \quad (138)$$

The velocity of a particle whose undisturbed or mean abscissa is x is

$$\begin{aligned} u = \frac{d\xi}{dt} &= Aa \cos (at - lx + \alpha) - \frac{3}{4} A^2 al^2 x \sin 2(at - lx + \alpha) \\ &\quad - \frac{1}{8} A^2 al \cos 2(at - lx + \alpha) + P. \end{aligned} \quad (139)$$

The fact that ζ contains only simple cosine or sine terms shows that its mean value must be zero. The velocity can not follow exactly a similar simple law. For, at the time of flood the depth is greater than it is at the time of ebb, and so, in order that the same amount of water shall cross a given cross section on the ebb as on the flood, the ebb current must average the stronger. In order to represent the decrease in the velocity on the flood, and the increase on the ebb, with neither increase nor decrease at the time of mean river level, it is natural to add a term proportional to the square of the velocity and finally determine P . Take a section so near to the mouth of the river that the term having x in its coefficient may be neglected. Then

$$u = Aa \cos (at - lx + \alpha) + Ku^2,$$

or

$$-\frac{1}{8} A^2 al \cos 2(at - lx + \alpha) + P = Ku^2;$$

whence

$$K = -\frac{1}{4} \frac{l}{a}, \quad P = -\frac{1}{8} A^2 al.$$

The value of u thus becomes

$$u = Aa \cos (at - lx + \alpha) - \frac{3}{4} A^2 al^2 x \sin 2 (at - lx + \alpha) - \frac{1}{8} A^2 al \cos 2 (at - lx + \alpha) - \frac{1}{8} A^2 al. \quad (140)$$

This value of u is allowable because velocities are not, like heights, restricted to a simple periodic term where $x=0$. The velocity becomes zero wherever $at - lx + \alpha$ is an odd multiple of 90° , i. e., at the time of mean river level.

Because of relations (135), equation (134) may be written

$$\frac{\zeta}{h} = Al \cos (at - lx + \alpha) - \frac{3}{4} A^2 l^3 x \sin 2 (at - lx + \alpha), \quad (141)$$

where ζ is the height of the surface for a point whose undisturbed abscissa is x .

From (139) and (141) it is evident that the first approximate value of the velocity of the current at a point whose undisturbed abscissa is x , is

$$u = \frac{a}{l} \frac{\zeta}{h} = \zeta \sqrt{\frac{g}{h}}, \quad \text{since } \frac{a}{l} = \frac{\lambda}{\tau} = \sqrt{gh}. \quad (142)$$

The second approximate value of the velocity is

$$u = \frac{a}{l} \left(\frac{\zeta}{h} - \frac{\zeta^2}{4h^2} \right), \quad (143)$$

as can be seen upon substituting the value of $\frac{\zeta}{h}$ from (141) in (139).

If $x' = \xi$ be written for x in the expression for the velocity, it becomes

$$u = Aa \cos (at - lx' + \alpha) - \frac{3}{4} A^2 al^2 x' \sin 2 (at - lx' + \alpha) + \frac{3}{8} A^2 al \cos 2 (at - lx' + \alpha) + P, \quad (144)$$

which gives $P = \frac{3}{8} A^2 al$.

The magnitude of the coefficient of $\sin 2 (at - lx + \alpha)$ or $\sin 2 (at - lx' + \alpha)$ is readily computed by aid of the relation $l = a \sqrt{gh}$ where $a = 0.0001405$ radian per second for the M_2 -wave. Let $A' = Alh$ = original amplitude of the tide. Then

$$\frac{3}{4} A^2 l^3 x' = \frac{3}{4} A'^2 \frac{lx'}{h}; \quad (145)$$

and so for the mean lunar tide,

$$M_4 = \frac{3}{4} M_2^2 \frac{lx'}{h}. \quad (146)$$

The value of $l = 2\pi/\lambda$, is given in the last column of Table 50 for various depths. To find the l for any other component whose speed is b , multiply the tabular values by b/a .

The amplitude of the tide whose speed is $2a$, is $\frac{3}{4}A^2 l^3 h x'$, and the angle or phase is double the angle or phase of the fundamental—this double angle being increased by 90° if the second term of (134) is to be written as a positive cosine term: \therefore by equations (45) and (46), Part III, high water is accelerated $\frac{3}{2}Al^2 x'$ radians or degrees, according to the manner in which x' is expressed, or $\frac{3}{2}\frac{Al^2 x'}{a}$ hours; low water is retarded by the same amount. Hence the duration of rise is $3\frac{Al^2 x'}{a}$ hours less than $180^\circ/a$, or a quarter lunar day for the lunar wave, while the duration of fall is $3\frac{Al^2 x'}{a}$ greater than $180^\circ/a$. If the time origin be such that $\alpha = 0$, the time of high water at x' is

$$T = \frac{x'}{\sqrt{gh}} - \frac{3}{2} \frac{Alx'}{\sqrt{gh}}. \quad (147)$$

The reciprocal of $\frac{dT}{dx'}$, i. e., the rate of advance of the high-water phase, is

$$\sqrt{gh} \left(1 + \frac{3}{2} Al \right) = \sqrt{gh(1 + 3Al)} = \sqrt{gh(1 + 3\frac{A'}{h})}. \quad (148)$$

For the low-water phase the approximate rate of advance is

$$\sqrt{gh(1 - 3\frac{A'}{h})}. \quad (149)$$

For the mean lunar tide

$$M_4^0 = 2 M_2^0 - 90^\circ. \quad (150)$$

To approximately take into account the proper outward velocity of the stream (U), these rates of propagation should be diminished by U . Similarly, the value of the velocity of the current will be approximately $u - U$.

29. *Tides in a canal stopped by a barrier.**

Taking the mouth of the canal as the origin, and denoting its length by L , the approximate displacements are, by section 30, Part I,

$$\xi = \frac{A}{\cos lL} \sin [l(L-x)] \cos (at + \alpha), \quad (151)$$

$$\zeta = \frac{A lh}{\cos lL} \cos [l(L-x)] \cos (at + \alpha). \quad (152)$$

The dynamical equation taken in connection with the equation of continuity is

$$\frac{\partial^2 \xi}{\partial t^2} = gh \frac{\partial^2 \xi}{\partial x^2} \left[1 - 3 \frac{\partial \xi}{\partial x} + 6 \left(\frac{\partial \xi}{\partial x} \right)^2 - \dots \right].$$

* Cf. Airy, *Tides and Waves*, art. 309.

Substituting the value of $\frac{\partial \xi}{\partial x}$ as obtained from (151) in the second term in the brackets and disregarding the third and later terms, this equation becomes

$$\frac{\partial^2 \xi}{\partial t^2} - gh \frac{\partial^2 \xi}{\partial x^2} = -3gh \frac{A^2 l^3}{2 \cos^2 lL} \sin 2[l(L-x)] \frac{1 + \cos 2(at + \alpha)}{2}. \quad (153)$$

As a second approximation, assume

$$\begin{aligned} \xi = & -\frac{A}{\cos lL} \sin [l(L-x)] \cos (at + \alpha) \\ & - \frac{3}{16} \left(\frac{A}{\cos lL} \right)^2 l^2 (L-x) \cos 2[l(L-x)] \cos 2(at + \alpha) \\ & - \frac{3}{16} \left(\frac{A}{\cos lL} \right)^2 l \sin 2[l(L-x)]. \end{aligned} \quad (154)$$

This displacement becomes zero at $x=L$ for all values of t .

The result of applying $\frac{\partial^2}{\partial t^2} - gh \frac{\partial^2}{\partial x^2}$ to this value of ξ is

$$\begin{aligned} & -gh \frac{3}{4} \left(\frac{A}{\cos lL} \right)^2 l^3 \sin 2[l(L-x)] \cos 2(at + \alpha) \\ & - \frac{3}{4} gh \left(\frac{A}{\cos lL} \right)^2 l^3 \sin 2[l(L-x)]. \end{aligned}$$

This shows that (154) satisfies (153);

$$\begin{aligned} \therefore \frac{\partial \xi}{\partial t} = & -\frac{Aa}{\cos lL} \sin [l(L-x)] \sin (at + \alpha) \\ & + \frac{3a}{8} \left(\frac{A}{\cos lL} \right)^2 l^2 (L-x) \cos 2[l(L-x)] \sin 2(at + \alpha). \end{aligned} \quad (155)$$

This indicates that the velocity curve is composed of two simple harmonic waves. Hence, because of the difference in depth at different stages of the tide, the rise and fall can not be exactly of this character.

The corresponding value of ζh is, by (130),

$$\begin{aligned} & \frac{Al}{\cos lL} \cos [l(L-x)] \cos (at + \alpha) - \\ & + \frac{3}{8} \left(\frac{A}{\cos lL} \right)^2 l^3 (L-x) \sin 2[l(L-x)] \cos 2(at + \alpha) \\ & - \frac{3}{16} \left(\frac{A}{\cos lL} \right)^2 l^3 \cos 2[l(L-x)] \cos 2(at + \alpha) \\ & + \left(\frac{A}{\cos lL} \right)^2 l^2 \cos^2 2[l(L-x)] \cos^2 2(at + \alpha) \\ & - \frac{3}{8} \left(\frac{A}{\cos lL} \right)^2 l^2 \cos 2[l(L-x)]. \end{aligned} \quad (156)$$

At the mouth of the canal, where $x=0$, the rise and fall is not exactly in agreement with the tide outside, owing to the fact that at the head of the canal the horizontal motion is assumed to be zero, thus implying a reflection to each pulse travelling inward.

If $x' - \xi$ be substituted for x in the expression for ζ/h , the term in \cos^2 goes out. For a short canal, the harmonic term of greatest importance is

$$-\frac{3}{16} \left(\frac{A}{\cos lL} \right)^2 l^2 \cos 2 [l(L-x')] \cos 2 (at + \alpha). \quad (157)$$

For such cases

$$M_1^0 = 2M_2^0 \pm 180^\circ. \quad (158)$$

30. *Approximate results, there being a permanent current in deep water.**

To a first approximation the equation of continuity and of motion may be written

$$\frac{\partial \xi}{\partial x} = -\frac{\zeta}{h}, \quad (159)$$

and

$$\frac{\partial^2 \zeta}{\partial t^2} = gh \frac{\partial^2 \zeta}{\partial x^2}. \quad (160)$$

Equation (160) is satisfied by any function of $\sqrt{gh}t - x$. Consequently one may write

$$\zeta = \phi \left(\frac{\sqrt{gh}t - x}{\sqrt{gh} - U} \right) \quad (161)$$

If U denotes the constant outward velocity of the permanent current, the true abscissa of the section is

$$x' = x - Ut + \xi,$$

or

$$x' = x - Ut$$

if the displacement due to wave motion be neglected, as may be done in the argument of the function upon which the height depends.

$$\therefore \zeta = \phi \left(t - \frac{x'}{\sqrt{gh} - U} \right). \quad (162)$$

If $x'=0$, $\zeta = \phi(t)$ expresses the law of rise and fall at the river's mouth. The coefficient of t divided by the coefficient of $-x'$ gives the rate of wave propagation, which therefore is $\sqrt{gh} - U$ up the stream.

The velocity of the current at x' in the flood direction is $\frac{\partial x'}{\partial t}$.

From equation (159)

$$\xi = \frac{\sqrt{gh} - U}{h} \int \phi(w) dw + F(x) \quad (163)$$

* Cf. Maurice Lévy, *Leçons sur la théorie des marées*, Chap. IX.

where w is written temporarily for $\frac{\sqrt{gh}t-x}{\sqrt{gh}-U}$ or $t - \frac{x'}{\sqrt{gh}-U}$;

$$\therefore \frac{\partial x'}{\partial t} = -U + \frac{\sqrt{gh}-U}{h} \phi \left(t - \frac{x'}{\sqrt{gh}-U} \right) \quad (164)$$

since

$$\begin{aligned} \frac{\partial \xi}{\partial t} &= \frac{\partial \xi}{\partial w} \frac{\partial w}{\partial t} \\ \therefore \frac{\partial x'}{\partial t} &= -U + \frac{\sqrt{gh}-U}{h} \zeta = \zeta \sqrt{\frac{g}{h}} - \left(1 + \frac{\zeta}{h}\right) U. \end{aligned} \quad (165)$$

This expresses the velocity of the current at any stage of the tide, x' is positive for the upstream direction and ζ for heights above mean water level.

Where there is no permanent current

$$\dot{\xi} = \frac{\partial \xi}{\partial t} = \zeta \sqrt{\frac{g}{h}}. \quad (166)$$

31. *Exact results, the cross section being rectangular.*

As before, let x' denote the actual abscissa of a moving slice of the liquid and x the abscissa of the slice before disturbance, both reckoned from any assumed origin; and so the displacement $\xi = x' - x$. Then the equations of continuity, velocity, and acceleration are

$$\frac{\partial x'}{\partial x} = \frac{h}{z} = \frac{1}{1 + \frac{\zeta}{h}} \quad (167)$$

or

$$\frac{\partial \xi}{\partial x} = -1 + \frac{1}{1 + \frac{\zeta}{h}}, \quad (168)$$

and

$$\frac{\partial x'}{\partial t} = u,$$

$$\frac{\partial^2 x'}{\partial t^2} = -\frac{g}{\gamma} \frac{z \partial \rho}{h \partial x} = -g \frac{z \partial z}{h \partial x} = -g \left(1 + \frac{\zeta}{h}\right) \frac{\partial \zeta}{\partial x} = g h \frac{\partial^2 \xi}{\partial x^2} \left(1 + \frac{\partial \xi}{\partial x}\right). \quad (169)$$

Differentiating (169) with respect to x , we have

$$\begin{aligned} h \frac{\partial^3 x'}{\partial x \partial t^2} &= -\frac{\partial}{\partial x} g z \frac{\partial z}{\partial x} = -g \left(\frac{\partial z}{\partial x} \right)^2 - g z \frac{\partial^2 z}{\partial x^2} \\ &= -g h \left[\frac{\partial^2 \zeta}{\partial x^2} + \frac{\zeta \partial^2 \zeta}{h \partial x^2} + \frac{11}{h} \left(\frac{\partial \zeta}{\partial x} \right)^2 \right] \end{aligned} \quad (170)$$

and differentiating (167) with respect to t ,

$$\begin{aligned} h \frac{\partial^3 x'}{\partial x \partial t^2} &= -\frac{\partial}{\partial t} h^2 \frac{\partial z}{z^2 \partial t} = -\frac{h^2 \partial^2 z}{z^2 \partial t^2} + 2 \frac{h^2}{z^3} \left(\frac{\partial z}{\partial t} \right)^2 \\ &= h \left[-\frac{1}{h} \frac{\partial^2 \zeta}{\partial t^2} + \frac{2 \zeta \partial^2 \zeta}{h^2 \partial t^2} + \frac{2}{h^2} \left(\frac{\partial \zeta}{\partial t} \right)^2 \right] \end{aligned} \quad (171)$$

whence

$$\frac{\partial h^2}{\partial t} \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} g^2 \frac{\partial z}{\partial x} \quad (172)$$

The complete solution of (172) for an advancing series of waves of any form is

$$z = F_0(x - t g^2 h^{-1} z^2). \quad (173)$$

To prove this, assume a slightly more general form for z , viz,

$$z = F_0(x - \alpha t) \quad (174)$$

where α is a function of z . Then regarding x as constant and t and z as variables,

$$\frac{\partial z}{\partial t} = - \frac{\alpha F_0'(x - \alpha t)}{1 + t F_0'(x - \alpha t) \frac{d\alpha}{dz}}; \quad (175)$$

similarly regarding t as constant and x and z as variables

$$\frac{\partial z}{\partial x} = \frac{F_0'(x - \alpha t)}{1 + t F_0'(x - \alpha t) \frac{d\alpha}{dz}}. \quad (176)$$

In either case the accent denotes, as usual, differentiation with reference to the quantity within the parenthesis. Hence

$$\frac{\partial z}{\partial t} = - \alpha \frac{\partial z}{\partial x}. \quad (177)$$

If β is also a function of z , then

$$\frac{\partial}{\partial t} \beta \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \alpha^2 \beta \frac{\partial z}{\partial x}. \quad (178)$$

For, from (177),

$$\beta \frac{\partial z}{\partial t} = - \alpha \beta \frac{\partial z}{\partial x}, \quad (179)$$

and so

$$\frac{\partial}{\partial t} \beta \frac{\partial z}{\partial x} = - \frac{\partial}{\partial t} \alpha \beta \frac{\partial z}{\partial x}.$$

Again,

$$\frac{\partial}{\partial t} \alpha \beta \frac{\partial z}{\partial x} = \alpha \beta \frac{\partial^2 z}{\partial t \partial x} + \frac{\partial(\alpha \beta)}{\partial z} \frac{\partial z}{\partial t} \frac{\partial z}{\partial x}, \quad (180)$$

$$\frac{\partial}{\partial x} \alpha^2 \beta \frac{\partial z}{\partial x} = \alpha^2 \beta \frac{\partial^2 z}{\partial x^2} + \beta \frac{\partial(\alpha^2 \beta)}{\partial z} \left(\frac{dz}{dx} \right)^2. \quad (181)$$

But

$$\frac{\partial(\alpha \beta)}{\partial z} = \beta \frac{\partial \alpha}{\partial z} + \alpha \frac{d\beta}{dz}, \quad \frac{\partial(\alpha^2 \beta)}{\partial z} = 2\alpha \beta \frac{d\alpha}{dz} + \alpha^2 \frac{d\beta}{dz}. \quad (181)$$

Because of (177), and these relations, the sum of the last terms of (180) and (181) becomes

$$\alpha\beta\frac{d\alpha}{dz}\left(\frac{\partial z}{\partial x}\right)^2.$$

From (177)

$$-\frac{\partial^2 z}{\partial t \partial x} = \frac{\partial}{\partial x} \alpha \frac{\partial z}{\partial x} = \frac{\partial \alpha \partial z}{\partial x \partial x} + \alpha \frac{\partial^2 z}{\partial x^2}. \quad (182)$$

This equation taken in connection with

$$\frac{\partial \alpha}{\partial x} = \frac{\partial \alpha}{\partial z} \frac{\partial z}{\partial x}$$

shows that the sum of the second members of (180) and (181) is zero, therefore the sum of the first members is zero, and so the given equation (172) is satisfied by the assumed form of z .

In particular, if one puts $\alpha^2\beta = gz$, and $\beta = \frac{h^2}{z^2}$, equation (178) becomes identical with (172).

If $F_1 \equiv F_0^{-1}$, i. e., if F_1 is such a function that

$$F_1 [F_0 (\quad)] = (\quad), \quad (183)$$

then

$$x = F_1 (z) + tg^{\frac{1}{2}} h^{-1} z^{\frac{1}{2}}. \quad (184)$$

From (167)

$$\partial x' = \frac{h}{z} \partial x$$

and so

$$\begin{aligned} x' &= \int \frac{h}{z} \frac{\partial F_1(z)}{\partial z} dz + tg^{\frac{1}{2}} h^{-1} \int \frac{h \partial z^{\frac{1}{2}} dz}{z \partial z} + \text{funct. } t, \\ &= F_1(z) + 3tg^{\frac{1}{2}} z^{\frac{1}{2}} + kt \end{aligned} \quad (185)$$

if the flow independent of the tide is assumed to be constant.

If $F \equiv F_2^{-1}$,

$$\begin{aligned} z &= F [x' - (3g^{\frac{1}{2}} z^{\frac{1}{2}} - k) t] \\ &= f \left(at - \frac{ax'}{3\sqrt{gz-k}} \right). \end{aligned} \quad (186)$$

$$z = F(x') = f \left(\frac{-ax'}{3\sqrt{gz-k}} \right) \quad (187)$$

is the equation of the wave surface when $t=0$

$$u = \frac{dx'}{dt} = \frac{\partial x'}{\partial z} \frac{\partial z}{\partial t} + \frac{\partial x'}{\partial t} \frac{\partial t}{\partial t}. \quad (188)$$

From (167)

$$\frac{h}{z} = \frac{\partial x'}{\partial x} = \frac{\partial x'}{\partial z} \frac{\partial z}{\partial x'} \quad (189)$$

and so by (175), (176)

$$\frac{\partial x'}{\partial z} \frac{\partial z}{\partial t} = \frac{h}{z} \frac{\partial z}{\partial t} \bigg/ \frac{\partial z}{\partial x} = -\sqrt{gz}; \quad (190)$$

and by (185)

$$\frac{\partial x'}{\partial t} = 3\sqrt{gz} + k.$$

$$\therefore u = 2\sqrt{gz} - k. \quad (191)$$

From this it is evident that in a river without a permanent current the greatest velocity of the tidal current occurs at the time of high or low water.

In case of a tidal river, let the space origin be taken at its mouth and the time origin at the time of high water there; then

$$z - h = \zeta = A' \cos at$$

represents the tide at the river's mouth and

$$\zeta = A' \cos \left[at - \frac{ax'}{3\sqrt{g(h+\zeta)} - k} \right] \quad (192)$$

the elevation at any section distant x' from the mouth.

From this it is seen that the rate of advance is

$$\frac{\lambda}{\tau} = \frac{a}{l} = 3\sqrt{g(h+\zeta)} - k. \quad (193)$$

To determine k , make $\zeta = 0$; then the rule found in the preceding section becomes accurate, i. e., the rate of advance up the river is \sqrt{gh} less U the velocity of the permanent current;

$$\therefore 3\sqrt{gh} - k = \sqrt{gh} - U \quad (194)$$

or

$$k = 2\sqrt{gh} + U$$

\therefore rate of advance

$$= \sqrt{gh} \left(1 + \frac{3}{2} \frac{\zeta}{h} - \frac{3}{4} \frac{\zeta^2}{h^2} + \dots \right) - U = \sqrt{gh} \left(1 + \frac{3}{2} \frac{\zeta}{h} \right) - U \quad (195)$$

velocity of current

$$= 2\sqrt{g(h+\zeta)} - 2\sqrt{gh} - U = \sqrt{gh} \left(\frac{\zeta}{h} - \frac{\zeta^2}{4h^2} + \dots \right) - U. \quad (196)$$

Hence the velocity of the current at a given point depends only upon the height of the tide at that point.

If there be no permanent current, $k=2\sqrt{gh}$ and (192) becomes approximately

$$\zeta = A' \cos \left(at - \frac{ax'}{\sqrt{gh}} + \frac{3}{2} \frac{ax'}{\sqrt{gh}} \frac{\zeta}{h} \right). \quad (197)$$

Expanding by Taylor's theorem and using for ζ its first approximate value we have

$$\zeta = A' \cos \left(at - \frac{ax'}{\sqrt{gh}} \right) - \frac{3}{4} \frac{A'^2 ax'}{\sqrt{gh} h} \sin 2 \left(at - \frac{ax'}{\sqrt{gh}} \right), \quad (198)$$

which agrees with equation (136).

Differentiating (192) with respect to t , the duration of fall becomes

$$\frac{1}{2}\tau + x' \left(\frac{1}{3\sqrt{g(h-A')}} - k - \frac{1}{3\sqrt{g(h+A')}} - k \right) \quad (199)$$

or, approximately, taking $k=U+2\sqrt{gh}$ and neglecting higher power of A'/h ,

$$\frac{1}{2}\tau + x' \frac{3\sqrt{gh}}{(\sqrt{gh}-U)^2} \frac{A' h}{h} \quad (200)$$

The duration of rise is

$$\frac{1}{2}\tau - x' \frac{3\sqrt{gh}}{(\sqrt{gh}-U)^2} \frac{A' h}{h} \quad (201)$$

Hence, the difference between these two intervals increases directly with x' or A' ; it is also greater, the greater the velocity (U) of the permanent current.

Since $u=2\sqrt{g(h+\zeta)}-k$, it follows that the times of slack after flood and after ebb are given by the equation

$$2\sqrt{g(h+\zeta)} \doteq 2\sqrt{gh} + \zeta \sqrt{\frac{g}{h}} = k = 2\sqrt{gh} + U \quad (202)$$

$$\therefore \pm \frac{1}{2} \zeta \sqrt{\frac{g}{h}} \doteq U$$

The flood slack occurs at a time when the height of the river is $\sqrt{\frac{h}{g}}U$ above the undisturbed level. Therefore by (192)

$$t = \frac{1}{a} \cos^{-1} \frac{\zeta}{A'} \doteq \frac{1}{a} \left(\frac{\pi}{2} - \frac{\zeta}{A'} \right) = \frac{1}{a} \left(\frac{\pi}{2} - \frac{1}{A'} \sqrt{\frac{h}{g}} U \right), \quad (203)$$

where ζ has the above value, gives the time of slack water after the time of high water. Similarly, the time of the ebb slack reckoned from high water is

$$t = \frac{1}{a} \cos^{-1} \left(\pi - \frac{\zeta}{A'} \right) \doteq \frac{1}{a} \left(\frac{3\pi}{2} - \frac{\zeta}{A'} \right) = \frac{1}{a} \left(\frac{3\pi}{2} + \frac{1}{A'} \sqrt{\frac{h}{g}} U \right). \quad (204)$$

The duration of flood is

$$\frac{1}{2}\tau - \frac{2}{A' a} \sqrt{\frac{h}{g}} U \quad (205)$$

and of ebb

$$\frac{1}{2}\tau + \frac{2}{A'a}\sqrt{\frac{h}{g}}U \quad (206)$$

When the cross section of the channel is of any given form, its area, which is assumed to be constant, may be written $\psi(z)$. The equation of continuity then becomes

$$\frac{\partial x'}{\partial x} = \frac{\psi(h)}{\psi(z)}$$

One can proceed as before, considering α and β to denote suitable functions of z . For this and other more general developments, see McCowan's paper already referred to and from which the essentials of the work just given have been taken.

For the equations in ζ , we have from (170) and (171)

$$\frac{\partial^2 \left(1 + \frac{\zeta}{h}\right)^{-1}}{\partial t^2} + \frac{g h}{2} \frac{\partial^2 \left(1 + \frac{\zeta}{h}\right)^2}{\partial x^2} = 0, \quad \frac{\partial^2 \zeta}{\partial t^2} - g h \frac{\partial^2 \zeta}{\partial x^2} = \frac{1}{h} \frac{\partial^2 \zeta^2}{\partial t^2} + \frac{g}{2} \frac{\partial^2 \zeta^2}{\partial x^2}. \quad (207)$$

This is satisfied by putting

$$\zeta = \phi\left(t - \frac{x'}{\omega}\right) \quad (208)$$

where

$$\omega = \sqrt{g h} \left(1 + \frac{3}{2} \frac{\zeta}{h}\right) - U.*$$

For (207) is the equivalent of (172) and this latter is satisfied by

$$z = \phi\left(t - \frac{x'}{3\sqrt{g z - k}}\right) \quad (209)$$

where

$$k = 2\sqrt{g h} + U,$$

as has just been shown, and

$$3(g h + g \zeta)^{\frac{1}{2}} - k = 3\sqrt{g h} \left(1 + \frac{1}{2} \frac{\zeta}{h} - \dots\right) - 2 g h - U. \quad (210)$$

32. The equation of motion where there is a natural slope to the bed.

Let the variable upward slope of the surface (in going up the river) be denoted by I_s , the resistance per unit area by F , and the heaviness of the liquid by γ . For a broad stream the hydraulic mean depth is approximately equal to z . The acceleration, or force per unit mass, is

$$\frac{\partial^2 x'}{\partial t^2} = -g \sin I_s - \frac{g F}{\gamma z}, \quad (211)$$

or

$$\frac{\partial^2 x'}{\partial t^2} = -g \sin I - g \frac{\partial z}{\partial x'} - \frac{g F}{\gamma z}, \quad (212)$$

if I be the uniform upward slope of the bed.

* Cf. Levy, l. c., § 128.

Since

$$x' = x - Ut + \xi, \quad z = h + \zeta,$$

the above equation multiplied by $\frac{\partial x'}{\partial x}$ becomes

$$\left[\frac{\partial^2 \xi}{\partial t^2} + g \sin I + \frac{g}{\gamma} \frac{F}{h} \left(1 + \frac{\zeta}{h} \right)^{-1} \right] \left(1 + \frac{\partial \xi}{\partial x} \right) = -g \frac{\partial \zeta}{\partial x}. \quad (213)$$

The equation of continuity is

$$\Omega \frac{\partial x'}{\partial x} = \Omega_0, \text{ or } b' z \frac{\partial x'}{\partial x} = bh, \text{ or } \frac{b'}{b} \left(1 + \frac{\partial \xi}{\partial x} \right) = \left(1 + \frac{\zeta}{h} \right)^{-1} \quad (214)$$

where Ω_0 and Ω denote the areas of the cross sections and b and b' the breadths at x and x' , respectively.

For a nontidal stream

$$g \sin I + \frac{g F_0}{\gamma h} = 0 \quad (215)$$

where F_0 denotes the resistance per unit area for the permanent stream.

Equation (213) may, for a canal of uniform breadth, be written

$$\frac{\partial^2 \xi}{\partial t^2} + g \sin I + \frac{g F}{\gamma h} \left(1 + \frac{\zeta}{h} \right)^{-1} = -\frac{g h}{2} \frac{\partial \left(1 + \frac{\zeta}{h} \right)^2}{\partial x} \quad (216)$$

since

$$\frac{\partial x'}{\partial x} = 1 + \frac{\partial \xi}{\partial x} = \left(1 + \frac{\zeta}{h} \right)^{-1};$$

or, approximately,

$$\frac{\partial^2 \xi}{\partial t^2} + g \sin I + \frac{g F}{\gamma h} \left(1 - \frac{\zeta}{h} \right) = -\frac{g h}{2} \frac{\partial \left(1 + \frac{\zeta}{h} \right)^2}{\partial x}. \quad (217)$$

In this equation F is supposed to be a known function of u where

$$u = \frac{\partial x'}{\partial t} = -U + \frac{\partial \xi}{\partial t}.$$

Expressions for F are given in sections 7 and 8.

This equation is treated in some detail by Lévy.* His results have more especial reference to the assumption that the resistance is directly proportional to the velocity.

Supposing the slope of the bed to be negligible and the amplitude of the wave small in comparison with the depth, equation (213) becomes, approximately,

$$\frac{\partial^2 \xi}{\partial t^2} + \frac{g F}{\gamma h} = g h \frac{\partial^2 \xi}{\partial x^2}, \quad (218)$$

which equation is obvious upon recalling that γ/g = density or mass per unit volume, and h = depth or volume per unit area of bottom.

* L. c., secs. 138-141.

If

$$F = \pm \zeta' u^3 \frac{\gamma}{2g} + \bar{\zeta} u \frac{\gamma}{2g}, \quad (219)$$

then

$$\frac{\partial^2 \xi}{\partial t^2} \pm \frac{\zeta'}{2h} \left(\frac{d\xi}{dt} \right)^2 + \frac{\bar{\zeta}}{2h} \left(\frac{d\xi}{dt} \right) = gh \frac{\partial^2 \xi}{\partial x^2} \quad (220)$$

where the upper sign goes with the flood stream and the lower with the ebb, ξ being positive in the direction of the flood.

If the bed has a slope, the corresponding equation is

$$\frac{\partial^2 \xi}{\partial t^2} + \frac{gF}{\gamma h} + g \sin I = gh \frac{\partial^2 \xi}{\partial x^2}. \quad (221)$$

If the cross section has a variable width, the equation is

$$\frac{\partial^2 \xi}{\partial t^2} \pm \frac{\zeta'}{2h} \left(\frac{\partial \xi}{\partial t} \right)^2 + \frac{\bar{\zeta}}{2h} \frac{\partial \xi}{\partial t} + g \sin I - \frac{gh}{b} \left(\frac{\partial^2 b \xi}{\partial x^2} - \frac{1}{b} \frac{\partial b}{\partial x} \frac{\partial b \xi}{\partial x} \right) = 0 \quad (222)$$

since the equation of continuity for long-wave motion is then

$$\zeta = - \frac{h}{b} \frac{\partial b \xi}{\partial x} \quad (223)$$

and the force of restitution per unit mass is

$$gh - \frac{\partial \left(\frac{1}{b} \frac{\partial b \xi}{\partial x} \right)}{\partial x} \quad (224)$$

It is evident that a solution of (222) involves the law or function connecting b and x , i. e., connecting the breadth and length of the river.

33. *To find the form of an estuary the undisturbed depth of water being constant, and the tide wave progressive.*

The energy contained in a wave length, the amplitude being A' at a given cross section distant x from the sea, is

$$\frac{1}{2} \gamma A'^3 b \lambda, \quad (225)$$

where b denotes the breadth at the same cross section.

For, the potential energy of a wave length is

$$\gamma \lambda \int_0^{\zeta} z b dz = \frac{1}{2} \gamma \lambda \zeta^2 b \quad (226)$$

where ζ denotes the elevation of the free surface; or $\frac{1}{4} \gamma \lambda A'^3 b$ if the oscillation is harmonic.

The kinetic energy is

$$\frac{1}{2} \frac{\gamma}{g} b h \int_{x=0}^{x=\lambda} u^2 dx \quad (227)$$

or $\frac{1}{4} \frac{\gamma}{g} \lambda b h A^2$ if the oscillation is harmonic. Since $u = \zeta \sqrt{\frac{g}{h}}$, and so $A = A' \sqrt{\frac{g}{h}}$, the kinetic energy is $\frac{1}{4} \gamma \lambda A'^2 b$.

The total energy of a wave length is $\frac{1}{2} \gamma \lambda A'^2 b$.

This shows that if the energy of a long wave remains constant, the amplitude of the tide in a canal of slowly varying section will vary inversely as the square root of the breadth and inversely as the fourth root of the depth.

The energy contained in a wave length whose amplitude is A' , at a cross section distant x , from the sea is, of course, $\frac{1}{2} \gamma \lambda A'^2 b$, where b , denotes the breadth at the upper cross section.

The resistance for a slice dx in length is

$$\zeta' \frac{\gamma}{2g} u^2 b dx, \quad (228)$$

This quantity multiplied by the distance the water particles of the slice travel during the time dt , or $u dt$, gives the energy consumed upon this elementary slice during the time dt , viz.,

$$\zeta' \frac{\gamma}{2g} u^3 b dx dt \quad (229)$$

or

$$\frac{4}{3\pi} \zeta' \frac{\gamma}{2g} A'^2 b dx dt \quad (230)$$

if the oscillation is harmonic. Replacing A by $A' \sqrt{\frac{g}{h}}$ and integrating from $t=0$ to $t=\tau$, and from $x=x$ to $x=x''$ the energy consumed during the time τ on the portion of the estuary above x and below x'' is

$$\frac{\tau 4}{3\pi} \zeta' \frac{\gamma}{2} \int_{x=x}^{x=x''} A'^2 b \frac{g^{\frac{1}{2}}}{h^{\frac{3}{2}}} dx. \quad (231)$$

Since the energy of a wave where the breadth of the channel is b must equal the energy consumed between $x=x$ and $x=x''$ during the time τ , plus the energy of a wave where the breadth is b'' , it follows that

$$A'^2 b \lambda = \frac{\tau 4}{3\pi} \zeta' \int_{x=x}^{x=x''} A'^2 b \frac{g^{\frac{1}{2}}}{h^{\frac{3}{2}}} dx + A''^2 b'' \lambda. \quad (232)$$

$$A'^2 b = \frac{4}{3\pi} \frac{\zeta'}{h^{\frac{3}{2}}} \int_{x=x}^{x=x''} A'^2 b dx + A''^2 b'' \quad (233)$$

since $\lambda/\tau = \sqrt{gh}$. A'' , b'' are the values of the amplitude and width at $x=x''$.

Assuming the depth to be constant, the above equation shows how A' , A'' , b , and b'' , are related. Hence, if the amplitudes of the tide at two cross sections and one of the widths of the channel be given or one amplitude and two of the widths, the remaining width or amplitude can be approximately inferred.

Special cases.—Suppose A' =constant; then, differentiating with respect to x , we have

$$\frac{1}{b} \frac{db}{dx} = -\frac{4}{3} \frac{\zeta'}{\pi h^2} A'. \quad (234)$$

The general solution of this is

$$\log b = -KA'x + \text{constant}$$

or

$$b = C e^{-KA'x} \quad (235)$$

where C =constant and $K = \frac{4}{3} \frac{\zeta'}{\pi h^2}$.

Where $x=0$,

$$b = b_0, \text{ and so } C = b_0; \\ \therefore \log b_0 - \log b = KA'x. \quad (236)$$

Since $A = A' \sqrt{\frac{g}{h}}$, it follows that the average velocity of the tidal stream is the same for all cross sections. This assumption seems to be reasonable or even fundamental in the formation of estuaries; for, if at any point the velocity were unusually great, the banks would there be more eroded than elsewhere by tidal action, and so the width would there increase and the velocity be reduced in the same ratio. At any rate, for purposes of commerce, the velocity and depth should be made as uniform as possible.

Wheeler, on page 183 of his treatise on tidal rivers, gives an empirical formula for the breadth of a tidal river at different parts of its course. He applies his formula to the Humber River, the width being known at each end, i. e., at Spurn Point and Goole. His results are quoted here, and to them are added the values obtained by equation (236) using the same assumed data.

River Humber.

Distance from Spurn Point	Mean low water width	Width by Wheeler's for- mula	Width calcu- lated by aver- age rate of in- crease	Width by formula (236)
<i>Stat. miles</i>	<i>Feet</i>	<i>Feet</i>	<i>Feet</i>	<i>Feet</i>
0	18 000	18 000	18 000	18 000
5	10 800	12 935	16 100	12 942
15	7 392	6 505	12 309	6 690
20	4 950	4 687	10 411	4 810
25	3 300	3 441	8 514	3 458
40	1 320	1 281	2 821	1 285
45	924	924	924	924

In this computation K was determined from the known or assumed end widths and the distance between these two sections.

Theoretically, only one width is necessary for the determination of all others.

In case of the Humber River, the low-water width at the mouth is 18 000 feet; required the width at a distance of 45 statute miles up the river. The average value of A' is 7 feet; the depth h , about 35 feet; and ζ' may be taken as 0.007. These make K 0.000 002 43. The required width is, according to computation, 316 feet, which is about one-third of the measured value. Of course only a very rough agreement between theory and fact could be expected in such cases—hardly more than an agreement in order of magnitude.

Suppose the width of the stream to be constant. The equation (233) becomes

$$A'^2 = K \int_{x=x}^{x=x''} A'^3 dx + A''^2. \quad (237)$$

$$\therefore \frac{1}{A'} = + \frac{K}{2} x + \text{constant}, \quad (238)$$

where constant = $1/A_0$. This shows that the relation between the amplitude of the tide and the distance x can be represented by a hyperbola.

For a short distance, $x'' - x$, the above integral becomes

$$\begin{aligned} A'^2 - A''^2 &= KA'^3 (x'' - x); \\ \therefore A'' &\doteq A' - \frac{K}{2} A'^3 (x'' - x), \end{aligned} \quad (239)$$

Starting with A' at the mouth of the river, the value of A'' can be computed by this formula for a section a few miles higher up the river; starting with this value of the amplitude of the tide, and proceeding as before, the value for a section still higher up can be found; and so on.

34. *Harmonic constants for shallow water components, and related quantities.*

The following table contains the constants for the principal shallow-water tides. The second set of values of S_4 , MS , S_4^0 , and MS^0 are inferred in accordance with section 48, Part II, and are given for the purpose of testing this mode of inference. The high-water and low-water intervals have been computed from M_2 and its harmonics in accordance with Part III. The tidal or cotidal hour is not the time of the M_2 tide alone, but is the hour obtained by using the high-water interval. It should, for each place given, agree with the tidal hour taken from a chart of cotidal lines (Part IV B).

		Geographic position				M ₂ ⁰							
No.	Station	Latitude	Longitude		M ₂	De- grees		S ₂	S ₂ ⁰	M ₄	M ₄ ⁰	M ₆	
			Arc	Time		Lunar hours							
EAST COAST OF AMERICA													
		° /	° /	<i>h. m.</i>	<i>Fl.</i>	°	<i>h.</i>	<i>Fl.</i>	°	<i>Fl.</i>	°		
		<i>North</i>	<i>West</i>										
6	Beechey Island, Barrow Str.....	74 43	92 00	6 08	2.00	347	11.57	0.69	34	0.024	268	
7	Port Leopold, Barrow Str.....	73 50	90 25	6 02	2.00	338	11.27	0.64	29	0.015	202	
15	Kingua Fiord.....	66 36	67 20	4 29	7.43	159	5.30	2.67	202	
20	Godthaab.....	64 12	51 44	3 27	4.46	193	6.43	1.54	229	
25	Fort Conger, Discovery Har.....	81 44	64 44	4 19	1.96	335	11.17	0.89	19	0.018	322	
28	St. Johns, Newfoundland.....	47 34	52 41	3 31	1.17	209.6	6.99	0.48	254	0.020	48	0.020	
32	Quebec.....	46 49	71 12	4 45	5.80	186.9	6.23	1.37	236	0.901	283	0.228	
36	St. Paul I., Gulf of St. Lawrence...	47 14	60 08	4 01	0.98	245.4	8.18	0.33	287	0.017	176	0.003	
40	Halifax.....	44 40	63 35	4 14	2.04	223.5	7.45	0.45	258	0.116	25	0.013	
42	St. John, New Brunswick.....	45 14	66 04	4 24	10.04	324.7	10.82	1.62	4	0.119	146	0.092	
44	Eastport.....	44 54	66 59	4 28	8.58	326.1	10.87	1.40	6	0.208	180	0.171	
45	Pulpit Harbor.....	44 09	68 53	4 36	4.89	320.4	10.68	0.77	355	0.025	143	0.120	
46	Portland.....	43 40	70 14	4 41	4.34	323.6	10.79	0.68	0	0.034	75	0.042	
50	Boston.....	42 22	71 03	4 44	4.44	335.4	11.18	0.71	14	0.056	164	0.189	
58	Newport.....	41 29	71 20	4 45	1.66	217.5	7.25	0.38	237	0.179	120	0.011	
59	Bristol.....	41 40	71 16	4 45	1.90	222.4	7.41	0.43	233	0.292	134	
60	Providence.....	41 49	71 24	4 46	2.02	228.1	7.60	0.44	252	0.344	150	0.087	
63	New London.....	41 21	72 05	4 48	1.14	274.1	9.14	0.21	288	0.064	62	0.039	
66	Willels Point.....	40 48	73 46	4 55	3.65	328.6	10.95	0.64	352	0.096	211	0.210	
67	New York, Governors Island.....	40 42	74 01	4 56	2.12	231.0	7.70	0.41	257	0.087	332	0.076	
70	Sandy Hook, The Horseshoe.....	40 27	74 00	4 56	2.22	217.6	7.25	0.43	246	0.026	336	0.054	
77	Philadelphia, Washington Ave...	39 56	75 09	5 01	2.46	43.5	1.45	0.34	82	0.368	7	0.112	
85	Old Point Comfort.....	37 00	76 18	5 05	1.22	248.4	8.28	0.23	269	0.039	244	0.016	
90	Washington Navy-Yard, D. C.....	38 52	76 59	5 08	1.43	227.9	7.60	0.20	271	0.083	350	0.021	
95	Baltimore, Fells Point.....	39 17	76 35	5 06	0.57	190.2	6.34	0.08	225	0.011	329	0.006	
98	Wilmington, N. C.....	34 14	77 57	5 12	1.15	292.1	9.74	0.10	344	0.183	149	0.026	
101	Charleston, Custom-house wharf...	32 46	79 56	5 20	2.48	213.6	7.12	0.43	240	0.090	242	0.025	
103	Savannah Entr., Tybee I. Light...	32 02	80 51	5 23	3.22	209.5	6.98	0.59	235	0.058	287	0.021	
105	Fernandina, Dade St.....	30 41	81 28	5 26	2.85	228.3	7.61	0.51	258	0.030	295	0.032	
113	Key West, Fort Taylor.....	24 33	81 48	5 27	0.56	260.3	8.68	0.17	280	0.036	235	0.011	
114	Tortugas Harbor Light.....	24 38	82 53	5 32	0.48	278.1	9.27	0.17	292	0.010	297	0.004	
119	Cedar Keys.....	29 08	83 02	5 32	1.06	24.4	0.81	0.42	51	0.054	276	0.012	
120	St. Marks Light, Apalachee Bay...	30 04	84 11	5 37	1.12	43.5	1.45	0.44	73	
125	Warrington Navy Yd., Pensacola B.	30 21	87 16	5 49	0.06	317.0	10.57	0.03	315	0.008	318	0.001	
128	Biloxi Light.....	30 24	88 54	5 56	0.11	11.3	0.37	0.09	32	0.019	138	0.003	
129	Cat Island Light.....	30 14	89 09	5 57	0.12	11.0	0.37	0.07	24	
131	Port Rads, South Pass, Miss. R....	29 01	89 10	5 57	0.06	316.5	10.53	0.04	298	0.006	91	0.001	
140	Galveston, Doswell's wharf.....	29 19	94 47	6 19	0.22	124.5	4.15	0.04	134	0.002	128	0.004	
143	Tampico.....	22 16	97 49	6 31	0.08	62.6	2.09	0.03	73	0.004	63	
145	Vera Cruz.....	19 12	96 08	6 25	0.20	74.6	2.49	0.06	355	0.009	247	0.002	
154	Nassau, Bahamas.....	25 05	77 21	5 09	1.24	213.4	7.11	0.21	237	0.017	65	0.006	
155	Great Harbor, Culebra Island.....	18 18	65 17	4 21	0.29	241.2	8.04	0.04	266	0.017	40	0.009	
156	San Juan.....	18 29	66 07	4 24	0.49	246.3	8.21	0.07	267	0.007	84	0.007	
157	Ponce, P. R.....	17 59	66 40	4 27	0.03	280.0	9.33	0.02	264	0.007	57	0.001	
		<i>South</i>											
175	Pernambuco (Recife Arsenal).....	8 04	34 54	2 20	2.49	133.6	4.45	0.87	151	0.050	243	0.011	
184	Montevideo.....	34 53	56 12	3 45	0.19	34.2	1.14	0.04	318	0.034	146	0.012	
186	Buenos Ayres, La Plata R.....	34 36	58 22	3 53	0.81	184.7	6.16	0.17	266	0.073	90	0.018	
190	South Georgia (Royal Bay).....	54 31	36 01	2 24	0.74	213	7.10	0.38	236	0.01	308	
195	Port Louis, Berkeley Sound.....	51 29	58 00	3 52	1.54	157	5.23	0.49	195	0.068	357	0.012	

M_0^0	S_4	S_4^0	MS	MS ⁰	$\frac{S_2}{M_2}$	$\frac{M_4}{M_2}$	$\frac{M_6}{M_2}$	$2M_2^0 - M_0^0$	$3M_2^0 - M_0^0$	$M_4 \left(\frac{S_2}{M_2} \right)^2$ (= S_4)	$M_4 M_2$ (= MS)	$M_4^2 + 2S_2^0 - 2M_2^0$ (= S_4^0)	$M_4^2 + S_2^0 - M_2^0$ (= $M_2 S_0$)	HWI	LWI	Co-tidal hour	No.
					0.35	0.012		66		0.003	0.017						6
	0.007	257			0.32	0.008		114		0.002	0.010	304					7
					0.36												15
					0.35												20
					0.45	0.009		348		0.004	0.016						25
344					0.41	0.018	0.017	12	285	0.003	0.016			7 18	1 08	10.57	28
258	0.053	35	0.478	337	0.24	0.155	0.039	91	303	0.052	0.432	21	332	6 04	0 52	10.61	32
308			0.004	21	0.34	0.017	0.003	314	68	0.002	0.012	260	218	8 30	2 12	12.23	36
72	0.021	313	0.060	154	0.22	0.057	0.006	62	238	0.006	0.051	94	60	7 33	1 46	11.53	40
173	0.010	208	0.047	179	0.16	0.012	0.009	143	81	0.003	0.034	225	185	11 08	4 59	3.16	42
242					0.16	0.024	0.020	113	16	0.005				11 09	5 05	3.24	44
62	0.005	9	0.021	325	0.16	0.005	0.025	138	179	0.001	0.008	212	177				45
71					0.16	0.008	0.010	212	180	0.001				11 11	4 56	3.49	46
262					0.16	0.013	0.043	146	25	0.001				11 28	5 18	3.81	50
127					0.23	0.108	0.007	315	166	0.009				7 44	0 49	12.22	58
					0.23	0.154		310		0.015							59
264					0.22	0.170	0.043	307	60	0.017							60
134					0.18	0.056	0.034	126	328	0.002				9 27	3 30	1.93	63
84					0.18	0.026	0.058	87	182	0.003				11 09	5 22	3.69	66
89					0.19	0.041	0.036	130	244	0.003				8 04	2 05	12.73	67
353					0.19	0.012	0.024	99	300	0.001				7 35	1 27	12.26	70
206					0.14	0.149	0.045	80	284	0.007				1 22	8 34	6.34	77
191					0.19	0.032	0.013	253	194	0.001				8 44	2 15	1.52	85
82			0.024	207	0.14	0.058	0.015	106	242	0.0016	0.0232	76	33	7 52	2 04	12.73	90
185					0.14	0.019	0.011	51	26	0.0002	0.0031	39	4	6 29	0 23	11.36	95
278			0.033	201	0.09	0.159	0.023	76	239	0.0014	0.0318	252	201	9 37	4 47	2.49	98
311					0.17	0.036	0.010	185	329	0.0027	0.0311	295	268	7 25	1 10	12.50	101
286					0.18	0.018	0.007	132	343	0.0019	0.0212	338	312	7 11	1 05	12.32	103
8	0.028	12			0.18	0.011	0.011	161	317	0.0010	0.0107	355	325	7 54	1 43	1.07	105
180					0.30	0.064	0.020	285	240	0.0033	0.0219	275	255	9 20	2 36	2.47	113
183					0.35	0.021	0.008	259	291	0.0013	0.0071	325	311	9 44	3 21	2.94	114
90					0.40	0.051	0.011	133	343	0.0085	0.0428	329	302				119
					0.39												120
359					0.50	0.133	0.017	316	232	0.0020	0.0080	314	316	11 19	4 08	4.75	125
205					0.82	0.173	0.027	244	189	0.0127	0.0312	180	159				128
					0.58												129
46					0.67	0.100	0.017	182	184	0.0027	0.0080	54	72				131
29					0.18	0.009	0.018	121	345	0.0001	0.0007	147	138	4 18	10 33	10.47	140
					0.38	0.052		62		0.0006	0.0032	84	74	2 00	8 34	8.45	143
337					0.30	0.045	0.010	262	246	0.0008	0.0005	88	168				145
279	0.004	319			0.17	0.014	0.005	1	1	0.0005	0.0058	112	89	7 22	1 09	12.27	154
331					0.14	0.059	0.032	83	32	0.0003	0.0048	89	64	8 04	2 14	12.14	155
308					0.14	0.013	0.013	49	71	0.0001	0.0019	125	105	8 23	2 15	12.50	156
58					0.67	0.233	0.033	143	63	0.0031	0.0093	25	41	8 28	3 37	12.63	157
301					0.35	0.020	0.004	24	99	0.0061	0.0349	278	260	4 33	10 50	6.73	175
333					0.21	0.018	0.063	283	129	0.0015	0.0145	353	69	1 41	5 46	5.38	184
292					0.21	0.090	0.022	280	262	0.0032	0.0307	252	171	6 50	12 21	10.49	186
	0.004	39			0.51	0.001		118		0.0026	0.0103	354	331				190
76	0.007	64			0.32	0.044	0.008	317	35	0.0069	0.0432	73	35				195

No.	Station.	Geographic position				M ₂	M ₂ ⁰		S ₂	S ₂ ⁰	M ₁	M ₁ ⁰	M ₁
		Latitude	Longitude		De- grees		Lunar hours						
			Arc	Time									
	WEST COAST OF AMERICA	° ' "	° ' "	h. m. s.	Fl.	° ' "	h. m. s.	Fl.	° ' "	Fl.	° ' "		
201	Cape Horn, Orange Bay, Chile	55 31	68 05	4 32	1.93	104	3.47	0.30	134	0.016	197	0.017	
205	Valparaiso, Chile	33 02	71 39	4 47	1.65	279.2	9.31	0.47	300	0.007	147	0.004	
		North											
210	Panama (Naos I.)	8 55	79 32	5 18	5.93	86.7	2.89	1.66	144	0.218	358	0.041	
215	Mazatlan	23 11	106 27	7 06	1.08	265.2	8.83	0.74	254	0.014	294	0.012	
217	Magdalena Bay	24 38	112 09	7 29	1.59	244	8.13	1.01	253				
218	San Juanico Bay	26 15	112 28	7 30	1.72	246	8.20	1.02	252				
219	Ambrejos Pt., Ballenas Bay	26 43	113 34	7 34	2.14	261	8.70	1.07	275				
221	San Diego, La Playa, Cal	32 42	117 14	7 49	1.70	276.6	9.22	0.70	275	0.026	186	0.010	
224	San Francisco Entr., Fort Point	37 50	122 28	8 10	1.70	330.7	11.02	0.38	335	0.086	32	0.012	
225	Sausalito	37 51	122 29	8 10	1.57	336.7	11.22	0.35	348	0.033	55	0.021	
227	Astoria, Oregon	46 11	123 50	8 15	2.97	8.6	0.29	0.77	39	0.100	317	0.034	
230	Port Townsend	48 07	122 45	8 11	2.22	105.6	3.52	0.55	130	0.131	290	0.033	
235	Victoria Harbor	48 25	123 23	8 14	1.22	68.8	2.29	0.33	86	0.057	317	0.042	
241	Sand Heads, Fraser River	49 05	123 16	8 13	2.81	142.0	4.73	0.69	171	0.008	297	0.025	
246	Seymour Narrows, Discovery Pt.	50 08	125 23	8 22	2.93	70.9	2.36	0.62	100	0.363	253	0.140	
248	Port Alice	55 48	133 36	8 54	4.04	1.3	0.04	1.30	34	0.048	41	0.018	
250	Sitka, Baranof Island	57 03	135 20	9 01	3.58	2.8	0.09	1.14	34	0.013	140	0.002	
251	Sergius Narrows	57 25	135 38	9 03	4.93	11.7	0.39	1.66	45	0.100	230	0.046	
251.5	Hooniah	58 07	135 47	9 03	5.97	14.2	0.47	2.03	48	0.069	218	0.016	
252	Port Althorp	58 07	136 17	9 05	3.61	353.5	11.78	1.13	35	0.018	329	0.058	
252.5	Granite Cove	58 12	136 24	9 06	4.01	5.9	0.20	1.29	38	0.036	204	0.029	
255	Kokinhenic I.	60 18	145 03	9 40	1.12	11.9	0.40	0.28	51	0.319	13	0.052	
256	Pete Dahl Slough	60 23	145 24	9 42	3.52	12.7	0.42	1.05	46	0.188	296	0.084	
259	Orca, Prince William Sound	60 34	145 41	9 43	4.52	357.7	11.92	1.61	40	0.167	138	0.087	
260	Orca, Cape Whitshed	60 28	145 55	9 44	4.42	8.4	0.28	1.56	44	0.363	231	0.143	
261	Camp April	60 32	146 00	9 44	4.54	356.0	11.87	1.53	32	0.061	138	0.023	
262	Valdez Arm	61 07	146 27	9 46	4.51	353.7	11.79	1.52	25	0.090	141	0.029	
265	Kodiak (St. Paul), Kodiak Id.	57 48	152 21	10 09	3.23	7.7	0.26	1.08	40	0.038	97	0.032	
267	Peterson Bay	54 23	162 38	10 51	1.92	354.8	11.83	0.73	18	0.053	290	0.063	
268	Tigalda Bay	54 05	165 10	11 01	0.38	60.1	2.00	0.28	5	0.026	265	0.007	
269	Unalga Bay	54 00	166 10	11 05	0.61	105.2	3.51	0.13	304	0.042	260	0.043	
270	Dutch Harbor	53 54	166 32	11 06	0.86	111.5	3.72	0.07	350	0.009	280	0.005	
271	Kaskega Bay	53 28	167 05	11 08	0.71	95.5	3.18	0.11	92	0.024	15	0.013	
280	St. Michael	63 29	162 02	10 48	0.55	235.4	7.85	0.12	338	0.042	150	0.018	
285	Port Clarence	65 14	166 24	11 06	0.47	213.4	7.11	0.03	346	0.097	301	0.028	
295	Point Barrow	71 18	156 40	10 27	0.17	336.2	11.21	0.07	16	0.003	319	0.003	
	EAST COAST OF ASIA		East										
342	Yokohama (Nishihatoba)	35 27	139 39	9 19	1.57	154.3	5.14	0.73	185	0.048	98	0.012	
392	Nagasaki	32 44	129 51	8 39	2.84	228.9	7.63	1.17	259				
420	Chemulpho, inner harbor	37 29	126 37	8 26	9.43	107.8	3.59	3.84	187	0.278	67	0.079	
425	Tientsin Entr., Taku Lightship	38 55	117 52	7 51	3.47	94.4	3.15	0.53	157	0.281	99		
430	Shanghai, Wusung Bar	31 21	121 30	8 06	3.11	30.3	1.01	1.03	77	0.700	331		
435	Amoy, inner bar	24 23	118 10	7 53	6.12	1.2	0.04	1.34	57	0.042	92		
436	Swatow, China	23 23	116 39	7 47	1.35	23	0.77	0.32	86	0.228	154	0.053	
438	Whampoa	23 05	113 26	7 34	2.18	32	1.07	0.67	64	0.160	313		
440	Hongkong	22 18	114 10	7 37	1.43	266	8.87	0.56	292	0.076	322	0.014	
490	Singapore	1 17	103 51	6 55	2.60	300	10.00	1.07	348	0.053	264	0.035	

M_4^0	S_4	S_4^0	MS	MS ⁰	$\frac{S_2}{M_2}$	$\frac{M_4}{M_2}$	$\frac{M_6}{M_2}$	$2M_2^0 - M_4^0$	$3M_2^0 - M_6^0$	$M_4 \left(\frac{S_2}{M_2} \right)^2$ (= S_4)	$M_4 \frac{2S_2}{M_2}$ (= MS)	$M_4^0 + 2S_2^0 - 2M_2^0$ (= S_4^0)	$M_4^0 + S_4^0 - M_2^0$ (= MS ⁰)	HWI	LWI	Cot- tidal hour	No.
313					0.16	0.008	0.009	11	359	0.0004	0.0048	257	227	3 35	9 49	8.00	201
107					0.28	0.004	0.002	51	10	0.0006	0.0039	189	168	9 37	3 26	2.07	205
276					0.28	0.037	0.007	176	343	0.0171	0.1221	112	55	2 59	9 13	8.18	210
30					0.69	0.013	0.011	236	46	0.0066	0.0192	272	283				215
					0.64												217
					0.59												218
					0.50												219
112					0.41	0.015	0.006	7	358	0.0044	0.0214	183	185	9 32	3 20	5.03	221
342					0.22	0.051	0.007	269	290	0.0043	0.0385	41	36	11 39	5 03	7.42	224
338					0.22	0.021	0.013	258	312	0.0016	0.0145	78	66				225
106		0.054	340		0.26	0.033	0.011	60	280	0.0068	0.0517	18	347	0 15	6 42	8.50	227
233		0.067	313		0.25	0.059	0.015	281	84	0.0081	0.0635	339	314	3 47	9 32	11.84	230
176					0.27	0.046	0.035	181	30	0.0041	0.0306	351	334	2 17	8 31	10.44	235
11		0.002	268		0.25	0.003	0.009	347	55	0.0005	0.0039	356	326				241
132					0.21	0.124	0.048	248	81	0.0163	0.1537	312	282				246
257					0.32	0.012	0.044	322	107	0.0050	0.0307	105	73	0 03	6 12	8.95	248
94					0.32	0.004	0.001	225	275	0.0013	0.0083	203	172	0 07	6 18	9.13	250
84					0.34	0.020	0.009	153	306	0.0011	0.0671	297	264	0 25	6 41	9.45	251
20					0.34	0.012	0.003	171	23	0.0080	0.0469	285	252	0 29	6 42	9.52	251.5
326					0.31	0.005	0.016	18	16	0.0018	0.0113	51	10	12 10	5 58	8.84	252
48					0.32	0.009	0.007	168	330	0.0038	0.0232	268	236	0 13	6 26	9.31	252.5
346					0.25	0.284	0.047	11	50	0.0199	0.1600	91	52	0 14	6 42	9.89	255
355					0.30	0.053	0.024	89	43	0.0168	0.1126	3	329	0 11	6 45	9.88	256
30					0.36	0.037	0.019	218	323	0.0212	0.1190	222	180	0 05	6 07	9.80	259
11					0.35	0.082	0.032	145	14	0.0454	0.2564	303	267	0 04	6 36	9.80	260
129					0.34	0.013	0.005	214	219	0.0070	0.0411	210	174	12 20	6 04	9.65	261
190					0.34	0.020	0.006	206	151	0.0102	0.0604	204	173	12 13	5 56	9.57	262
239					0.33	0.012	0.010	279	144	0.0042	0.0254	161	129	0 17	6 23	10.42	265
4					0.38	0.028	0.033	59	340	0.0077	0.0404	337	313	12 13	6 10	10.65	267
83					0.74	0.068	0.019	215	97	0.0141	0.0381	155	210	0 08	8 55	11.15	268
203					0.21	0.069	0.070	291	112	0.0020	0.0184	317	118	3 28	8 56	2.43	269
238					0.08	0.010	0.006	303	96	0.0001	0.0014	37	159	3 51	9 59	2.82	270
235					0.15	0.034	0.018	176	51	0.0006	0.0074	7	11	3 12	9 27	2.22	271
266					0.22	0.076	0.032	321	80	0.0020	0.0182	356	253	8 07	1 27	6.64	280
212					0.06	0.206	0.060	125	68	0.0004	0.0124	207	74	6 10	1 10	5.06	285
106					0.41	0.018	0.018	354	183	0.0005	0.0025	37	358	11 37	5 22	9.67	295
109					0.46	0.031	0.008	211	354	0.0105	0.0449	159	129	5 24	11 29	7.90	342
					0.41									7 49	1 41	10.90	392
7		0.226	146		0.41	0.030	0.008	148	317	0.0461	0.2268	226	147				420
		0.086	161		0.15	0.081		90		0.0066	0.0860	224	162				425
		0.465	18		0.33	0.225		90		0.0772	0.4648	64	18	0 13	8 06	4.11	430
					0.22	0.007		270		0.0020	0.0184	204	148	0 04	6 13	4.18	435
172	0.025	216	0.103	200	0.24	0.169	0.039	252	257	0.0128	0.1081	280	217	1 53	6 39	6.04	436
	0.003	16	0.144	359	0.31	0.073		111		0.0152	0.0986	17	345	0 48	7 34	5.21	438
140	0.007	37	0.067	301	0.39	0.053	0.010	210	298	0.0117	0.0596	14	348	9 23	2 56	1.45	440
43					0.41	0.020	0.014	336	137	0.0090	0.0437	0	312	10 20	4 02	3.07	490

No.	Station	Geographic position				M ₁ ⁰		S ₂	S ₃ ⁰	M ₄	M ₄ ⁰	M ₆
		Latitude	Longitude		M ₂	De- grees	Lunar hours					
			Arc	Time								
	OCEANICA	° ' "	° ' "	h. m. "	Fl.	°	h.	Fl.	°	Fl.	°	
590	Boeloengan, Borneo	North 2 50	East 117 22	7 49	0.93	336	11.20	0.49	291			
		South										
591	Samarinda, Borneo	0 30	117 08	7 49	1.39	209	6.97	0.86	261	0.076	113	
593	Moera Djawa, Borneo	0 37	117 18	7 49	1.61	198	6.60	1.05	256	0.052	292	
594	Bay of Balik Papan, Borneo	1 16	116 48	7 47	1.89	153	5.10	1.64	204			
598	Macassar	5 08	119 24	7 58	0.27	70	2.33	0.36	194			
600	Donggala	0 40	119 44	7 59	1.55	159	5.30	1.30	208			
		North										
602	Tontoli	1 00	120 53	8 04	1.38	161	5.37	1.16	199			
633	Maimbun	5 55	121 01	8 04	1.32	178	5.94	0.76	221	0.038	325	0.029
634	Iloilo, Point Gimalik	10 42	122 35	8 10	1.35	332.6	11.09	0.64	18	0.109	248	0.042
634.5	Cebu	10 18	123 54	8 16	1.37	334.3	11.14	0.75	22	0.014	73	0.005
635	Tacloban	11 15	125 00	8 20	0.53	220.6	7.35	0.13	269	0.066	355	0.045
635.5	Santa Elena	11 26	124 54	8 20	0.49	312.3	10.41	0.34	30	0.017	317	0.026
636	Santa Rita Island	11 26	124 57	8 20	1.18	347.8	11.59	0.76	50	0.042	39	0.025
636.5	Catbalogan	11 47	124 52	8 19	1.50	341.1	11.37	0.90	36	0.044	78	0.040
637	Calbayog	12 07	124 38	8 19	1.11	342.7	11.42	0.74	42	0.052	335	0.025
637.5	Halsey Harbor	11 48	119 57	8 00	0.78	311.1	10.37	0.33	4	0.028	239	0.013
640	Manila	14 36	120 57	8 04	0.72	310.2	10.34	0.30	29	0.016	347	0.010
641	Olongapo	14 49	120 17	8 01	0.50	292.9	9.76	0.20	325	0.002	114	0.009
642	Santa Cruz	15 46	119 54	8 00	0.38	271.0	9.03	0.06	324	0.032	341	0.013
643	Bolinao	16 24	119 56	8 00	0.32	278.3	9.28	0.12	306	0.020	357	0.028
644	Sual	16 04	120 06	8 00	0.29	275.9	9.20	0.09	311	0.018	348	0.018
645	Tabaco	13 22	123 44	8 15	1.75	174.7	5.82	0.77	199	0.035	222	0.004
		West										
660	Honolulu, Oahu Island	21 18	157 52	10 31	0.52	109.4	3.65	0.16	109	0.001	28	0.002
		South										
670	Apia, Upolu Island	13 46	171 44	11 27	1.26	186.0	6.20	0.29	184			
		East										
675	Finschhafen	6 35	147 50	9 51	0.22	75.2	2.51	0.31	124	0.03	211	0.02
680	Port Russell, Bay of Islands	35 16	174 08	11 37	2.54	215.9	7.20	0.39	276	0.197	37	0.102
680.2	Auckland	36 51	174 48	11 39	3.78	204.8	6.83	0.63	265	0.20	74	0.10
680.5	Wellington	41 17	174 46	11 39	1.60	137.1	4.57	0.09	325	0.045	332	0.015
681	Port Chalmers	45 50	172 30	11 30	2.39	99.0	3.30	0.27	96	0.044	239	0.052
681.5	Port Darwin	12 23	130 37	8 42	6.56	144	4.80	3.44	193	0.013	279	0.006
682	Cooktown	15 27	145 15	9 41	1.87	282	9.40	0.79	258			
682.5	Cairns Harbor	16 55	145 47	9 43	1.96	282	9.40	1.12	245			
683	Brisbane Bar	27 31	153 00	10 12	2.20	290	9.67	0.58	315			
683.5	Ballina	28 52	153 33	10 14	1.08	262	8.73	0.28	275	0.058	121	0.025
684	Newcastle	32 57	151 44	10 07	1.60	249	8.30	0.39	265	0.027	233	0.018
687	Melbourne (Williamstown)	37 53	144 55	9 40	0.81	69.4	2.31	0.10	164	0.021	49	
689	Port Adelaide (Semaphore)	34 51	138 30	9 14	1.70	120.0	4.00	1.68	181	0.02	174	0.01
689.5	Princess Royal Harbor	35 08	118 00	7 52	0.16	339	11.30	0.26	342	0.005	16	0.002
690	Freemantle, Swan River Entrance	32 03	115 45	7 43	0.16	286.0	9.53	0.14	292	0.010	260	0.007
	INDIAN OCEAN.	North										
710	Mergui (Bay of Bengal)	12 26	98 36	6 34	5.50	310.0	10.33	2.92	349	0.120	133	0.072
720	Amherst, Moulmein River	16 05	97 34	6 30	6.32	67.3	2.24	2.71	102	0.324	43	0.131
722	Moulmein, Moulmein River	16 29	97 37	6 30	3.79	113.5	3.78	1.36	149	0.896	172	0.094
725	Elephant Point, Rangoon River	16 30	96 18	6 25	5.90	103.0	3.43	2.38	140	0.281	88	0.244
726	Rangoon, Rangoon River	16 46	96 10	6 25	5.78	131.4	4.38	2.09	170	0.432	170	0.220

M_0^0	S_4	S_4^0	MS	MS^0	S_2 M_2	M_4 M_2	M_4 M_2	$2M_2^0 - M_0^0$	$3M_2^0 - M_0^0$	$M_4(S_2^0)^2$ ($=S_4$)	$M_4^2 S_2^0$ M_2 ($=MS$)	$M_4^0 + 2S_2^0 - 2M_2^0$ ($=S_4^0$)	$M_4^0 + S_2^0 - M_2^0$ ($=MS^0$)	HWI	LWI	Co- tidal hour	No.
					0.53												590
					0.62	0.054		305		0.0289	0.0936	217	165				591
					0.65	0.033		104		0.0223	0.0685	48	350				593
					0.87												594
					1.33												598
					0.84												600
					0.84												602
81					0.58	0.028	0.022	31	93	0.0124	0.0431	52	8	5 58	12 17	9.70	633
186					0.47	0.081	0.031	58	92	0.0245	0.1032	338	293	11 06	5 22	2.56	634
315					0.55	0.010	0.003	236	328	0.0404	0.0148	168	121	11 35	5 18	2.92	634.5
230					0.25	0.125	0.084	86	72	0.0040	0.0325	93	44	6 53	1 25	10.32	635
122					0.70	0.034	0.054	308	95	0.0081	0.0235	113	35	10 35	4 05	1.89	635.5
234					0.64	0.035	0.021	297	89	0.0173	0.0536	163	100	12 00	5 31	3.26	636
295					0.60	0.029	0.026	244	9	0.0159	0.0529	188	133	11 50	5 28	3.12	636.5
240					0.67	0.047	0.022	351	68	0.0230	0.0589	93	34	11 45	5 26	3.04	637
152					0.42	0.036	0.016	23	61	0.0050	0.0235	345	292	10 37	4 30	2.26	637.5
274					0.42	0.022	0.014	274	297	0.0028	0.0135	144	66	10 51	4 29	2.42	640
127					0.36	0.003	0.015	112	32	0.0002	0.0011	178	146	10 03	3 52	1.69	641
250					0.16	0.085	0.035	201	203	0.0008	0.0102	87	34	9 49	3 05	1.48	642
203					0.38	0.063	0.086	200	272	0.0028	0.0151	53	25	10 21	3 44	2.00	643
239					0.31	0.063	0.064	203	238	0.0018	0.0114	59	24	10 20	3 33	1.98	644
265					0.44	0.020	0.002	128	259	0.0069	0.0312	270	246	5 59	12 19	9.53	645
69					0.31	0.002	0.004	191	259	0.0001	0.0006	27	28	3 48	10 00	2.19	660
					0.23									6 25	0 12	5.65	670
222	0.01	272			1.41	0.136	0.091	299	4	0.0597	0.0085	309	260	2 47	8 32	4.84	675
62					0.15	0.078	0.040	35	226	0.0047	0.0607	157	97	7 26	1 55	7.56	680
67					0.17	0.053	0.026	336	187	0.0056	0.0668	195	134	7 11	0 44	7.29	680.2
135					0.06	0.028	0.009	302	277	0.0001	0.0051	348	160	4 52	10 54	5.05	680.5
334					0.11	0.018	0.022	319	323	0.0006	0.0100	233	236	3 31	9 39	3.90	681
167					0.52	0.002	0.010	9	265	0.0036	0.0136	377	328				681.5
					0.42									9 44	3 31	11.72	682
					0.57									9 44	3 31	11.69	682.5
					0.26									10 00	3 48	11.46	683
133	0.003	246	0.043	199	0.26	0.054	0.023	43	293	0.0039	0.0302	147	134	9 02	3 07	10.49	683.5
74	0.006	289	0.062	252	0.24	0.017	0.011	265	313	0.0016	0.0132	265	249	8 42	2 22	10.29	684
					0.12			90		0.0003	0.0052	238	144				687
259	0.03	186	0.09	99	0.99	0.012	0.006	66	101	0.0195	0.0396	296	234	4 04	10 22	6.70	689
227	0.012	204	0.015	268	1.62	0.031	0.012	302	70	0.0131	0.0162	22	19	11 43	5 18	3.45	689.5
277	0.004	72			0.88	0.062	0.044	312	221	0.0077	0.0175	272	266				690
237	0.044	233	0.157	176	0.53	0.022	0.013	127	333	0.0338	0.1274	211	172				710
252	0.095	114	0.318	75	0.43	0.051	0.021	92	310	0.0596	0.2780	112	78				720
204	0.068	228	0.708	213	0.36	0.236	0.025	55	137	0.1156	0.6433	243	208				722
336	0.084	176	0.291	127	0.40	0.048	0.041	118	333	0.0455	0.2265	162	125				725
86	0.084	259	0.404	212	0.36	0.075	0.038	93	309	0.0566	0.3128	247	208	4 26	11 07	9.87	726

No.	Station	Geographic position				M ₂ ⁰		S ₂	S ₂ ⁰	M ₄	M ₄ ⁰	M ₆
		Latitude	Longitude		M ₂	De-grees	Lunar hours					
			Arc	Time								
	INDIAN OCEAN—continued	° /	° /	h. m.	Fl.	°	h.	Fl.	°	Fl.	°	
		North	East									
735	Akyab	20 08	92 54	6 12	2.56	278.1	9.27	1.13	308	0.007	274	0.023
740	Chittagong	22 20	91 50	6 07	4.44	35.2	1.17	1.57	69	0.406	343	0.140
745	Dublat, Hoogly River	21 38	88 06	5 52	4.61	290.8	9.70	2.11	328	0.088	149	0.011
746	Diamond Harbor, Hoogly River	22 11	88 12	5 53	5.16	344.5	11.48	2.23	26	0.752	247	0.150
747	Calcutta (Kidderpore)	22 32	88 20	5 53	3.63	57.7	1.92	1.50	100	0.740	37	0.154
748	False Point	20 25	86 47	5 47	2.25	269	8.97	1.01	302	0.035	229	0.010
755	Vizagapatam	17 41	83 17	5 33	1.47	253.7	8.46	0.65	286	0.013	320	0.005
756	Cocanada	16 56	82 15	5 29	1.51	252.6	8.42	0.64	286	0.030	106	0.016
763	Madras	13 06	80 18	5 21	1.03	250.8	8.36	0.44	280	0.007	193	0.008
765	Negapatam	10 46	79 51	5 19	0.71	251.2	8.37	0.27	283	0.022	79	0.011
770	Pamban Pass, Rāmesvaram Island	9 16	79 12	5 17	0.58	47.2	1.57	0.37	92	0.016	194	0.010
772	Tuticorin	8 48	78 09	5 13	0.66	43.4	1.45	0.47	84	0.025	156	0.015
773	Trincomalee Ceylon	8 33	81 13	5 25	0.58	241.0	8.03	0.20	265	0.012	224	0.005
775	Point de Galle, Ceylon	6 02	80 13	5 21	0.53	56.9	1.90	0.36	94	0.012	164	0.003
776	Colombo, Ceylon	6 57	79 51	5 19	0.58	49.9	1.66	0.39	95	0.016	170	0.004
780	Port Blair, Andaman Islands	11 41	92 45	6 11	2.00	280.0	9.33	0.96	316	0.011	132	0.004
785	Cochin	9 58	76 15	5 05	0.73	332.1	11.07	0.26	29	0.026	75	0.009
787	Beyppore	11 10	75 48	5 03	0.94	328.3	10.94	0.33	17	0.021	38	0.008
793	Kārwār	14 48	74 06	4 56	1.74	301.8	10.06	0.62	335	0.055	17	0.011
795	Goa or Mormugōa	15 25	73 48	4 55	1.81	300.2	10.01	0.64	332	0.047	6	0.011
800	Bombay	18 55	72 50	4 51	4.04	330.3	11.01	1.61	4	0.130	329	0.010
802	Bhāvnagar	21 48	72 09	4 49	10.9	134.2	4.47	3.47	176	0.894	152	0.239
805	Port Albert Victor	20 58	71 33	4 46	2.88	58.3	1.94	1.13	82.8	0.212	176	0.126
807	Porbandar	21 37	69 37	4 38	2.13	292.8	9.76	0.78	323.6	0.033	124	0.029
809	Okha Point and Bet Harbor	22 28	69 05	4 36	3.82	347	11.57	1.22	14	0.136	107	0.007
810	Navanar	22 44	69 43	4 39	6.04	24.4	0.81	1.89	55	0.109	273	0.065
811	Hanstal	22 56	70 21	4 41	6.85	45.6	1.52	1.93	85	0.727	330	0.305
815	Karachi	24 48	66 58	4 28	2.54	293.7	9.79	0.95	323	0.028	7	0.048
820	Minikoi Light	8 17	73 03	4 52	0.86	329.4	10.98	0.35	20	0.009	67	0.002
825	Bushire	29 00	50 52	3 23	1.06	211.1	7.04	0.39	261.9	0.025	337	0.008
830	Maskat	23 37	58 35	3 54	2.07	276.2	9.21	0.78	306	0.006	65	0.006
840	Aden	12 47	44 59	3 00	1.57	226.5	7.55	0.69	246	0.006	313	0.005
845	Suez	29 56	32 33	2 10	1.84	342.5	11.42	0.46	8	0.029	127	0.010
850	Perim	12 38	43 24	2 54	1.20	226.4	7.55	0.56	243	0.024	13	0.006
		South										
870	Port Louis, Mauritius Island	20 08	57 29	3 50	0.43	23	0.77	0.33	26	0.004	296	0.005
880	Betsy Cove, Kerguelen Island	49 09	70 12	4 41	1.42	9	0.30	0.80	52	0.03	289
890	Durban, Port Natal	29 53	31 04	2 04	1.72	115	3.83	0.95	150
	WEST COAST OF AFRICA AND EUROPE.											
900	Cape Town, Table Bay	33 54	18 25	1 14	1.60	44.5	1.48	0.67	88	0.039	96	0.013
		North										
908	Duala (Kamerun) Africa	4 03	9 40	0 39	2.45	156.3	5.21	0.81	195	0.236	139	0.062
923	Valetta Harbor, Malta	35 55	14 30	0 58	0.20	93	3.10	0.12	100	0.003	350	0.001
925	Toulon	43 07	5 56	0 24	0.20	246	8.20	0.09	250	0.014	352	0.001
926	Marseilles	43 18	5 23	0 22	0.22	228	7.60	0.08	247	0.019	0
		West										
932	Lisbon	38 41	9 06	0 36	3.83	59.1	1.97	1.50	83	0.233	196	0.032
938	Socoo	43 24	1 40	0 07	4.37	89.1	2.97	1.56	121	0.097	323	0.017

M_0^0	S_4	S_4^0	MS	MS ⁰	$\frac{S_2}{M_2}$	$\frac{M_4}{M_2}$	$\frac{M_6}{M_2}$	$2M_2^0 - M_0^0$	$3M_2^0 - M_0^0$	$M_1 \left(\frac{S_2}{M_2} \right)^2$ (= S_4)	$M_4 \frac{2S_2}{M_2}$ (= MS)	$M_4^0 + 2S_2^0 - 2M_2^0$ (= S_4^0)	$M_4^0 + S_2^0 - M_2^0$ (= $M(S)$)	HWI	LWI	Co-tidal hour	No.
123	0.007	196	0.014	274	0.44	0.003	0.009	282	352	0.0014	0.0062	334	304				735
190	0.055	61	0.346	23	0.35	0.091	0.032	87	275	0.0508	0.2874	51	17				740
221	0.016	223	0.074	170	0.46	0.019	0.002	73	291	0.0185	0.0806	223	186				745
108	0.123	327	0.706	287	0.43	0.146	0.029	82	206	0.1414	0.6512	330	289				746
322	0.092	113	0.673	80	0.41	0.204	0.042	78	211	0.1265	0.6127	122	79	1 14	9 40	7.31	747
78	0.008	320	0.040	268	0.45	0.016	0.004	309	9	0.0071	0.0314	295	262				748
69	0.004	50	0.011	356	0.44	0.009	0.003	187	332	0.0025	0.0115	29	352				755
97	0.006	134	0.022	136	0.42	0.020	0.011	39	301	0.0054	0.0254	173	139				756
157	0.002	233	0.006	254	0.43	0.007	0.008	309	236	0.0013	0.0060	252	222	8 35	2 26	2.94	763
130	0.005	135	0.019	99	0.38	0.031	0.016	63	264	0.0032	0.0167	143	111				765
42	0.003	261	0.018	292	0.64	0.028	0.017	260	100	0.0065	0.0205	284	239				770
19	0.006	234	0.012	258	0.71	0.034	0.023	290	111	0.0127	0.0355	238	197				772
112	0.006	217	0.011	242	0.35	0.021	0.009	258	251	0.0014	0.0083	272	248				773
341	0.003	235	0.007	255	0.68	0.023	0.006	310	190	0.0055	0.0163	238	201				775
27	0.005	236	0.009	253	0.67	0.028	0.007	289	123	0.0072	0.0214	261	215	1 47	7 47	8.41	776
116	0.004	358	0.012	229	0.48	0.006	0.002	68	4	0.0025	0.0106	204	168				780
37	0.006	175	0.020	140	0.36	0.036	0.012	229	190	0.0033	0.0185	189	132				785
133	0.005	135	0.010	74	0.35	0.022	0.008	258	132	0.0026	0.0147	136	87				787
224	0.010	98	0.026	67	0.36	0.032	0.006	227	322	0.0070	0.0393	83	50				793
249	0.009	92	0.024	48	0.35	0.026	0.006	234	291	0.0059	0.0333	70	38				795
86	0.012	256	0.138	30	0.40	0.032	0.002	332	185	0.0207	0.1040	36	3	11 27	5 07	6.21	800
125	0.121	235	0.661	196	0.32	0.082	0.022	117	278	0.0902	0.5686	235	194				802
129	0.025	266	0.158	213	0.39	0.074	0.044	301	46	0.0326	0.1666	225	200				805
307	0.004	277	0.028	204	0.37	0.016	0.014	102	211	0.0044	0.0242	186	155				807
270	0.013	117	0.064	111	0.32	0.036	0.002	227	51	0.0137	0.0868	161	134				809
41	0.013	360			0.31	0.018	0.011	136	32	0.0107	0.0682	334	304	0 46	7 04	8.09	810
246	0.021	62	0.351	12	0.28	0.106	0.044	122	251	0.0578	0.4100	48	9	1 24	8 20	8.67	811
206	0.010	20	0.031	320	0.37	0.011	0.019	221	315	0.0039	0.0209	65	36	10 14	3 58	5.42	815
64	0.003	87	0.007	49	0.41	0.010	0.002	232	204	0.0015	0.0073	168	117				820
330	0.007	104	0.027	34	0.37	0.024	0.008	85	304	0.0034	0.0184	78	28				825
24	0.004	333	0.014	305	0.38	0.003	0.003	127	85	0.0009	0.0045	125	95				830
342	0.005	283	0.017	157	0.44	0.004	0.003	140	338	0.0010	0.0048	352	333	7 48	1 37	4.54	840
66	0.004	171	0.016	158	0.25	0.016	0.005	198	241	0.0018	0.0146	178	152				845
2	0.005	328	0.016	83	0.47	0.020	0.005	79	318	0.0052	0.0224	47	30				850
94	0.003	116			0.77	0.009	0.012	110	335	0.0024	0.0061	302	299				870
					0.56	0.021		89		0.0095	0.0339	15	332				880
	0.02	70			0.55	0.0											890
296					0.42	0.024	0.008	353	198	0.0069	0.0327	183	140	1 34	7 45	0.28	900
127					0.33	0.096	0.025	173	342	0.0260	0.1562	217	178	5 24	11 40	4.57	908
26	0.001	37			0.60	0.015	0.005	196	253	0.0011	0.0036	364	357				923
145	0.002	288			0.45	0.070	0.005	140	233	0.0028	0.0126	0	356				925
	0.003	277			0.36	0.086		96		0.0025	0.0138	38	19				926
284			0.195	228	0.39	0.061	0.008	282	254	0.0359	0.1827	244	220	2 04	7 46	2.60	932
112	0.004	92	0.174	67	0.36	0.022	0.004	215	155	0.0124	0.0695	27	355	3 07	9 14	3.13	938

No.	Station	Geographic position				M ₂ ⁰		S ₂	S ₂ ⁰	M ₄	M ₄ ⁰	M ₆
		Latitude	Longitude		M ₂	De- grees	Lunar hours					
			Arc	Time								
	WEST COAST OF AFRICA AND EUROPE—continued	° ' North	° ' West	h. m.	ft.	°	h.	ft.	°	ft.	°	
942	Boyard	46 00	1 13	0 05	5.82	92.3	3.08	2.11	126	0.915	356	0.080
943	Rochelle	46 09	1 09	0 05	5.72	92.3	3.08	2.03	126	0.804	2	0.088
944	Saint Nazaire	47 16	2 12	0 09	5.67	101	3.37	1.98	136	0.518	43	0.079
946	Brest	48 23	4 29	0 18	6.76	99.2	3.31	2.47	139	0.182	85	0.116
948	St. Malo	48 39	2 02	0 08	12.45	173.7	5.79	4.80	225	0.855	271	0.069
950	Cherbourg	49 39	1 37	0 06	6.16	225.3	7.51	2.26	269	0.450	345	0.081
			East									
952	Havre	49 29	0 06	0 00	8.74	285.5	9.52	2.89	333	0.786	85	0.574
			West									
954	Edinburgh	55 59	3 10	0 13	5.94	48.5	1.62	2.00	58	0.231	178	0.243
956	West Hartlepool	54 41	1 12	0 05	5.16	95.9	3.26	1.74	137	0.095	104	0.074
957	Hull	53 44	0 20	0 01	7.56	175.8	5.86	2.34	228	0.345	253	0.164
			East									
958	Sheerness	51 27	0 45	0 03	6.30	0.5	0.02	1.75	56	0.296	44	0.199
			West									
960	London Bridge	51 30	0 07	0 00	8.31	55.0	1.83	1.64	110	0.821	20
			East									
962	Ramsgate	51 20	1 25	0 06	6.14	343.7	11.46	1.88	36	0.548	248	0.164
964	Dover	51 07	1 19	0 05	7.20	336.1	11.20	2.07	28	0.743	229	0.173
			West									
966	Portland Breakwater	50 34	2 25	0 10	2.05	189.4	6.31	1.07	239	0.468	23	0.206
968	Pembroke	51 41	4 56	0 20	8.18	166.0	5.53	2.66	205	0.123	242
970	Helbre Island, Mersey River	53 22	3 18	0 13	9.76	312.9	10.43	3.13	356	0.479	200	0.070
971	Liverpool, Mersey River	53 24	3 00	0 12	9.98	320.7	10.69	3.16	6	0.691	211	0.196
978	Greenock	55 57	4 45	0 19	4.36	337	11.23	1.04	42	0.346	44
			East									
982	Ostende	51 14	2 55	0 12	5.92	12.3	0.41	1.80	64	0.364	345	0.232
982.2	Noord-Hinder, Light Ship	51 35	2 37	0 10	3.94	12.6	0.42	1.19	51	0.308	122
982.5	Schonwenbank, Light Ship	51 47	3 27	0 14	2.93	36.6	1.22	0.87	82	0.462	148
983	Hook of Holland	51 59	4 09	0 7	2.54	72	2.40	0.65	131	0.558	134	0.115
983.2	Maas, Light Ship	52 01	3 54	0 16	2.37	76.3	2.54	0.51	122	0.394	165
984	Ymuiden	52 28	4 34	0 18	2.20	113	3.77	0.58	180	0.620	154	0.177
984.8	Haaks, Light Ship	52 58	4 18	0 17	2.19	179.8	3.99	0.46	228	0.118	253
985	Helder	52 58	4 46	0 19	1.74	171	5.70	0.50	238	0.367	191	0.190
985.5	Terschellingerbank, Light Ship	53 27	4 52	0 19	2.81	213.0	7.10	0.68	257	0.295	308
986	Wilhelmshaven, Jade River	53 31	8 09	0 33	5.34	358.4	11.95	1.35	71	0.299	178	0.184
987	Rothen Sande	53 51	8 05	0 32	4.00	338.5	11.28	1.09	53	0.199	191	0.120
990	Helgoland Island	54 11	7 53	0 32	3.18	334.8	11.16	0.83	40	0.218	189	0.063
995	Copenhagen, Baltic Sea	55 42	12 36	0 50	0.20	277	9.23	0.09	249
1000	Christiania	59 55	10 44	0 43	0.37	128	4.27	0.12	88	0.062	6	0.023
1001	Oscarsborg	59 41	10 37	0 42	0.47	129	4.30	0.16	90	0.072	3	0.026
1002	Arendal	58 27	8 46	0 35	0.28	100	3.33	0.09	68
1003	Stavanger	58 59	5 44	0 23	0.48	282.5	9.42	0.22	332	0.033	252	0.049
1004	Bergen	60 24	5 08	0 21	1.44	297.5	9.92	0.52	334
1006	Bodoe	67 17	14 23	0 58	2.84	356.5	11.88	0.98	35	0.162	292
1007	Fineide	67 17	15 30	1 02	1.74	57.0	1.90	0.50	106	0.039	61	0.039
1008	Kabelvaag	68 13	14 30	0 58	2.98	3.5	0.12	1.08	44	0.141	323
1011	Vardoe	70 20	31 06	2 04	3.29	163.5	5.45	0.92	208	0.039	202

M_0^0	S_4	S_4^0	MS	MS^0	$\frac{S_2}{M_2}$	$\frac{M_4}{M_2}$	$\frac{M_6}{M_2}$	$2M_2^0 - M_4^0$	$3M_2^0 - M_6^0$	$M_4\left(\frac{S_2}{M_2}\right)^2$ ($=S_4$)	$M_4\frac{2S_2}{M_2}$ ($=MS$)	$M_4^0 + 2S_2^0 - 2M_2^0$ ($=S_4^0$)	$M_4^0 + S_2^0 - M_2^0$ ($=MS^0$)	HWI	LWI	Co-tidal hour	No.
309	0.029	247	0.555	82	0.37	0.157	0.014	189	328	0.1198	0.6625	63	30	3 27	9 22	3.42	942
316	0.043	259	0.513	88	0.35	0.141	0.015	183	321	0.1005	0.5692	69	36	3 23	9 26	3.35	943
50	0.027	268	0.384	122	0.35	0.091	0.014	159	253	0.0632	0.3616	113	78	944
325	0.005	301	0.264	107	0.37	0.027	0.017	114	333	0.0242	0.1327	164	124	3 23	9 45	3.57	946
3	0.006	90	0.257	321	0.39	0.069	0.006	76	158	0.1274	0.6601	14	323	5 43	0 04	5.66	948
91	1.838	74	0.078	60	0.37	0.073	0.013	106	224	0.0606	0.3303	72	29	7 30	1 54	7.35	950
301	0.006	227	0.407	170	0.33	0.090	0.066	126	195	0.0865	0.5202	180	132	9 03	4 14	8.74	952
284	0.34	0.039	0.041	279	221	0.0263	0.1555	257	218	2 05	7 54	2.23	954
43	0.022	174	0.044	122	0.34	0.018	0.014	88	245	0.0108	0.0642	186	145	3 19	9 41	3.29	956
211	0.31	0.046	0.022	99	316	0.0329	0.2132	357	305	5 59	0 05	5.80	957
60	0.28	0.047	0.032	317	301	0.0230	0.1652	155	99	0 14	6 16	0.18	958
.....	0.20	0.099	90	0.0277	0.3235	130	75	1 31	8 30	1.46	960
135	0.032	9	0.324	132	0.31	0.089	0.027	79	176	0.0513	0.3354	353	301	11 26	6 07	10.95	962
102	0.450	290	0.29	0.103	0.024	83	187	0.0616	0.4280	333	281	11 08	5 56	10.67	964
55	0.012	176	0.267	81	0.52	0.228	0.100	356	153	0.1278	0.4886	122	73	6 21	13 13	6.30	966
.....	0.33	0.015	90	0.0130	0.0800	320	281	968
15	0.030	298	0.280	254	0.32	0.049	0.007	66	204	0.0493	0.3075	286	243	10 37	4 47	10.47	970
331	0.057	302	0.406	258	0.32	0.069	0.020	71	271	0.0690	0.4367	301	256	10 56	5 16	10.76	971
.....	0.24	0.079	270	0.0198	0.1654	174	109	11 44	5 18	11.65	978
315	0.234	53	0.30	0.062	0.039	39	82	0.0339	0.2220	89	37	0 07	6 33	11.91	982
.....	0.30	0.078	264	0.0281	0.1860	198	160	982.2
.....	0.30	0.158	285	0.0407	0.2744	238	193	982.5
74	0.039	265	0.312	176	0.26	0.220	0.045	10	142	0.0365	0.2857	252	193	2 17	8 46	1.92	983
.....	0.22	0.166	347	0.0182	0.1694	256	211	983.2
241	0.023	293	0.338	204	0.26	0.282	0.080	72	98	0.0432	0.3274	288	221	2 48	11 04	2.40	984
.....	0.21	0.054	107	0.0052	0.0498	348	300	984.8
293	0.013	353	0.203	236	0.29	0.211	0.109	151	220	0.0302	0.2107	325	258	5 52	0 29	5.35	985
.....	0.24	0.105	118	0.0173	0.1428	36	352	985.5
30	0.026	51	0.242	274	0.25	0.056	0.034	179	325	0.0191	0.1513	323	251	0 03	6 14	11.50	986
15	0.017	97	0.115	244	0.27	0.050	0.030	126	280	0.0147	0.1083	340	266	987
334	0.124	239	0.26	0.069	0.020	121	310	0.0148	0.1138	319	254	11 24	5 36	10.48	990
.....	0.45	995
75	0.039	101	0.32	0.168	0.062	250	309	0.0066	0.0403	286	326	1000
75	0.033	109	0.34	0.153	0.055	255	312	0.0083	0.0490	285	324	1001
.....	0.32	1002
93	0.030	317	0.46	0.069	0.102	313	34	0.0069	0.0302	351	302	1003
.....	0.377	318	0.36	1004
.....	0.139	346	0.35	0.057	60	0.0193	0.1118	10	331	1006
179	0.061	62	0.29	0.022	0.022	53	352	0.0032	0.0224	159	110	1007
.....	0.154	28	0.36	0.047	44	0.0185	0.1021	44	4	1008
.....	0.094	173	0.28	0.012	125	0.0031	0.0218	291	246	1011

35. *Stations whose M_1 is due chiefly to propagation in shallow waters.*

The shallow waters are usually tidal rivers. The surmise that M_1 is due to propagation is confirmed by the fact that at places along their shores $2 M_2^0 - M_1^0 \doteq 90^\circ$. At the following places this difference from 90° does not exceed 30° and is generally much less:

(32) Quebec, (40) Halifax, (44) Eastport, (66) Willets Point, (77) Philadelphia, (90) Washington, (98) Wilmington, (256) Pete Dahl Slough, (280) St. Michael, (425) Tientsin Entrance, (430) Shanghai, Wusung Bar, (438) Whampoa, (593) Moera-Djawa, Borneo, (635) Tacloban, (687) Melbourne (Williamstown), (689) Port Adelaide, (720) Amherst, Moulmien River, (725) Elephant Point, Rangoon River, (726) Rangoon, (740) Chittagong, (745) Dublat, Hoogly River, (746) Diamond Harbor, Hoogly River, (747) Calcutta (Kidderpore), (765) Negapatam, (802) Bhávnagar, (807) Porbandar, (825) Bushire, (850) Perim, (880) Betsy Cove, Kerguelen Island, (926) Marseilles, (946) Brest, (948) St. Malo, (950) Cherbourg, (956) West Hartlepool, (953) Hull, (960) London Bridge, (962) Ramsgate, (964) Dover, (968) Pembroke, (970) Helbre Island, Mersey River, (971) Liverpool, (984) Ymuiden, (984.8) Haaks Light Ship, (985.5) Terschellingerbank Light Ship.

In a few of these cases, the M_1 may be of a less obvious origin, and where it is very small the value of M_1^0 given in the table may be unreliable.

36. *Stations whose M_1 is due chiefly to a stationary wave.*

By section 29 the values of M_2^0 and M_1^0 should in a short canal closed at one end approximately satisfy the relation $2 M_2^0 - M_1^0 = 180^\circ$. At the following places this relation is always satisfied to within 30° and generally much less. In many cases the stationary wave is the shallow marginal strip of the sea.

(101) Charleston, (105) Fernandina, (210) Panama, (251.5) Hooniah, (252.5) Granite Cove, (262) Valdez Arm, (271) Kaskega Bay, (440) Hongkong, (642) Santa Cruz, (643) Bolinao, (644) Sual, (660) Honolulu, (755) Vizagapatam, (845) Suez, (908) Duala (Kamerun), (942) Boyard, (943) Rochelle, (944) St. Nazaire, (986) Wilhelmshaven.

It will be seen from the table that, as might be expected, many stations are intermediate between these two cases: e. g., New York and Savannah; Hanstal Point, at the head of the Gulf of Kutch.

37. *Stations whose M_1 is due to a truncated low water.*

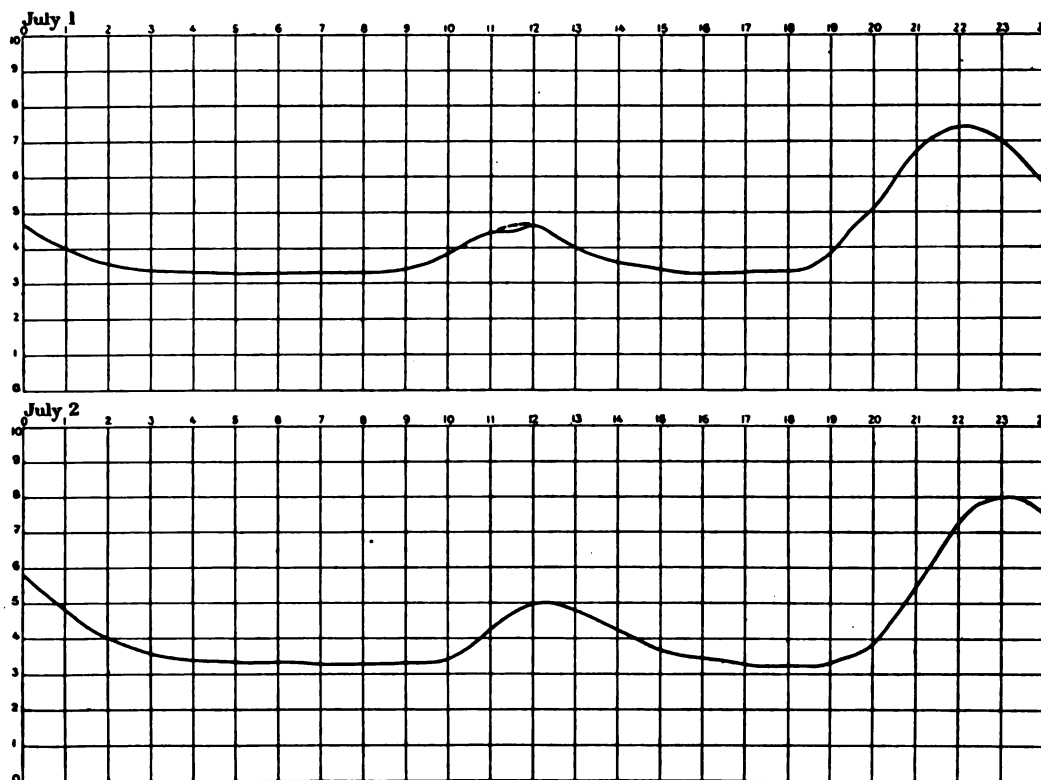
If the low waters are to be flattened while the high waters are undisturbed, it is evident that M_2^0 and M_1^0 should approximately satisfy the relation $2 M_2^0 - M_1^0 = 0$.

The occurrence of truncated low waters must be rare. Kokinhenic Island, Copper River Delta, (latitude $60^\circ 18' .1 N.$, longitude $145^\circ 03' W.$), is situated among extensive flats. At low water there is apparently little or no connection with the outside waters, for tides were there observed night and day during a period of two weeks (June 27 to July 11, 1898), by H. P. Ritter's hydrographic party, and the greatest variation in the height of low water during this time was less than 0.8 foot, notwithstanding the large low-water inequality along the outer coast (Fig. 2). See tables of harmonic constants given under section 19, Part IV B, and section 34, Part V. The truncated low waters should cause a large M_1 , whose epoch expressed in time should approximately agree with that of M_2 . Under No. 255, section 34, it is seen that $2 M_2^0 - M_1^0 = 11^\circ$, and that $M_1^0/M_2^0 = 0.284$.

38. *Stations where M_4 is due to displacement of level due to currents rounding a point.*

From section 12, Part IV A, it appears that if a current rounds a cape and so causes the particles to describe arcs of concentric circles with equal angular velocities as seen from the common center, the amount of transverse tilting resulting therefrom will vary as the square of the velocity of the stream. Hence the outer threads of a stream harmonic in time will produce a sharpened elevation at the time of the maximum velocity; and at the same time the inner threads will produce a similar depression. $\therefore M_4$ has its maximum or minimum at the time when the stream is swiftest. If the time of

No. 2.



Tide curve, Kokinhenic Island.

high water agree with the time of greatest flood velocity, then at the outer side of the curved stream $2 M_2^0 - M_4^0 = 0$ and at the inner side $2 M_2^0 - M_4^0 = 180^\circ$.

Off Victoria, Vancouver Island, the greatest maximum flood velocity occurs near the time of high water; being on the inner edge of the stream, $2 M_2^0 - M_4^0$ should lie near 180° . Observation makes this out to be 181° .

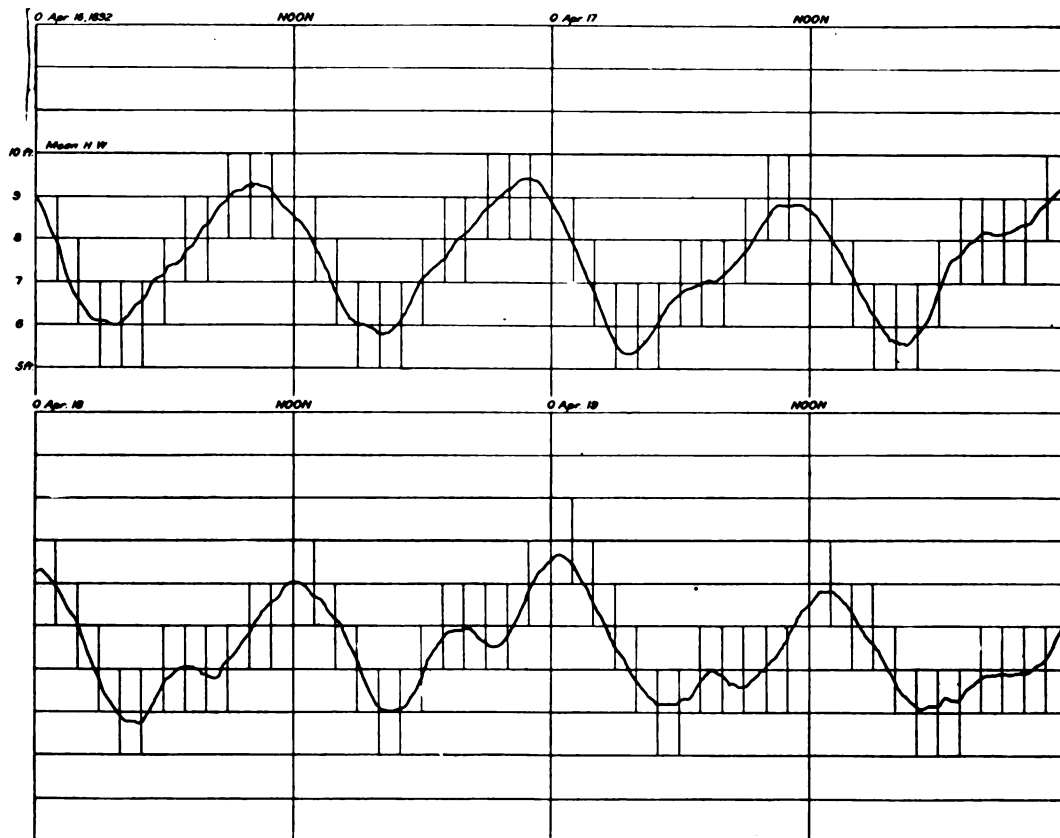
39. *Stations where the duration of rise exceeds the duration of fall.*

For tides of this character the relation $2 M_2^0 - M_4^0 = 270^\circ$ should be approximately satisfied. The M_4 in such cases seems generally to be due, in some way not easily explained, to the fact that the station is situated either upon a strait or at or near the

head of a bay or estuary. At the following stations this condition is satisfied to within 30 degrees or less:

(85) Old Point Comfort, (113) Key West, (114) Tortugas Harbor, (129) Biloxi, (145) Vera Cruz, (184) Montevideo, (186) Buenos Ayres, (224) Fort Point, (225) Sausalito, (230) Port Townsend, (246) Seymour Narrows, (265) Kodiak, (267) Unalga Bay, (435) Amoy, (436) Swatow, (636) Santa Rita I., (636.5) Catbalogan, (640) Manila, (675) Fenschhafen, (684) Newcastle, (735) Akyab, (770) Pamban Pass,

No. 3.



Tide curve, Providence.

(772) Tuticorin, (773) Trincomalee, (776) Colombo, (787) Beypore, (932) Lisbon, (954) Edinburgh, (978) Greenock, (982.2) Noord-Hinder Light Ship, (982.5) Schouwenbank Light Ship, (1000) Christiania, (1001) Oscarsborg.

40. *Stations whose M_4 is chiefly due to a fractional oscillating area.*

If the length of the arm of water upon which a station is situated approach $\frac{1}{4}\lambda$, where λ denotes the length of the M_4 -wave, and if the water lying outside of this arm be shallow for some distance, then the M_4 will be unusually large at the station. According to section 34, Part IV A, one would expect that the phase of the M_4 tide in the arm of water should, in the critical case, be one-fourth of a period behind that of

the M_4 tide outside. This, however, necessitates that shoal water exist at the outer end of the arm of water, in order that an inward progression shall arise. If everything is practically stationary immediately outside, then the arm will be in the same or in the opposite phase to the outside water according as its length is less or greater than $\frac{1}{4}\lambda$.

Consider Newport, Bristol, and Providence, upon Narragansett Bay. By the table $M_4 = 0.18, 0.29, 0.34$ foot. $M_4^0 = 120^\circ, 134^\circ, 150^\circ$, and $2 M_2^0 - M_4^0 = 315^\circ, 310^\circ, 307^\circ$, respectively.

If a body of water symmetrical to Narragansett Bay were located to the south and joined the latter along an east-and-west line across the mouth, the period of free oscillation of the body composed of these two would, on account of the narrowness of the connecting passes, be very nearly 6 hours. But, as there is little progression in the locality of the mouth of the bay, the stationary wave will extend some distance seaward, and so be somewhat more than $\frac{1}{4}\lambda$ long. The phase of the M_4 oscillation in the bay will therefore be 180° different from its phase outside. Also, $2M_2^0 - M_4^0$ should, because of the stationary character of the M_4 wave, be approximately 180° different in the bay from the corresponding value outside. Observations at Newport give $2 M_2^0 - M_4^0 = 315^\circ$, and so the inferred outside value is $315^\circ - 180^\circ = 135^\circ$. This value seems reasonable, as it lies between the value for a progressive and stationary wave; and such is the character of the tide between Narragansett Bay and deep ocean water. Because of the critical length of the bay, a small M_4 outside will cause a large M_4 at Providence.

The values of $2 M_2^0 - M_4^0$ in the bay indicates that the duration of rise must exceed that of fall; also, that there is a tendency to a double low water, which becomes conspicuous at Providence. (See Fig. 3.)

At Portland Breakwater, England, the value of $2M_2^0 - M_4^0$ is 356° , a value very favorable for double low waters. The cause of this peculiarity is not easily ascertained.*

41. *Stations where an apparent M_4 arises from want of free communication between the float box and outside water; or a real M_4 in a nearly inclosed harbor.*

As noted in section 9, Part IV A, an imperfect flow into and out of the float box will give rise to an M_4 whose epoch is connected with that of M_2 by the relation $3 M_2^0 - M_4^0 = 0$.

Upon looking over the table it will be seen that at few if any stations is there a large M_4 whose epoch is such that the above relation is satisfied. Consequently it seems fair to conclude that the observations upon which these values are based were so carried on as to be practically free from such errors as would result from too restricted openings through the walls of the float box.

No analyses giving M_4 are available for harbors having very narrow openings. Probably this sharpening of the high and low waters on the marigram at Melbourne, Port Phillip, could be shown to exist if the M-sums were analyzed for M_4 .

It will be noticed that those places given in the table where this relation is approximately satisfied, are in many instances land-locked harbors. It seems probable that this topographic feature explains the M_4 at such places.

* For a tide curve at this place, see Plate 26, Eruption of Krakatoa and subsequent Phenomena.

42. Stations where M_s is due to the hydraulic effect in a strait connecting two tidal bodies.

According to Section 37, Part I, the current curve for the strait has its maxima and minima flattened and so the duration of slack lessened, i. e., $3\dot{M}_s^0 - \dot{M}_s^0 = 180^\circ$.

While it is assumed that the tides some distance from either end of the strait are not affected by the current and tide in the strait, a like assumption does not apply to the waters just off its ends.

Suppose the case of a short strait having a large stationary tide wave off one end. At about the time of its low water the current of the strait will be at its strength in that direction. This velocity will be greatly diminished upon leaving the strait because of the sudden enlargement of cross section of the stream. Whatever tide is thus made will occur at about the time of the slackening of the stream in this direction. This corresponds to the case of a small opening into a float box, Fig. 7, Part IV A (see also case just described). Hence the considerable sharpening of the \bar{M}_s tide curve where the strait joins the larger body.

$\therefore 3\bar{M}_s^0 - M_s^0 = 0$ where \bar{M}_s denotes the tide due to the strait. Also, $\bar{M}_s^0 = \dot{M}_s^0 + 90^\circ$ where \dot{M}_s^0 refers to the current going toward the stationary wave.

$\therefore M_s^0 = 3(\dot{M}_s^0 + 90^\circ) = 3\bar{M}_s^0 - 90^\circ$.

East River joins Long Island Sound at Willets Point. The western end of the sound is largely surrounded by land, and so is influenced by the Hell Gate tide. Moreover, it is approximately low water in the western end of the sound at the time of maximum eastward velocity in East River. From Fig. 11 it is readily seen that for the eastern portion of East River $\dot{M}_s^0 = 6 \times 30^\circ = 180^\circ$. The relation $M_s^0 = 3\bar{M}_s^0 - 90^\circ$ gives $\bar{M}_s^0 = 90^\circ$. From observations made at Willets Point $M_s^0 = 84^\circ$. Here $3\bar{M}_s^0 - M_s^0 = 182^\circ$; hence the tendency to double high water and to double low water. (See Fig. 9, Pt. I.)

43. References to discussions of tides not simply harmonic:

E. Barlow: *An Exact Survey of the Tide*, pp. 149-153.

M. de Lalande: *Astronomie*, Vol. IV, sections 123, 151-154.

G. B. Airy: *Tides and Waves*, articles 503-520.

Wm. Ferrel: *Tidal Researches*, section 254.

M. Comoy: *Étude Pratique sur les Marées Fluviales*, et notamment sur le Mascaret.

O. Krümmel: *Handbuch der Ozeanographie*, Vol. II, pp. 256-275.

The bore:

G. B. Airy: *Tides and Waves*, articles 513, 514.

O. Krümmel: *Handbuch der Ozeanographie*, Vol. II, pp. 275-280.

V. Cornish: *London Geographic Journal*, Vol. 19 (1892), pp. 52-54.

W. B. Dawson: *Survey of Tides and Currents in Canadian Waters*, Report of Progress, 1899, pp. 22-25 and Plate II.

W. H. Wheeler: *A Practical Manual of Tides and Waves*, Chap. XII.

This manual, Part I, Fig. 19, sections 15, 65, 67, 73, 83; Part V, sections 86, 94.

CHAPTER IV.

COMBINATIONS OF MOTIONS.

44. *The combination of two progressive waves.*

Suppose the space origin to be situated at any convenient point, and suppose one progressive wave to move towards $+x$ and the other at some angle to this direction. Suppose the time to be reckoned from Greenwich or any other given meridian. Let T' denote the time of maximum positive velocity of the particles in the first wave at the origin and T'' that of the second at the origin. Then the maximum positive velocity may be written

$$u = A' \cos (at - lx - aT') = A' \cos (\theta - lx - \epsilon') \quad (240)$$

$$v = A'' \cos (at - ly - aT'') = A'' \cos (\theta - ly - \epsilon''), \quad (241)$$

where the axis of y is generally oblique to the axis of x . For convenience the dots which have been in many places used to indicate velocity, amplitudes, and epochs will be omitted in this chapter.

From $\frac{\partial (u^2 + v^2)}{\partial \theta} = 0$, we have, for finding the times of the resultant maximum and minimum velocities, if x and y are at right angles to each other,

$$\tan 2\theta = \frac{A'^2 \sin 2(lx + \epsilon') + A''^2 \sin 2(ly + \epsilon'')}{A'^2 \cos 2(lx + \epsilon') + A''^2 \cos 2(ly + \epsilon'')}. \quad (242)$$

For a given value of θ , this equation represents a cocurrent line in the xy -plane. If y coincides with x , then from

$$\begin{aligned} \frac{\partial (u + v)}{\partial \theta} &= 0, \\ \tan \theta &= \frac{A' \sin (lx + \epsilon') + A'' \sin (ly + \epsilon'')}{A' \cos (lx + \epsilon') + A'' \cos (ly + \epsilon'')}, \end{aligned} \quad (243)$$

$$\tan lx = \frac{A' \sin (\theta - \epsilon') + A'' \sin (\theta - \epsilon'')}{A' \cos (\theta - \epsilon') + A'' \cos (\theta - \epsilon'')}. \quad (244)$$

The time is referred to the time of maximum velocity (and also of high water) of the first wave at $x=0$ if $\epsilon'=0$, and to the time of maximum velocity (and also of high water) of the second wave at $y=0$ if $\epsilon''=0$. The amplitude of the velocity of the current at any point x, y , is obtained by substituting for θ in (240), (241), its value from (242), and afterwards taking the square root of $u^2 + v^2$.

45. *The combination of a stationary and a progressive wave, both lying in the same direction.*

Suppose the space origin to be situated at a loop of the stationary wave, and suppose the progressive wave to move towards $+x$. Suppose the time to be reckoned from Greenwich or any other given meridian. Let T' denote the time of the maximum

positive velocity of the first wave at (near) $x = 0$ and T'' that of the progressive wave at $x = 0$. Then the resultant velocity may be written

$$A' \sin lx \cos (at - aT') + A'' \cos (at - lx - aT''),$$

or

$$A' \sin lx \cos (\theta - \epsilon') + A'' \cos (\theta - lx - \epsilon''). \quad (245)$$

From $\frac{\partial (\text{velocity})}{\partial \theta} = 0$, we have, for finding the times of the resultant maximum and minimum velocities,

$$\tan \theta = \frac{A' \sin lx \sin \epsilon' + A'' \sin (lx + \epsilon'')}{A' \sin lx \cos \epsilon' + A'' \cos (lx + \epsilon'')}, \quad (246)$$

and for the position of maximum or minimum velocity at any given time or hour,

$$\cot lx = \cot (\theta - \epsilon'') - \frac{A' \sin (\theta - \epsilon')}{A'' \sin (\theta - \epsilon'')}. \quad (247)$$

The time is referred to the time of maximum positive velocity of the stationary wave if $\epsilon' = 0$, and of the progressive wave at $x = 0$, if $\epsilon'' = 0$. The former assumed time refers to mean water of the first wave and the latter to high water of the second. If ϵ' be replaced by $\epsilon' + 90^\circ$ in the above equations, the time will be referred to the time of high water at $x = 0$ of the stationary wave. The amplitude of the velocity at any point x is obtained by substituting for θ in (245) its value from (246).

46. *The combination of one stationary wave with another lying transversely to it.*

Suppose the space origin to be situated at a loop of each wave. Suppose the time to be reckoned from Greenwich or any other given meridian. Let T' denote the time of the maximum positive velocity at (near) $x = 0$ of the stationary wave whose motion is parallel to x , and T'' the time of the maximum positive velocity at (near) $y = 0$ of the stationary wave whose motion is parallel to y . Then

$$u = A' \sin lx \cos (at - aT') = A' \sin lx \cos (\theta - \epsilon'), \quad (248)$$

$$v = A'' \sin ly \cos (at - aT'') = A'' \sin ly \cos (\theta - \epsilon''). \quad (249)$$

$$\therefore u^2 + v^2 = A'^2 \sin^2 lx \cos^2 (\theta - \epsilon') + A''^2 \sin^2 ly \cos^2 (\theta - \epsilon'').$$

From $\frac{\partial (u^2 + v^2)}{\partial \theta} = 0$ we have, for finding the time of the resultant maximum and minimum velocities,

$$\tan 2\theta = \frac{A'^2 \sin^2 lx \sin 2\epsilon' + A''^2 \sin^2 ly \sin 2\epsilon''}{A'^2 \sin^2 lx \cos 2\epsilon' + A''^2 \sin^2 ly \cos 2\epsilon''}. \quad (250)$$

By assuming a value θ (i. e., to at), this equation gives a relation between x and y which is the equation of the cocurrent line from the assumed time.

From (250) we have

$$\frac{dy}{dx} = - \frac{A'^2 \cos 2lx \sin 2(\theta - \epsilon')}{A''^2 \cos 2ly \sin 2(\theta - \epsilon'')}, \quad (251)$$

which gives the direction of the cocurrent line whose characteristic is θ at any per-

missible point x, y . The ratio appears to be indeterminate where lx and ly are both odd multiples of 45° . The value is, however $= -\frac{A'^2 \sin 2(\theta - \epsilon')}{A''^2 \sin 2(\theta - \epsilon'')}$. For the lines $lx =$ an odd multiple of 45° , $\frac{dy}{dx}$ becomes generally zero, and for $ly =$ an odd multiple of 45° , it generally becomes infinite. Hence:

The cocurrent lines are normal to the lines in a square oscillating area drawn parallel to the sides and at a distance therefrom equal to one-fourth the length of one side.

At the center of the square where $lx = 90^\circ$, $ly = 90^\circ$,

$$\frac{dy}{dx} = -\frac{A'^2 \sin 2(\theta - \epsilon')}{A''^2 \sin 2(\theta - \epsilon'')} \quad (252)$$

From the fact that two stationary waves of different phases are combined together, and that at the centers of the sides the times of maximum velocity form a cycle of values, it is readily seen that the time of maximum current will assume all hours as we proceed around the square.

The equation

$$A'^2 \sin^2 ly \cos^2(\theta - \epsilon') + A''^2 \sin^2 lx \cos^2(\theta - \epsilon'') = (\text{constant})^2, \quad (253)$$

represents a line along which the velocity at any given time is constant, while the equation

$$\frac{A'' \sin ly \cos(\theta - \epsilon'')}{A' \sin lx \cos(\theta - \epsilon')} = \text{constant}, \quad (254)$$

represents a line along which the direction of motion is constant. This last expression is equivalent to the tangent of the angle formed by the direction of the motion and the x -axis.

If we eliminate θ between (253) and its derivative with respect to θ , the result will be the equation of a line of equal maximum velocity, i. e., of equal velocity amplitude. The result being the envelope of (253), it follows that all lines having one and the same velocity are tangent to the line of equal velocity amplitude.

If θ be eliminated between (254) and the derivative of (250), the result will be the equation of a line along which the direction of maximum velocity is constant.

The numerator and denominator in the expression for $\tan 2\theta$ will vanish, and so the time of maximum or minimum current becomes indeterminate if

$$\begin{aligned} A' \sin lx &= A'' \sin ly, \\ 2\epsilon' &= 2\epsilon'' + (2\nu + 1)\pi. \end{aligned} \quad (255)$$

These values substituted in the expression for u and v show that the component velocities are there equal in amplitude and differ in phase by 90° . Such points may be called "circular points." From them the cocurrent lines radiate, their direction for a given value of θ being the angle whose tangent is dy/dx as obtained from (251).

47. *The combination of a progressive wave with a stationary wave lying transversely to it.*

Suppose the space origin is so taken that $x=0$ at a loop of the stationary wave, and suppose the progressive wave to move towards $+y$, a direction transverse to the motion of the stationary wave. Suppose the time to be reckoned from Greenwich or any other given meridian. Let τ' denote the time of maximum velocity of the stationary wave at (near) $x=0$, and τ'' that of the progressive wave at $y=0$. The velocities may be written

$$u = A'' \sin lx \cos (at - a\tau') = A' \sin lx \cos (\theta - \epsilon'), \quad (256)$$

$$v = A'' \cos (at - ly - a\tau'') = A'' \cos (\theta - ly - \epsilon''). \quad (257)$$

From $\frac{\partial(u^2 + v^2)}{\partial \theta} = 0$ we have for finding the times of maximum and minimum velocities

$$\tan 2\theta = \frac{A'^2 \sin^2 lx \sin 2\epsilon' + A''^2 \sin 2(ly + \epsilon'')}{A'^2 \sin^2 lx \cos 2\epsilon' + A''^2 \cos 2(ly + \epsilon'')}. \quad (258)$$

By assigning a value to θ , (i. e., to at), this equation gives a relation between x and y , which is the equation of the cocurrent line for the assumed time.

The equation

$$A'^2 \sin^2 lx \cos^2 (\theta - \epsilon') + A''^2 \cos^2 (\theta - ly - \epsilon'') = (\text{constant})^2 \quad (259)$$

represents a line along which the velocity at any given time is constant, while the equation

$$\frac{A'' \cos (\theta - ly - \epsilon'')}{A' \sin lx \cos (\theta - \epsilon')} = \text{constant} \quad (260)$$

represents a line along which the direction of motion is constant. This last expression is equivalent to the tangent of the angle formed by the direction of the motion and the x -axis.

If we eliminate θ between (259) and its derivative with respect to θ , the result will be the equation of a line of equal velocity amplitude, as in the preceding case. Similarly for the line along which the direction of maximum velocity is constant.

The particles will describe rectilinear paths if v/u is a constant, i. e., free from θ .

At any point defined by the equations

$$\begin{aligned} A' \sin lx &= \pm A'', \\ \epsilon' &= ly + \epsilon'' + \left(\frac{2\nu \text{ or } 2\nu + 1}{2\nu + 1} \right) \pi, \end{aligned} \quad (261)$$

it is observed from (256) and (257) that $u=v$, or $v/u=1$.

The numerator and denominator in the expression for $\tan 2\theta$ will vanish, and so the time of maximum or minimum current become indeterminate, at the points

$$\begin{aligned} A' \sin lx &= A'', \\ 2\epsilon' &= 2(ly + \epsilon'') + (2\nu + 1)\pi. \end{aligned} \quad (262)$$

These values substituted in the expressions for u and v show that the component velocities are there equal in amplitude and differ in phase by 90° . Such points are also called "circular points." From them the cocurrent lines radiate, the direction for a given value of θ being the angle whose tangent is $\frac{dy}{dx}$ as obtained from (258).

48. *Currents in a marginal strip of shallow water.*

Given the rise and fall of tide and the varying depth of the water from the shore outward, required the current velocities at various distances from the shore upon the assumption that the tide in the region considered is a dependent stationary wave of sensibly constant range.

Let x be reckoned outward from the shore line and t from the time of high water. Let A' denote the amplitude of the tide and a its speed. Let $h=f(x)$ be the depth at mean-tide stage. Then the tidal volume, one unit broad and extending outward from the shore line a distance x is

$$Ax \cos at.$$

The cross section at x of unit width has for its area

$$f(x) + A \cos at.$$

Assuming the flow to take place in vertical slices, the velocity at any given time of tide is

$$\begin{aligned} u &= -\frac{1}{f(x) + A \cos at} \frac{\partial (Ax \cos at)}{\partial t}, \\ &= \frac{Aax \sin at}{f(x) + A \cos at} \end{aligned} \quad (263)$$

This is not strictly harmonic. The last half of the flood and first half of the ebb will appear, upon a plotting of the current, more nearly like a straight line than would have been the case for simple harmonic motion, while the last half of the ebb and first half of the flood will have more curvature because of this irregularity.

Ignoring the effect of the rise and fall upon the cross section,

$$u = \frac{Aax \sin at}{f(x)},$$

which is simply harmonic.

For M_2 , $a = m_2 = 0.0001405$ radian per second.

If $f(x) = \mu x + H$, H being the depth at the shore and μ the downward slope of the bottom, then

$$u = \frac{Aax \sin at}{\mu x + H};$$

and if $H=0$,

$$u = \frac{Aa}{\mu} \sin at.$$

Hence, the off or on shore velocity for a uniformly sloping bottom, the depth of water being zero at the shore, is independent of the distance from the shore.

If $\mu=0$,

$$u = \frac{Aax \sin at}{H};$$

that is, the off or on shore velocity for a shallow strip of water of uniform depth is directly proportional to the distance from the shore.

The cases found in nature often lie between the two just considered.

It is important to observe that for a uniformly sloping bottom the reflux under currents due to winds diminish from the shore in going outward.

Let τ' denote the time of high water of the stationary wave at $x=0$ and τ'' that of the progressive wave at $x=0$. Then the velocity due to a stationary wave normal to the shore line may be written

$$\frac{A'a}{\mu} \sin (at - a\tau')$$

where A' is amplitude of the stationary portion of the tide. The velocity for the progressive wave near the origin and along its line of advance is, equation (38), Part I,

$$A'' \sqrt{\frac{g}{h}} \cos (at - a\tau''),$$

A'' being the amplitude of the progressive portion of the tide. If this progress shoreward, the total offshore velocity is

$$u = \frac{A'a}{\mu} \sin (at - a\tau') - A'' \sqrt{\frac{g}{h}} \cos (at - a\tau''). \quad (264)$$

This becomes zero when

$$\tan at = \frac{\alpha \sin a\tau' + \beta \cos a\tau''}{\alpha \cos a\tau' - \beta \sin a\tau''} \quad (265)$$

where $\alpha = \frac{A'a}{\mu}$ and $\beta = A'' \sqrt{\frac{g}{h}}$.

If $\tau'' = \tau' = 0$,

$$\tan at = \frac{A''}{A'} \frac{\mu}{a} \sqrt{\frac{g}{h}}, \quad (266)$$

\therefore if $A''=0$, slack water occurs at the time of high or low water, but if $A'=0$, three hours before or after.

Progressions result from such irregularities in the shore line as estuaries and narrow openings into bays, also from certain irregularities in the bottom, although the rise and fall of the outside waters is, of course, the cause of all such motion. The time of tide on the general shore line depends little upon the existence or absence of estuaries and straits leading inland. Hence, the time of high water in the entrance to such estuaries or straits can not differ much from the time of tide for the same shore line devoid of such irregularities. The difference generally becomes greater as the size of the estuary or other shallow arm of the sea increases; but it seldom exceeds one hour and is generally much less. Because of the existence of an antecedent wave (sec. 8, Part IV B), one can assume that somewhere not far off the mouth of the estuary or

strait, the time of tide is the same as it would have been had no such opening been present. For this point the time of turning of the current will approximately satisfy (266); and conversely if the time of turning is known, the ratio A''/A' can be estimated. In fact, $\frac{\mu}{a}$ is often in the neighborhood of 15, and, for the shelving areas here considered, $\sqrt{\frac{g}{h}}$ generally ranges from one-fourth to unity or more. Since the observed delay of turning outside the estuary seldom exceeds one hour, it follows that A'' is there a small fraction of A' .

As the estuary is approached and entered, the τ'' terms in (264) become dominant and t approaches the values $\tau'' \pm 3^h$. On the other hand, equation (265) shows that when h becomes sufficiently great, $t = \tau'$ or $\tau' + 6^h$, as was to be expected.

49. *The combination of a progressive stream with a stationary one lying transversely to it.*

Let the velocities of the stationary and progressive motions be written

$$\begin{aligned} u &= A' \cos \theta, \\ v &= A'' \cos (\theta - l''y - \epsilon''); \end{aligned}$$

then

$$\frac{\partial(u^2 + v^2)}{\partial \theta} = 0$$

leads to the equation

$$\tan 2\theta = \frac{A''^2 \sin 2(l''y + \epsilon'')}{A'^2 + A''^2 \cos 2(l''y + \epsilon'')}, \quad (267)$$

which gives the time of maximum or minimum current at any point x, y after the time of the maximum current in the stationary wave; A' and A'' are supposed to be functions of x, y , suited to the problem in hand.

There exists a point at which the component velocities are equal in amplitude and differ in phase by an odd multiple of 90° . For, the point x, y will cause the numerator of (267) to vanish if $l''y + \epsilon'' = 0$ or $(2\nu + 1)\frac{\pi}{2}$. This second value of y substituted in the denominator will cause it also to vanish, and so the time of maximum current to become indeterminate, if $A' = A''$. The value of x thus determined is the remaining coordinate of the point in question and which, as already noted, may be styled a "circular point."

In particular, suppose $A' = \text{constant}$ and suppose A'' to be some such function of x as $A' e^{-\rho^2 x}$ or $\frac{L-x}{L} A'$. Taking the circular point as the space-origin it is evident that all along the line $x=0$, the component velocities are equal. Since for simplicity ϵ' has been taken as zero, $\epsilon'' a$ denotes the excess of the current hour of the progressive wave at this point over that of the stationary wave. The velocity ellipse at any part x, y , will have its y -semiaxis numbered $l''y + \epsilon''$ in excess of the numbering of the x -semiaxis. The latter number is the current hour of the stationary wave multiplied by a .

50. *Given the two component velocities, to find the time of occurrence of the maximum velocity, together with its value and direction.*

Let the velocities be written

$$\begin{aligned} u &= A' \cos \theta, \\ v &= A'' \cos (\theta - E), \end{aligned}$$

where $E = l'y + \epsilon''$. Then the time is given by the equation

$$\tan 2\theta' = \frac{A''^2 \sin 2E}{A'^2 + A''^2 \cos 2E}. \quad (268)$$

If we have

$$z = A \cos \theta + B \cos (\theta + \beta),$$

the time of the maximum or minimum value of z is given by the equation

$$\tan \theta = -\frac{B \sin \beta}{A + B \cos \beta}$$

where β is the phase of B relatively to the phase of A , or

$$\tan \theta = -\frac{B \sin E}{A + B \cos E}$$

if $E = -\beta$. Or,

$$-\tan \theta = \tan \theta' = \frac{B \sin \beta}{A + B \cos \beta}. \quad (269)$$

Hence Table 14 can be used directly as it stands provided we make the following substitution

For A , A'^2 ;
for B , A''^2 ;
for θ' , 2θ ;
for β , $2E = 2(l'y + \epsilon'')$.

For distinction, θ' is here used to denote the θ of sect. 4, Part III.

Upon substituting in u and v the value of θ thus found, the value of the maximum current, $\sqrt{u^2 + v^2}$, and of its direction, $\tan^{-1} \phi = \frac{v}{u}$, become known.

51. *Given the times and the magnitudes of two components at right angles to each other, to find the times, directions, and magnitudes of the maximum and minimum velocities.*

Let

$$u = A' \cos (at - aT') = A' \cos (\theta - \epsilon'), \quad (270)$$

$$v = A'' \cos (at - aT'') = A'' \cos (\theta - \epsilon''). \quad (271)$$

The required times are given by the equation

$$\tan 2\theta = \frac{A'^2 \sin 2\epsilon' + A''^2 \sin 2\epsilon''}{A'^2 \cos 2\epsilon' + A''^2 \cos 2\epsilon''}. \quad (272)$$

From the harmonic analysis of the observations the component amplitudes or H 's and the local epochs or κ 's are supposed to be known. Upon substituting these for the A 's and ϵ 's in the above formula, the required times, referred to the time of transit of the tidal body across the local meridian, become known.

Take one of the four values of θ and substitute it in the expression for u and v . The signs of u and v will show the quadrant of the corresponding maximum and minimum velocity; $\sqrt{u^2 + v^2}$ will be the value of the velocity, and $\frac{v}{u}$ the tangent of its direction with the x -axis.

Tables 14 and 15 can be used in this connection if we take as time origin the time of maximum velocity in the x -direction and make the following substitution:

For A , A'' ;
for B , A''' ;
for ϵ' , 2θ ;
for β , ϵ'' .

52. *Given the times, directions, and magnitudes of the maximum and minimum velocities to find the velocity at any other time.*

Suppose that here u , v , A' , A'' , ϵ' , ϵ'' , refer to the given principal directions, and not to the arbitrary north and east or y - and x -directions.

The equations (270), (271), (272) still apply, but with the additional restriction

$$\epsilon'' = \epsilon' \pm 90^\circ.$$

The component velocities along the principal directions are the values u and v with the given value of t or θ substituted.

To find the velocity at any given time graphically, describe a circle with twice the maximum velocity as diameter. Divide the circumference into twelve equal hour spaces. Mark the required time and draw an ordinate to the diameter through this point. Diminish this ordinate in the ratio $\frac{\text{minimum velocity}}{\text{maximum velocity}}$. The line drawn from the center to the extremity of this diminished ordinate represents the velocity for the given time in both magnitude and direction.

53. *Given the times, directions, and magnitude of the maximum and minimum velocities of the periodic current also the velocity of the permanent current to find the times, directions, and magnitudes of the resulting maximum and minimum velocities.*

Let the velocity ellipse of the periodic current be written

$$\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1, \text{ or } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad (273)$$

and let x' , y' denote the extremity of the vector representing the velocity of the permanent stream taken in the reverse direction. Then the distance D from this point to any point on the ellipse satisfies the equation

$$(x' \sim x)^2 + (y' \sim y)^2 = D^2. \quad (274)$$

• Substituting in the expression for D^2 the value of y from (273) and equating the x -derivative to zero there results the equation in x

$$(a^2 - b^2)x + \frac{by'x}{\sqrt{1 - \frac{x^2}{a^2}}} - a^2x' = 0. \quad (275)$$

Writing $\sin \theta$ for $\frac{x}{a}$ this equation becomes

$$(a^2 - b^2) \sin \theta + by' \tan \theta - ax' = 0, \quad (276)$$

which is of the same form as equation (28), Part III; this is tabulated in Tables 17 and 44.

Having thus found the value of x , the value of y becomes known from (273); the required maximum and minimum velocities then become known in both magnitude and direction.

The velocity ellipse may be denoted either by the single equation (273) or by the two equations (270), (271), provided A' , A'' , ϵ' , ϵ'' , in the latter refer to principal axes. Hence the angle θ becomes known through them for any particular point, x , y or u , v . Consequently the times become known when the velocities attain their maximum or minimum values.

54. *Rectilinear tidal currents combined with a permanent current.*

By means of the parallelogram of velocities it is easily seen that the time of maximum tidal current is the time of maximum resultant current (sometimes of minimum current, if the resultant direction is more than 90° away from the tidal current considered). Hence, the observed time of maximum resultant current is the time of maximum tidal current.

Given the resultant or observed maximum flood and ebb arrows to find those representing the true flood and ebb and the permanent current.

Join the ends of the observed arrows and bisect the connecting line; join the origin and the point of bisection. This line represents the permanent current in position, magnitude, and direction. The two portions of the bisected line represent the true flood and ebb in magnitude and direction, but should be transferred to the origin, in order that the position may be correct. The value of an ordinary maximum current (slack) is the perpendicular distance of the bisected line from the origin. If ω represents the angle between the permanent current and the true ebb, then the time of slack before flood will be accelerated over its position midway between the two strengths by

$$\frac{1}{a} \sin^{-1} \left(\frac{C \cos \omega}{A} \right) \quad (277)$$

hours, A being the amplitude of the tidal-current velocity and C the velocity of the permanent stream.

If $\omega = 0$, as in a tidal river, then this value becomes $\frac{1}{a} \sin^{-1} \frac{C}{A}$. If in such cases F denote the numerical value of the strength of flood and E that of ebb, then

$$\begin{aligned} A &= \frac{1}{2}(E + F), \\ C &= \frac{1}{2}(E - F). \end{aligned} \quad (278)$$

Where the tidal currents are not rectilinear, but elliptical, the heads of the radiating arrows representing the observed velocities (at each hour, say) will lie upon the perimeter of an ellipse. The major semiaxis found from the plotting represents in magnitude and direction (but not in position) the maximum tidal velocity, and the minor semiaxis, the minimum. The permanent current is represented by a line drawn from the origin of the plotting to the center of the ellipse.

55. *Circular points.*

Fig. 4 represents the case where

$$A'' = \frac{L-x}{L} A', \quad \epsilon' = 0, \text{ and } \epsilon'' = 90^\circ.$$

$$\therefore u = A' \cos \theta,$$

$$v = \frac{L-x}{L} A' \cos (\theta - l''y - 90^\circ) = \frac{L-x}{L} A' \sin (\theta - l''y).$$

With a given θ and x, y , it is easy to find upon the diagram the u and v of the maximum velocity.

For a minimum, use the same equation but decrease or increase θ by 90° according as the velocity wanted is before or after the maximum. It will be noticed that in the figure the rotation of the current arrows is counterclockwise.

From (267) and the assumptions just made

$$\begin{aligned} \tan 2\theta &= \frac{\left(\frac{L-x}{L}\right)^2 \sin 2(l''y + 90^\circ)}{1 + \left(\frac{L-x}{L}\right)^2 \cos 2(l''y + 90^\circ)} \\ &= -Ll'' \frac{y}{x} + 2l''y, \end{aligned} \quad (279)$$

near the origin. In this vicinity

$$\frac{dy}{dx} = -\frac{K-2}{Ll''-2l''x} \tan \phi$$

where K is written for $\tan 2\theta$. At the origin

$$\begin{aligned} \frac{dy}{dx} &= -\frac{K}{Ll''} = \tan \phi \\ \therefore \tan \phi &= -\frac{1}{Ll''} \tan 2\theta. \end{aligned} \quad (280)$$

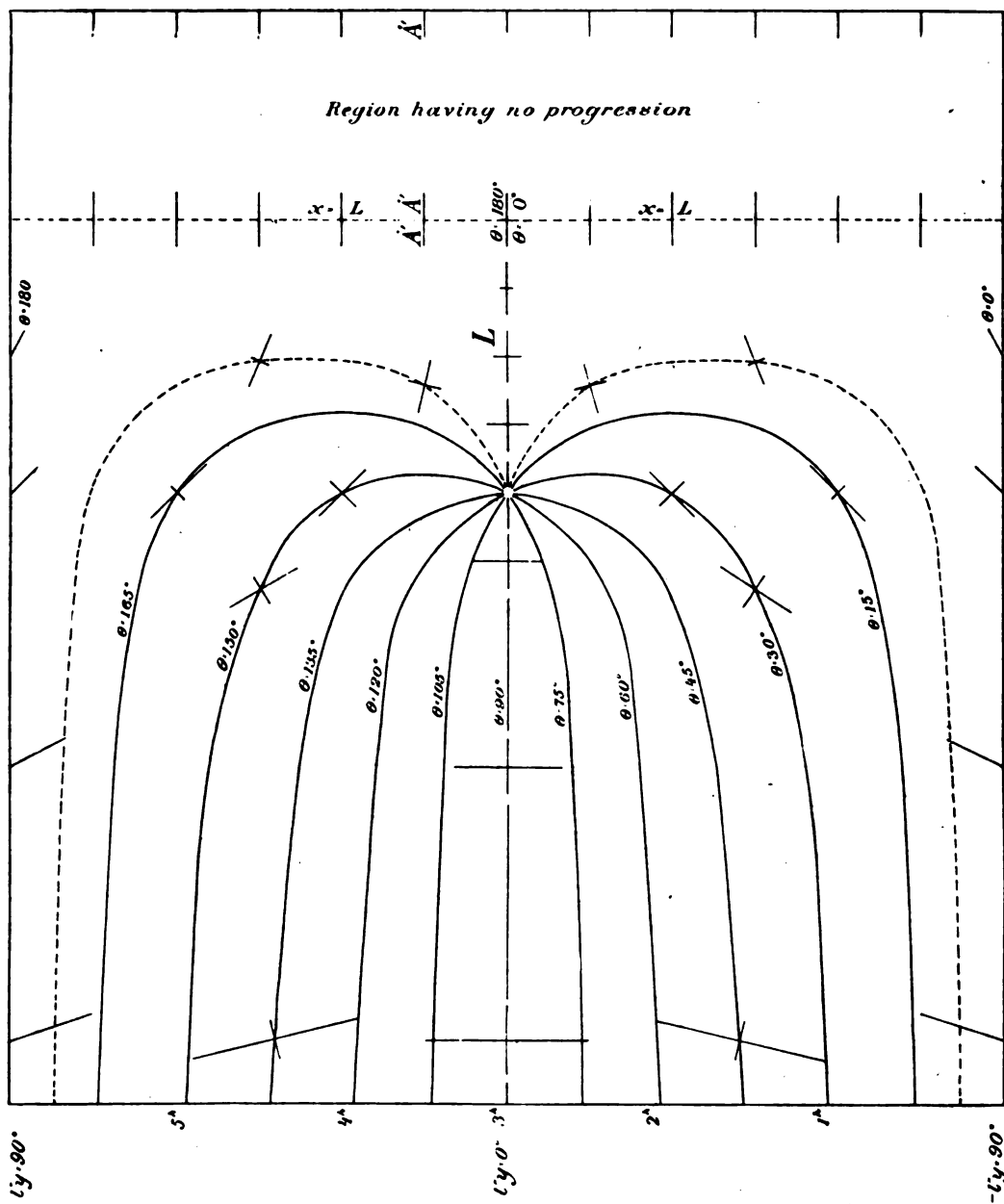
The cocurrent lines radiate from the origin; their numbering is clockwise and ranges from 0 to 6 hours.

At the origin

$$\begin{aligned} u &= A' \cos \theta, \\ v &= A' \cos (\theta - 90^\circ); \end{aligned}$$

consequently the rotation of the current is counterclockwise, the period being 12 hours.

No. 4.



Suppose $x=0$; then from (279)

$$\tan 2\theta = \frac{-\sin 2l''y}{1 - \cos 2l''y}. \quad (281)$$

Fig. 4 shows a strip such that

$$-90^\circ < l''y < 90^\circ.$$

For negative values of y , within these limits, $\tan 2\theta$ is positive, and for positive values of y , it is negative. For $l''y = -90^\circ$, $\theta = 0$, and for $l''y = 90^\circ$, $\theta = 180^\circ$.

If

$$A'' = A' e^{-p^2 x}, \quad \epsilon' = 0, \quad \text{and} \quad \epsilon'' = 90^\circ,$$

$$\tan 2\theta = -\frac{e^{-2p^2 x} \sin 2l''y}{1 - e^{-2p^2 x} \cos 2l''y}. \quad (282)$$

Near the origin

$$e^{-2p^2 x} \doteq 1 - 2p^2 x$$

$$\frac{dy}{dx} \doteq -\frac{2Kp^2 - 2l''p^2 y}{l'' + 2Kl''^2 y} \doteq -\frac{Kp^2}{l''}.$$

$$\therefore \tan \phi = -\frac{2p^2}{l''^2} \tan 2\theta.$$

If

$$A' = C e^{p^2 x}$$

$$A'' = D \frac{L-x}{L} e^{q^2 y^2},$$

the stationary portion becomes dominant as x increases in value, while the progressive portion becomes dominant as x increases numerically in the negative direction.

$$\tan 2\theta = \frac{D^2 \left(\frac{L-x}{L}\right)^2 e^{2q^2 y^2} \sin 2(l''y + 90^\circ)}{C^2 e^{2p^2 x} + D^2 \left(\frac{L-x}{L}\right)^2 e^{2q^2 y^2} \cos 2(l''y + 90^\circ)}. \quad (283)$$

If $A' = C$, i. e., if $p = 0$, also if $D = C$; then the line of equal component velocities (i. e., where $u = v$) is not the y -axis, but is a curve tangent to the y -axis at the origin, and the distribution or radiation is the same as that shown in the figure.

Circular points may frequently be regarded as resulting from the crossing of two tidal streams, the phase of one being nearly constant in respect to distance, the phase of the other varying rapidly (i. e., l'' is much greater than l , which depends directly upon the depth); the former may be spoken of as a stationary stream and the latter as a progressive stream. The velocity amplitudes in either or both may vary according to any regular law. At a circular point the component-velocity amplitudes must be equal and lie perpendicularly to each other, also the phases in these two directions must differ by 90° or three hours.

If two such streams intersect (not necessarily at right angles) there will generally be a line along which the two component-velocity amplitudes in these directions are

equal to each other. For, the ratio of these two amplitudes may be supposed to vary in the region covered by the streams according to some law, and in particular, near any point at which they happen to be equal. Along a certain line drawn through this point the amplitude ratio will vary most rapidly, and along some one line it will not vary at all from unity.

Lay off from any point two arrows, each equal to the amplitude of either component velocity, their direction being the directions of the two streams. But the simply harmonic motions (displacements or velocities) along two intersecting paths, give as the resultant path or hodograph, an ellipse, whose center is this point and whose major or minor axis bisects the angle between the two paths. By varying the phase of one motion relatively to the phase of the other, all forms of ellipses will be obtained and all of these will be inscribed in a rhombus, the centers of two of whose sides are the outward extremities of the given paths. One of these ellipses must be a circle. Note the relative phase corresponding to the circle. If it is $\pm 90^\circ$, the point is a circular point. If not, go along the line of equal amplitudes until the relative phase for the circular paths is $\pm 90^\circ$.

56. *To find the time and amount of the greatest height difference for a strait connecting two tidal bodies.*

Let $\theta = at$ and suppose the time origin to be the time of high water at the end characterized by the single subscript. Then $z, -z, \text{ or } A, \cos \theta - A, \cos (\theta - \epsilon)$ is to be a maximum or minimum. The time after the A , high water when this will occur is given by the equation

$$-\tan at = -\tan \theta = \frac{A, \sin \epsilon}{A, - A, \cos \epsilon} = \frac{A, \sin \beta}{A, + A, \cos \beta} = \tan v' \quad (284)$$

where $\beta = 180^\circ - \epsilon$ or $\epsilon = 180^\circ - \beta$. § 4, Part III.

Now Table 15 gives the angle v' , and the "HW phase" is the angle β . Hence if we take the argument β from 180° the tabular value of the table will, when reduced to hours, be the time whereby the time of the A ,-tide must be diminished in order to give the time when the A ,-height most exceeds the A ,,-height.

If the time is reckoned from the time of the A ,,-high water instead of to the time of the A ,-tide, the A ,,-height will most exceed the A ,-height at times given by entering Table 15 with $\epsilon = \beta - 180^\circ$, using the "HW phase" or β written at the bottom of the table. In this way Table 60, showing the time of maximum slope, has been constructed.

Either of these values of ϵ substituted in $A, \cos \theta - A, \cos (\theta - \beta)$ gives $A, \cos \theta + A, \cos (\theta + \beta)$ which is of the same form as the height due to two simple tide waves, equations (19), Part III. Hence if the columns of Table 16 be taken in the inverse order these results follow immediately. Table 61 shows the difference in surface elevation for the two ends of the strait.

In accordance with sections 35, 102, Part IV A, the time of strength of flood or ebb in a strait of length L connecting two tidal bodies is given by the equation

$$\tan at''' = \frac{A, \cos l(L-x) \cos \alpha, - A, \cos lx \cos \alpha,,}{A, \cos l(L-x) \sin \alpha, - A, \cos lx \sin \alpha,,} \quad (285)$$

If LL is but a small fraction of 2π , the above equation becomes

$$\tan at'' = \frac{A_1 \cos \alpha_1 - A_{11} \cos \alpha_{11}}{A_1 \sin \alpha_1 - A_{11} \sin \alpha_{11}} \quad (286)$$

Let $\alpha = 0$, then $\alpha_{11} = -\epsilon$, and

$$\tan at''' = \frac{A_1 - A_{11} \cos \epsilon}{A_{11} \sin \epsilon}; \quad (287)$$

$\therefore at'''$ is 90° greater than at in (284),

and so the greatest velocity occurs three hours after the time of greatest slope. This is what would be expected in small slope motion through a straight sufficiently long for oscillatory motion to occur, and connecting two deep tidal bodies whose horizontal motions can be ignored. (Cf. sec. 11, Part IV A.)

Where the strait is so short and narrow that the motion is due to hydraulic effects, the time of maximum velocity approaches the time of greatest slope. (See secs. 34-37, Part I; sec. 106, Part IV A.)

The section of the strait at which the range becomes a minimum, upon the assumption that its instantaneous surface is a plane, may be found in the following manner:

Let L denote the length of the strait; x the distance of the required section from the end where the amplitude of the tide is A_1 . The height at any section in the strait is (sect. 106, Part IV A)

$$\zeta = \frac{\zeta_1 (L-x) + \zeta_{11} x}{L} = \frac{(L-x)A_1 \cos at + A_{11} x \cos (at + \alpha_{11})}{L} \quad (288)$$

the time being reckoned from the time of high water at the first end

From $\frac{\partial \zeta}{\partial t} = 0$, we have

$$\tan at = - \frac{A_{11} x \sin \alpha_{11}}{A_1 (L-x) + A_{11} x \cos \alpha_{11}} \quad (289)$$

and from

$$\frac{\partial \zeta}{\partial x} = 0$$

$$\tan at = \frac{A_{11} \cos \alpha_{11} - A_1}{A_{11} \sin \alpha_{11}}; \quad (290)$$

$$\therefore \frac{x}{L} = \frac{A_1 (A_1 - A_{11} \cos \alpha_{11})}{A_1^2 + A_{11}^2 - 2 A_1 A_{11} \cos \alpha_{11}} \quad (291)$$

These values of x and t when substituted in the expression for ζ give the amplitude of the tide where it becomes a minimum.

In the expression for x/L it will be noticed that the denominator is the square of the side of a triangle opposite the angle α_{11} , whose including sides are A_1 and A_{11} . The factor $A_1 - A_{11} \cos \alpha_{11}$ in the numerator is the distance from the outward extremity of A_1 to a point upon A_1 below the point marking the outward extremity of A_{11} .

Since $\frac{x}{L} < 1$, it follows that $A_{11} > A_1 \cos \alpha_{11}$, if a minimum occurs in the strait.

CHAPTER V.

OBSERVATION AND REDUCTION OF TIDAL CURRENTS.

57. Observations of the directions and velocities of currents are usually attended with considerable difficulty and expense; for, the work being carried on in boats, is liable to interruptions caused by unfavorable weather and in many instances by the passing of boats. The installation and maintenance of a fixed self-registering current meter is probably out of the question; and so the amount of observation must depend upon the time during which the observers are on duty.

In whatever manner current observations are to be made, great care should be taken to ascertain and to give in the record the location of each station, not only by angles between three or more objects marked upon charts and hydrographic sheets, but also upon a tracing, sketch, or fragment of a chart, which should always accompany the record. Generally the position upon both flood and ebb should be shown. The work of the field party is not complete until the directions, azimuths, or bearings of all objects sighted upon in connection with the observations and all directions of the observed currents have been ascertained and given in the record. For locating stations, objects not too far away should be sighted upon, and these may be quite numerous; but for determining the direction of the current it is advantageous to use objects rather remote and few in number. If at some distance from land, positions should be given by latitude and longitude with as much precision as the means at hand will permit. In all cases soundings should be frequently made, as this aids in identifying the station and in judging of the probable nature of the current.

The record should be given in such a form as to show readily directions and velocities of the current, the depths at which the observations have been taken, the variation and deviation of the compass.

The kind of time used should always be specified, and care should be taken to write "a. m." or "p. m." at the top of each page and at the beginning of each half day.

The purpose of the survey will govern the distribution of stations and the length of time during which they are to be occupied. Owing to irregularities produced by the wind and the discharge of fresh water, each station should be occupied for several days, if possible. For such stations as may be chosen as principal stations in a hydrographic survey, the time of occupation should be 15 or 30 days. For determining the non-tidal currents, the same stations should be occupied at different seasons of the year.

58. *Floats.*

Current observations are usually made either by means of floats, or by means of meters having revolving vanes or cups.

The float or log is usually a cylindrical body 2, 3, or more inches in diameter and from 1 to 4 fathoms in length. If hollow, the amount of weight necessary to cause it to float vertically, and to project a small distance above the water is easily applied. In case the float is a solid log, then the loading is accomplished by pouring lead into a hollow extending upward from the bottom, or by tacking sheet lead around the outside.

If double floats are used for obtaining the velocity below the surface, the lower one should be large in comparison with the surface or upper one. The lower float (if it may be so called) may consist of a sphere, cylinder, or two intersecting plane sheets of galvanized iron. In the last instance it is desirable to have air cavities at the upper edges of the intersecting vanes, and to attach leaden weights to the lower edges. In this way the proper tension upon the connecting cord or wire can be secured and the vanes will keep a vertical position. The upper float may be either cylindrical or spherical.

Let the observed common velocity of the two floats be denoted by v_c , the observed surface velocity by v_s , then the velocity of the lower body (v_l) will be

$$v_l = \frac{v_c(R_s + R_l) - v_s R_s}{R_l} \quad (292)$$

where R_s and R_l denote resistances of impact upon the two bodies found by placing them successively in the same stream or drawing them through still water with the same velocities. The only assumptions implied in this formula are that the ratio of the force of resistance of the two bodies remains the same for all velocities (see secs. 12-14) and that the wire or cord connecting the two bodies is small.

The line, when thoroughly wet, should be divided by means of leather straps, suitably marked by perforated holes into divisions each 50.67 feet in length, representing knots. Each of these spaces should be subdivided by knotted cords into spaces 5.07 feet in length. If the interval used is 30 seconds, the number of large divisions run off will represent the velocity in knots per hour and the numbered smaller divisions the decimals of knots. If a run of 28 seconds be used, the principal divisions should be 47.29 feet in length. There should be at least 60 feet of stray line between the float and the zero division, in order that the float may drift beyond the influence of the vessel before the measurements commence. The length of the line should be frequently tested.

If a watch instead of a glass be used as the timepiece, the graduations of the line may be omitted. In using such a line the number of seconds consumed in paying out a given length is ascertained, preferably by aid of a stop watch. If 100 feet is the length of line so run off, the velocity in feet per second will be $\frac{100}{t}$, where t denotes the number of seconds consumed. But

$$(\text{feet per second}) \times \frac{45}{76} = \text{knots per hour},$$

and so for 100 feet of line

$$v = \frac{45}{76} \frac{100}{t} = \frac{59.21}{t} \text{ knots}$$

\therefore for a line $100 \times \frac{76}{45} (= 168.9)$ feet long

$$v = \frac{100}{t}, \text{ and for a line 84.45 feet long, } v = \frac{50}{t};$$

and so on.

That is, by taking a line of suitable length the velocity becomes the reciprocal of t multiplied by a simple number like 100 or 50.

59. *Observations by means of floats.*

There are numerous ways of observing currents by means of floats, some of which consist in (1) noting the number of knots or marks upon a log line which are paid out in 30 seconds of time; (2) noting the time required for a given length of line to be paid out; (3) following up a free float with a boat and fixing the positions from time to time by angles upon three or more objects; (4) measuring angles between the float and fixed objects by means of two theodolites located upon the shore, or watching the float pass two parallel ranges a known distance apart; (5) ascertaining the amount by which a vessel has been displaced through the action of a current. As a rule observations upon free floats are objectionable because of the great labor connected with their reduction, and because they do not refer to a single point or station.

In making observations with a line and float (first method) one man holds the reel while another notes the time. When the log is cast, and the stray line is out so that the initial mark has appeared and has reached the fixed reference point on or near the reel, the person holding the reel cries "now;" the person with the watch notes the position of the second-hand, and when thirty seconds have elapsed, cries "stop." The person at the reel now notices what division of the line has reached the reference point. This is recorded, and if the length of the line be sufficient, it may be allowed to run out for another thirty seconds after any particular division or marking of the line has reached the reference point on or near the reel. A stop watch will give greater precision to all work of this kind. After the velocities have been noted, the log is still allowed to drift until its direction is ascertained. This may be done either by measuring with a sextant the angle between a fixed object and the float, or by directing the sights of a compass toward the float. If the float is to the right of the object of reference, the angle is marked "*R*," if to the left, "*L*." This rule should be invariably followed in the record. If observations are made with a compass, the direction should be recorded in degrees (not in points) and corrected for variation and deviation as indicated in the form for record. If circumstances permit, it is best to carry on simultaneously both sextant and compass observations. During the day, the float should carry a wind vane, small flag, or slender rod; and during the night, a lantern.

In comparatively narrow bodies of water the velocity of the current is often obtained by aid of two ranges transverse to the stream. The float is set adrift some distance above the upper range and picked up below the lower range. Observers stationed on the shore note the time when each range is passed. Greater precision can be attained if observers stationed on the shore make simultaneous observations upon the float by means of two theodolites. This is important wherever the lines of flow are not known, or are not fixed in position for the varying phases of the tide.

For convenience of reduction care should be taken to make current observations upon the exact hours and half hours.

The time of the current's turning (middle of slack) should be carefully observed and recorded wherever slack water occurs.

The direction and force of the wind, together with the appearance of rips, eddies, and other interesting phenomena, should be observed and recorded.

60. *Current meters.*

A current meter usually consists of a rotating meter wheel actuated by the impact of the water; a framework for supporting the wheel; a vane for causing the instrument to lie parallel to the lines of motion of the current; a counterpoise for causing the suspended meter to lie in a nearly horizontal position; a worm gear and wheels for recording the number of revolutions or preferably an electric connection with a recording apparatus located in the boat from which the meter is suspended.

A float may be used for determining the direction when observations are made near the surface. One of the chief requisites of current meters is uniformity of operation. Hence, all bearings and gears belonging to the meter proper should be tightly inclosed, if possible, thus placing them beyond the reach of the water and sediment.

Woltmann's mill, and the meters of Fteley, Revy, and Moore, have counting devices in the instrument proper. The Henry, Price, and Haskell meters are electrically connected with the observing or recording apparatus.

For making observations at considerable depths, the meter must be provided with some means of determining the direction of the stream, and it is important that this may be done without the necessity of hauling up the meter after each reading.

Pillsbury's meter gives both velocity and direction, but has to be hauled up for reading. The Ritchie-Haskell direction meter has a registering apparatus in the boat, but requires great care in manipulation.

If a revolving meter could be so constructed as to have its cups or small blades move with a velocity equal to that of the stream, then

$$v = 2 n \pi r,$$

where n is the number of revolutions per second and r the radius or distance from the axis of rotation to the centers of the small cups or blades. The cup anemometer and the Price current meter partially fulfill these hypothetical requirements.

If small oblique vanes were placed upon the circumference of a skeleton wheel so that the whole wheel would in some respects resemble a wind mill, and if I denote the inclination of a vane to a line drawn parallel to the axis upon which the wheel revolves, then v denoting the velocity of the stream, the velocity of the rim of the wheel should be, in the case of no resistance, $v \tan I$. Here v does not equal $2 n \pi r$, but $2 n \pi r \cot I$.

These simple illustrations show why one should expect to find current velocities to vary almost linearly with the number of revolutions. In practice other small terms come into the expression for v or n , and so it is reasonable to assume that v is of the form

$$v = \alpha + \beta n + \gamma n^2$$

61. *Remarks on the use of meters.*

Shortly before using a meter it must be carefully rated for various velocities, and it should be tested from time to time. This is usually accomplished by driving it at uniform rates through still water, the meter being attached to the prow of the boat and well submerged. The course over which the boat is driven being accurately known, and the various times and readings for each run being noted, it is not difficult to com-

pute a curve representing the rating. To do this, assume that observations or runs have been made giving directly the revolutions per second in each case, viz., n_1, n_2, \dots, n_m , and from the known times of going over the known course, the velocities in feet per second, viz., v_1, v_2, \dots, v_m . For convenience, write a, b, c for 1 ($=n^0$), n, n^2 , then if we assume that the velocity and number of revolutions per second are already connected by the relation

$$v = \alpha + \beta n + \gamma n^2 \quad (294)$$

where α, β, γ are constants to be determined, we have three linear equations for their determination, viz.

$$\begin{aligned} \sum_{i=1}^{i=m} a_i(a_i\alpha_i + b_i\beta_i + c_i\gamma_i - n_i) &= 0, \\ \sum_{i=1}^{i=m} b_i(a_i\alpha_i + b_i\beta_i + c_i\gamma_i - n_i) &= 0, \\ \sum_{i=1}^{i=m} c_i(a_i\alpha_i + b_i\beta_i + c_i\gamma_i - n_i) &= 0. \end{aligned} \quad (295)$$

If the velocity is to be measured at a depth of only a few feet below the surface, the meter is attached to a pole; if at a considerable depth, it must be suspended by a strong, slender cord or cable, and to the lower extremity of the meter sufficient weight should be attached for keeping the axis of the meter in a nearly horizontal position.

In recording the observations the same form as that used in recording float observations can still be used. The number of revolutions should be written a little above the line with the corresponding velocity on the line. The depth at which the observation is taken should be written in the column of remarks.

For details concerning the rating of meters, see "Accuracy of Stream Measurements," by E. C. Murphy, pages 80 et seq. The current meter devised by Lieut. J. E. Pillsbury, U. S. N., is described in the U. S. Coast and Geodetic Survey Reports, 1885, pages 495-501; 1890, pages 459-620. It consists, in part, of a cup meter and an enclosed compass, both placed within an open frame. The frame terminates in a heavy ball and is hung in gimbals, so that the weight of the ball secures the uprightness of the instrument at all times. The revolutions are counted by means of a worm gear and wheel register. The compass needle is locked as soon as hoisting of the instrument begins.

Further references to measurements of the Gulf Stream made by this meter are the Survey reports for 1886, 1887, and 1889.

There is little or no difficulty connected with the construction and operation of an electric meter which measures the velocity only, for an insulated wire can be carried down inside of or alongside of the suspending cable to the axis of the meter wheel. A projection on the wheel near its axle comes into contact with this pole or end of the

wire, one or more times for each revolution. This closes the circuit and causes the movement of an armature on an electro-magnet; the armature in turn actuates the recording apparatus. The meters of Price and Haskell are of this character.

Many greater difficulties arise in the determination of the direction of the current, especially if the meter is not to be hoisted in making the reading. The problem is to ascertain by electrical means the magnetic direction of the current. One means of accomplishing this requires that a suitably constructed needle, swinging over a horizontal circle or ring, be inclosed in the body of the instrument. One electro-magnet in the body of the instrument is in circuit with the observing apparatus in the boat. The armature of this magnet, when the latter is magnetized and demagnetized by the observer in the boat, mechanically moves or rotates the horizontal ring under the needle until a contact is made between a metallic point situated upon it and a similar point on the needle or the case inclosing it. This contact closes a second circuit, which in turn opens the first. The position of a pointer in the receiving apparatus shows the direction of the needle at the time of observation. The moving horizontal ring or circular arc is brought back to its initial position by means of a spring when the observation is ended. In a general way this describes the direction current meter devised by Ritchie and Haskell. A brief description of the meter is given on pages 343-345 of the U. S. Coast and Geodetic Survey Report for 1891 (II).

A meter for measuring the directions and velocities of ocean currents at various depths, invented by Prof. O. Pettersson, is briefly described in Vol. I, Svenska Hydrografisk Biologiska Kommissionens Skrifter.

62. *Miscellaneous apparatus and methods for observing currents.*

A method of observing currents below the surface has been employed by W. Bell Dawson in St. Lawrence Bay.* A fan consisting of two sheets of galvanized iron intersecting at right angles, suspended by sounding wire. The direction and inclination to the vertical of the wire near the surface are noted. The value of the velocity becomes known from a table constructed from actual observation.

The velocity of the current can be measured by noting the angle of deviation of a pendulumlike body from the vertical. An apparatus adapted to this mode of measurement is called a hydrometric pendulum.

Pitot's tube is an instrument which measures the velocity of the stream by means of the head of water sustained by the impulse of the stream. More or less elaborate types of this instrument are described in treatises on hydraulics.

In case of very low velocities, such as those found at sea, it seems probable that floats are the only reliable apparatus.

For a comparison of results obtained by means of different instruments and by different methods, reference may be made to Doctor Murphy's paper entitled "Accuracy of Stream Measurements," and especially to pages 47-59.

63. *Nonharmonic reduction of currents.*

Unless the current observations extend over several weeks of time, it is generally advantageous to plot them upon cross-section paper. This is done by taking the times as abscissæ and the velocities as ordinates of a curve. At the foot of each ordinate the direction is written, unless a second curve is constructed having times for abscissæ

* Survey of Tides and Currents in Canadian Waters; Report of Progress, 1897.

and directions for ordinates. Near the upper margin of the sheet containing the plotting or plottings, the times of high and low waters at some station to which it is proposed to refer the currents are indicated. In case no near-by or suitable station is available, the currents may be referred to the moon's transits, and these should be indicated near the upper margin of the sheet. For tidal rivers, straits, sounds, and other narrow bodies of water it is generally sufficient to tabulate the times of the maximum velocities and of the slack waters. In more open bodies of water, or wherever the current may be rotary in character, hourly values before and after the times of the high and low waters, or before and after the times of the moon's transit, should be tabulated. See tabulations in sections 95 and 96.

By using multiples of twenty-four hours for the length of the series tabulated, the diurnal inequalities will be nearly eliminated and the mean values of flood, ebb, etc., will be those pertaining to the semidaily wave.

If there is no permanent current, the semidaily values obtained can be reduced to their mean values by means of the factor Mn/Mn' , where Mn denotes the true mean range of tide at the reference station and Mn' the mean range at the reference station for the period covered by the current series. In a short series, use in place of Mn' , the actual range of each particular tide at the reference station. If there is a permanent flow, this should be taken out in accordance with section 53 before applying this factor to the tidal current; it may then be restored. In the tables of current data, sections 83 and 90, both kinds of current are to be kept together as has generally been done. Only the harmonically analyzed currents are free from the permanent flow wherever such flow exists. On the maps (Figs. 5-17) the two kinds are kept separate.

64. *Harmonic reduction of currents.*

The advantage of harmonic over nonharmonic methods is even greater in the reduction of currents than in the reduction of tides, for current observations are much more irregular than tidal observations, and the quantities sought are frequently so small as to be completely hidden from view.

A fairly good analysis can be made for a series of hourly observations extending over fifteen or twenty-nine days. These may be plotted on cross-section paper in the manner already described, but omitting the times of the high and low waters and the transits. Whether plotted or not the hourly values of the current should be resolved into north-and-south and east-and-west component velocities. This can be done either by means of an ordinary traverse table or graphically, using cross-section paper upon which is drawn a large circle divided into degrees and over which moves a graduated arm, rotating about the center of the circle. The north-and-south values are written upon one sheet or set of sheets and the east-and-west upon another. To avoid negative numbers, one or more whole knots may be added to the true values.

Each set of sheets is then summed for the current constituents in the same manner as would have been done had they contained tidal ordinates. (See sec. 77, Part II.) The harmonic analysis as carried out here differs in no respect from that applied to the tides.

When a series is only a few days in length, the most that analysis can give directly is a semidiurnal part, a diurnal part, and possibly quarter daily part; these may be denoted by \dot{J}_{2n} , \dot{J}_{2e} , \dot{J}_{1n} , \dot{J}_{1e} , $\dot{J}_{1/2n}$, $\dot{J}_{1/2e}$. Now, about the only reasoning available is that in each of these waves the ratios between constituent amplitudes and the relative phases

or ages are the same as the corresponding ratios and ages in the tide wave. Assuming these quantities to be known for the tide, they become available for inferring constituents of the currents from the observed \dot{A} 's.

We are thus led to the problem: Given the harmonic tidal constituents for a given station, required the amplitude and time or phase of the semidaily wave and of the daily wave for a particular day or for several days.

These can be readily obtained from reliable predictions or from tidal record covering the period of current observation by summing and analyzing as if for currents. If no predictions are available, they can be made by some of the methods mentioned in sections 57-67, Part III. For this purpose the predictions need not be very elaborate.

Having found the ζ and R of any tidal constituent, C , the epoch and amplitude of the corresponding current constituent, \dot{C} , are given by the relations

$$\dot{C}_n = \zeta (\dot{C}_n) + C^n - \zeta (C), \quad (296)$$

$$\dot{C}_e = \zeta (\dot{C}_e) + C^e - \zeta (C), \quad (297)$$

$$\dot{C}_n = C \frac{12}{12} \frac{R (\dot{C}_n)}{R (C)}, \quad (298)$$

$$\dot{C}_e = C \frac{12}{12} \frac{R (\dot{C}_e)}{R (C)}, \quad (299)$$

For obtaining the maximum and minimum velocities from the north and south components, see section 51.

65. *Prediction of currents.*

If the tidal currents have but a small diurnal inequality, they can generally be predicted by applying differences to the predicted times of tides at a near-by station and suitable factors to the heights reckoned from mean sea level. At such stations the times of currents can be predicted by applying intervals (varying somewhat during the synodic half month) to the times of the moon's transit.

Where the diurnal inequalities are considerable, the currents do not generally correspond well with the tides. If sufficient observations are available, they may be tabulated according to two arguments, viz., the moon's transit and the day of year. Such tabulations are available at once for making predictions. (Cf. sec. 59, Part I.)

From suitable harmonic analyses the entire current ellipse for stations having rotary currents can be predicted by means of tide predicting machines. (Sec. 78, Part II.) An ordinary machine will give at one setting one of the component velocities for as long a period of time as may be desired. Another setting will give the other component velocity. The two can be combined without difficulty by means of a right-angled triangle.

If the machine have two sets of cranks and pulleys, and the two sets of cranks differ in phase by 90° , then by using principal directions the two motions can go on at the same time. If the summation chains after passing over the pulleys cross each other perpendicularly, the distance between two fixed points upon them can be made to represent the velocity of the resultant current in both magnitude and direction.

CHAPTER VI.

DESCRIPTIONS OF TIDAL CURRENTS.

66. *Remarks on current charts.*

Synoptical charts are referred to in the description of the tidal currents around the British Isles. They aim to show, without computation, the direction and velocity of the currents for the region covered, at each tidal hour. The fact that at least 6 or 12 charts are thus required, is in itself, a serious drawback. They are not convenient for showing how the times of the turning of the tide at different places compare with one another; for, several charts would have to be consulted, and hourly values do not give such differences very closely. Such charts are, however well suited to the wants of the navigator.

Mr. John Ross, of this Survey, has devised a form of chart for presenting current and tidal data, not over areas, but along a given path of commerce. The distance from an assumed point is one coördinate and the time with reference to a near-by tide the other coördinate. The tabular values are the velocity and sometimes the direction of the current; also, the time of local high and low water. A simple diagram accompanies each chart; the object of the diagram is to enable the navigator to readily apply the values found upon the chart. Examples of such charts are given in the Coast and Geodetic Survey Tide Tables, and Chart No. 1610 published by the United States Hydrographic Office.

A current hour is the Greenwich lunar time of a particular phase of the tidal current, say, of the maximum flood.

Generally speaking, the flood may be taken as the stream whose maximum velocity occurs after local low water and before or at local high water. In some rare cases where the maximum velocity occurs shortly after local high water, the stream may still be regarded as the flood.

In many localities a given stream if followed some distance will change its name from flood to ebb—the change occurring where the time of maximum flood is simultaneous with the time of local high water; similar remarks apply to the ebb.

The necessity for changing the designation of a stream from flood to ebb, or vice versa, shows the advantage of using not exactly the times of actual maximum velocities, but the times of the maximum of the regular semidaily velocities; in other words, the times of the M_2 current stripped of the disturbing influences of the M_4 - and M_6 -currents. Some values of this regular semidaily current will be found in the table of harmonic constants given under section 97. These have been brought out by analyzing hourly values of the velocities generally resolved into north-and-south and east-and-west directions. A good approximation to the M_2 -current hour can be obtained by taking the mean between the observed flood-current hour and the ebb-current hour, having first increased or decreased the latter by 6. Another way is to take the mean between

two consecutive slack-current hours. A still closer approximation is to take the mean between the flood-current hour, the ebb-current hour, increased or decreased by 6; one slack hour, increased or decreased by 3; and the other by 9.

The ebb arrows are not generally shown upon the cocurrent charts, because they are simply the reverse of the flood arrows; when shown they are distinguished by a single barb. The permanent current is represented by the blunt-headed arrows. Where rotary currents are represented, the flood arrow is drawn in the usual way and the minimum velocity following the flood by 3 hours is shown by a line without barbs. Its length compared with the length of the flood arrow bears the ratio of the observed velocities. For convenience all flood arrows upon the same chart are of equal length; so, also, are the blunt arrows representing permanent streams. The velocities in knots in these two cases may be written upon or near the arrows.

The tabular values given under sections 95 and 96 contain whatever permanent current was running at the time when the observations were taken.

When the current at a given station is referred to a tide at some point more or less distant, it is to be assumed that a common time is used for both places.

When the current is referred to a transit of the moon, it may be assumed that the transit refers to the local meridian.

67. Common characteristics of currents.

The effect of the combination of a nearly stationary stream with one rapidly progressive has been considered in section 49. The former is often an on-and-off shore stream, the latter, one flowing nearly parallel to the shore line. Circular points occur most frequently where the stream divides, although not simultaneously, and much less frequently where two streams come together.

If the current turn clockwise, the order of the cocurrent lines is counterclockwise, and vice versa.

Having a reliable map of cotidal lines, the time of turning, or of maximum velocity, of the tidal current through a strait of moderate length can generally be inferred with considerable accuracy. The two cases which lend themselves to computation most readily are: 1. A narrow strait of varying cross section and whose length is only a small fraction of λ (λ being reckoned according to the depth of the strait), and wherein the motion is hydraulic because the velocity is so great that too many particles leave the strait proper for permitting the motion to become oscillatory. 2. A strait of such considerable length and of sufficiently great cross section (implying reduced velocity), for preventing more than a small fraction of the particles from leaving the strait (i. e., the strait proper and its approaches), and so causing the motion to become oscillatory.

The motion pertaining to a narrow strait dies out rapidly in either tidal body connected, while the motion of a broader strait is shared by the water for some distance beyond one or both ends of the strait.

Where the current is hydraulic, the greatest velocity toward the body having temporarily the lower level occurs when the downward slope in that direction is the greatest; such times can be taken directly from Table 60.

Where the current is oscillatory, the greatest acceleration toward the body having temporarily the lower level occurs when the downward slope in that direction is the greatest; such times can be taken directly from Table 60, but the maximum velocity in

the given direction occurs $\frac{1}{4}\tau$ (or three hours for a semidaily tide) later than the time given in Table 60. Many examples of these cases will be given in the descriptions of tidal streams.

A long shallow strait, especially if connecting shallow bodies, may have a progressive wave or current passing through it. If a strait have such dimensions that its current is partly progressive and partly stationary, the computation of the times of greatest velocity at various points along the strait becomes more difficult and unsatisfactory.

In small, sharp bays, in bays connected with the sea by means of tolerably broad straits, or in other small arms of the sea wherein the rise and fall of tide is nearly simultaneous with that outside, the slacks must occur at approximately the times of the tides. (See Chap. VIII, Part IV A.)

Another motion of a local character may be described here. This is what may be described as hydraulic-slope motion. If a stationary wave have an amplitude increasing (say) as one proceeds along a coast line, there is a tendency for any water along the shore not fully participating in the oscillatory motion, because of shore impediments, to seek its level. Moreover, the gravitational action tending to produce motion is direct; it acts upon the littoral strip of water and produces motion in this strip, and differs from hydraulic motions in straits where gravity acts upon the bodies connected. The motion thus set up in the strip may be simultaneous notwithstanding the shallowness of the water and so the slow rate with which pulses would travel; this is so because gravity acts similarly all along the strip and produces motion throughout such a body simultaneously. The effect of this action is to cause the water along the lateral boundaries of a stationary wave to turn earlier than the water along the axis or central portion.

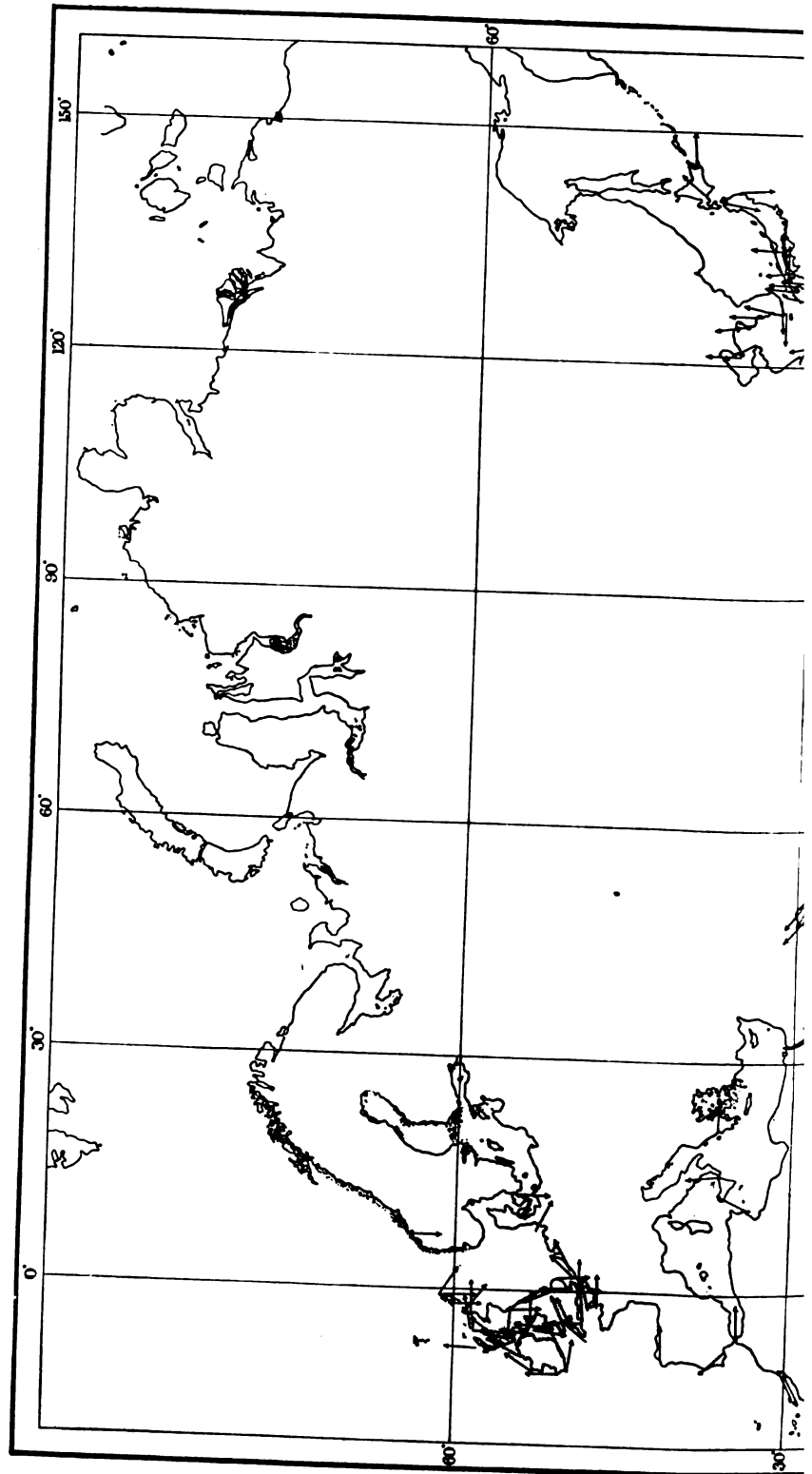
Along the end boundaries of stationary waves (whether dependent or not), the current is weak and normal to the coast line.

68. *Tidal currents for the world at large.*

Tidal currents have been observed only in comparatively shallow waters, such as the marginal strips next the coast lines, the waters overlying continental shelves and shoals, in sounds and other dependent bodies. For this reason the information regarding tidal currents is very meager. In Figs. 5 and 6 an attempt has been made to bring together, chiefly from the Admiralty charts, the principal observational data relating to the direction of the ocean tidal currents.

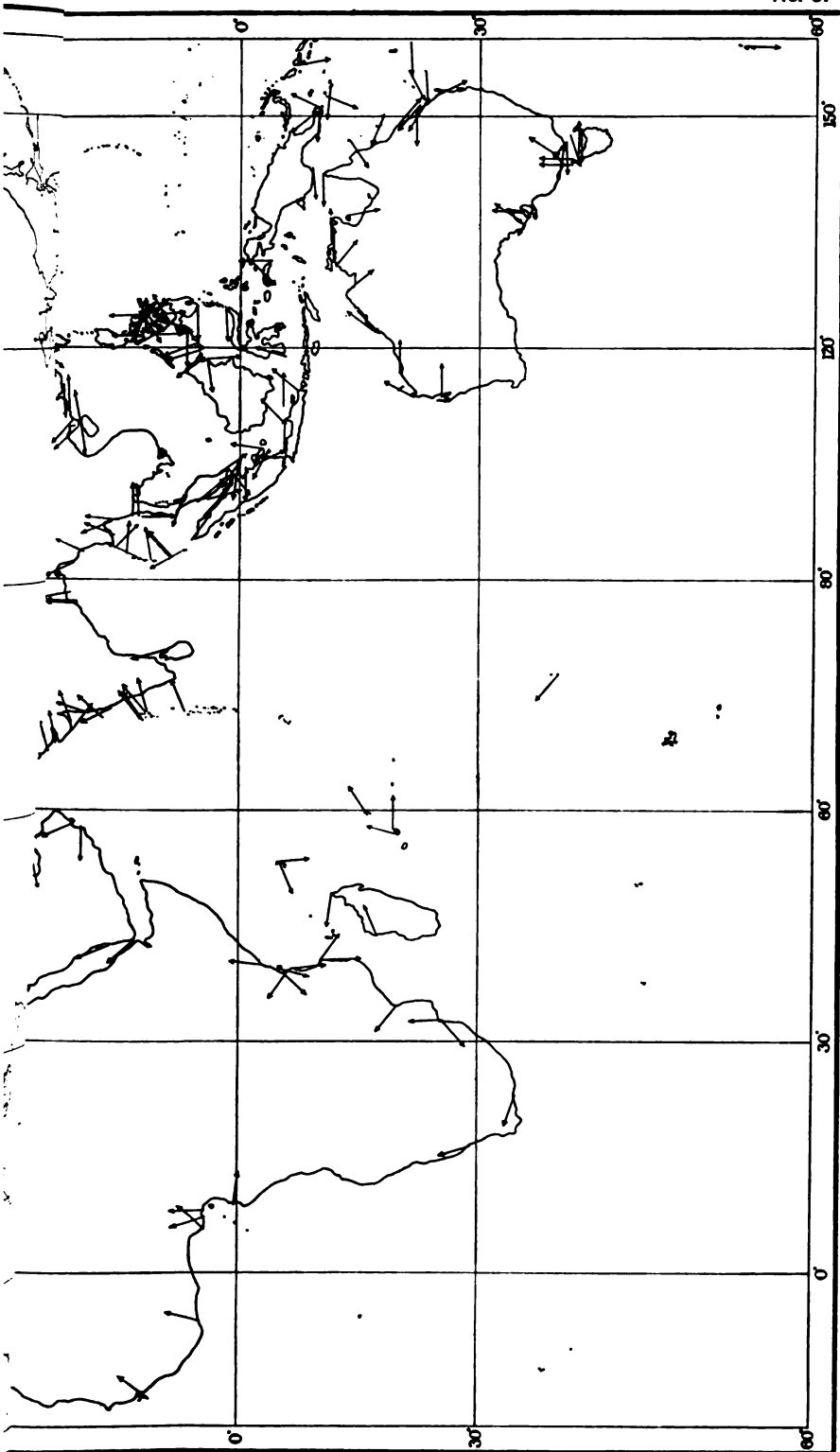
As remarked in section 30, Part IV B, the flood flows northerly at the Baluchistan end, and southerly at the Madagascar end of the half-wave area extending between these coasts. It was also noted that the flood flows southeasterly through the Seychelle Archipelago, indicating the superposition of an east-and-west motion upon the north-and-south motion of the area just mentioned. The currents of the Bay of Bengal are nearly normal to the coast, indicating the stationary character of the tide. The same is true for the waters northwest of Australia, south of Cape Colony, southeast of Brazil, southeast of the United States, northeast of Brazil, off Panama, off Alaska, and probably south of Australia. Some further evidence bearing upon this will be noted below, especially in connection with quotations from the Admiralty Coast Pilots.

Many localities have the controlling tidal streams running parallel to the shore. Such shores may be lateral boundaries of either a stationary or a progressive wave.

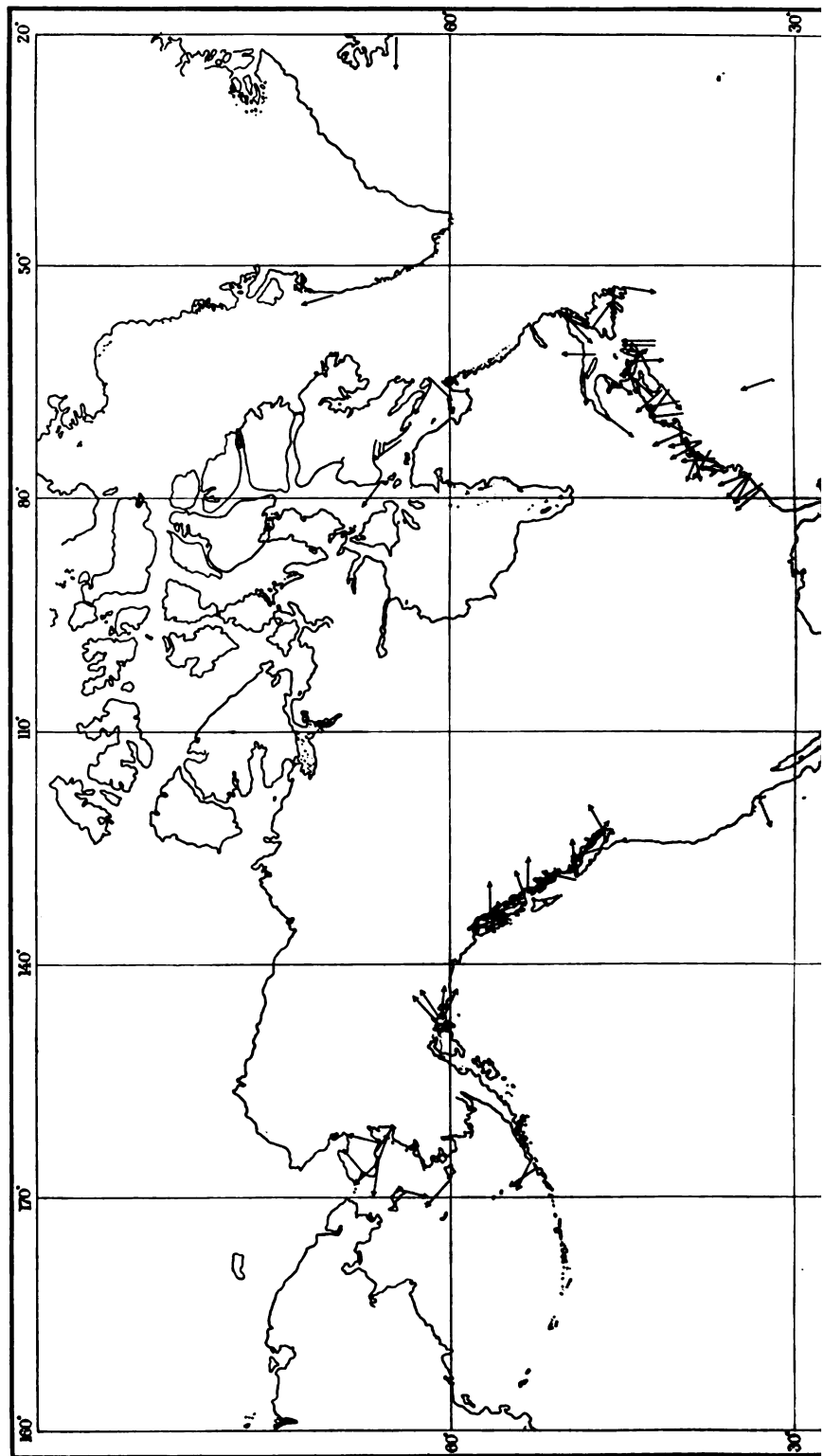


TIDAL C

No. 5.

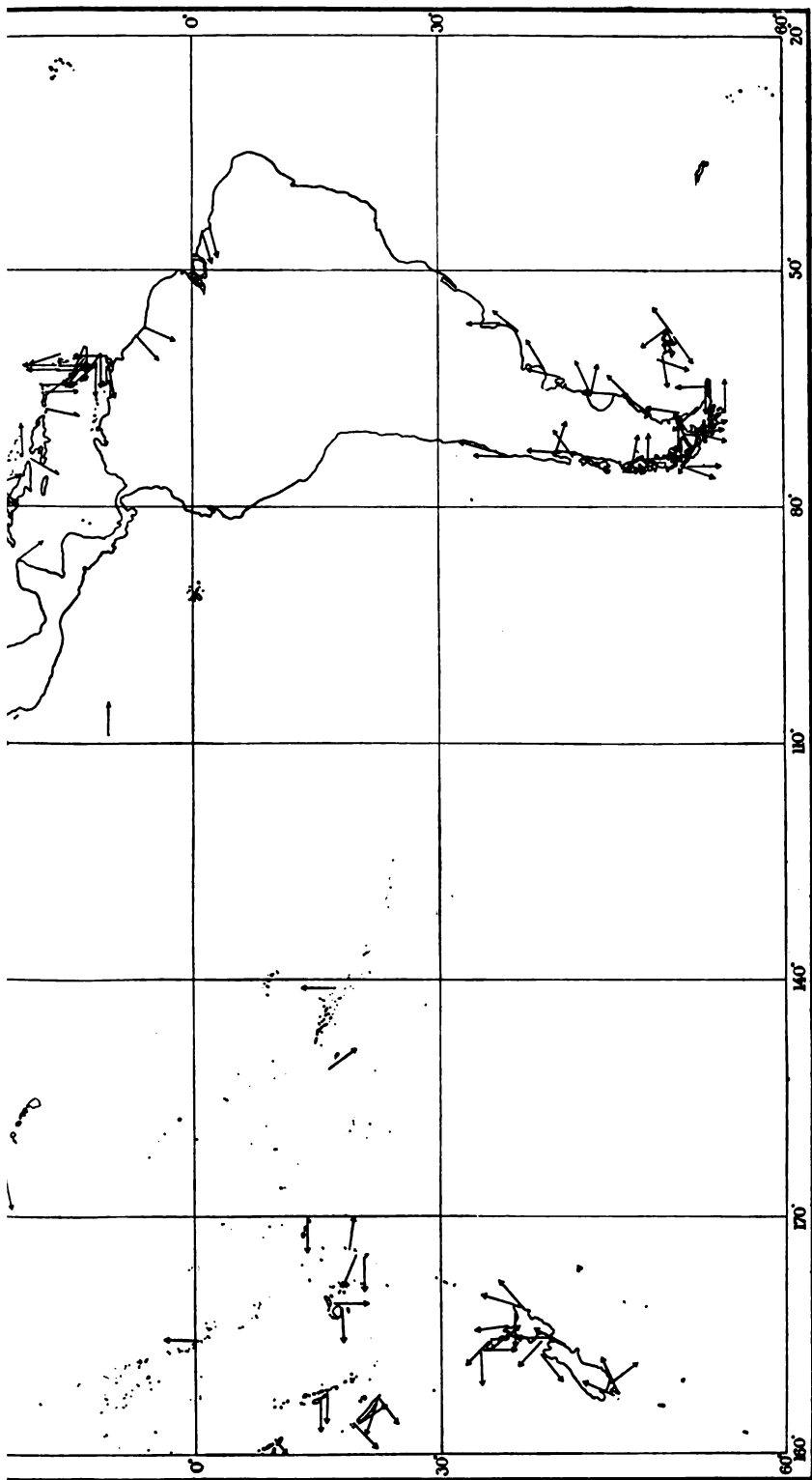


URRENTS.



TIDAL C

No. 6.



IENTS.

The eastern coast of Ceylon and Somaliland, and the outer coast of the Virgin Islands and California—all lying near nodal lines—are examples of the first case; while portions of western Europe, southern South America, western Oceanica, and many arms of the sea extending inland are examples of the second.

The harmonic constants of stations off the Atlantic coast of the United States (sec. 97) show how very feeble the tidal currents become away from land and near the end of a stationary wave; also how progressions into neighboring dependent arms of the sea cause transverse velocities often comparable with the velocities of the main oscillation.

69. *Coasts of Africa.*

On account of the regular outline of Africa and the comparative absence of off-lying shoals and chains of islands, it is fair to assume that the movements of the ocean extend to the very coast of the continent. The tidal currents as well as the tide should therefore be some index as to the character of the motion in the surrounding deep waters. The quotations given below are taken from Parts I (1899), II (1901), and III (1897), of the *African Pilot* published by the Admiralty. The matters relating to the times and heights of the tides are generally omitted, as such information can be obtained from the charts in Part IV B, this manual.

Evidence of the half-wave area extending from Mozambique Channel to Baluchistan and India is afforded by the southwesterly set of the flood current at the northern end of the channel.

The fact that at the southern end of the channel flood sets northerly indicates that here is a loop of one or more stationary waves extending in a southerly direction. (See Fig. 23, Part IV A, and Fig. 1, Part IV B.)

Evidence of an area whose motion is north and south with a loop near Cape Colony is afforded by the quotations which indicate small tidal streams around Table Bay and eastward.

The smallness of the stream in the Gulf of Guinea is in accord with the rather small ranges of tide and the fact that this gulf does not partake of the general oscillatory movement of the South Atlantic system.

The smallness of the currents off French Guinea and Sierra Leone indicates the proximity of a loop of the South Atlantic system.

The considerable northeasterly flood through the Canary Islands indicates the existence of the North Atlantic system.

A similar flood through the Azores indicates northerly progression due to openings northwest of Europe.

At the entrance to the Strait of Gibraltar the streams are weak while the range of tide is considerable; they turn at nearly the times of high and low waters.

The quotation relating to the Kongo River shows that the varying density of the water near the river's mouth has its influence upon the currents.

70. *Coasts of Africa, quotations.**

On the north shore of Mohilla the flood sets to the westward, but changes before the water has done rising, as does the stream to the eastward before low water.

* In the Admiralty Pilots, Directories, etc., the bearings are magnetic unless otherwise stated.

St. Lazarus Bank: A regular tide was observed when at anchor on the bank, the flood setting E.S.E. about 4 hours, and the ebb W.N.W. about 7 hours, with about half an hour slack water; the strength at springs was 2 knots; the rise and fall, approximately, 12 feet.

In the northern part of Pemba channel, near the coast of Pemba island, the flood stream setting to the southward neutralizes and at times overcomes the constant north-going current, and the ebb accelerates it.

It is high water, full and change, in Kokotoni harbor, at 4h. 10m.; springs rise 15 feet, neaps 10 feet. As a rule the flood runs southward, and the ebb northward.

Zanzibar Channel: The tidal streams, as a rule, are as follows: The flood runs southward in the northern part of Zanzibar channel, and in a contrary direction at the southern end, thus meeting at high water at a point near the center, the position of which depends much upon the wind.

Dar-es-salaam Bay: As a general rule, the flood runs north-westward, and the ebb in the contrary direction, but amongst the islands and reefs the streams will often be found setting to the opposite points.

Mafia Channel: The tidal streams are strong in all the channels, and along the shore; the flood stream running southward and towards the shore, the ebb to the northward and off the shore.

The direction of the tidal stream northward of Ras Kisimani is ebb to the northward and eastward, and flood to the southward.

It is high water, full and change, at Kilwa Kivinje, at 4h. 0m.; springs rise about 12 feet. There is but little tidal stream at the anchorage.

It is high water, full and change, in Lindi river, at 4h. 5m.; springs rise 11 feet. The tidal streams in the bay, outside the bank of soundings are not strong.

It is high water, full and change, at Mgau Mwanja at 3h. 45m.; springs rise 12 feet; off the entrance the flood runs to the northward, and the ebb to the south-eastward, with a force of from 2 to 3 knots.

It is high water, full and change, in Mikindani harbor at 3h. 50m.; springs rise 12 feet. The tidal stream in the harbor is scarcely perceptible.

It is high water in Rovúma bay, full and change, at 4h. 10m.; springs rise 12 feet; the ebb running to the northward and flood to the south-eastward.

In Maiyapa bay, the flood sets north-westward, and the ebb south-eastward at the rate of 2 to 4 knots at springs.

Nyuni Pass: The flood sets north-westward from 2 to 3 knots at springs, but is scarcely perceptible at neaps.

Nameguo Pass: The tidal streams within the outer reefs are irregular.

It is high water at Kero Nyuni, full and change, at 4h. 15m.; springs rise 13 feet.

It is high water, full and change, at Mozambique, at 4h. 15m.; springs rise 12 feet. The streams run strong in the harbour—the flood to the westward, the ebb to the eastward.

The tidal rise in the mouths of the Zambezi is about 12 feet at springs; this amount is reduced to about 5 feet at Mchenga, situated about 25 miles above the entrances and 5 miles above the junction of the Chinde; the time of high water at Mchenga is $2\frac{1}{2}$ hours later than at the mouth of the Chinde, or 6h. 50m., full and change.

Inhamissengo Mouth: It is high water, full and change, at 4h. 30m.; springs rise about 12 feet. The ebb tide at springs runs 4 to $4\frac{1}{2}$ knots in the entrance.

Pungue River: The tidal streams are very strong, especially when the river is high; as much as 5 knots at springs have been observed at the junction of the Pungue and Buzi.

It is high water, full and change, at Innambán at 5h. 38m.; springs rise 11 feet, neaps 7 feet. The stream runs strong in the river; off the town it sometimes amounts to 4 knots an hour.

Delagoa Bay: Seaward of the shoals, the flood sets to the northward at the rate of 2 knots.

Port Natal: In the port, the time of high water at full and change is 4h. 30m., springs rise 6 feet. The velocity of the ebb at springs is about 3 miles an hour in the Bluff channel and of the flood about $2\frac{1}{2}$ miles.

In the road, outside the bar, the flood stream sets nearly north and the ebb in the opposite direction.

Kei River: It is high water, full and change, at Kei river at about 4h. 0m.; springs rise about 5 feet. The flood stream sets north-eastward close in shore, and the ebb south-westward.

It is high water, full and change, at Port Elizabeth at 3h. 10m., and the rise is 6 feet; the tides are often irregular, being acted upon by the wind. The surface stream is uncertain in direction and inappreciable.

It is high water, full and change, in Table bay at 2h. 40m.; springs rise 5 feet, neaps $3\frac{1}{2}$ feet. The duration of slack at high water varies considerably, and greatly depends on the prevailing wind; the water is never stationary more than 30 minutes, and frequently it begins to fall immediately on reaching high water. There is no sensible stream of tide, either in the bay or on the adjacent coast. The time of high water and its rise is nearly the same at Simons bay, and at all the bays along the coast from the Cape of Good Hope to Cape Agulhas.

Congo River: The observations appeared to show that the fresh water of the Congo extends from the surface to the bottom until the head of the Congo cañon just below Kissanga, when it encounters a body of salt water filling this deep gully. It then runs over this denser water with decreased depth and increased velocity, the layer of fresh water being deeper with the ebb tide and shallower with the flood, both decreasing the broader the river becomes, until, from being from 3 to 5 fathoms deep just below Bull island, it is only a few feet deep after passing Bulambemba point.

This deep body of salt water is either perfectly still, or has a very slight tidal flow (two-tenths to half a knot per hour) up river with the flood, and down with the ebb tide.

Ambas Bay: The tidal streams appear to run both ways for an equal period; the flood setting to the south, and the ebb to the north out of Ambas bay between Ambas island and Pirate rocks.

New Calabar River: The ebb stream sets over Baleur bank, the flood stream sometimes sets over the western shoals, but if the outside current is setting strongly to the eastward, the flood stream will be but little felt. Near Sand island both ebb and flood streams set toward that island.

Off Bonny town the ebb stream runs from 3 to $3\frac{1}{2}$ knots an hour in a S.W. by W. direction.

It is high water, full and change, in Ramos river at 4h. 20m.; springs rise 5 feet. The ebb runs for 9 hours.

Forcados River: On the bar, the flood tide sets across it to the northward, the ebb in the contrary direction, which must be allowed for.

Benin River: The tidal stream in the river is said to sometimes run at a rate of 4 to 5 knots an hour.

On the bar, the ebb stream sets to the westward and the flood about E. by N., but a set, toward the northern breakers, has been experienced on the flood.

Banana Islands: The flood sets East and E.S.E.; the ebb W.S.W. and W. by N.; rate 1 to $1\frac{1}{2}$ knots.

Isles do Los: The flood stream sets to the N.E., and the ebb in contrary direction, with a rate of from $1\frac{1}{2}$ to $1\frac{1}{2}$ knots an hour at spring tides.

At the entrance of the Nunez river the flood sets N.E. and ebb S.W. at rates of from 2 to 3 knots an hour. West of Talabuncha point the flood sets in a northerly, and ebb in a contrary, direction.

Orange Channel: The ebb stream seldom exceeds $2\frac{1}{2}$ knots and the flood $1\frac{1}{2}$ knots an hour in velocity, except after heavy freshets in the rivers, the general direction of the flood stream being to the N.E., and the ebb to the S.W.

The ebb stream in the Cacheo, as also off it, sets to the N.E., and the flood to the S.W.

It is high water, full and change, in the Kasamanze river at 9h. 55m.; springs rise $5\frac{1}{2}$ feet.

In the Great pass the flood stream has a tendency to set towards the north, and the ebb toward the south bank; the rates vary from 2 to 3 knots an hour.

The tidal streams are felt so far as point Piedras, about 60 miles from the bar.

Salum River: At Kaolack it is high water, full and change, at 6h. 5m. the rise and fall of tide is about $3\frac{1}{2}$ feet.

The tidal streams run very strongly on the bar and in the narrow parts of the river, the stream of both flood and ebb continuing from 2 to $2\frac{1}{2}$ hours after high and low water, during which period there is no appreciable rise and fall until the stream turns. The flood tide entering at West pass sometimes attains a velocity of 3 knots an hour, and splits after crossing the bar.

It is high water, full and change, between Sta. Lucia and Branca islands at 7h.; springs rise about 5 feet.

The flood stream sets to the westward, and the ebb to the eastward with a velocity of 2 knots an hour during springs.

Near the shore, at Cape Verde, the tidal streams are very appreciable, the flood dividing into two branches, one setting to the northward, the other toward Almadi point; the flood stream occasions strong and irregular currents in Yof bay; the ebb stream sets off shore.

Senegal River: Abreast of this river, and for a space of several miles to seaward, the powerful tidal streams, both in and out, affect the general uniformity of the southerly current, and are often so strong as to bring vessels, anchored in the outer road, with their broadsides to the wind in the strongest breezes. These outer tidal streams have no very regular set; the flood stream, however, generally runs E.N.E. and the ebb W.N.W.

It is high water, full and change, in Ouro river at oh. om.; springs rise 8 or 9 feet.

Off the entrance the flood stream sets nearly east, and the ebb west, with a velocity of about $2\frac{1}{2}$ knots an hour.

Lanzarote Island: The flood streams run to E.N.E. and ebb stream in a contrary direction, the rate, at spring tides, being about 1 knot an hour.

Lobos Island: The flood stream sets E.N.E. and ebb W.N.W.

It is high water, full and change, in Santa Cruz bay at 1h. 30m.; springs rise 8 feet, neaps 6 feet.

The tidal streams set by the Dezerta islands during spring tides, at the rate of $1\frac{1}{2}$ to 2 miles per hour; the flood in the direction of N.E. by E., and the ebb S.W. by W. Springs rise 7 feet.

It is high water, full and change, in Funchal bay at oh. 48m.; springs rise 7 feet.

The tidal wave strikes these [Madeira] islands nearly at the same time as the Azores, the flood stream running to the north-eastward at the rate of $1\frac{1}{2}$ miles per hour at spring tides, and in the narrow channels between Dezerta islands and off San Lourenzo point, it sometimes attains the velocity of 2 miles per hour.

It is high water, full and change, in Mogador harbour at 1h. 18m.; springs rise 10 or 12 feet. The tides are generally regular in their rise and fall, but the direction of the tidal stream varies with the wind, and its strength is at all times weak.

The flood stream in the offing north of Tangier bay runs from east to west, and the ebb in the reverse direction, turning in mid-channel at high and low water by the shore.

In Fayal channel the flood stream sets N.E., and the ebb S.W., with a velocity of from 1 to 2 knots an hour.

It is high water, full and change, at Corvo and Flores islands at oh. 20m.; springs rise $3\frac{1}{2}$ feet.

The stream of flood tide runs to N.E. by N. and the ebb S.W. by S., with a velocity of $1\frac{1}{2}$ knots per hour at springs; these tidal streams, when opposed by gales, create a most confused sea off the north and south extremes of both islands.

The currents in the Straits of Gibraltar and Messina, and the Euripus, are described in sections 84-86, Part IV A.

On account of the large rise and fall around the coasts of Spain, Portugal, and France, the currents are very strong in the tidal rivers. For brief accounts of these currents see *Sailing Directions for the West Coasts of France, Spain, and Portugal*, published by the Admiralty; also *Étude Pratique sur les Marées Fluviales*, by M. Comoy.

71. *The British Isles and the North Sea.*

The behavior of the tidal currents in this region has long been understood. For instance, Barlow, on a map facing page 140 of his *Exact Survey of the Tide* (1717), shows the general direction taken by the flood stream in these waters, although he here aims at showing the advance of the tidal wave. A similar map for the northern part of the Irish Sea occurs between pages 152 and 153 of his treatise.

The charts of Captain Beechey are briefly referred to in section 126, Part I, and two of these are reproduced as Figs. 27 and 28, Part IV A.

The character of the tide wave, whether progressive or stationary, is indicated upon a chart opposite page 125 of a book entitled "The Tides;" Society for Promoting Christian Knowledge (1857).

In connection with the below descriptions it will be found advantageous to frequently consult section 44 and Figs. 20-22, Part IV B, in order to see the relations between the currents and the tides.

The English Channel and the Irish Sea contain stationary waves whose particles are at elongation inward at about the time of high water at Dover, or at about XI Greenwich lunar time (Fig. 20; Part IV B). At this time the water is slack over nearly all of the Irish Sea, including the axis of the North Channel, and over a portion of the English Channel north of Cherbourg. Three (lunar) hours before high water at Dover the flood is running strongly in the regions just mentioned, and three hours after high water the ebb is running strongly. The tendency of the apparently progressive part of the wave between the northwestern corner of France and the southern part of Ireland is to cause the inward stream to have its maximum velocity at IV instead of at XI-3 or VIII. As a matter of fact, the greatest inward velocity here occurs about 4 hours after Dover high water, or at III. This indicates that the tidal wave in this region is partly stationary, as is generally the case in a marginal strip of shallow water lying between the deep water of the ocean and the land. In both the Bristol Channel and the Gulf of St. Malo, there is evidence from the cotidal chart of a stationary wave; for, in either case the rate of advance is greater than that due to depth. But toward the inner ends of these arms of water, where the tidal hour is VI, the greatest velocity of the flood stream occurs at about $2\frac{1}{2}$ hours before the time of high water there, or at about III $\frac{1}{2}$, Greenwich time. The effect of Bristol Bay upon the currents off the Scilly Islands is evidently to stop the northeasterly flood stream before the easterly stream slackens at this part of the English Channel. Hence the currents are here rotary in character and clockwise in rotation. The reverse must be true of the currents north of Cotes du Nord. The following quotation and table are taken from the Admiralty Tide Tables for the British and Irish ports for the year 1907, and illustrate what has just been said:

Off the mouth of the English Channel the stream, although materially influenced by the indraft and outset of the Channel, will be found running to the *northward and eastward*, while the water is *falling* at Dover; and to the *southward and westward* while it is *rising* at that port. The particular direction given to the stream in this part of the sea, by the meeting of the Channel and of the offing tides, will be shown in the table [1st below]; and it is only necessary to mention here, that to the southward of the parallel of Scilly, the tidal streams of the Channel and offing blend together with varying force and direction, and occasion the direction of the stream to be constantly changing, and in some places even to make the entire circuit of the compass in one tide, without ever remaining long upon any one point; so that any written description of their course is rendered almost impossible, and the table alone must be consulted for the direction at any particular hour. From this rotatory motion of the stream, it has been asserted that a vessel can never be carried far in any direction by it. Such, however, is not the case; for, although it may be true that while at anchor in a particular spot the vessel's head will turn to every point of the compass, yet directly she is loose she will be carried away upon a rhomb depending upon the state of the tide at Dover.

Westward of a line joining Ushant and the Land's End.

Hours	North side of latitude 49° N.						South side of 49° N.	
	West part	Rate	Near Scilly	Rate	Seven stones L. V.	Rate	West part	Rate
H. W.						<i>Knots</i>		
After high water, Dover	1 NW. by W.	Greatest rate, springs, 1½ knots	1 NW. by N.	Greatest rate, springs, 1½ knots	WSW. to WNW.	0 to 1	Greatest rate, springs, 1½ knots
	2 N. by W.		2 N. by W.		NW. to NNW.	0 to 1	WNW.	
	3 NE.		3 N. by E.		N. by E.	0 to 1½	NNW.	
	4 ENE.		4 NE.		NE. by N.	0 to 1½	ENE.	
	5 ENE.		5 NE.		NE.	¼ to 1½	ENE.	
	6 Ely.		6 E.		ENE. to ESE.	0 to 1	NE. by E.	
Before high water, Dover	5 ESE.	Greatest rate, springs, 1½ knots	Greatest rate, springs, 1½ knots	SE. to SSE.	¼ to 1	Ely.	Greatest rate, springs, 1½ knots
	4 S.		Sly.		S.	¼ to 1½	SE.	
	3 SSW.		SW.		SSW.	¼ to 1½	Draining.	
	2 SWly.		SWly.		SW. ½ S.	¼ to 1½	SSW.	
	1 WSW.		SWly.		SW. by W.	¼ to 1½	SW. by S.	

Entrance of Gulf of St. Malo on a line joining Brehat Island and southwest end of Guernsey.

Hours	Westward from Sept Isles		12 miles from Brehat Island		12 miles from Guernsey		Near SW. point Guernsey		4 miles W. by S. from Casquets		4 miles WNW. of Cape La Hague	
	Course	Rate	Course	Rate	Course	Rate	Course	Rate	Course	Rate	Course	Rate
H. W.	WNW.	Greatest rate, springs, 2 knots	NNW.	Greatest rate, uncertain	WNW.	Greatest rate, uncertain	Wly.	Greatest rate, uncertain	W. by N.	Greatest rate, springs, 5 to 7 knots	WSW.	Greatest rate, springs, 5 to 7 knots
After high water, Dover	1 W. by S.		WNW.		Wly.		Wly.		W. by S.		SW. by W.	
	2 WSW.		SWly.		SWly.		SSW.		SW.		SW. by W.	
	3 SW. by W.		Sly.		Sly.		SSW.		SW.		SW. by W.	
	4 SSW.		SE.		SE. by S.		SE.		S.		SW. by S.	
	5 S.		SE.		ESE.		ESE.		SE. by S.		SW. by S.	
	6 SE.		SE.		ESE.		ESE.		SE. by E.		NE. by E.	
Before high water, Dover	5 E. by S.	Greatest rate, springs, 2 knots	SE. by E.	Greatest rate, uncertain	ESE.	Greatest rate, uncertain	{ ESE. } { E. by N. }	Greatest rate, uncertain	E. by N.	Greatest rate, springs, 5 to 7 knots	NE. by E.	Greatest rate, springs, 5 to 7 knots
	4 E. by N.		NEly.		Ely.		{ ESE. } { E. by N. }		NE. by N.		NE. by E.	
	3 NEly.		NW.		NNE.			NE. by N.		NE. by N.	
	2 Turning.		NW.		NNW.		NNW.		NE. by E.		NE. by N.	
	1 NW. by N.		NW. by W.		W. by N.		NNW.		N.		NE. by N.	

It should be remembered that the directions in the quotations from the Admiralty Tide Tables are magnetic.

Concerning the streams near the Channel Islands the Tide Tables say:

Near Guernsey and to the northward of that island the true Channel stream prevails; the great body of the water running about E. by N. whilst the tide is *rising* at Dover, and about W.S.W. when it is *falling* at that place; but near Roches Douvres to the southward, the stream sets S.E. into the Gulf of St. Malo, from 2 hours after high water at Dover to 4 hours before high water there, and N.W. during the remainder of the tides.

Thus what is called *tide and half tide* prevails at Guernsey and amongst the islands to the northward; whilst at Jersey and along the southern shore of the gulf, and out to the westward toward Roches Douvres, the stream is more uniform and regular; the former resulting directly from the action

of the Chandel stream, the latter from an interruption of the southern portion of that stream by the coast of France, and its diversion into the Gulf of St. Malo.

The center of Deroute channel (between Roches Douvres and Guernsey) may be considered to mark the separating boundary of these two streams; for along this line and to the eastward they successively run together side by side, blend, and separate in alternating direction and force, depending on the state of the tide.

It should here be noted that the tidal stream around and between the Channel islands has a rotatory motion (evidently caused by the different action of the above-described two streams and the peculiar form of the shores of the gulf) from right to left, going right round the compass in little more than 12 hours.

* * * * *

In the offing westward of Guernsey, the stream seldom attains a rate of 3 knots until the island is approached within 4 or 5 miles, where it increases to $4\frac{1}{2}$ knots; in the Russel channels it exceeds 5 knots, and it runs about the same rate between Jersey and the Minquiers; in the center of Deroute channel, between Jersey and Sark, its strength is barely 4 knots, and 3 knots farther westward between Guernsey and Roches Douvres; near Roches Douvres the rate appears to be $3\frac{1}{2}$ knots; in the offing north of Alderney and the Casquets, $5\frac{1}{2}$ knots is not an uncommon rate for an ebb spring tide, and on similar occasions the Race and Swinge streams run more than 7 knots.

The rapidity with which the tides rise and fall and their velocity are greatly influenced by strong north-eastern and south-western gales; the former retarding and the latter accelerating their progress in a remarkable degree; the latter will also cause the Race stream to run three-quarters of an hour longer to the north-eastward than usual, although the former has not a similar effect upon the stream when running to the south-westward.

About one mile south of the bill of Portland, at half flood by the shore, or $4\frac{1}{2}$ hours after high water at Dover, the stream sets from S.S.E. to S.E. by E., and the opposite stream about W.S.W.; the velocity of both streams, at springs, being from 5 to 6 knots; but although they run with such violence near the Race, about one mile S.W. of the bill they are weak.

Off Portland Bill the easterly velocity of the main stationary wave should have its greatest value at about XI—3 or VIII. But the observed tidal hour for Portland Bill is VI. As a matter of fact, the greatest easterly velocity occurs at about VII $\frac{1}{2}$, according to the Admiralty Tide Tables.

The times of the greatest eastward velocity through Dover Strait can be inferred from the tides by noting the tidal hours and ranges off either end of the strait proper and adding 3 hours to the values given by Table 60. The tide off the west end of the strait occurs at $X\frac{1}{2}$ hours, the mean range being 20 feet; the corresponding quantities for the east end are XI $\frac{1}{2}$ and 12. Hence the time of maximum eastward downward slope is $X\frac{1}{2} - 1.066 = 9.434$; this increased by 3 hours gives XII.43 for the time of the maximum eastward current in the strait, which practically agrees with the results of observation.

As already remarked, the principal current in the Irish Sea has its maximum inward velocity at XI—3 or VIII. Just off St. David's Head this velocity is from 2 to 4 knots and around the Skerries from 2 to 5. West of the Isle of Man this velocity is very small.

At the northern end of North Channel the tidal hour and mean range in feet are V.5 and 7, while for the southern end they are X.7 and 12. Using these values in Table 60 the time of maximum downward northward slope is X.98, and so, subtracting 3 hours from this value, the result is VII.98 for the time of the maximum southerly stream. Observation gives VIII as the current hour of the stream.

The cotidal lines in the North Channel, and also those in the Irish Sea and English Channel, are much influenced by the transverse slope due to the deflecting force of the earth's axial rotation. Not so with the cocurrent lines; for, the transverse velocity is small in comparison with the velocity along the strait or channel. Hence, *in a channel*

only moderately wide, and having a current of some magnitude, the cocurrent lines are frequently more simple in character than are the cotidal lines. The reverse of this is often true along the irregular borders of open bodies of water and where the current is generally small.

The following quotations from the Admiralty Tide Tables partially describe the tides in the Irish Sea and adjacent waters:

72. In the Irish channel, as before observed, experiments have shown that, notwithstanding the variety of times of high water throughout the channel, the turn of the stream over all that part which may be called the fair navigable portion of the channel is nearly simultaneous; that the northern and southern streams in both channels commence and end in all parts (practically speaking) at nearly the same time; and that that time happens to correspond nearly with the time of high and low water on the shore at the entrance of Liverpool and of Morecambe bay,* a spot remarkable as being the point where the opposite streams coming round the extremities of Ireland terminate. So that it is necessary only to know the times of high and low water at either of these places, to determine the hour when the stream of either tide will commence or terminate in any part of the channel. For this purpose the Liverpool tide table may be used, subtracting a quarter of an hour from the times there given, in consequence of the high water at George pier being later than the point which is considered as the head of the tide.

The tidal undulation from the Atlantic enters the Irish channel by two channels; of which Carnsore point, the S.E. point of Ireland, and St. Davis head, the S.W. point of Wales, are the limits of the southern one; and Rathlin and the Mull of Cantyre the boundaries of the northern.

The axis of the in-going stream runs nearly in a line from a point midway between the Tuskar and the Bishops, to a position 16 miles due west of Holyhead; beyond which it begins to expand eastward and westward; but its main body preserves its direction straight forward toward the Calf of Man, which it passes to the eastward with increased velocity as far as Langness point, and then at a more moderate rate on toward Maughold head. Here it is arrested by the southern stream from the North channel coming round the point of Ayr, and is first turned to the eastward by it, and then goes with it at an easy rate direct from Morecambe bay; thus changing its direction nearly eight points.

* * * * *

The western part of the stream, after passing the Saltees, runs nearly in the direction of the Tuskar, sets sharply round it, and then takes a N.E. direction, setting fairly along the coast, but over the banks skirting the shore, so that vessels tacking near the inner edge of the sands with the northeast-going stream, and on the outer edge on the opposite stream, have been carried upon them and lost, especially upon the Arklow and Codling banks. Abreast of Arklow is situated that remarkable spot in the Irish channel, where the tide scarcely either rises or falls. The stream notwithstanding sweeps past it at the rate of 4 knots at springs, and reaches the parallel of Wicklow head. Here it encounters an extensive projection of Codling bank; and while the outer portion takes the circuit of the bank, the inner stream sweeps over it, occasioning an overfall and strong rippling all round the edge, by which the bank may generally be recognized. Beyond this point the streams unite and flow on toward Howth and Lambay, growing gradually weaker as they proceed, until they ultimately expend themselves in a large space of still water situated between the Isle of Man and Carlingford. There we have not been able to detect any stream; for there another remarkable phenomenon occurs—the water rising and falling without apparently any perceptible stream. This space of still water is marked by a bottom of blue mud. Such is the course of the flowing water of the Southern channel.

In the North channel the stream enters between the Mull of Cantyre and Rathlin island simultaneously with that passing the Tuskar into the Southern channel, but flows in the contrary direction. It runs at the rate of 3 knots at springs, increasing to 5 knots near the Mull, and to 4 near Tor point, on the opposite side of the channel. The eastern branch of this stream turns round the Mull toward Ailsa

*The entrance of Liverpool and of Morecambe bay are, as before stated, 18 minutes earlier in their time of high water than those given for Liverpool in the tide tables.

At N. W. L. V., Liverpool bay, the flood stream sets in an ESE. direction, with a maximum rate at springs of $2\frac{1}{2}$ knots; the ebb WNW., 2 knots. At neaps the flood sets at the rate of 1 knot, the ebb three-quarters of a knot.

and the Clyde, a portion passing round Sanda up Kilbrennen sound and loch Fyne. The main body sweeps to the S. by E., taking nearly the general direction of the channel, but pressing more heavily on the Wigtonshire coast. Near the Mull of Galloway the stream increases in velocity to 5 knots; the eastern portion turns sharply round the promontory toward the Solway, and splits off St. Bees head, one portion running up the Solway and the other toward Morecambe bay.

73. The currents on the west coast of Scotland turn soon after the times of local high water, indicating that the wave is there chiefly stationary. Through the larger channels the current is nearly oscillatory, but through the short and narrow straits it is nearly hydraulic. (See Chap. VIII, Part IV A.)

For example, the tidal hour just off the western end of Corrievrekin Strait is V.4 and that just off the southeastern end IV.5; the mean ranges off these two ends are 7 and 4 feet, respectively. Using these values in Table 60, it follows that the greatest southeastward downward slope occurs at VI.2; observation gives about VI.5 as the hour of the southeastward stream. Hence, the Corrievrekin is nearly hydraulic in character. The velocity of the strength of current varies from 4 to 8 knots.

Before making very satisfactory computations of this kind, it will be necessary to have the times and the heights of the tide carefully observed.

The currents in the Little Minch are oscillatory. The tidal hours off the north and south ends of the strait are VI.5 and V.5, while the ranges in feet are 11 and 8. By Table 60 the greatest southward downward slope occurs at VIII. Subtracting 3 from this, the theoretical hour for the north-going stream is V. Observation gives V½.

Scott, in *The Lord of the Isles*, thus speaks of the currents among the islands along the western coast of Scotland:

O'er look'd dark Mull! thy mighty Sound,
Where thwarting tides, with mingled roar,
Part thy swarth hills from Morven's shore.

* * * * *
All day with fruitless strife they toil'd,
With eve the ebbing currents boil'd
More fierce from strait and lake;
And midway through the channel met
Conflicting tides that foam and fret,
And high their mingled billows jet,
As spears, that, in the battle set,
Spring upward as they break

* * * * *
Or that your eye could see the mood
Of Corryvrekin's whirlpool rude,
When dons the Hag her whiten'd hood.

The following quotations from the Admiralty Tide Tables refer to the streams along the west coast of Scotland:

While the laws of the streams are thus of more importance than the laws of the rise and fall of the tide, they are also much more simple. The times of high and low water are very different at different parts of the coast, while the times of slack water are nearly the same throughout the whole region in question. In a great part of this region the stream has no distinct title to be considered either a flood or an ebb stream, although at any point it generally flows for six hours in one direction, and for six hours in the opposite direction.

* * * * *
Between the Mull of Cantyre and the north-east coast of Ireland, the most westerly part of the north-going stream turns to the west, and runs through the sound of Rathlin along the north coast

of Ireland; the central part flows to the north-west past the Rhynns of Islay; the easterly part, which has flowed partly through the sound of Sanda, turns sharply round the Mull of Cantyre, and flows to the northward, pouring with great velocity through the narrow openings in the chain of islands, viz.: the sound of Islay, between Islay and Jura, the gulf of Coirebhreacain between Jura and Scarba, the little Coirebhreacain between Scarba and Lunga, the Slate isles and Cuan sound; of these, the little Coirebhreacain is quite impassable; and Coirebhreacain and Cuan sound are seldom attempted except near slack water.

Great complication arises from describing the time of change of the stream by reference to the time of high and low water on the shore; thus we should have to say that in the sound of Sanda, the ebb stream begins two hours before high water; at the Mull of Cantyre, one hour before high water; a little north of this, again two hours before high water. Southward of Gigha, we might say indifferently, that the flood tide runs to the south and begins three hours before low water, or that it runs to the north and begins three hours after low water; in the sound of Islay and in the gulf of Coirebhreacain that it begins an hour before low water; and in describing the streams along the north coast of Ireland we have even greater complication.

The direction of the tidal streams on the rest of the West coast of Scotland may be thus described: Outside of Islay and Iona the streams turn at the time of high and low water at Liverpool, running southward with the rising and northward with the falling tide at that place. At the northern end of the passage of Tiree the streams change at $1\frac{1}{2}$ hours, at the southern end of the Little Minch at from three to four hours, and at the northern end at from four to five hours after high and low water at Liverpool; the time of the turn of the streams being thus gradually retarded as we proceed north. In the sound of Mull there is the same retardation, the streams turning at the southern end at one hour before, and at the northern end half an hour after, high and low water at Liverpool, and flowing in the same direction as above mentioned.

Round the north end of the island of Lewis, the stream bends into the Minch and meets the stream from the southward, the course of both streams being nearly the same as if there were an embankment from loch Shell in the island of Lewis to Ru Rea on the coast of Ross-shire. At the same time, another branch of the stream which has rounded Ardnamurchan point flows through Sleat sound, and being an hour earlier than the tide which has rounded the north end of Skye, it pours with great velocity through Kyle Rhea, but owing to the undulations round Skye meeting near Kyle Akin there is very little stream through that narrow opening; the flood stream, as it is stated, sometimes flowing in one direction and sometimes in the other, according to the prevailing winds.

74. The tide wave along the eastern coast of Scotland and England as far south as Flamborough Head is progressive in character as might be inferred from the cotidal charts, Figs. 21, 22, Part IV B. Now, observation shows that the currents follow this coast and attain their maximum flood velocities at about the times of local high water.

The corner of the sea lying between England and Holland and Belgium is characterized by a stationary oscillation (sec. 44, Part IV B) which is very apparent in the currents. Being an arm of an almost tideless sea sustained by the rise and fall in the Dover Strait, the northeastern stream should occur at XI + 3, or II, a fact agreeing with observation.

The tidal hours of the estuaries of the Schelde and Meuse ranges from XII to II. The east-going stream in the offing occurring at II, it follows that since the estuaries must contain tide waves, stationary in part, the times of greatest influx must be somewhat earlier than the times of tide. Observation shows that the greatest influx occurs from XII to I o'clock, or from 2 to 1 hours earlier than the greatest easterly stream in the offing.

Van Der Stok finds that at Schouwenbank L.V. the currents are rotary and counter-clockwise as they should be in a case of this kind. (See table under sec. 97.)

The southeastern corner of the North Sea contains a tide wave stationary in part. This might be inferred from Fig. 22, Part IV B; but the greatest flood stream around Helgoland Island occurs $1\frac{3}{4}$ hours before local high water.

The tidal streams in the approaches to the Baltic Sea are everywhere weak.

The famous Malström or Moskenström in the Lofoten Islands can possess no very unusual strength, because the mean range of tide is there only 6 feet, and the time of high or low water must be about the same for the whole region concerned in the production of the whirlpool. (Fig. 23, Part IV B.) Barlow gives a map (opposite p. 196 of his treatise) showing the streams of this region and the whirlpool itself. The Malström is described at some length in Part II of the Norway Pilot, 2d ed., pp. 400-401, 422-424.

References to recent charts and discussions of the currents around the British Isles and in the North Sea.

Hydrographic Office, Admiralty: Tidal Streams, English and Irish Channels (1899); Tidal Streams, Coasts of Scotland (1899); Tidal Streams, North Sea (1899); Tide Tables, Charts, and Pilots, or Directories or Sailing Directions.

M. Hédouin: Current charts published by the Service hydrographique de la marine, 1891.

J. P. Van der Stok: Études des phénomènes de marées sur les Côtes Néerlandaises, Koninklijk Nederlandisch Meteorologisch Institut, 1905.

Deutsche Seewarte: Atlas der Gezeiten und Gezeitenströme für das Gebiet der Nordsee und der Britischen Gewässer, 1905.

75. *The Gulf of St. Lawrence and outside waters.*

As already explained in section 73, Part IV A, and sections 34-40, Part IV B, the tides along the coast of America from Cape Race, Newfoundland, to Florida, result from a stationary wave extending from this coast in a southeasterly direction. The tidal hour of the American loop of this wave being XII, it follows that well off the coast the current hour should be IX. As the northwestern edge of the ocean basin is approached this hour changes little or considerable, according to circumstances. From New York entrance to Savannah, observations made a few miles off the coast indicate, with scarcely an exception, that the northwesterly or westerly component of the current is running with the greatest velocity at a little after IX. Stations off New Jersey, Maryland, Virginia, North Carolina, and South Carolina are given in the accompanying table. Since the first nodal line of the stationary tidal oscillation terminates near Guadeloupe Island, it follows that the times of tide for the northern coasts of Cuba, Haiti, Porto Rico, and the Virgin Islands can not differ very much from XII. (See Fig. 11, Part IV B.)

If the Gulf of Mexico and the Caribbean Sea be regarded as tideless bodies, it follows that the greatest southward downward slope through Florida Strait, Windward Passage, and Mona Passage must occur at about the time of high water outside. Straits of these dimensions, especially if connecting deep bodies of water, must contain stationary waves. Consequently the maximum southward velocities must occur about 3 hours after the time of high water off their northern openings. The inward current hour (ebb) should therefore be in the neighborhood of XII+3, or III. Observation gives for Florida Strait off Fowey Rocks II.3, and near the western end of the strait III.0 as the current.

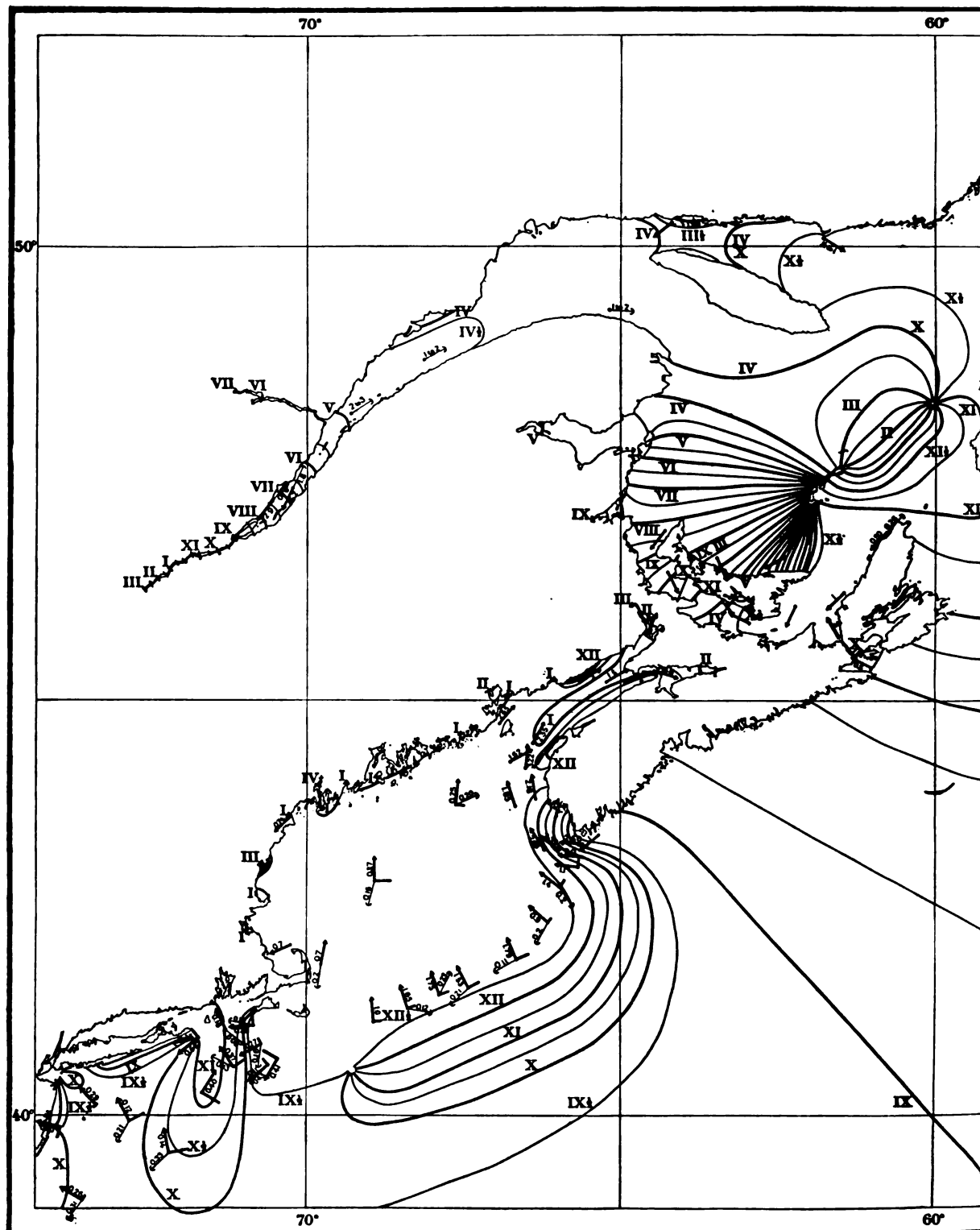
hours. For Windward Passage the observed hour is III.4, for Mona Passage II.7, and for Vieques Passage III.1. The same thing is nearly true of the tide in the Lesser Antilles. Between Barbados and Tobago the current hour of the westerly stream is XI.78 and between St. Vincent and St. Lucia the hour is X.34. The tidal hour is VII. This being increased by 3 hours gives X for the current hour, assuming the oscillation to be stationary.

The principal part of the tide in the Gulf of St. Lawrence is a fractional stationary wave whose node is north of the Magdalen Islands and whose loop is the St. Lawrence Estuary. (See sec. 91, Part IV A.) The tidal hour of this loop is about VI; hence if there were no progression the current hour for the in-going stream should be III. An inspection of the map (Fig. 13, Part IV B) will show that high water for the northeastern arm of the gulf occurs at a trifle before II; consequently water must be most rapidly flowing northeastward at a little before XI, or II - 3; hence the current hour shown on Fig. 7. Just outside of Cabot Strait the ocean wave is partly stationary, with a tendency to a maximum northwesterly velocity at IX, and partly progressive, owing to the irregularities in the shore line and the progression existing in the gulf, with a tendency to a maximum velocity at XII. The observed maximum velocity in the strait occurs at XI. But, as already stated, the westerly velocity of the stationary wave north of Magdalen Islands is about III. So far as velocities are concerned, a progressive wave affects a stationary wave least near a nodal line, and most at the end from which the progressive wave seems to come. Hence the tendency for the cocurrent lines to bunch up toward Cabot Strait rather than at the node. The hours for the west-going stream change from XI at Cabot Strait to a little less than III off the Magdalen Islands. The rapidly changing stream is intersected by the stationary stream filling or emptying the northeastern corner of the gulf. They intersect at right angles, have equal velocities, and phase difference of 3 hours at the point shown in Fig. 7, from which the cocurrent lines radiate. The hour of the stationary stream is $X\frac{1}{2}$ or $IV\frac{1}{2}$, and of the progressive $I\frac{1}{2}$.

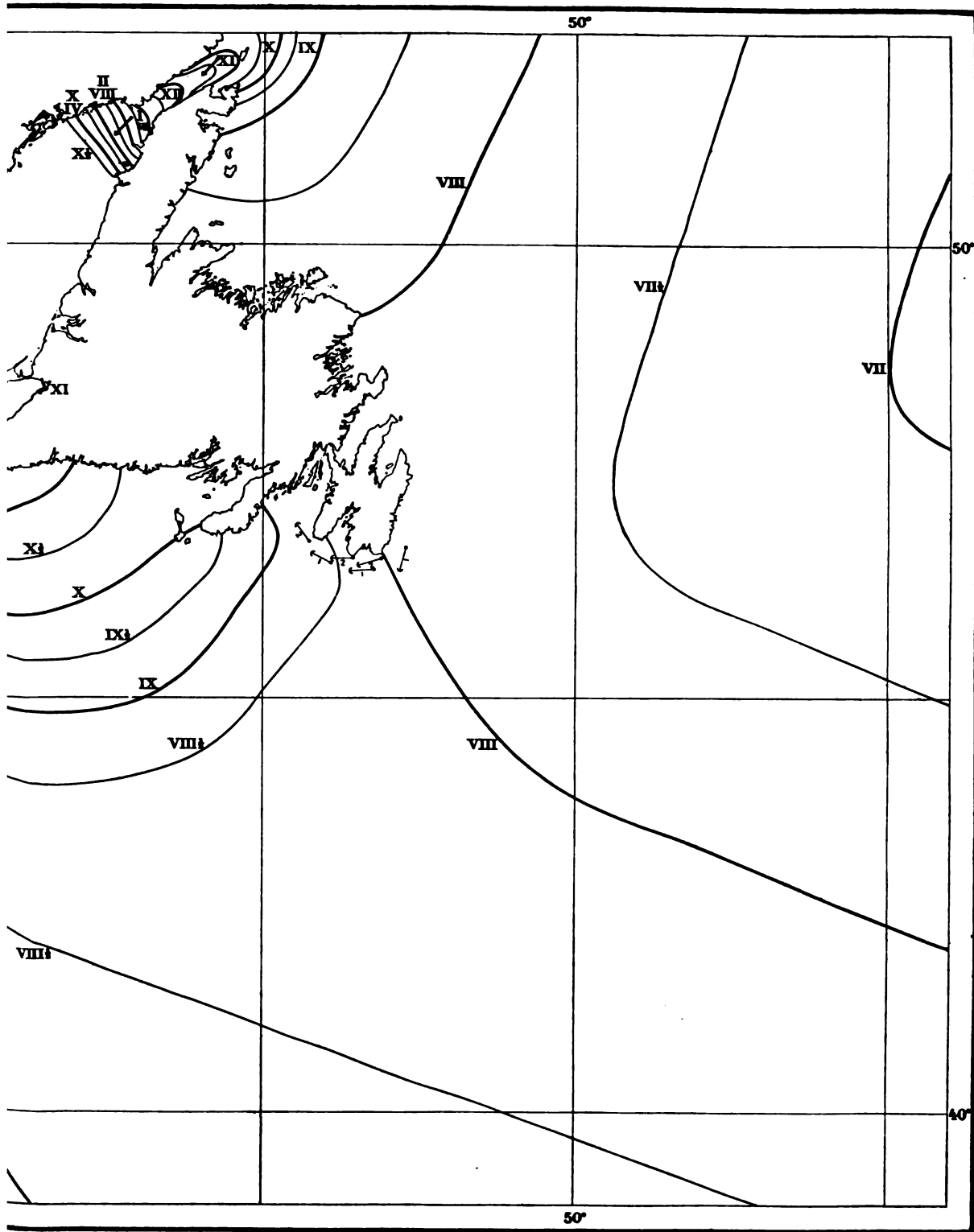
The current in the Strait of Belle Isle is oscillatory. At the outer end the tidal hour is X and at the inner end II; the mean ranges are $2\frac{1}{2}$ and $3\frac{1}{2}$ feet, respectively. These values, substituted in Table 60, give, when increased by 3 hours, XI.8 o'clock for the time of maximum inward velocity. Observation makes the hour off Armour Point a trifle over $XII\frac{1}{2}$. As already noted, the stream flowing out of the northeast angle of the gulf in a southwesterly direction is swiftest at $IV\frac{1}{2}$; consequently off the inner end of the strait the current hour must change from I to $IV\frac{1}{2}$ if the west-going stream be followed. The mean maximum velocity off Armour Point is about $1\frac{1}{4}$ knots.

Above its estuary, the St. Lawrence River has a tidal wave nearly progressive in character, and so the flood continues a considerable time after local high water and the ebb, after local low water.

In comparing the chart of cotidal lines (Fig. 13, Part IV B) with the cocurrent chart (Fig. 7, Part V) it must be remembered that here and elsewhere the cotidal lines refer to actual (not the M_2) high water, while the cocurrent lines refer to the time obtained by using the maxima of both flood and ebb (adding 6 lunar hours to the latter) or to the M_2 -current. Where river stations are involved, an M_2 cotidal map would be preferable to one referring to actual high water for making comparisons with the currents. At Quebec, the M_2 high water is nearly one-half lunar hour behind the actual high water.



COCURRENT LINES FOR THE GULF
Tidal hours: Quebec, X. 61, V. 58; St. Jol



OF ST. LAWRENCE AND VICINITY.
 n, III. 16, IX. 22; Boston, III. 82, IX. 86.

The spring velocity of the tidal current in the narrows of the South Traverse is $7\frac{1}{2}$ knots.

The tidal current between Anticosti Island and the mainland north of it is a stationary oscillatory stream. The tidal hours off the east and west ends of this strait are II.5 and V.6, respectively, while the mean ranges of tide are 3 and 5 feet. The time ascertained by Table 60 when decreased by 3 hours gives III.6 as the current hour of the west-going stream.

The current in the narrow part of Northumberland Strait is also oscillatory, as can be seen by noting the tidal hours and ranges at the ends and applying Table 60. The values II $\frac{1}{2}$ and 5 for the east end and XII $\frac{1}{2}$ and 2 for the west end give, upon subtracting 3 hours from the value in Table 60, XII o'clock for the time of greatest eastward current. Near the narrowest part of the strait the flood stream turns to ebb and the cocurrent lines, if drawn, would bunch up together. The numbers between XI and IV have been omitted on the map. North of Prince Edward Island the transition from flood to ebb is less sudden.

Through the Gut of Canso the streams are hydraulic. The hours and ranges in feet for the two ends being XI.5, XII.7, and 4, $2\frac{1}{2}$; the value from Table 60 is X.3. Off Cape Porcupine the rate of flood or ebb is 4 knots.

References to tidal currents in Canadian waters:

Admiralty: Newfoundland and Labrador Pilot; Sailing Directions S. E. Coast of Nova Scotia and Bay of Fundy; St. Lawrence Pilot.

W. B. Dawson: Survey of Tides and Currents in Canadian Waters, Reports 1901, 1902; The Currents at the Entrance of the Bay of Fundy, Ottawa. 1905.

76. Gulf of Maine.

As explained in sections 34, 91, Part IV A, and section 39, Part IV B, the ocean tide wave supports an oscillation of approximately critical length in the Gulf of Maine and Bay of Fundy by means of an intermediate progressive wave which is in evidence at Georges Bank. Here, where the range of the stationary dependent wave is small, the progressive wave greatly influences the times of the resultant observed tides. In approaching this bank from deep water, and by the time the maximum flood reaches the center of it, the current hour changes from IX $\frac{1}{2}$ to XII $\frac{1}{2}$, the change taking place chiefly at the loop of the oceanic oscillation.

Observation shows that the time of current increases less than half an hour between Georges Bank, where it is XII $\frac{1}{2}$, and the regular shore line 175 miles to the northwest. This is shown in Fig. 7. In the central portion of the Bay of Fundy the current hour is I. Along the shores of the bay the current turns earlier because its motion being there partially arrested by the irregularities of the shore line, the hydraulic tendency to flow into or out of the bay in accordance with the surface slope comes into existence.

Because of the narrow channel above the city of St. John, the tides produce two falls each way daily, the slack waters occurring when the river and the bay are upon the same level. The flow is strong in consequence of there being an average maximum head of nearly 10 feet. (See Fig. 13, Part IV B.)

In the center of the passage to the Basin of Mines, the spring maximum velocity is 5 to 6 knots, while near Cape Split it is 7 or 8 knots.

In so far as the currents in the Gulf of Maine are rotary, the rotation is clockwise. East of Cape Cod Peninsula the cold permanent current setting southward is apparent.

77. Nantucket Shoals to Narragansett Bay.

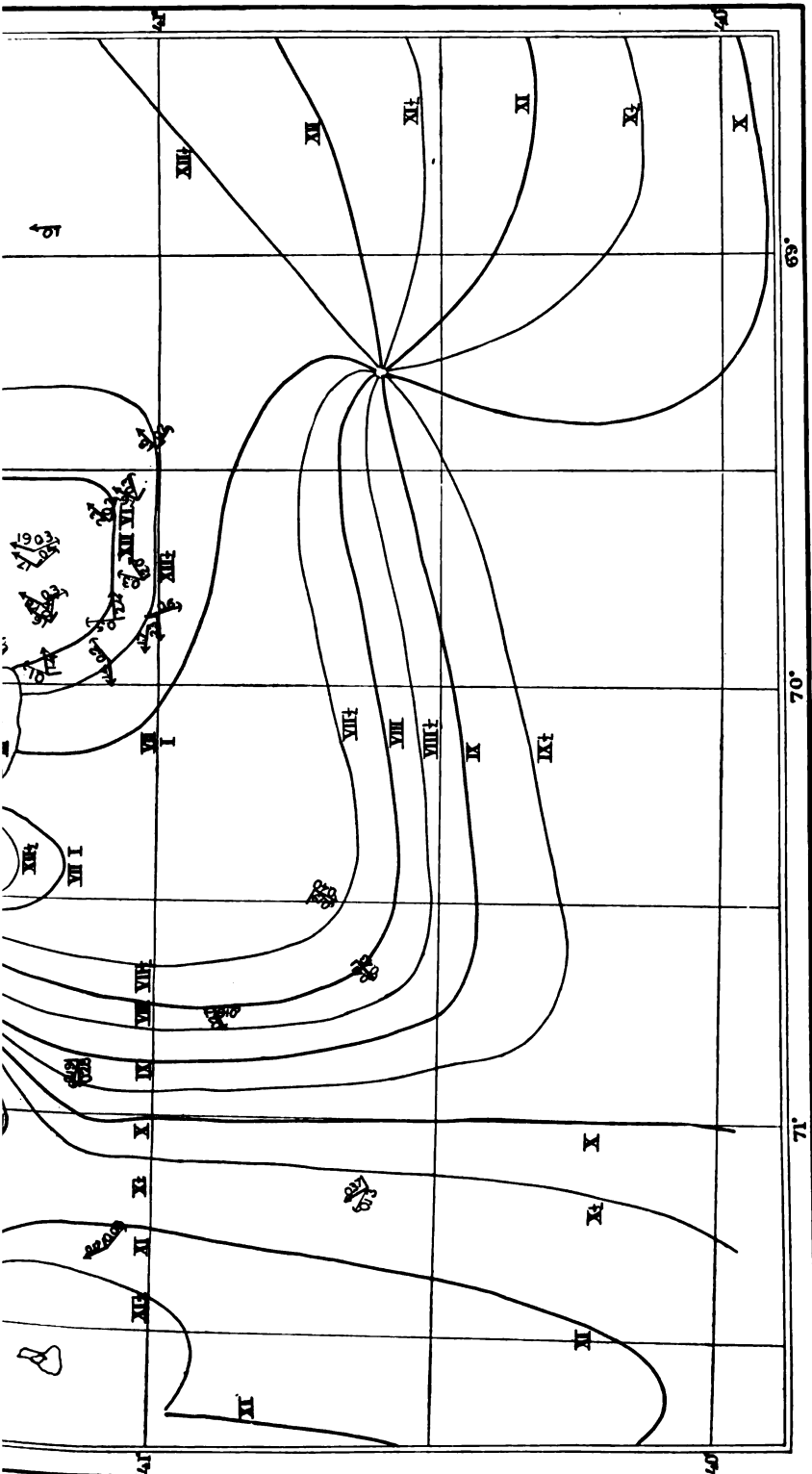
From what has been said concerning the westerly ocean currents at about IX o'clock and the northerly current into the Gulf of Maine at about XII½, it is reasonable to expect a circular point in the neighborhood of Nantucket Light Vessel (or where the stream, in a sense, divides), and that the rotation of currents on Nantucket Shoals should be clockwise as a rule. So far as observations have been made they go to verify these conclusions. Easterly from this point, the progression of the hours is northward, and westerly from the point, the progression is southward. This indicates that the progression upon Georges Bank controls in the first region, while the progression over the shoals southeast of Nantucket controls in the second. In other words, we are to imagine about half of the scheme illustrated by Fig. 4 to be in each locality; in either case the north and south motions are simultaneous across the lines marked XII and IX. All observed currents off Nantucket which are rotary turn clockwise.

The fact that the current intervals increase in going southward and southwestward along the coast of Nantucket Island, while the tidal intervals increase rapidly in the opposite direction, was noted by Schott more than 50 years ago, on page 163 of a paper relating to the currents on Nantucket Shoals and published in the Coast Survey Report for 1854.

Upon referring to Fig. 8 it will be seen that the current hour for the easterly going stream increases through Vineyard Sound to a late region northeast of Cape Poge; farther east the hour of the main stream continues to decrease to a little east of Great Point; still farther east the hour of the main current increases, but the direction of flood veers to northward. Along the eastern shore of Monomoy Peninsula is a region about 2½ hours earlier than that east of Great Point. This is caused by the hydraulic effect along the coast, the downward northerly slope of the water being greatest at about the time of low tide at Provincetown or IX½. One may therefore go westerly from this early region to the late region marked I, northeast of Cape Poge, and find the current hour invariably increasing. Other early regions are located inside of Great Point and of Monomoy Point. These bays are simply filled and drained nearly simultaneously with the rising and falling of the tide in the main body of the sound. The currents (filling) have their greatest velocity 2 or 3 hours before the times of high water. The west-going stream of the sound occurs only 2 or 3 hours after these times of influx into the dependent arms or bays. Consequently, their times of most rapid filling must gradually merge into the times of the swiftest west-going stream in the main body of the sound.

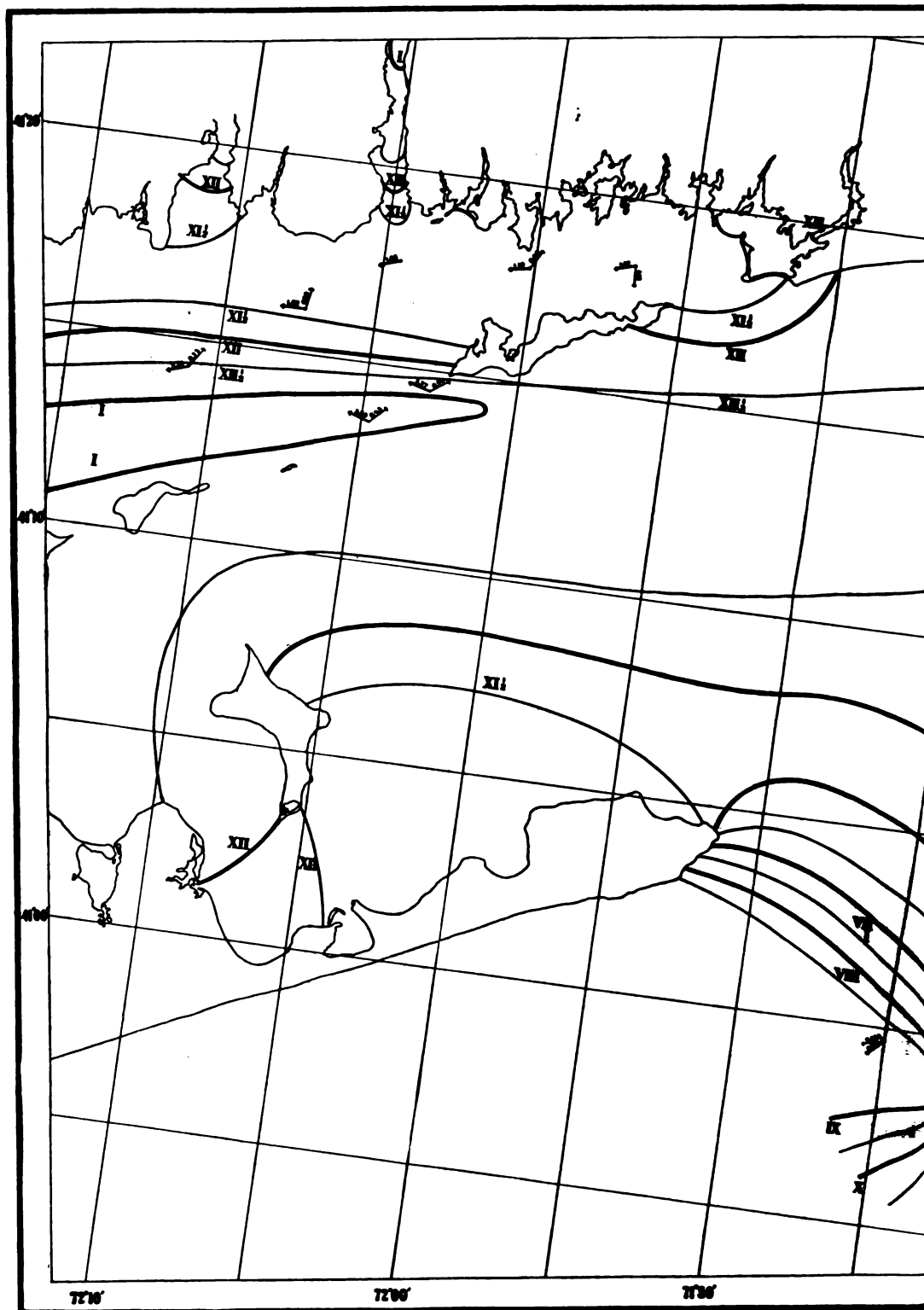
The currents through Muskeget Channel and Edgartown Harbor (when open to the south) are hydraulic, as can be seen upon substituting the ranges and tidal hours off the ends of these channels, as found on Fig. 15, Part IV B, in Table 60. The tidal hours for Muskeget Channel are XII.5 and IV.2, while the ranges are 3 and 2½ feet; therefore the north-going stream should be XI.5. Observation gives XII—. In Quicks Hole and other openings between Buzzards Bay and Vineyard Sound, the currents are hydraulic. Because of these openings and the proximity to the entrance to Vineyard

No. 8.



CKET SHOALS AND VICINITY.
 XII. 23, V. 54; Sandy Hook, XII. 26, VI. 33.

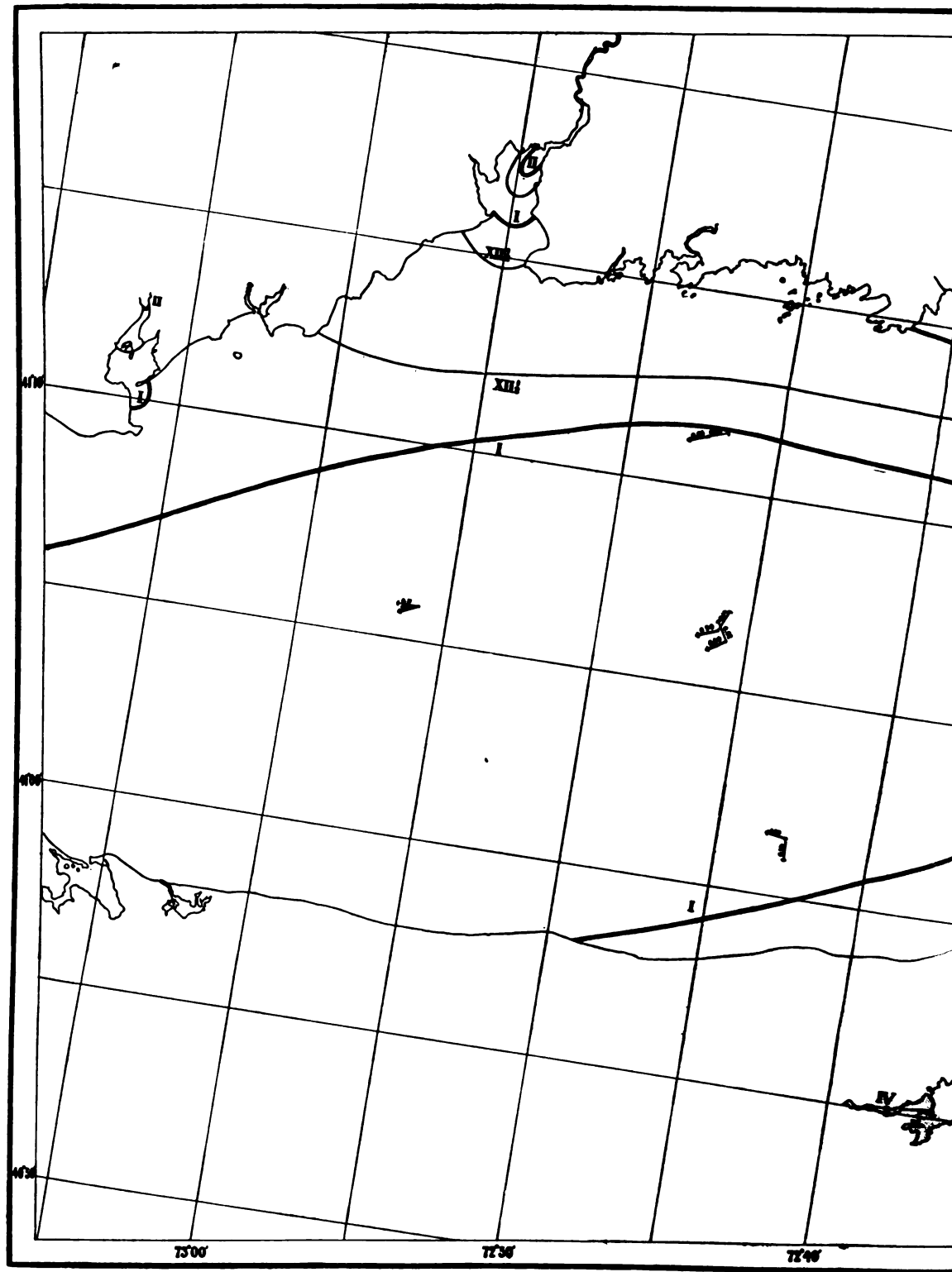
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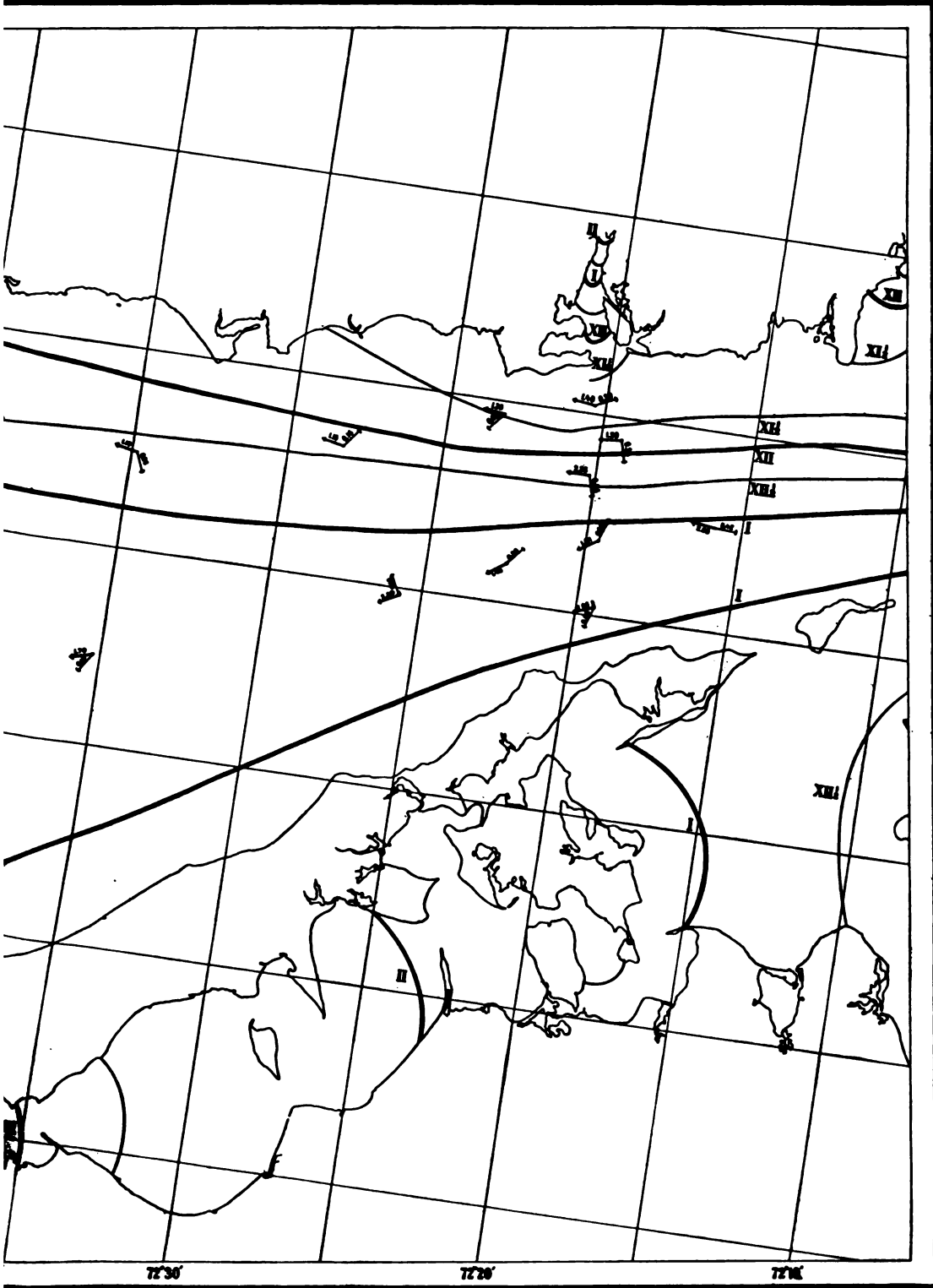
COCURRENT LINES FOR THE EAST
Tidal hours: New London, I. 92, VIII. 19: Falkne

land, III. 38, IX. 31; Sandy Hook, XII. 26, VI. 33.

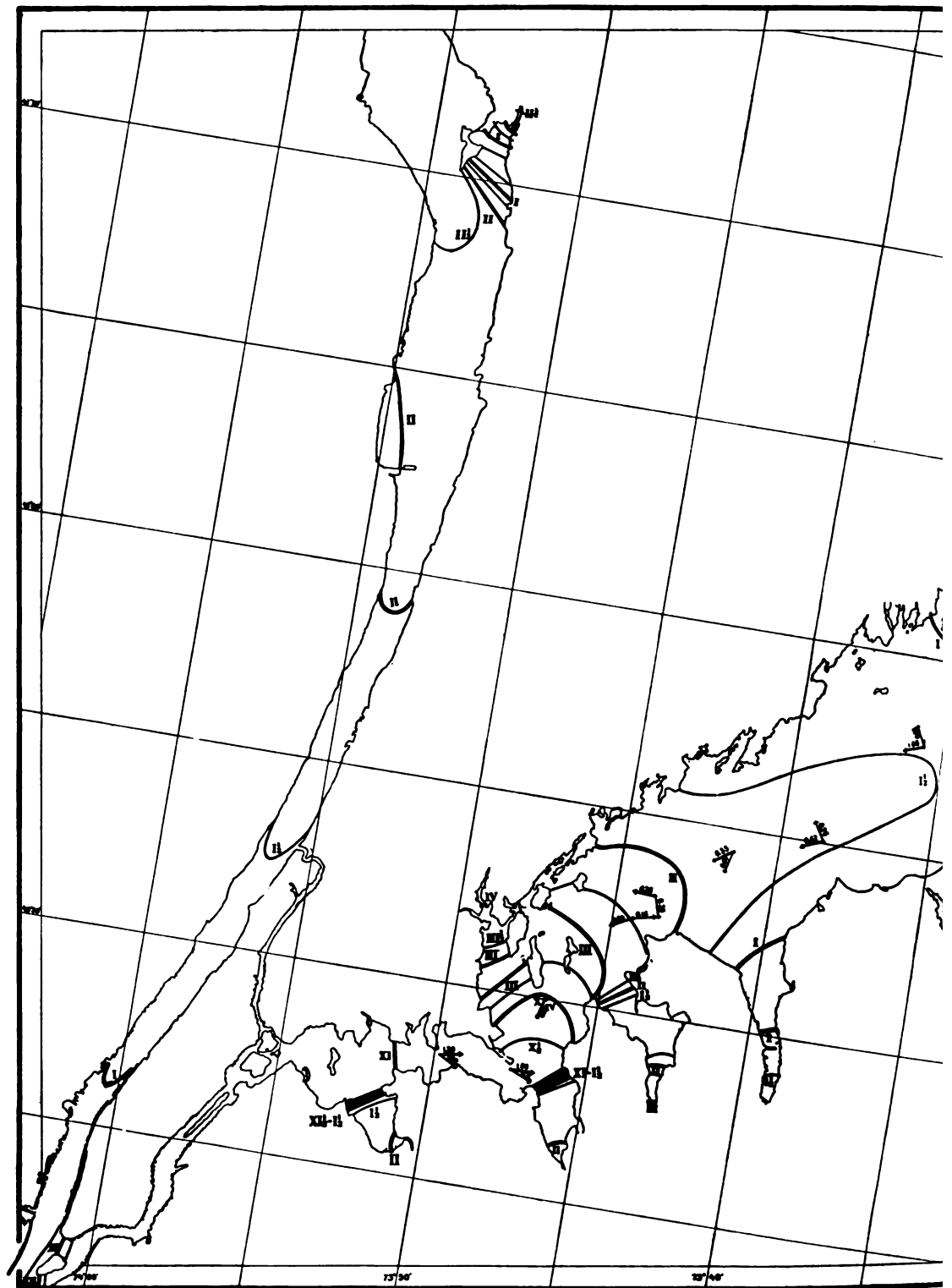




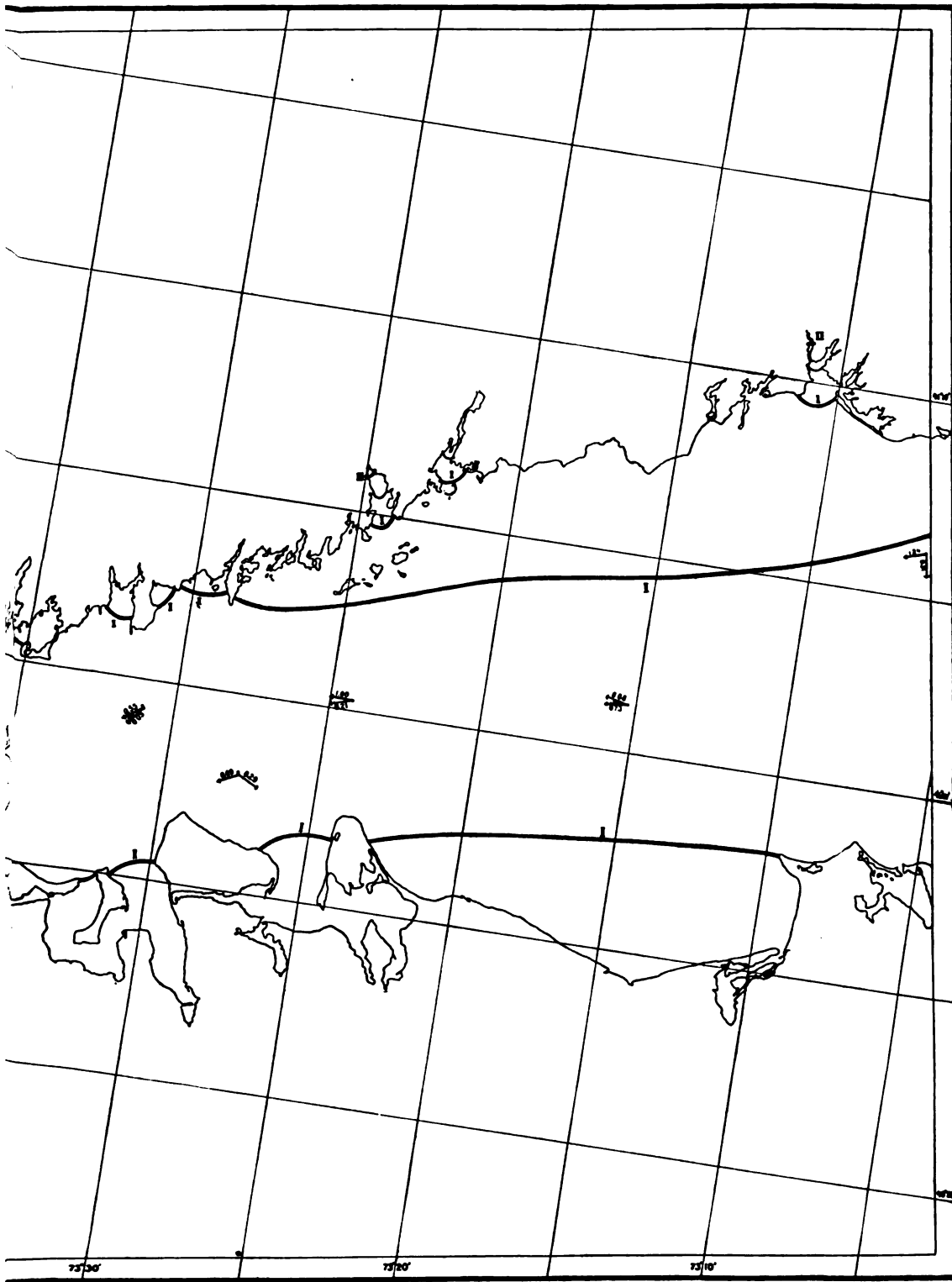
COCURRENT LINES FOR THE CENTRAL
Tidal hours: New London, I. 92, VII



PORTION OF LONG ISLAND SOUND.
19; Sandy Hook, XII. 26, VI. 88.



COCURRENT LINES FOR THE WESTERN
Tidal hours: Stamford, III. 61, IX. 70; Willets Point



IN PORTION OF LONG ISLAND SOUND.
 at, III. 69, X. 10; Governors Island, XII. 73, VI. 95.

Sound, the current hour increases from the northwestern shore of Buzzards Bay, where it is about $IX\frac{1}{2}$, toward the southeastern shore, consisting of islands, where the hour varies from X to XI.

Narragansett Bay has a nearly stationary tide wave, and has few belating dependencies or connections; hence the time of current must be about 3 hours before the time of high water, and this observation shows it to be the case; the tidal hour for the bay varies from XII to $XII\frac{1}{2}$ and the current hour from IX to $IX\frac{1}{2}$.

78. *Long Island Sound.*

The tide of Long Island Sound consists chiefly of a stationary wave approximately $\frac{1}{4}\lambda$ long. The tidal hour for this wave as inferred from the time of tide near the loop is IV. Observation shows that the current hour for the sound is I or a trifle over. The transition of the stationary Atlantic wave to the dependent wave of the sound is apparent many miles southward of Montauk Point, because here the ocean is comparatively shallow and because the time difference between the ocean and sound is considerable. The late current entering the sound joins, near the nodal line, two early stationary waves or currents, one being that of the ocean to the south, the other, that of Narragansett Bay to the northeast. Hence the two circular points near the entrance to the sound. Near the southern point the rotation of the current is clockwise, and near the northern point counterclockwise.

It may be noted that even in the region south and southeasterly from Montauk Point, where the current is as late as XI o'clock, there is a westerly component at about IX; this part of the current belongs to the Atlantic oscillation.

A cocurrent line marked IX extends from southern Nova Scotia southeasterly probably to a point eastward from Porto Rico. Observations made east of Désiderade Island show a northwesterly current at VIII.29. This agrees with the horizontal motion belonging to the stationary wave which causes the tides along the Atlantic coast of the United States. A line marked $IX\frac{1}{2}$ extends from Nova Scotia, outside of Georges Bank, to the circular point off Virginia, and is shown in Figs. 7 and 17. The character and extent of the cocurrent lines between the lines just referred to and the land, can be ascertained by means of the accompanying maps. The belating effect of Long Island Sound is very apparent, while the similar effect for New York Harbor is not great, because the time of the tide in the Lower Bay is not much later than that of the ocean.

At the eastern end of Long Island Sound the current is considerable, because a north-and-south line $\frac{1}{4}\lambda$ from the head of the sound would fall not far to the east of Montauk Point. Moreover, Plum, Great Gull, and Fishers Islands partially obstruct the passageway, thus necessitating increased velocities for maintaining a given rise and fall at the head of the sound. In the Race the velocity is 3 knots. The current hour for the greater part of the sound lies between I and $I\frac{1}{2}$. Near the northern shore the current hour is less than I. In Fishers Sound it is $XI\frac{1}{2}$. This acceleration is due to the direct action of gravity upon the shore waters when possessing an eastward or westward slope. The similar effect is not equally great near the south shore of the sound, because the regularity of the shore line there permits the oscillation to extend very near to the land; but in the bays on that side the current is early, owing to the fact that they must be filling most rapidly 3 hours before high water.

In entering tidal rivers where the tide wave soon becomes nearly progressive the current hour must change rapidly. (See Figs. 9-11.)

79. *East River.*

The tidal hour at and just east of Throgs Neck is III.8; in New York Upper Bay the tidal hour is XII.7. The mean range in the former locality is 7.2 feet, in the latter 4.4. By means of Table 60 it follows that the maximum eastward current should occur at X.8. Observation shows that from Throgs Neck to the eastern side of Governors Island the time of current changes by only 1 hour, viz., from $X\frac{1}{2}$ to $XI\frac{1}{2}$. Just east of the line marked X and IV, the former for the east-going stream, the latter for the one going west, the stream changes in name from flood to ebb, or vice versa, according as in passing beyond this line the maximum stream follows high water or low water instead of preceding it, as a true flood or ebb is supposed to do. Off either shore of the northerly end of Blackwells Island the velocity of flood or ebb is 4 knots. Eddies occur in Pot Cove, Astoria Cove, and Wallabout Bay.

The times of current in the sharp bays bordering the East River are governed by the times of their high and low waters, the current preceding the tide by 3 hours.

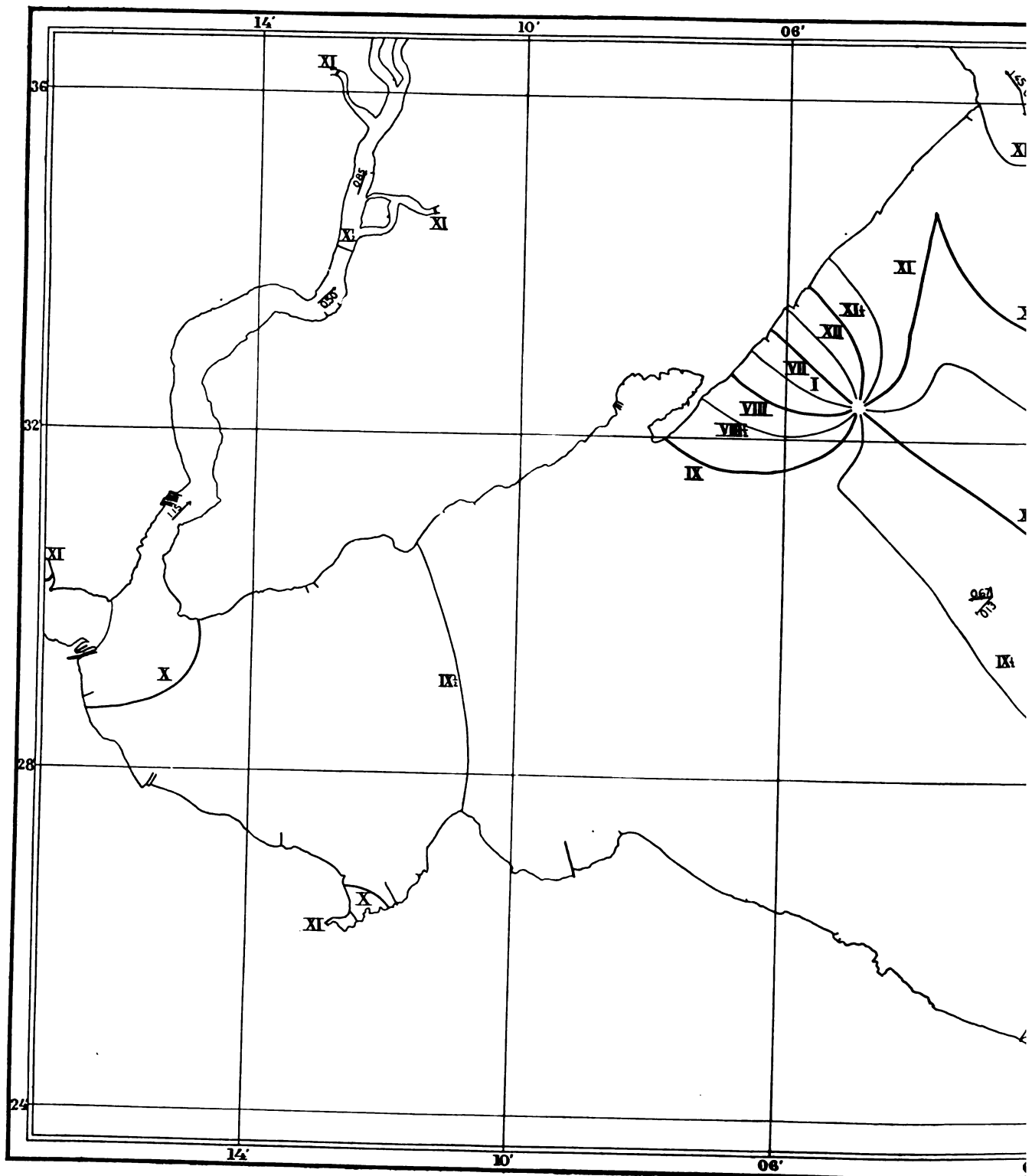
80. *New York Harbor and Hudson River.*

Both coasts of Sandy Hook have early currents (even earlier than IX), due to the direct effect of gravity tending to cause the greatest motion when the slope of the surface of water along the coasts is greatest. There is probably a circular point very near the New Jersey coast north of Barnegat Inlet, at which the IX and $IX\frac{1}{2}$ lines lying off the coast terminate. The X line from the circular point off Virginia probably also terminates here.

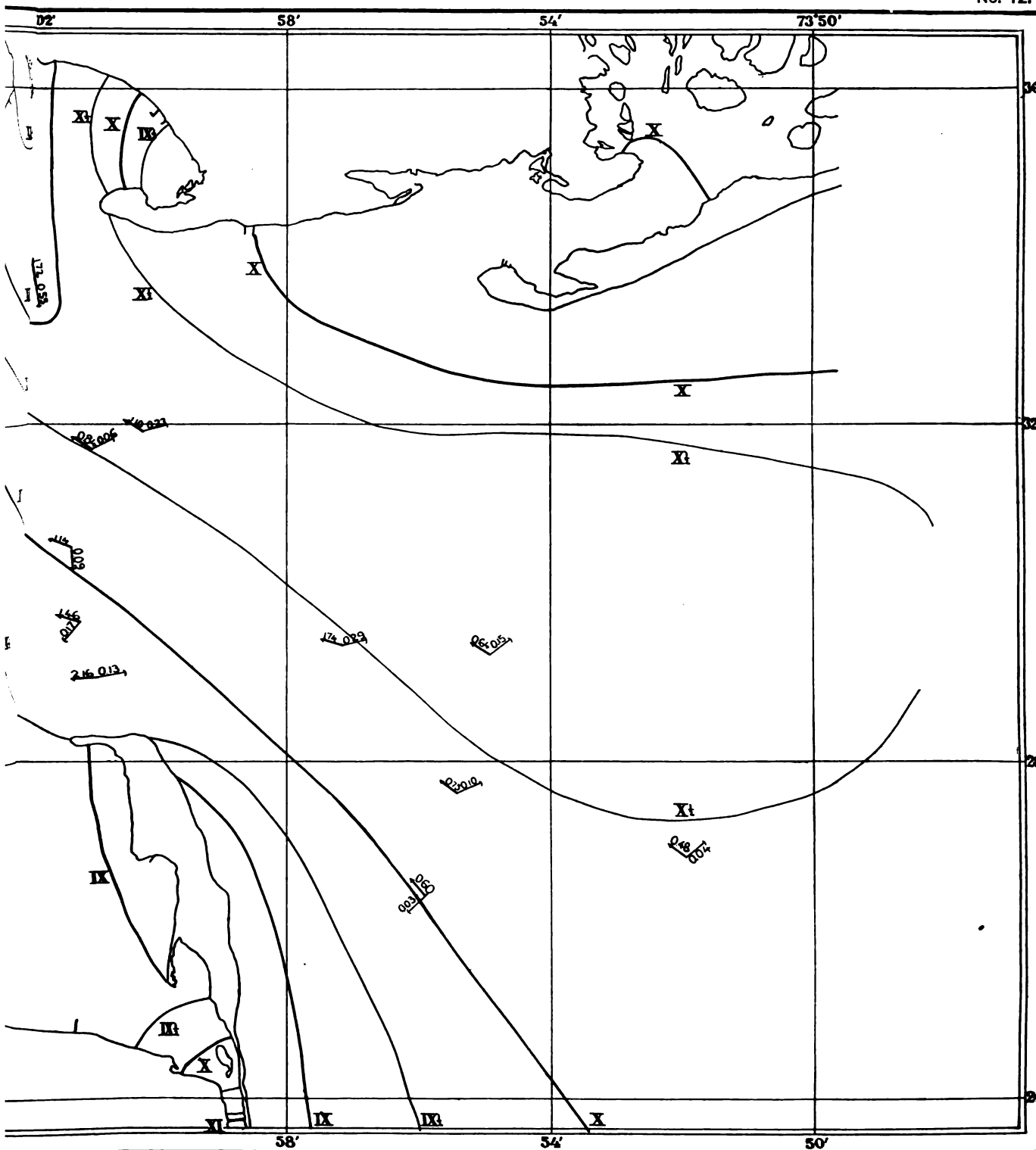
In passing through Fire Island Inlet the current hour changes from less than IX outside to more than X at the inner end of this strait. But, as can be seen from Fig. 12, the current hour changes little in going through Rockaway Inlet into Jamaica Bay. Because of rapid change in time in going through Fire Island Inlet and the opposite directions of the flood current without and within the inlet, the complete representation of the current at the mouth of the inlet requires a circular point around which the numbering of the lines is counterclockwise, and so the rotation of the near-by currents must be clockwise. At the entrance to Rockaway Inlet the current simply splits, there being a wedge of dead water between the two branches.

No recent observations have been made close to the point of Sandy Hook; but Mitchell found there an almost continuous outward tidal current caused by the fact that an eddy was formed just within the hook on the flood stream, but none was formed there on the ebb. It seems probable that a similar condition exists now notwithstanding the variations which the hook has since undergone.

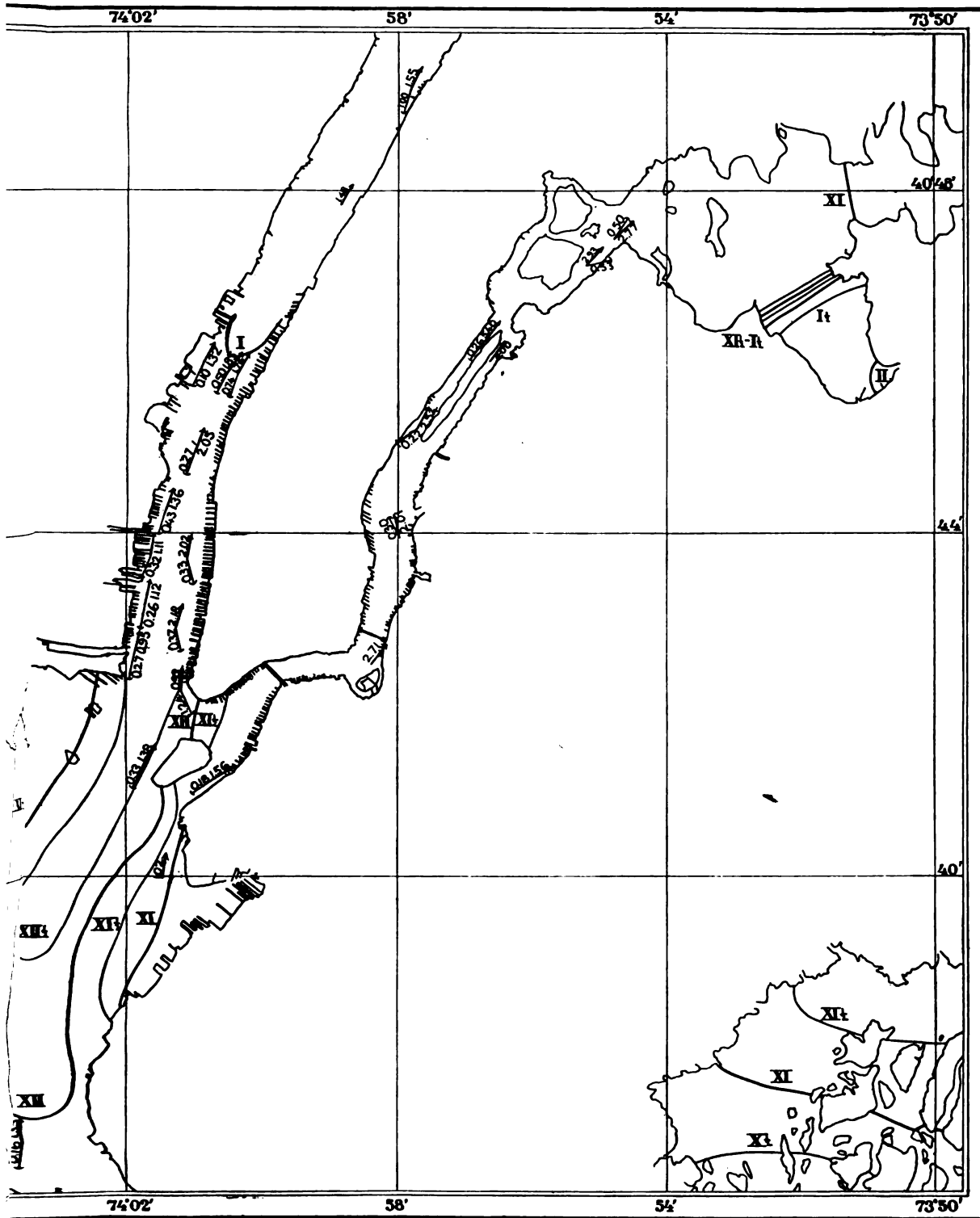
The tide in Raritan Bay consists chiefly of a stationary wave increasing in range to the westward. Hence the greatest downward and northeastward slope along the Staten Island shore occurs not far from the time of high water. The direct effect of gravity is to produce a northeasterly going stream at about this time, or a little later. The stationary character of the Raritan Bay tide causes maximum flood to occur at about $IX\frac{1}{2}$ while the progressive character of the Upper Bay and Hudson River tide causes current hour off Coney Island to be XI. Hence, the times of the northeasterly going stream must vary from XI to $III\frac{1}{2}$ along the Staten Island shore, while the onshore stream occurs at X; hence, the circular point.



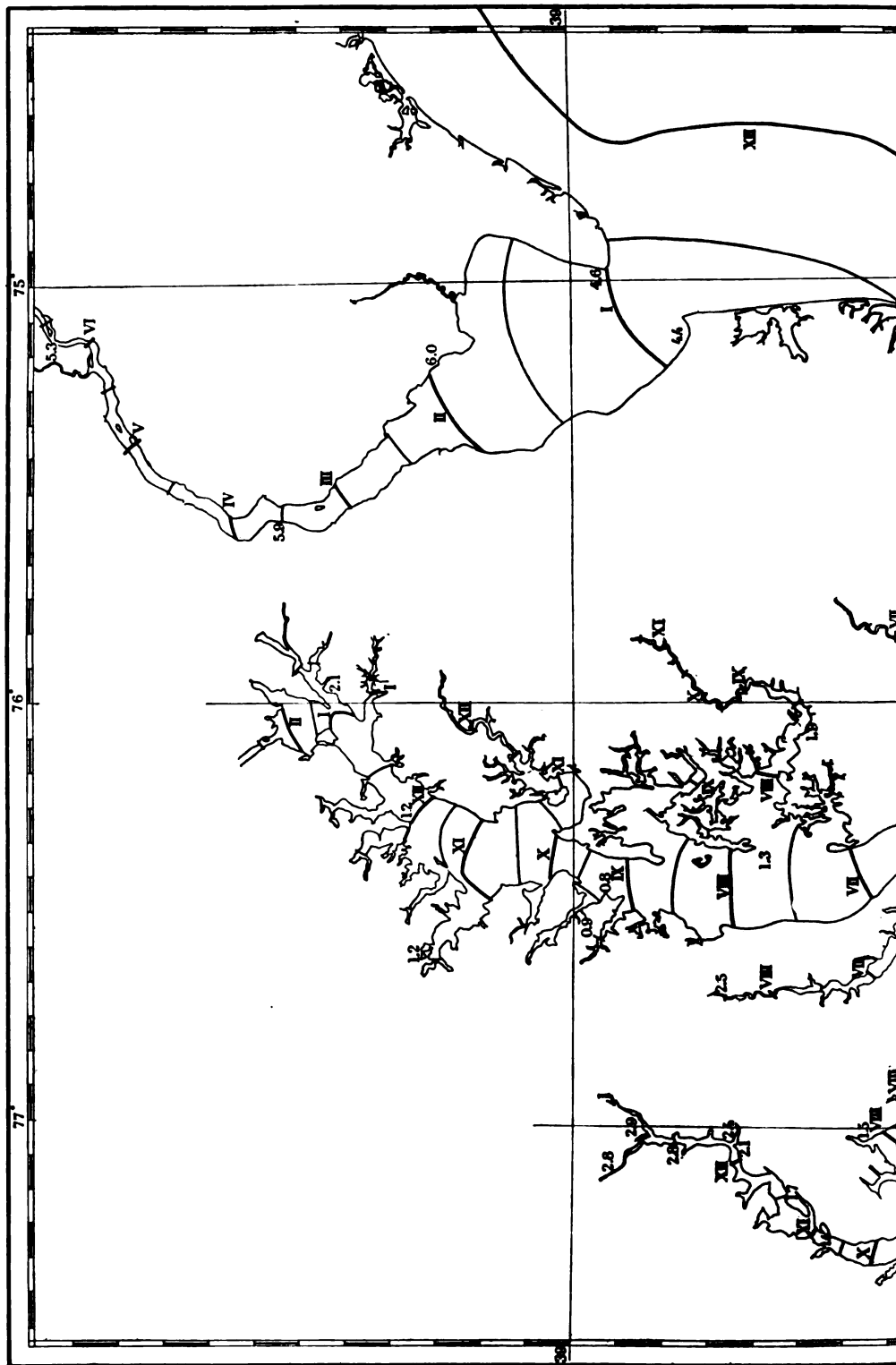
COCURRENT LINES FOR THE EN
Tidal hours: Sandy Hook, XII. 26, V.



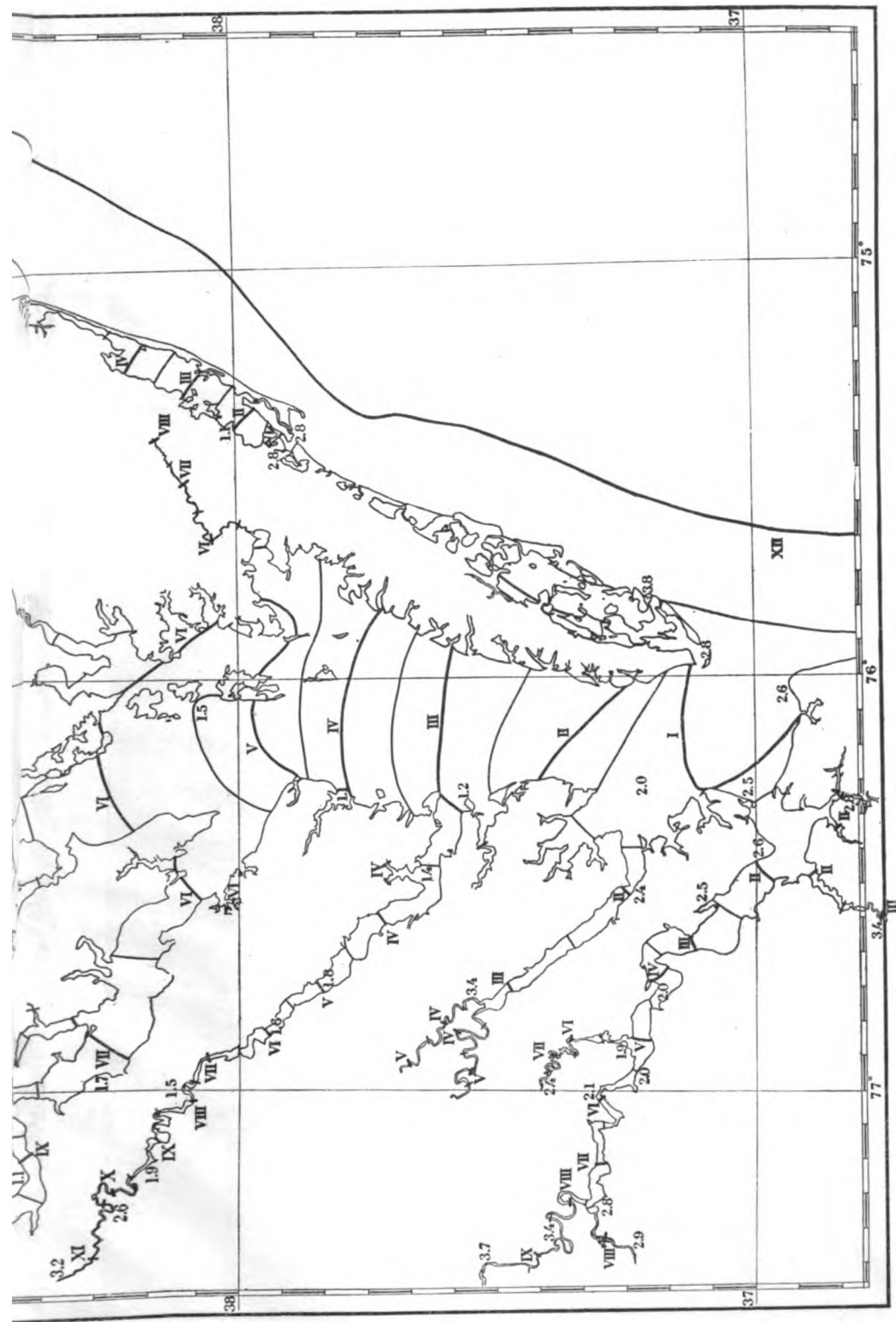
ANCE TO NEW YORK HARBOR.
 ; Governors Island, XII. 73, VI. 95.



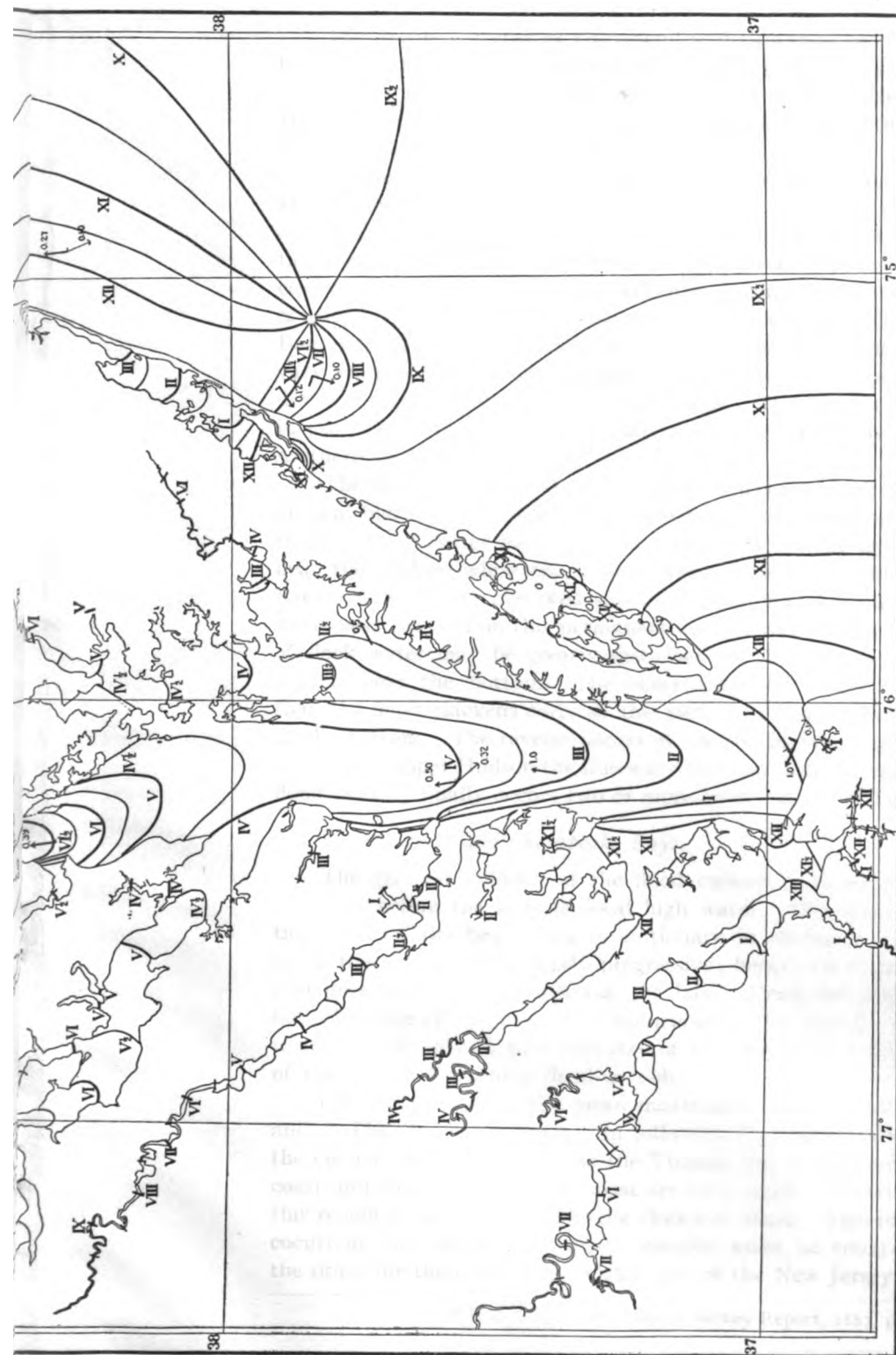
ORK UPPER BAY AND VICINITY.
Island, XII. 73, VI. 95; Willets Point, III. 69, X. 10.



COTIDAL LINES FOR DELAWARE



RE AND CHESAPEAKE BAYS.



ARE AND CHESAPEAKE BAYS.

VII. 26; Baltimore, XI. 45, V. 85; Washington, XII. 69, VII. 08.

The feeble currents in Gravesend and Gowanus bays are earlier by from 1 hour to 2 hours than are the currents in the middle of the channel.

Just south of Battery Park is a wedge of nearly tideless water, lying between the Hudson and East rivers. Because of this fact, the permanent flow of the Hudson generally dominates the current.

In the Kill van Kull the current is $2\frac{1}{2}$ hours earlier than in the channel of the Hudson opposite the mouth of this stream. The fact is of some importance with reference to any sewerage which may be discharged into the Kill van Kull on the ebb tide; for, it will, in the first place, be carried southward a considerable distance. The currents through both kills are hydraulic as can be seen upon noting times and ranges of tide in Newark Bay and the waters outside of it. The circular point in Newark Bay indicates that, following one stream, the times (current hours) of this stream vary much within a short distance. In other words, the streams coming from opposite directions are approximately simultaneous. The currents of Newark Bay, following in the general direction of the eastern and western shores has $X\frac{1}{2}$ as its hour in the southern part of the bay.

The flood current in the lower Hudson has its greatest velocity at about the time of local high water, indicating the progressive character of the tide wave. Where the fresh and salt water meet there is a tendency for the light fresh water to lie at and near the surface while the more dense salt water occupies the lower layers of the stream. Hence the observed surface current may have a strong permanent current, while near the bottom this permanent current will be much weaker. Hence the times of slack water may be greatly disturbed at the surface and remain comparatively regular near the bottom. The observations of Mitchell and Marindin have shown that the flood slackens early at the surface in comparison with its time of slackening at the bottom. The reverse occurs at the slackening of the ebb stream.*

In the upper Hudson the tide wave becomes partially stationary so that the greatest flood velocity finally occurs two or more hours before the time of local high water.

81. *Delaware and Chesapeake Bays.*

The greatest velocity of the flood current in lower Delaware Bay occurs about $2\frac{1}{2}$ hours before the time of local high water. This gives further evidence that the tide wave of the bay is nearly stationary in character. In the river, on the other hand, the tide must be nearly progressive; hence, the crowding together of the cocurrent lines in the upper portion of the bay. Along the sides of the lower part of the bay, the time of the current is influenced by the direct effect of gravity upon these waters which only in part partakes of the oscillatory motion. Hence, its early time of turning or of running flood or ebb.

Off Cape Henlopen the mean maximum velocity is 1.8 knots, at New Castle 2.6, and at Philadelphia it is 1.4. In following the coast southward from Cape Henlopen the current becomes later. As the Virginia line is approached the velocity along the coast and that normal to the coast are very small. Observation shows the current in this region to be rotary and in the clockwise sense. This implies that the order of the cocurrent lines about the near-by circular point be counterclockwise, or opposite to the order for the circular point just east of the New Jersey coast.

* U. S. Coast and Geodetic Survey Report, 1887, pp. 301-312

The IX½ line setting out from the circular point of the Virginia coast passes near the outside of Georges Bank thence to southern Nova Scotia.

On account of the progressive character of the tide in Chesapeake Bay, the times of the currents are delayed for some considerable distance outside of the capes. Between the capes the current-hour is XII½, while in the water beyond its influence the hour is probably a little under IX½. Hence the crowding up of the cocurrent lines in approaching the entrance to the bay. The bay being a propagative body, the maximum flood velocity occurs at nearly the time of local high water. Many of the dependent arms of water intersecting the general shore line must have tides of a partially stationary character. Hence, there occur many localities such that the current hour increases toward the bay, and generally inland as well. These may be styled "early regions." They are made apparent through the convention of always writing the numbers on the later sides of the lines. The following are examples: Pocomoke Sound; Fishing Bay; Honca River; Choptank River, near entrance; Eastern Bay; Chester River, near entrance; Sassafras River, near entrance; Gunpowder River; Patapsco River; Magothy River, near entrance; Severn River; South River; Herring Bay; Patuxent River, near entrance; St. Mary River; Great Wicomico River, near entrance; Rappahannock River, near entrance; Piankatank River, near entrance; Mobjack Bay; entrance to York River; Hampton Roads; Elizabeth River.

The intervals and ranges indicate that near the head of the bay the tide wave is in part stationary. The early currents at this end of the bay give further confirmation of this hypothesis. At Baltimore harbor the tide wave is entirely stationary; for, current observations made by F. A. Kummell near Fort McHenry show that the north-westerly velocity is there greatest 3 hours before the time of high water.

In the acute angle between the axes of the Patapsco River and of upper Chesapeake Bay the currents are rotary; but this fact has not been represented upon the map because of the small extent of the locality in question.

The current near the heads of the James, Rappahannock, and Potomac Rivers is made early by the sudden changes in cross section and elevation of the river beds. The same is true of the lower Susquehanna, and of the Delaware at Philadelphia.

The velocity of the current between the Capes is about 1 knot. Off the mouth of the Rappahannock the velocity is 0.50 knot; off the Patuxent, 0.39 knot; off Herring Bay, 0.32 knot; off Sandy Point, 0.66 knot.

82. *Florida Strait.*

As already noted, the current in Florida Strait of the semidaily tide indicates that the tide wave is here partly stationary. The diurnal tide should here be a stationary wave, and with greater reason, because the length of the strait is a smaller fraction of λ in the latter case than in the former. Consider the diurnal tide at Fernandina and Key West, Fla.

From section 97, Part IV A, we have at the two places

$$K_1 = 0.34, K_1^0 = 120^\circ; K_1 = 0.27, K_1^0 = 274.$$

The O_1 's at the two places compare in about the same manner and will, for convenience, not be considered. The longitude of Fernandina is 5.43 hours and of Key West 5.45. The K_1 tidal hours at the two places are therefore XIII.43 and XXIII.72, respectively.

These differ by 10.29 hours. Hence, there should be almost a nodal line for the K_1 -tide in Florida Strait. An harmonic analysis of 191½ days at Cape Florida, beginning February 15, 1857, gives the following constants, not corrected for imperfect elimination:

$$\begin{aligned} K_1 &= 0.0857 \text{ foot, } K_1^\circ = 189^\circ.09, & K_2 &= 0.0408 \text{ foot, } K_2^\circ = 286^\circ.89. \\ M_2 &= 0.7654 \text{ foot, } M_2^\circ = 244^\circ.36, & O_1 &= 0.0714 \text{ foot, } O_1^\circ = 205^\circ.91, \\ S_2 &= 0.1337 \text{ foot, } S_2^\circ = 279^\circ.47. \end{aligned}$$

It is thus seen that at this point the diurnal tide is small in comparison with the semidiurnal. This fact was anticipated before the analysis was undertaken.

To infer the K_1 -current, take the tidal hours and amplitude ratio for Fernandina and Key West, and make use of Table 60. The time of greatest southward, downward slope is thus found to be XII.7; consequently the tidal hour of the maximum south-going stream is XII.7+6=XVIII.7. From the reduction for the observations off Fowey Rocks, longitude 5.33 K_1° (north) is $14^\circ = 0.93$ hour. Hence, the observed current-hour for the south-going K_1 current is 5.33+0.93+12=XVIII.26, which agrees well with inference.

The diurnal current flowing into the Gulf of Mexico through the Yucatan Channel should have its greatest velocity 6 hours before diurnal high water over the Gulf, or at II-6=XX. (See Nos. 113-148, Section 97, Part IV A.) Observation gives 248°.6 for the epoch of the K_1 north-going current at a point whose longitude is 5.75 hours. Hence, the current hour is XXII.32.

83. *Papers in Coast Survey Reports relating to tidal currents of the Atlantic Coast of the United States:*

C. A. Schott: On the currents of Nantucket Shoals, Report 1854, pp. 161-166; Currents in Muskeget Channel and off the northeast coast of Martha's Vineyard, Report 1854, pp. 166-168; Tidal currents of Long Island Sound and approaches, Report 1854, pp. 168-179.

H. Mitchell: Tides and tidal currents of New York Harbor and its dependencies, Report 1856, pp. 264-266; Tides and currents in Nantucket and Martha's Vineyard sounds, and in East River at Hell Gate with remarks on the revision of levelings on the Hudson River, Report 1857, pp. 350-354.

A. D. Bache: Tidal currents of New York Harbor near Sandy Hook, Report 1858, pp. 197-203.

H. Mitchell: Currents in the East River at Hell Gate and Throg's Neck, the sub-currents of New York Bay and Harbor and levelings on the banks of the Hudson River, Report 1858, pp. 204-207; Tides and currents of Hell Gate, N. Y., Report 1867, pp. 158-169; harbor of New York, 1873, Report 1871, pp. 109-133; Middle-ground Shoal, New York Harbor, Report 1872, pp. 257-261; Circulation of the sea through New York harbor, Report 1886, pp. 409-432; On the movements of the sands at the eastern entrance to Vineyard Sound, Report 1887, pp. 159-163; Report on the results of the physical surveys of New York Harbor, Report 1887, pp. 301-311.

H. L. Marindin: Tide levels and flow of currents in New York Bay and Harbor, Report 1888, pp. 405-408; Tides and currents in the harbor of Edgartown and in Katama Bay, Martha's Vineyard, Report 1892, pp. 225-241.

84. *Coasts of South America.*

Like Africa, South America is exposed to the direct action of the deep ocean tide; and so observations made upon tides and currents have an important bearing upon the general problem of the tides.

The quotations given below are taken from the Admiralty Pilots for South America, Part I (1893), Part II (1895), and for the West Indies, Vol. I (1893).

They indicate the ending of one stationary wave against the southeastern coast of Brazil, and another against the northern coast. They also indicate a loop of a stationary wave in the Gulf of Panama.

They show how in dependent stationary arms of the sea, like those along the eastern coast of Patagonia, the velocity of the tidal streams may be small while the range of tide is exceptionally great.

There is good evidence from the tides, and possibly some evidence from the currents, of the ending of a stationary wave along the Chilean coast. Both currents and tides indicate the southerly and southeasterly direction of the flood as Cape Horn is approached.

Both currents and tides indicate that the general direction of the flood along the eastern coast of South America from Staten Island to Rio de la Plata is northerly. One remark concerning the tides off the mouth of the Rio Negro indicates that their establishments change rapidly in going a comparatively short distance. (Cf. Fig. 29, Part IV B.)

85. *Southeast coast of South America, quotations.*

It is high water, full and change, at cape St. Roque at 4h. 14m.; springs rise from 8 to 10 feet. In the St. Roque channel the flood sets to the south, and the ebb to the north, at about one mile an hour.

The establishment of the whole eastern shore of Brazil varies but little as the coast lies nearly in a straight line, and parallel to the tidal wave which traverses the Atlantic ocean from E. S. E. to W. N. W.

It is high water, full and change, at Bahia, at 4h. 26m., and the spring rise is 8 feet. The flood runs 5 hours to the north-ward, and the ebb 7 hours to the south-ward. The velocity of the tide is about $1\frac{1}{2}$ miles an hour, increasing to $2\frac{1}{2}$ and 3 miles during springs.

It is high water, full and change, at Caravellas, at 4h. 15m., spring rise about 10 feet. The tidal stream varies from 2 to 3 knots, the flood sets to the south, and the ebb to the north, outside the bar; but this direction varies very much with the locality, and force and direction of the wind.

It is high water, full and change, at Rio de Janeiro at 3h.; springs rise 4 feet and neaps 3 feet. The usual rate of the tide is about three-quarters of a mile an hour, springs run $1\frac{1}{2}$ miles.

It is high water, full and change, off Estrella bay at 0h. 30m.; springs rise 5 feet, and neaps 4 feet and at Parati 1h. 43m., springs rise $5\frac{1}{2}$ feet. There is little or no stream.

Santos Harbor: It is high water, full and change, at 2h. 50m., springs rise 5 feet. The tides are strong, particularly the ebb.

It is high water, full and change, at São Francisco at 2h. 30m. a. m.; springs rise 7 feet, and neaps 5 feet. At springs, the stream from the river runs from 3 to 4 miles an hour, and is only overcome by the strength of the flood, soon after resuming its course; this is called half tides (*meias marés*).

It is high water, full and change, at Anhatomirim islet at about 2h. 45m.; springs rise 6 feet and neaps $4\frac{1}{2}$ feet. The tides are tolerably regular in Santa Catherina channel; they enter from the north-ward and southward at the same time, and meet off the town, where they also separate. The strength seldom exceeds a third of a mile an hour, but near springs it sometimes runs $1\frac{1}{2}$ miles.

There are no appreciable tides in Maldonado bay.

Monte Video: It is high water, full and change, at 2h. 30m. (approx.); astronomical tides range about 18 inches.

Rio de la Plata: In the vicinity of the Cuirassier and Chico light vessels in ordinary weather, the average rise and fall is 4 feet, the ebb setting to the southeast at the rate of from one-half to 3 miles an hour, and the flood to the north-west at from one-half to $1\frac{1}{2}$ miles an hour.

It is high water, full and change, off Andres head at 10h., rise 8 feet. The flood sets to the northward and the ebb to the southward.

It is high water, full and change, in El Rincon about 5h. The tidal streams set strongly, the flood to the north, the ebb to the south, nearly 6 hours each way; off Asuncion point the flood sets to the eastward.

It is high water, full and change, in Union bay at 3h. 10m.; springs rise 12 feet, neaps 9 feet. The flood-tide at the entrance sets to the northward across the banks about 2 miles an hour.

Rubia Head: The tides run along this coast with dangerous strength, from 2 to 4 miles an hour.

Rio Negro: It is high water, full and change, on the bar, during settled weather, at 11h.; springs rise 14 feet, neaps 10 feet. In the offing, it is 3 hours later. The tidal stream runs parallel to the coast from 2 to 4 miles an hour.

The tidal wave comes up the coast from the southward, and rushes round Valdes peninsula with much strength, causing violent and dangerous overfalls off Valdes creek and Norte point. Part of the body of water thus going northward, separates, and runs round Norte point; thence to the port of San Josef the tide sets strongly, with riplings and races, dangerous for boats, or very small vessels. The main body continues its progress to the northward, inclining to the west, until near Belen bluff, when it divides; one stream running to the north-west, the other to the eastward. Eastward of Belen bluff, the ebb sets faintly to the south or south-eastward; westward of the bluff it sets to the south-eastward.

West of the meridian of Norte point, the south point of entrance to the gulf of San Matias, and northward of latitude $41^{\circ} 50'$ S., but little stream of tide is felt; though the water rises 24 feet. With a weather tide there is a very cross short sea in the entrance of the gulf.

Between Villarino point and the Reparo bank the tide runs from 3 to 5 miles an hour.

It is high water, full and change, within port San Josef at 10h. 0m. The tide rises from 20 to 30 feet, and the stream rushes between the heads from 3 to 5 miles an hour.

It is high water, full and change, at port Melo at 3h. 40m., springs rise 15 feet. The tides off this part of the coast are strong, running along the land at the rate of 2 or 3 miles an hour. Off the projecting points, and in confined passages, their strength is of course increased, and causes heavy riplings when opposed to the wind.

At full and change, the flood or northerly tide ceases in the offing about 4h. 15m., but near cape Blanco and among the shoals, the tides may be less regular; they produce strong riplings and set from 3 to 4 miles an hour round cape Three Points.

It is high water, full and change, at port Desire at 0h. 10m., springs rise $18\frac{1}{2}$ feet. The tides set in and out of the port with regularity, and at the rate of 5 knots an hour.

It is high water, full and change, in Sea Bear bay at 0h. 45m., rise 20 feet. The tide off the entrance is very rapid. The flood sets to the N. N. E., and has been observed as much as 3 knots against a strong northerly wind. The ebb sets nearly in the opposite direction and about the same rate. Off Penguin island the northerly stream ceases at about 4 hours after high water by the shore.

It is high water, full and change, in the river Santa Cruz, at 9h. 30m.; springs rise 40 feet, neaps rise 29 feet, with a velocity of from 3 to 6 miles an hour. In the offing the tides flow regularly 6 hours each way, but turn 2 hours later than the time of high water in-shore. The flood runs to the north-east ward and the ebb to the south-westward.

It is high water, full and change, in the entrance of port Gallegos at 8h. 50m.; springs rise 46 feet, the stream runs at the rate of 5 miles an hour.

General description: Along that dreary and almost unbroken coast, extending from cape Corrientes to Bahia Blanca, the stream of the tide is very weak, although the water rises and falls about 10 feet. The great tidal wave from the southward here appears to end, after sweeping along the southern half of South America. In the archipelago of Tierra del Fuego the flood-tide comes from the N. W., passes round cape Horn, and through the strait of Le Maire, and then, from cape St. John, sets strongly

to the eastward and north-eastward. From thence the flood runs to the north-east, along the north side of Staten island and Tierra del Fuego, occasions very high tides at the entrance of Magellan strait, where it unites with the stream which has come directly through the strait, and passing onward along the coast of Patagonia, produces high water at each place in succession until it is lost near cape Corrientes.

Near the coast between the dangerous banks of San Blas and Bahia Blanca, the flood and ebb streams set nearly north and south, from one to 4 miles an hour, according to the wind and the age of the moon. Between the banks of San Blas and the Rio Negro, the tides are regular, running a little more than 6 hours each way, if not affected by the wind, with a velocity of 2 to 5 miles an hour; these strong and dangerous tides are not much felt at the distance of 15 miles from the land. Between San Blas and cape Bermeja the tidal stream sets N.E. and S.W., about equally strong each way.

In the depth of the gulf of San Matais there is very little stream of tide, but a rise and fall of from 20 to 30 feet.

In the gulf of St. George there is not much stream of tide. Off capes Dos Bahias and Blanco, particularly the latter, the tides are again strong, and there are races off cape Blanco almost as dangerous as those off the peninsula of San Josef.

Off the peninsula of San Josef there are dangerous tidal races; and so high and so violent are the waves at particular times of tide that a small vessel might be most seriously injured if not totally destroyed by getting into them.

East Falkland Island: The flood runs to the north-east, past the Wolf rock, and becomes stronger as it approaches cape Pembroke, round which its rate is from 2 to 3 miles, according to the age of the moon. The flood runs directly to the northward of the Seal rocks to Volunteer point, while very little tide is felt within the heads of port William or Berkeley sound. The ebb runs with equal strength to the southward, and when there is a strong breeze, a heavy tide rip extends 2 miles off shore.

The tide sets to the westward during the flood along the whole south shore of East Falkland; its strength is from one to 2 miles an hour, but near Porpoise point, the south-west horn of the bay of Harbours, it is nearly 3 miles, and with westerly gales forms a strong race. The stream turns when it is high water by the shore.

It is high water, full and change, on the shore at Race point, at the northern entrance of Falkland sound, at 6h. 45m.; the velocity of the tide here is about 4 miles an hour, but in Grantham sound its rate diminishes to about $1\frac{1}{2}$ miles. At the southern entrance of the sound it is high water, full and change, at 7h.

The time of high water, full and change, in the harbors in Falkland sound, is given on the chart.

The tides in both entrances of the sound, and between the islands, run from 3 to 5 knots at springs, but in the wider portions they are moderate. The stream of tide at the north entrance makes into the sound about 3 hours before high water on the shore, or about 4 hours at full and change. Among the islands in the south-eastern part of the sound the tides are very irregular in their set and velocity.

There appears to be tide and half tide all through Falkland sound. The flood stream commences by running to the northward when it is half ebb by the shore, and runs until half flood; it then turns and runs to the southward until it is half ebb again. But the tides among these islands require further investigation; Captain Fitz-Roy states that the tide flows into both ends of Falkland sound, and that the two streams meet near the Swan island.

It is high water, full and change, in Pebble sound at 8h. 45m.; springs rise 8 feet. Running along the north coast of the islands to the westward, part of the flood rushes through Tamar and Whaler passes, and part sweeps round the West Pebble islet into Keppel sound, filling that sound, and port Egmont, 2 hours before it has ceased running to the westward. This latter portion rushes eastward through the North-west pass at the rate of 5 to 8 miles an hour; it sweeps through a part of Pebble sound, meeting the flood-tide that comes in with equal velocity through Tamar pass, and thus causes whirls and eddies in several quarters. The water having attained its height remains quiet only a little while, and then ebbs with similar fury.

Biscoe Islands: From the observations which have been made, it is inferred that the flood and ebb streams in moderate weather run eastward and westward for a distance of about 6 miles from the outer points of the land, taking the sweep of the bays.

86. *Northeast Coast of South America, quotations.*

Paranahyba River: The tides run at the rate of 4 or 5 miles an hour in the passage; outside the bar the ebb sets to the northward.

Within 3 or 4 miles of Santa Anna reefs the tidal influence from Rio Preha is felt, the flood sets to the south-west and the ebb north-east. It is high water, full and change, at 5h. 45m, rise on the reefs 13 feet.

It is high water, full and change, in San Luiz harbour at the custom-house quay, at 7h. om.; springs rise $16\frac{1}{2}$ feet, and neaps $10\frac{1}{2}$ feet. At the anchorage outside the harbour, the flood sets S.S.W. and the ebb N.N.E.

It is high water, full and change, at Manoel Luiz reef, at 5h.; and the rise is 12 feet. The tide runs regularly six hours each way, the flood to the S.W., and the ebb to the N.E., one mile an hour.

From Maranhão to the Para River: The flood tide generally runs S.W. near the coast and W.S.W., or more westerly, at some distance from it; it has a mean rate of $2\frac{1}{2}$ miles an hour near the land, which diminishes as the distance from the coast increases. The ebb tide sets about E.N.E. near the coast at the rate of $1\frac{1}{2}$ miles an hour.

From the Para to Cape North: The flood tide, which runs to the S.S.W. near the mouth of the Amazon, inclines toward the S.W. and W.S.W. in proportion to the distance from the land; and the ebb tide, which sets first N.E., inclines toward the N. and N.W. before it is united with the general current. A difference of 2 or 3 hours in the establishment of two places far from land and only 12 miles apart, and a rise of only $6\frac{1}{2}$ feet 12 miles from a point, where at the preceding tide 29 feet had been observed, are two anomalies in the tides quoted as most remarkable among others less striking, though numerous on this coast.

The Bore or Pororoca is a tidal phenomenon which sometimes occurs in the western branch of the Amazon at about spring tides.

The bore confines itself to the shallows and affluents, and is not felt in depths over 4 fathoms, except by an increase in the velocity of the stream, so that there is no danger to vessels keeping the main or deep channels.

When it makes its appearance, which is at the lowest of the tide, a roaring sound is heard at a distance of from 3 to 6 miles; as it approaches the noise increases, and soon a head of water, estimated to vary from 5 to 12 feet in height with a breaking face, is seen occupying the whole of the shallow water off Maraca island and Araguay river out to about a depth of 4 fathoms. Its velocity is estimated at from 10 to 15 miles an hour, being strongest and most dangerous in the months of January to June, and at the equinoxes, when the wind is north-eastward, and it carries away in its course everything that is opposed to it.

It is high water, full and change, at the entrance of Cayenne river at 4h. 37m. At the equinoxes the rise is 10 feet, at ordinary springs 7 feet, and at neaps from 4 to 5 feet. Near the coast the flood stream, combined with the current, sets north-westward with a velocity of 2 to 4 knots an hour, varying with the seasons, being greatest during the summer months; the ebb stream sets north-eastward with a velocity of one knot an hour.

The flood stream outside the bar of Maroni river sets north-westward; the ebb stream in the river has a tendency to set towards Dutch bank.

Surinam River: In the offing, the flood stream sets to the westward, the ebb to the eastward.

It is high water, full and change, at the entrance to Berbice river at 4h. 30m.; springs rise 8 to 10 feet, and neaps 5 to 6 feet. The flood stream in the river sets about S.W. and the ebb North.

The flood and ebb streams off the mouths of Demerara and Essequibo rivers set S.W. by S. and N.E. by N., respectively, and extend to the distance of about 20 miles from the land, or to a depth of about 10 fathoms, on the inner edge of the permanent north-westerly current. The flood stream at this distance runs with and somewhat increases the strength of the current, whilst the ebb retards it.

Waini or Guayma River: The flood stream sets south-westward across the entrance, and the ebb straight out of the river.

Orinoco River: The tidal streams for a short distance off the land run about 6 hours each way, the flood to the westward.

87. *Western Coast of South America, quotations.*

Le Maire Strait: It is high water, full and change, in Good Success bay, at 4h. 3m.; springs rise 6 to 8 feet; it is slack water in Le Maire strait at or near the time of high and low water in Good Success bay. In Le Maire strait the flood stream makes to the northward about one hour after low water, and the ebb to the southward about the same time after high water, and the strength of the stream is from 2 to 4 knots near cape San Diego and from 1 to 3 knots in mid-channel; more or less, according to the wind.

In Barbara channel the flood-stream was found to set to seaward, or to the southward, as was also the case in Cockburn channel; but the whole system of tides in this great archipelago requires a careful and patient investigation.

To the northward of cape Virgins the streams set north-west and south-east along the coast; the same will be found on the outer edge and outside the Sarmiento bank. The north-west-going stream, which runs from 3 hours before to 3 hours after high water at cape Virgins, appears to sweep up the eastern shore of Tierra del Fuego to the south end of Sarmiento bank, where it divides, one stream running into the strait, while the other continues northward along the outer edge of the bank. In the same way the east-going stream is met coming out of the strait, and turned to the southward, by the stream sweeping down the coast to the south-east, and across the entrance in the same direction. The seaman must not be deceived by this, which makes the west-going stream on the north end of the bank appear to run out of the strait, while toward the south end it varies with the time of tide.

Thus it will be seen that in the vicinity of capes Virgins and Espiritu Santo, it is high water, full and change, between 8h. 30m. and 9h. A. M., while the west-going stream is still running into the strait and to the northward past cape Virgins. The main stream continues running to the westward at full and change until near noon, though the water is falling everywhere. About noon the direction of the stream changes (there being no appreciable slack water in the channel), and until near 3h. P. M. the water continues falling, while the stream of tide is running to the eastward until after 6 o'clock.

Magellan Strait: It is high water, full and change, in the First narrows at 8h. 57m.; and the strength of the stream is from 5 to 8 knots; there is no slack water. The stream changes 3 hours after high and low water.

It is high water, full and change, in Philip bay at 9h. 29m.; springs rise 17 feet. The western-going stream makes three hours before high water by the shore and runs till three hours after.

It is high water, full and change, in Laredo bay at 11h.; springs rise 7 feet. When to the southward of Laredo bay, the tidal streams are scarcely felt; but to the northward they are strong, and must be carefully guarded against during the night, or in light winds.

It is high water, full and change, at port Famine, at noon; springs rise 6 feet, the ebb setting to the northward, and the flood to the southward.

It is high water, full and change, in this part of the strait at 1h. 40m. The rise in port Tamar is 6 feet, and a little less in port Churruca. The flood stream sets to the eastward, and may attain a rate of $1\frac{1}{2}$ knots.

The flood stream sets to the southward, or to seaward in Cockburn channel, but was not found to run with sufficient strength to affect a vessel working through. The rise is 6 or 8 feet at spring tides.

In Barbara channel the flood stream was found to set to seaward, or to the southward, as was also the case in Cockburn channel.

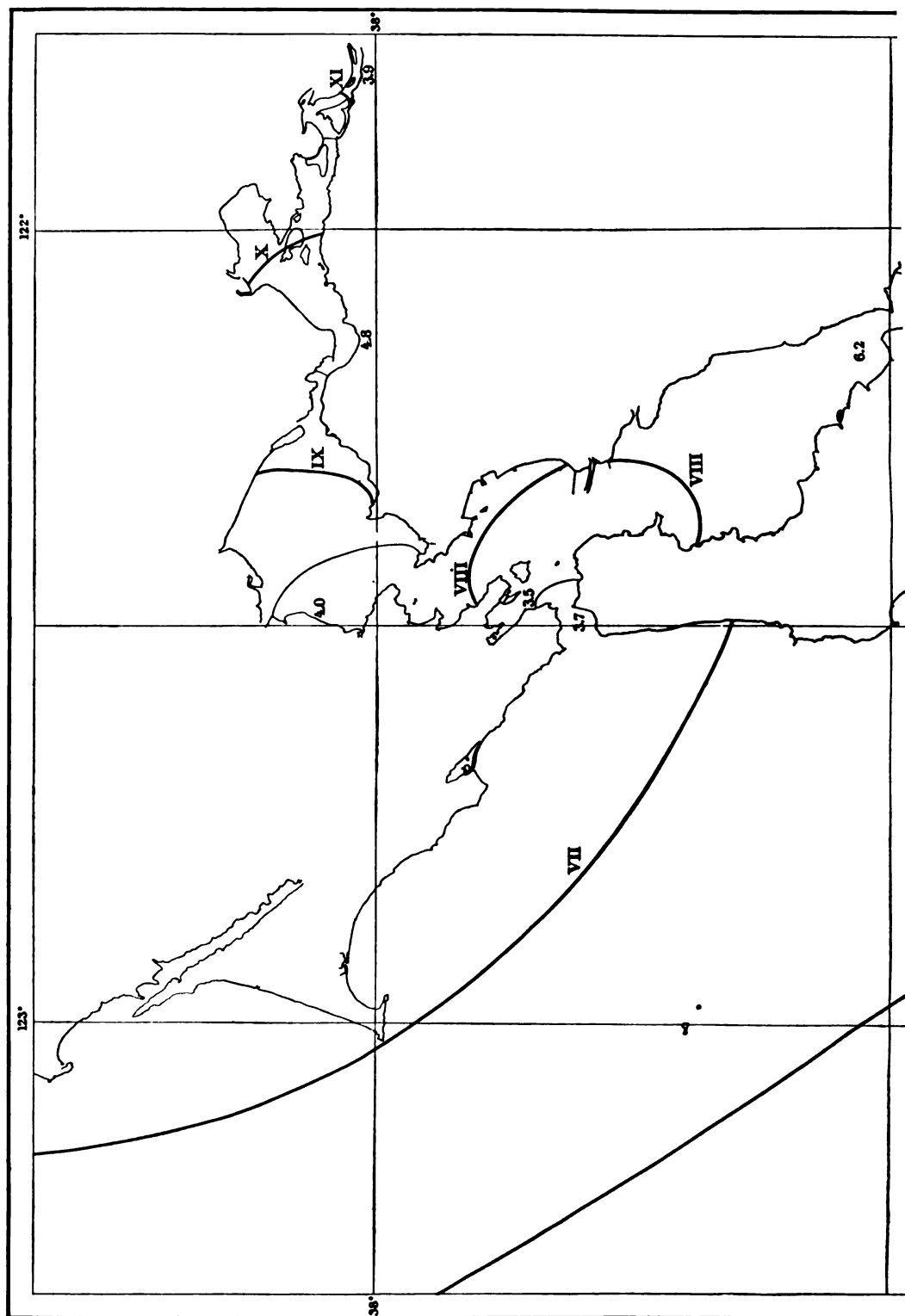
Guia Narrows: It is high water, full and change, at 12h. 0m.; springs rise 8 feet. The flood stream in the Narrows runs to the eastward, ebb to the westward, at the rate of $2\frac{1}{2}$ to $3\frac{1}{2}$ knots an hour at springs.

It is high water, full and change, in Brassey pass about noon. At spring tides the streams run through the pass $1\frac{1}{2}$ knots per hour; flood to the eastward, and ebb to the westward.

It is high water, full and change, in Alert harbor at 12h. 15m.; springs rise 7 feet. No stream is felt.

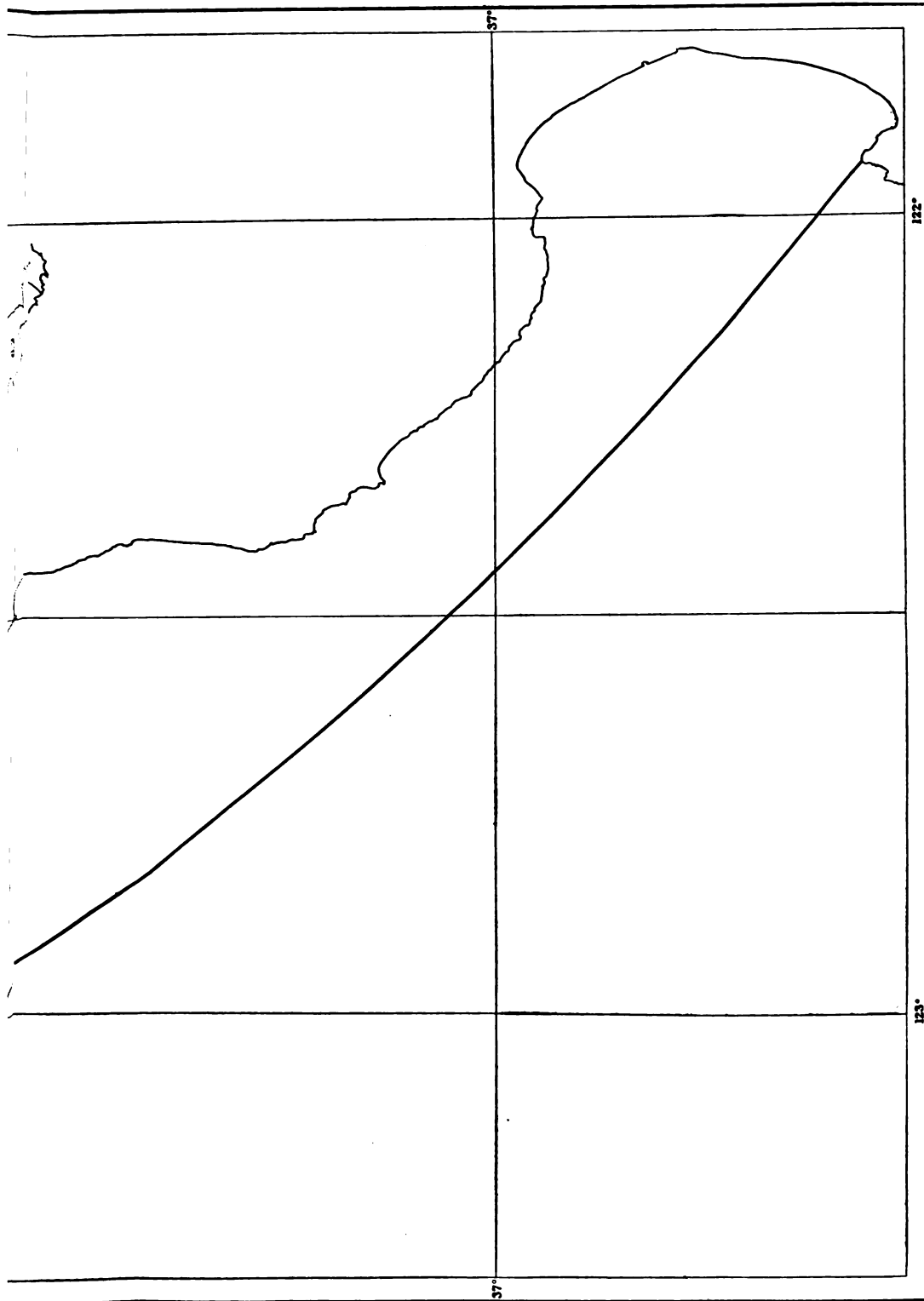
It is high water, full and change, in Trinidad channel about noon; springs rise about 6 feet. The flood-stream runs to the eastward and the ebb to the westward.

It is high water, full and change, in Eden harbour, at 0h. 15m.; springs rise 6 feet. The flood sets S.S.E., the ebb N.N.W.

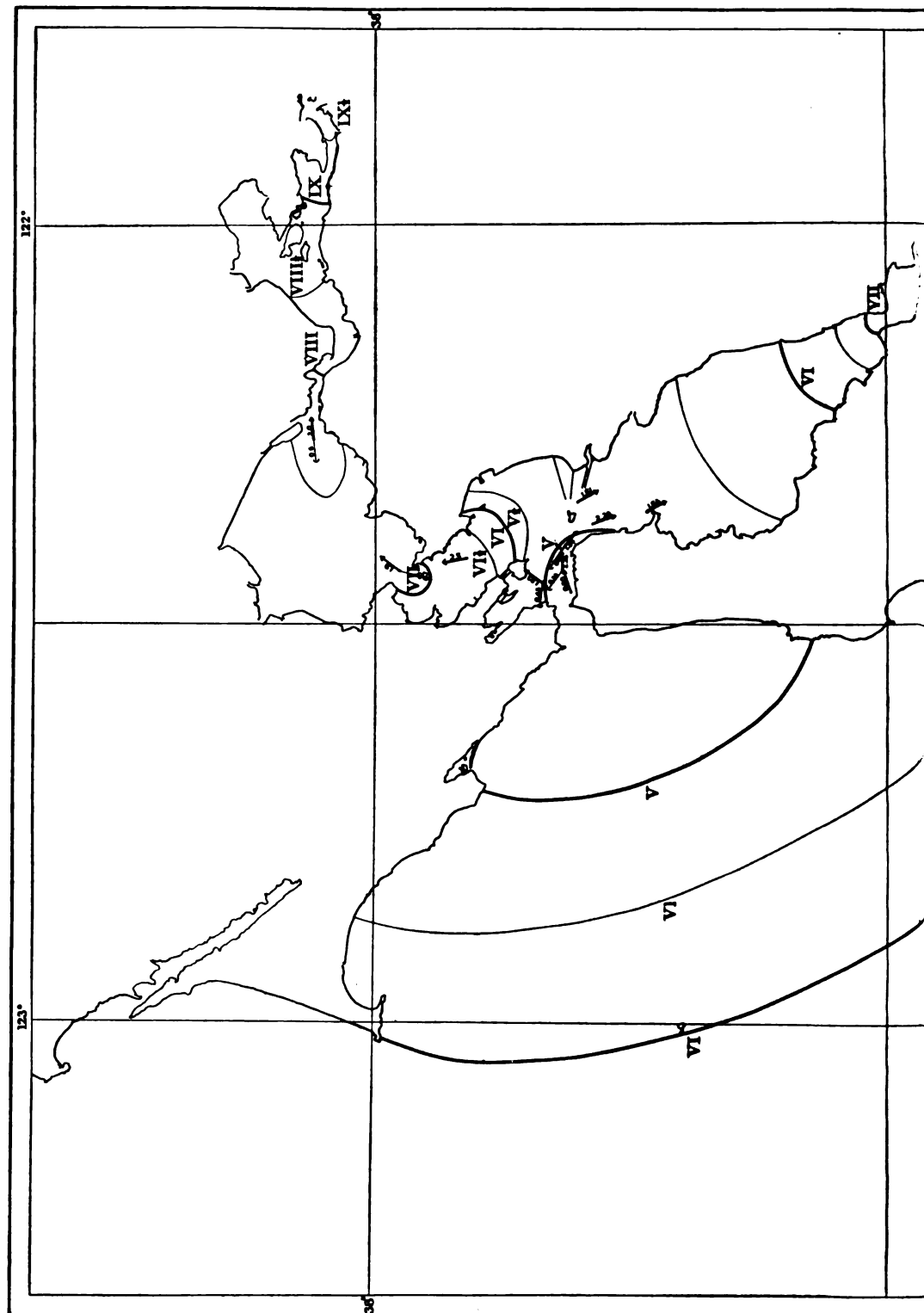


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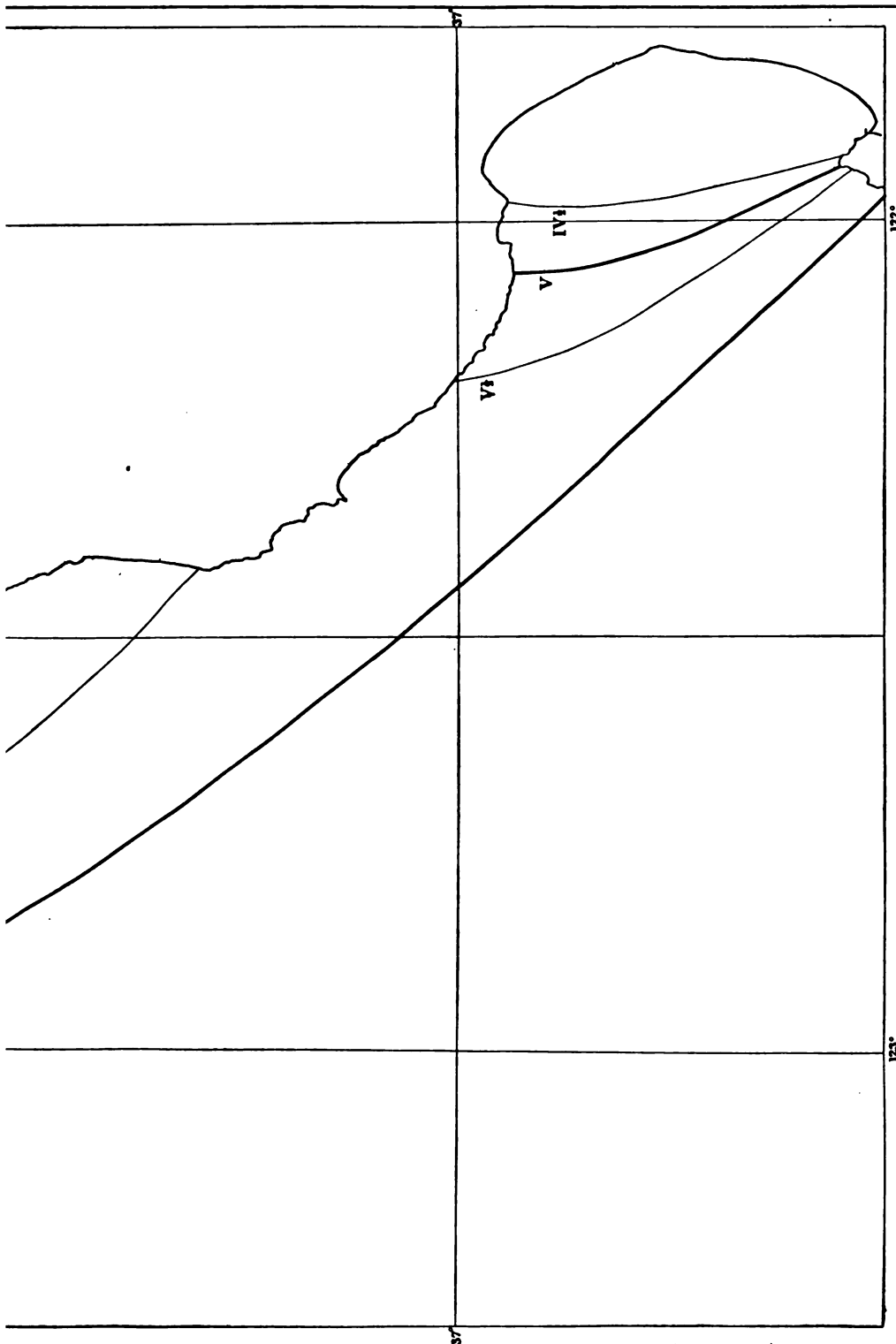


IN FRANCISCO BAY.



COCURRENT LINES FOR
Tidal hours: Fort Point, VII. 42, I.

No. 17.



FRANCISCO BAY.

Mission Street, VII. 87, I. 54.

Magellan Strait to Cape Tres Montes: High water on most parts of this coast takes place within half an hour on either side of noon, at full and change. The stream is inconsiderable, and the rise and fall rarely more than 6 feet.

Chonos Archipelago: In the channels which have an east and west direction, the flood sets to the eastward, and ebb to the westward. In those having a north and south direction the flood generally sets to the northward, and ebb to the southward.

It is high water, full and change, in Chacao narrows at oh. 50m.; springs rise 16 feet, neaps 7 feet. The strength of the streams between Huapacho and Dona Sebastiana islet is from 3 to 4 knots, and gradually increases in rate until the maximum of 9 knots at springs is attained off Remolinos point.

In Chacao bay an eddy sets in a contrary direction to the stream in the narrows from 1 to 3 knots; the line of separation is that drawn from Tres Cruces point to Remolinos point.

It is high water, full and change, at port Huite at oh. 55m.; springs rise 20 feet, neaps 13 feet. The flood tide off the port sets to the northward, and strongly at springs.

Chatham Island: At Wreck bay springs rise $8\frac{1}{2}$ feet. Across the entrance to the bay the flood stream sets N.E. by N. at $1\frac{1}{2}$ knots, and the ebb S.W. by S. at $1\frac{1}{2}$ knots an hour.

Pedro Gonzales Island, Gulf of Panama: It is high water, full and change, in Perry bay at 3h. 50m.; springs rise 16 feet. The tidal stream is not felt in the anchorage, but there is a considerable set off the island, the flood running to the northward, the ebb to the southward, the latter being generally the stronger.

It is high water, full and change, in Panama road at 3h. 50m.; springs rise from 15 to 22 feet, neaps from 10 feet to 16 feet. The ebb sets south from one to $1\frac{1}{2}$ miles an hour, and is stronger than the flood, which runs to the North-west.

From charts published by the United States Hydrographic Office the following velocities of the tidal currents are obtained: Off Cape San Diego, La Marie Strait, 3 to 4 knots (tide race); off Cape Virgins, 2 knots; First Narrows, Magellan Strait, 5 to 8 knots; Second Narrows, Magellan Strait, 3 to 6 knots; entrance to Nuevo Gulf, 2 knots; east of Valdes Peninsula, 3 knots (overfalls).

The charts indicate, by the absence of arrows, the probably small velocities of the tidal currents along the unbroken coasts against which stationary waves have endings.

88. *Pacific Coast of North America.*

The Gulf of California has a tide wave stationary in the main. Through passages formed by the islands in the gulf, the currents are very strong. Between Tiburon Island and the main land the flow is hydraulic, with a velocity of perhaps 7 knots.

Off Point Fermin, San Pedro Bay, California, a few current observations indicate that the tidal flow is for the most part along the Californian coast and not perpendicular to it. Moreover, the observed current hour for the western stream is V, which about agrees with the time of northwesterly current in the stationary oscillation, one of whose loops is in the Gulf of Alaska, where the tidal hour is IX (see Fig. 23, Part IV A); that is, the flow along the coast is probably much greater than the flow normal to it due to the South Pacific system (sec. 75, Part IV A).

Knowing the time of tide for San Francisco Bay to be about VIII, it follows that time of greatest influx must be about V. Observation shows that this inference is nearly correct. Northeasterly from the city the current is rotary in the counterclockwise sense. This agrees with Airy's rule in such matters; for, observations upon both tides and currents prove that the tide wave of the southern branch of the bay is nearly stationary, while considerable progression occurs toward San Pablo Bay. Near the mouth of Richardson Bay, on the northern side of the strait, the currents are rotary in the opposite sense and for a similar reason. Although not based directly upon observation,

there are good reasons for believing that beyond the influence of the bay the tidal streams must flow about parallel to the coast and here have VI for the hour of maximum flood.

If San Francisco Bay were only a deep reservoir whose level surface rose and fell with the water outside it would be reasonable to expect that for equal ranges of tide the diurnal velocities through the Golden Gate would be one-half those of the semidiurnal.

By comparing the values as \dot{K}_1/K_1 , \dot{O}_1/O_1 with \dot{M}_2/M_2 it will be seen that for San Francisco the ratio is even less than half. This indicates that more wave motion goes on with reference to the semidiurnal tide than with respect to the diurnal, and this would naturally be expected in so limited body of water as San Francisco Bay.

Off the southeastern extremities of Vancouver Island the current has a velocity of from 3 to 6 knots.

The currents are strong at the entrance to Burrard Inlet.

The tidal currents in Seymour Narrows, Discovery Passage, have a velocity of from 4 to 8 knots (see Fig. 32, Part IV A). They are chiefly hydraulic, as can be seen by substituting the tidal hours and ranges taken from Fig. 32, Part IV A, in Table 60, and comparing with the observed times of turning. According to Lieut. Commander E. K. Moore, U. S. Navy, the slack waters in the Narrows occurs $3^h 48^m$ after Sitka high water and $3^h 51^m$ after Sitka low water, in absolute time. The former slack is that preceding the north-going stream; the latter, the south-going.

The same authority gives as the time by which slack water in Sergius Narrows precedes the high and low waters at Sitka, $2^h 00^m$ and $1^h 41^m$, respectively.

That currents are also hydraulic in Sergius Narrows and in Clarence Strait, can be seen upon consulting the maps just referred to and substituting the values taken from them in Table 60.

The turning of currents soon after the high and low waters in most of the canals of Alaska indicate waves chiefly stationary.

Through the narrow passes of the Aleutian Islands the currents are hydraulic.

For further details concerning the tidal currents in Alaskan waters, see Pacific Coast Pilot, Alaska, published by the Coast and Geodetic Survey.

89. *New Zealand.*

The tidal streams (Fig. 6) indicate a species of progressive wave traveling entirely around this group of islands. As has been explained in Part IV A, the origin of this movement is the loop of a stationary wave north of the North Island. The fact that a tide wave progresses through Cook Strait in a northwesterly direction from a region where the mean range is only 4 feet to a region where it is 8, suggests at once the probability of the oscillation in the strait being stationary.

From Fig. 40, Part IV B, it is seen that the tidal hour and range for the southeastern entrance to Cook Strait, New Zealand, are IV.4 and 4 feet, while the corresponding quantities for the northwestern entrance are IX.5 and 8 feet. Table 60 gives IX.8 for the hour of greatest southeastward downward slope through the strait. This increased by 3 hours gives XII.8 as the ebb (southeasterly) current hour. This value agrees fairly with the values given upon B. A. Chart No. 695. This chart shows overfalls and heavy tide ripples in the strait due to the uneven bottom. The flow through Tory Channel is probably hydraulic.

The following quotations are from the New Zealand Pilot (1901):

90. *Coasts of New Zealand, quotations.*

On the eastern coast of North island the flood stream runs to the northward, and the ebb to the southward, at the rate of about one knot per hour; but in Hauraki gulf they take a contrary direction, the flood running south and the ebb north. The body of the flood stream, entering from the southward between cape Barrier and cape Colville, separates about False head on the west side of Great Barrier island, and sweeps round to the southward, filling the Thames and Waitemata rivers through the different channels leading to Auckland. The ebb tide runs from one to $1\frac{1}{2}$ knots to the southeast between Great Barrier island and cape Colville, but inshore much stronger. The range of tide in Hauraki gulf is from 4 to 10 feet.

In Whangaparaoa passage the tides run from one to 2 knots; in Waiheke channel, half a knot; but from 2 to 3 knots in the adjoining narrow channels.

It is high water, full and change, at Long point at 6h. om.; springs rise 5 feet; neaps 4 feet. The tides in Hawke's bay are slack in the bay, but strong in the river mouths. The flood sets inward to the northward, and the ebb to the southward.

It is high water, full and change, at port Napier at 6h. 15m.; rise, 3 to 4 feet. The flood stream runs in $1\frac{1}{2}$ hours after high water, and at the time of slack has fallen from $1\frac{1}{2}$ to 2 feet; similarly with the ebb, the water begins to rise $1\frac{1}{2}$ hours before the outward stream ceases.

In the narrow part of the channel the ebb runs 6 to 7 knots, and from the entrance sets north-eastward. In the bay the flood comes in from south-eastward, and the ebb runs out northeastward.

It is high water, full and change, in Lambton harbor, port Nicholson, at 4h. 17m., springs rise $3\frac{1}{2}$ feet, neaps $3\frac{1}{4}$ feet; the strength of tide in the Narrows at the entrance of the port, is from half a knot to 2 knots, but within it is much less.

The flood stream outside the entrance sets to the northward, and the ebb to the southward; each tide runs about six hours.

It is high water by the shore at the southern entrance of Cook strait at 6h. om.; but the flood or northerly stream commences at 3h. om., or three hours before, and runs until three hours after high water by the shore.

The narrowest part of Cook strait is formed by cape Terawhiti and Wellington head, the latter bearing from the former N. 80° W. 12 miles. It is high water, in the center of the strait here, on full and change days at 8h. om.; the flood or northerly stream commences at 4h. om. and runs until 10h. om., the strength of the tide varying from one to $3\frac{1}{2}$ knots.

Heavy tide rippings are experienced in the central part of the strait between these two heads, where there is uneven bottom, the depths varying from 80 to 122 fathoms sand. Tide rippings also extend off cape Terawhiti 2 miles, and for nearly 3 miles off Karori rock; eastward of Sinclair head these rippings cease.

Wanganui Heads: Off the adjacent coast the flood runs to the northward, and the ebb to the southward, from 1 to 2 knots.

Between Wanganui and cape Egmont the flood tide sets to the westward, and the ebb to the eastward, at a rate of from one to 2 knots per hour.

It is high water, full and change, at New Plymouth at 9h. 30m.; springs rise 12 feet, and neaps 9 feet. The flood sets to the westward, the ebb to the eastward about one knot.

It is high water, full and change, at the east entrance of Tory channel at 8h. 15m.; springs rise 8 feet and neaps 6 feet. At the eastern entrance to the channel the tidal streams run 5 to 7 knots, opposite Jackson bay 2 to 4 knots, and in the rest of the channel one to 3 knots.

The flood stream begins to run at $1\frac{1}{2}$ h. after low water by the shore, and continues for 5h. 35m.; the ebb stream begins at $1\frac{1}{2}$ h. after high water, and runs 6h. 25m.

Off Stephens island it is high water at 8h. om.; the flood or north-westerly stream begins $3\frac{1}{4}$ hours after low water; and the ebb or south-easterly $3\frac{1}{4}$ hours after high water.

French Pass: It is high water, full and change, at 10h. om.; the rise is 5 to 12 feet. The tide streams in French pass run 5 to 7 knots, and instead of setting directly through the narrow channel, set across more in a line from Rock Cod point to Channel point, and the contrary, and a tidal irregularity which though not of rare occurrence is especially remarkable in this pass, viz., that the ebb stream running to the eastward commences at 2 hours before high water by the shore, the tide at the same time rising in

Current basin and French pass; the extraordinary nature of the bottom, in connection with the narrowness of the channel, is quite sufficient to account for the whirling of the current, the depth varying from 7 to 53 fathoms, without reference to the distance from the shore or rocks.

Kaipara Harbor: The tides outside the harbor follow the direction of the coast, the flood running south and the ebb north.

Manukau Harbor: Tidal streams above Puponga, both in the Wairopa and Waiuku channels, average $2\frac{1}{2}$ knots at springs. In the narrow part of the channel off Paratutai they run 4 knots, and on the bar outside from one to 2 knots; on the outer coast the flood sets to the south and the ebb to the north.

It is high water, full and change, on the bar of Whaingaroa harbor at 9h. 50m.; springs rise 12 feet, neaps 9 feet.

The strength of the tides between the heads is from 4 to 6 knots; a mile above, from $2\frac{1}{2}$ to 3 knots; and at the anchorage off Matakokako point from $1\frac{1}{2}$ to 2 knots.

Newhaven Harbor: It is high water, full and change, at 2h. 30m.; springs rise 7 feet, neaps $5\frac{1}{2}$ feet, strength of tide 2 to 3 knots, both ebb and flood. The flood stream runs in for 50 minutes after high water.

The flood tide sets through Foveaux strait from west to east, and is strongest between Bluff harbour and Ruapuke island; its influence is felt as far as Long point, 45 miles eastward of that island. Between Ruapuke and Stewart island it sets to the south-eastward, running parallel with the shores of the latter. The ebb takes an exactly contrary direction.

It is high water, full and change, in the western entrance of Foveaux strait, that is, between the north point of Stewart island and Pahia point, at 12h. 15m.; the flood stream commencing from half an hour to 2 hours after low water, according to the winds, being earlier with those from the westward.

Both the ebb and flood streams run for 6 hours. At the eastern entrance of the strait, it is high water at 1h. 0m.; the flood stream commencing at 10h. 0m., or 3 hours after low water.

Along the northeast side of Stewart island the flood or south-easterly stream runs one hour and 20 minutes after it is high water at port William, or until 2h. 0m., on fall and change days. The strength of the tide varies from one half to $2\frac{1}{2}$ knots; in the narrow part of the strait, between Ruapuke and the Bluff, it is 3 knots.

The flood tide coming from southward strikes the south end of Stewart island and divides, one part running northward along its western side, and then eastward through Foveaux strait; the other runs north-east along the south-east side of the island, as far as port Adventure, where the streams meet again and flow eastward.

The strength of the tides off the coast is from one half to $1\frac{1}{2}$ knots, except in the narrow passages.

Chatham Island: Tidal streams are felt over an area of from 10 to 15 miles from the islands. The flood splits at the south point and runs north along the east and west sides, to join again at the north end; similarly the ebb divides at the north and rejoins off the south end. A single tide has been known to take a sailing vessel from off the southeast point of Chatham island into the vicinity of The Sisters.

Auckland Islands: The tide-rips off the north point of Enderby island extend a long way to the north-east, at times to a distance of 12 miles, and to a stranger have a most alarming appearance. The flood sets north-north-east and the ebb to the southward.

91. *Coasts of Australia.*

The tidal currents along the eastern coast of Australia are, as a rule, not strong. Those within the Great Barrier Reef seem, from the nature of the case, to be of no great importance in connection with the general problem of ocean tides; but any observed facts relating to the outer edge of the reef, or to the outlying islands must be of value. These indicate that the tides of the Coral Sea belong to a dependent wave nearly stationary in character.

Through Torres Strait the tidal streams are considerable.

Near the head of the Gulf of Carpentaria the semidaily tide nearly disappears.

The easterly and southeasterly direction of the tidal streams northwest of Australia indicate the stationary character of the tide. In the comparatively shallow waters bordering this coast, the currents sometimes have a rotary character.

There is probably little current along the western coast of Australia between Houtmann Rocks and Geographe Bay.

The current is probably small in the Great Bight.

Near the heads of Spencer Gulf and of the Gulf of St. Vincent the currents are weak and irregular, as is usually the case at the end of a stationary dependent wave.

On the southwestern coast of Tasmania the streams are weak, indicating a partial end boundary of the oceanic stationary oscillation from the southwest.

In the western end of Bass Strait the currents are strong (see Fig. 34, Part IV A).

The following quotations are taken from the Australian Directory, Vols. I (1897), II (1898), and III (1895):

92. *Coasts of Australia, quotations.*

It is high water at St. Paul island, full and change, at 11 h. om., springs rise 3 feet.

At the outer anchorage in 30 fathoms, with Ninepin rock bearing W.N.W., distant 8 cables, the stream sets N.W. from low water to 2 hours ebb on the shore, or for 8 hours; and sets S.E. from 2 hours ebb until low water.

Cape Arid: The tides are very weak and inconsiderable in this neighborhood, and are much influenced by the wind.

Coffin Bay: At the bar the streams make an hour after low and high water, respectively.

It is high water, full and change, in Boston bay at 1 h. 50 m.; springs rise 6 feet. There is very little tidal stream in any part of port Lincoln. At 2 or 3 miles off the coast outside, the stream sets to the northward during the rising tide, and to the southward during the falling tide, its greatest strength being from $1\frac{1}{2}$ to 2 knots an hour.

Spencer Gulf: It is high water, full and change, at the entrance to Franklin harbour at 4 h. om.; springs rise 5 feet 6 inches. The streams begin a few minutes after high and low water respectively.

It is high water, full and change, in port Victoria, at 2 h. 40 m.; springs rise 5 feet. The tidal streams set North and South; about $1\frac{1}{2}$ knots to the northward during the rising tide.

The stream sets N.N.E. during the rising tide, and S.S.W. during the falling tide, at the rate of 2 knots an hour over Tipara reef; outside it the streams set more North and South.

The stream divides off cape Spencer during the rising tide, one branch setting along shore, E.N.E., and the other to the north-westward and northward.

The tides in the northern part of Spencer gulf are very irregular.

Investigator Strait: During the rising tide the stream sets N.N.E. $1\frac{1}{2}$ miles an hour into Foul bay; and during the falling tide it sets S.W.

Kangaroo Island: The north-going tidal stream runs during the rising, and the south-going during the falling tide.

It is high water, full and change, in port Wakefield, at 4 h. 40 m.; springs rise 11 feet; neaps 5 to 6 feet.

On the bar at the sea mouth of the Murray river high tide occurs in the night or morning from September to March; and from March to September in the day or afternoon. Also the time of high water only varies 2 hours from the time observed on full and change days (oh. 50 m.), ranging from 11 h. when the moon's age is 10 or 26 days; to 3 h. when the moon's age is 20 or 7 days.

In the Murray mouth the ebb tidal stream runs strongest at low water, the ordinary rate then being 3 knots on the surface in the deep part, and 4 knots on the bar.

There is no tidal stream in Lacedpede bay.

It is high water, full and change, at all places on this coast at nearly the same time, namely, Portland bay, oh. 30 m.; port Fairy, oh. 31 m.; Warrnambool, oh. 37 m.; New Year islands (King island), oh. 48 m.; Surprise bay (south part of King island), oh. 43 m.; Sea Elephant bay (King island), oh. 50 m.; springs rise 3 feet.

The tides and tidal streams are much affected by the winds and are uncertain.

It is high water, full and change, at New Year islands at oh. 48 m.; springs rise 3 feet. The stream turns, in fine weather, at high or low water, but is greatly affected by prevailing winds.

It is high water, full and change, in Sea Elephant bay at oh. 50m.; springs rise 3 feet. The flood stream runs to the northward and the ebb to the southward, at springs $1\frac{1}{2}$ knots.

Port Phillip: The streams turn from 2 to 3 hours after high and low water on the shore.

The in-going stream comes from the southward and eastward, increases in strength as it nears the heads, sets right into the entrance, across and through the reefs, with great force, and spreads toward Shortland bluff and point King.

Between the heads the stream runs from 5 to 7 knots; in the West and South channels between 2 and 3 knots; and about $1\frac{1}{2}$ knots in the broad space above the channels.

At the eastern part of the fairway of Bass strait, the flood stream sets to the southwest and the ebb to the northeast.

Kent Group: The flood stream comes from the N.E., the ebb from the S.W. In fine weather it is slack water at the time of high and low water.

At the eastern entrance of Franklin sound the flood streams meet, one coming from the N.N.E. and the other from S.E. The flood stream sets to the westward through Franklin sound, and from thence about W.N.W. on the north side, and W.S.W. on the south side of Chappell islands; and the ebb in the contrary direction. In the north channel the streams run 2 to $2\frac{1}{2}$ knots.

Banks Strait: The flood stream is the west-going stream, and the ebb the east-going; the streams are each of $6\frac{1}{2}$ hours duration at springs; but during neaps the flood runs 7 hours and the ebb $5\frac{1}{2}$ hours. The interval of slack water never exceeds a quarter of an hour; the west-going stream begins 30 minutes after low water at springs, and 50 minutes after it at neaps; the east-going stream begins 40 minutes after high water at springs, and 10 minutes before it at neaps.

In the narrowest part of the strait ($8\frac{1}{2}$ miles wide) between Swan isles and Clarke island, the tidal streams run at the rate of 3 knots at springs; westerly winds accelerate the east-going stream, which occasionally attains a rate of 5 to 6 knots.

The tidal streams are strong at port Frederick, attaining a rate of 5 to 6 knots an hour both with the flood and ebb.

Outside the port, the flood is the west-going stream; it is not felt beyond 5 miles from the coast.

Tidal streams set through midchannel between King island and Hunter group from one to 3 knots the flood to the north-east, and the ebb to the south-west.

It is high water, full and change, at cape Grim at 10h. 30m.; springs rise 8 feet; the south-west-going stream has a rate at springs of 5 knots, and at neaps of 3 knots.

Macquarie Harbor: There is little or no tidal stream in Pine cove, and the rise and fall does not usually exceed $1\frac{1}{2}$ feet.

The tidal streams are weak and practically imperceptible, but after a heavy gale from the south-west a distinct set was felt into Frederick Henry bay, and in Flinders channel toward Norfolk bay.

In the Lachlan channels the flood stream runs to the north, the ebb to the south.

Swain Reefs: In the offing, 20 miles eastward from Hixson cay, the ebb sets East three-quarters of a knot, and the flood West one knot, the stream turning later than low water, and earlier than high water, on the reef.

At Claremont light-vessel the streams set, generally, to the south-westward during the rising tide and to the northward with the falling tide.

North of cape Sidmouth the flood runs to the northward, and the ebb to the southward.

It is high water, full and change, at Hannibal islands, at 9h. 50m. springs rise 10 to 12 feet, neaps 9 to 10 feet; neap range 6 feet. The flood stream sets north and the ebb south, but the result during the strength of the south-east trade or north-west monsoon, is generally to increase or diminish the prevailing current. The flood begins 4 or 5 hours before high water, and the ebb one to 2 hours after high water.

Tern Island: The tidal streams generally run parallel to the coast, flood to the northward, ebb to the southward.

It is high water, full and change, at Frederick point at 11h. 0m.; mean springs rise 10 feet, mean neaps rise $8\frac{1}{2}$ feet, and neaps range $6\frac{1}{2}$ feet. The diurnal inequality, amounting at times to 4 feet, chiefly affects the high waters.

The streams are rapid in Albany pass, attaining at springs a velocity of 5 knots an hour, and cause a confused sea when running in an opposite direction to the wind.

Off Fly point there is always a very heavy tide rip, dangerous to boats, on the flood at springs, caused by the stream from Newcastle bay meeting the stream on the east side of Ulfra rock.

The north-going stream runs until about 2 hours after high water by the shore, and the south-going stream until about $1\frac{1}{2}$ hours after low water.

In Adolphus channel the flood stream sets north-westward and ebb south-eastward, both attaining a velocity of from 2 to 4 knots at springs.

The channel is covered with rippings and swirls when the streams are at their strength, giving the appearance of shoal water, but Mid and Quetta rocks were the only dangers found.

The flood sets westward past Mount Adolphus islands, and meeting the stream through Adolphus channel causes heavy overfalls off the salient points. The streams attain great velocity at springs among the islands of the group.

Off Albany rock there is a heavy confused sea when the streams run strong.

It is high water, full and change, at Raine island at 8h. 10m., and the flood runs an hour and three-quarters later in the stream; springs rise 10 feet. The strength of the stream sometimes exceeds 2 knots, the flood coming from the eastward.

It is high water, full and change, at Possession island at 1h.; the rise at springs being $9\frac{1}{2}$ feet; the flood sets 7 hours S. 22° W., and the ebb 5 hours N. 22° E.

In Normanby sound and Thursday island harbour the flood stream sets to the westward, and the ebb to the eastward, the flood being strongest during the south-east trade.

In Normanby sound the streams run 3 to 5 knots in the direction of the channel. Between Vivien point and Prince of Wales island, both streams at times run 7 knots, but are less felt when Vivien point bears westward of N. 45° W.

On the north side of Thursday island harbour, the flood to the westward is from one to 3 knots, but with the ebb there is slack water. On the south side of the harbor the flood runs 2 to 4 knots, and the ebb along the edge of Madge reefs will sometimes reach 4 to 5 knots. The tide sets over Hovell rock with considerable strength.

In Flinders passage the tides run 3 to 5 knots, and cause overfalls between Horn island and Tuesday islets.

It is high water, full and change at Murray islands at 9h. 30m.; springs rise 10 feet. Close northward of the islands, the flood sets to the westward, and the ebb to the eastward, about 2 knots at springs. Between Maër and the two islets the tidal streams run with great force.

In the neighbourhood of Bramble cay, and in the south part of Bligh entrance, the flood runs in a westerly and the ebb in an easterly direction, $1\frac{1}{2}$ knots at springs; the flood runs 2 hours after high water. The neap tides are comparatively little, both in range and velocity.

The time of high water, full and change, does not appear to differ more than $1\frac{1}{2}$ hours throughout the whole length of the Great Barrier reefs, the average time of high water being at about 9h. 15m., and the rise of tide from 6 to 12 feet.

At Swain reefs the general direction of the flood through the reefs was found to be south-west, and the ebb to the north-eastward, the velocity between springs and neaps being from $1\frac{1}{2}$ to 2 knots; but the stream appeared to run with greater strength through the more confined channels.

Between Swain reefs and Lizard island the flood appeared to run in, and the ebb out, through the openings of the reefs, with a strength depending in great measure upon the breadth of the passage.

From Lizard island to lat. $12^{\circ} 30'$ or 13° S., the strength of the stream being confined to the openings, the velocity is increased or diminished according to the width of the channel.

From lat. $12^{\circ} 30'$ S. to Pandora entrance, in lat. $11^{\circ} 26'$ S., the velocity at springs increases to $2\frac{1}{2}$ and 3 knots, with a regular ebb and flow, except that the flood appeared to continue half an hour longer than the ebb.

At Raine island it is high water at 8h. 10m., the rise being 10 feet at springs; and it is in this vicinity that the strength of the stream increases materially. The flood rushes in through the smaller channels with great velocity; and a well-found merchant vessel, under full sail with a fair wind, has been barely able to effect an entrance against the strength of the ebb near Stead passage (in lat. $11^{\circ} 55'$ S.). The neap tides are comparatively weak; a reference to the phases of the moon therefore becomes a question of importance when navigating near this part of the Great Barrier reefs.

From Pandora entrance (in lat. $11^{\circ} 26' S.$) to the north-west extremity of the reefs, the sea being more confined between the coasts of Australia and New Guinea, the streams run with still greater velocity than further southward, the flood having been known to run 5 knots through Yule entrance (in lat. $10^{\circ} 23' S.$). Such a stream alone should deter a sailing-vessel from attempting to effect an entrance through any of the narrow gaps in this part of the barrier; but the strength of the stream diminishes very considerably as the distance from the reef is increased.

Gulf of Carpentaria: It was high water at the entrance of Van Diemen inlet, full and change, at 6h. 45m.; but in the upper part, the tides were $3\frac{1}{2}$ hours later. The duration of both tidal streams was 12 hours, and the direction of the rising stream from the northward, following the trend of the eastern shore of the gulf.

It is high water, full and change, at Norman river, at 7h. 30m. p. m. in January, and at 7h. 30m. a. m. in July, and are two hours earlier each successive month. There is only one tide in 24 hours.

Burketown: The rise and fall at springs is from 9 to 13 feet, and at neaps from 3 to 8 feet—the ebb running to the north-west, the flood to the south-south-east.

It is high water in Investigator road, full and change, at 8h. a. m.; springs rise 9 feet, but the neaps are very irregular.

The stream of rising tide sets to the southward, and the falling tide to the northward, from one to 2 knots at springs. The north-going stream makes from $1\frac{1}{2}$ to $2\frac{1}{2}$ hours before high water.

It is high water at Bountiful isles, full and change, at 7h. 45m.; the stream of rising tide sets south-westward at the rate of 2 knots.

Tidal streams run at times at the rate of 4 knots through Brown strait; that of the rising tide sets to the southward.

It is high water, full and change, at the Goulburn islands, at 6h., springs rise 6 feet; in the channels, the stream of rising tide sets to the eastward.

It is high water, full and change, in port Cockburn, at 5h. 45m.; springs rise about 14 feet. The streams run with a velocity of 2 to 4 knots in Apsley strait; the flood comes from the northward.

Baudin Island: The stream of rising tide sets to the south and west and begins about 20 minutes before low water. The stream of falling tide runs to the north and east, and begins to set 2 hours before high water.

It is high water, full and change, at the Montgomery isles, at noon; springs rise 36 feet; the flood stream sets to the southward, at the rate of 2 to $3\frac{1}{2}$ knots near the shore, and in the approach to Doubtful bay.

The streams run with a velocity of 7 to 8 knots through Sunday strait and the narrow channels in the entrance of King sound, and in the very narrow portions possibly stronger. In the fairway of the sound its rate is about 5 knots; near the western shore from 6 to 7 knots; and abreast Torment point approach to Fitz Roy river from 3 to 4 knots. Two of the boats of H. M. S. *Beagle* were nearly swamped in the entrance of Fitz Roy river, by the flood rushing in as a tidal bore; several feet in height.

King Sound: It is high water, full and change, in port Usborne, at 1h. 45m.; springs rise 34 feet. The tidal stream is scarcely felt.

Ashmore Reef: It is high water, full and change, at West islet between 10h. and 11h.; rise of tide 15 feet. The flood stream sets eastward and the ebb westward.

Browse Islet: At Browse islet, springs rise from 13 to 19 feet; neaps range about 4 feet. The tidal streams run strong. The stream of rising tide sets to the eastward and the falling tide to the westward.

It is high water, full and change, at Sandy islet, Scott reef, at about 11h., rise of tide 13 feet; the stream of rising tide sets to the eastward.

It is high water, full and change, at the Lacepede islands, at about noon; springs rise 20 feet. The stream of rising tide at the anchorage sets south-eastward, and of the falling tide north-westward. Inshore of the Lacepedes the streams set at the rate of from 2 to 3 knots an hour at springs.

Eighty Miles Beach: It is high water, full and change, at the northern Turtle isle, at 11h.; springs rise 28 feet. The stream of rising tide sets south-eastward at the rate of one to 2 knots.

Bedout Island: The stream of rising tide sets to the south-eastward, and of falling tide to the north-westward, rate one to 2 knots. The rise and fall is about 14 feet.

It is high water, full and change, at Depuch isle, at 10h. 40m.; springs rise 14 feet. At the anchorage the flood sets S. E. by E., and the ebb N.W. by W. from one to 2 knots.

It is high water, full and change, at port Robinson at 11h. 15m.; springs rise 19 feet. The tidal streams are not strong.

It is high water, full and change, in Gascoyne road, at about 10h. Springs rise 5 feet, neaps are irregular.

The stream of rising tide sets from East to S.E. and the falling tide N.W.; rate from one to 2 knots.

It is high water, full and change, in Champion bay at 9h.; springs rise $1\frac{1}{2}$ feet, and neaps $1\frac{1}{2}$ feet.

It is high water, full and change, in Warnbro sound at 9h.; springs rise 2 feet and neaps $1\frac{1}{2}$ feet; they are, however, very irregular, being greatly influenced by the prevailing wind.

93. *Eastern Coast of Asia, and the East Indies.*

The tidal currents along the eastern coast of Asia resemble in some respects those found along the coasts of Great Britain, France, Spain, and Portugal, or those along the northeastern coast of Brazil. Their variety is, however, greater owing to the presence of considerable diurnal tide or inequality in the China Sea and neighboring waters. Along this coast of Asia there are arms and belts of shallow water in which the tidal currents are in part rotary; there are several large tidal estuaries; the Bore of the Tsien-tang at Hang Chau is far famed; and in most of the rivers the duration of rise is much less than the duration of fall.

It will be noticed, from the statements quoted below, that the streams enter Formosa Strait from both ends; that there is a great crowding up of the cotidal lines near Wei-hai-wei; that there are strong currents southwest of Kiusiu; and that for the southeastern coast of Japan the flood sets southwesterly, as toward a loop of a stationary wave. Strong currents occur in the passages leading to the Inland Sea.

The following are a few quotations taken from the China Sea Directory, Vol. I (1896), Vol. II (1906), Vol. III (1904), Vol. IV (1894), and Eastern Archipelago, I (1890).

In regard to the tidal currents around the islands of the Pacific Ocean, it may be said that the information is very meager. Such as exists may be gathered from the sailing directions for the Pacific Islands Vols. I-III. A good portion of this matter, taken chiefly from Admiralty charts, is shown on the chart covering this ocean, (Fig. 6).

The same chart shows a portion of the currents around the East Indies. On account of the large diurnal wave, the arrows in this region are not always reliable. More detailed matter of a few localities will be found in Van der Stok's book entitled *Wind and Weather, Currents, Tides and Tidal Streams in the East Indian Archipelago* (1897).

Tidal currents for the Philippines are shown upon chart No. 1898, of the Hydrographic Office, United States Navy. One portion of the flood enters through the Sulu Archipelago. All along the northeastern and northwestern coasts of Borneo the flood stream probably progresses westward (Cf. Figs. 36, 37, Part IV B). Another branch passes through Balintang Channel and continues southward as far as Panay Island. Another branch enters through San Bernardino Strait.

Through San Juanico Strait the currents are hydraulic and strong.

As some account of the bearing of the tidal streams of the southern coast of Asia upon the tides has been given in sections 80, 81, Part IV A, and section 30, Part IV B, little will be done here except to refer to Bay of Bengal Pilot, Red Sea and Gulf of Aden Pilot, and Islands of the Southern Indian Ocean.

It will be seen from Fig. 5 that the tidal streams in the upper portions of the Arabian Sea and Bay of Bengal are nearly normal to the coast line. The flood arrow at the mouth of the Gulf of Suez pointing southerly on Fig. 5 instead of northerly is in accordance with the explanation of the tides of this gulf given in Parts IV A and IV B. The Bay of Bengal Pilot (1901) says—

Within a few miles of the Nicobars the flood tidal stream generally sets north-eastward and the ebb stream south-westward; the streams attain in the channels between the islands a rate of 3 to 4 knots.

This indicates that here the streams are strong, as if in the vicinity of a nodal line (see Fig. 23, Part IV A).

94. *Eastern Coast of Asia, quotations.*

Malacca Strait: It is high water, full and change, at Arang Arang, at 7h. approx.; springs rise 10 feet. In the harbour, the flood stream sets to the eastward and the ebb to the westward; but seaward of the 5-fathom bank, the streams set across the channel; the flood setting south-eastward from $3\frac{1}{2}$ hours before until $2\frac{1}{2}$ hours after high water by the shore, and the ebb north-westward.

It is high water, full and change, in Malacca road at 7h. 30m.; springs rise 11 feet, neaps $8\frac{1}{2}$ feet.

The tidal streams set S. E. by E. at the rate of $2\frac{1}{2}$ knots from 3 hours before to 3 hours after high water at One fathom bank.

It is high water, full and change, at Raffles lighthouse, at 11h., but the stream does not set to the eastward till two hours later, and it is then about half ebb by the shore.

The tidal streams from Malacca strait and from the China sea meet between Tree island and Tanjong Bulus, but no dependence can be placed upon them.

The tidal streams in Salat Sinki run with considerable strength, the flood to the westward and the ebb to the eastward.

In Sunda strait, it is high water, full and change, during the north-west monsoon, at 6h.; springs rise 3 feet. The flood sets north-eastward and the ebb south-westward with a rate at springs of $3\frac{1}{2}$ knots, but it should be observed that the tidal streams are much influenced by the prevailing winds outside the strait, so that as a consequence the set of the stream is mainly south-westward during the greater part of the year.

The tidal streams in Banka strait are strong but irregular, and are greatly influenced by the monsoons. The flood stream enters the strait at both ends, meeting near the Nangka islands.

Northwest coast of Borneo (lat., $5^{\circ} 3' N.$; long., $115^{\circ} 12' E.$). The flood stream on the Outer bar sets in $1\frac{1}{4}$ hours after low water, and the ebb stream runs out about $1\frac{1}{4}$ hours after high water, the rate at springs being from 2 to 3 knots. To seaward of the bar, the direction of the tidal streams has not been determined. Between the bar and Sapu point, the flood generally sets to the south-west; the ebb to the north-east.

Balabac Strait: The flood stream sets to the eastward and the ebb to the westward. The strength of the stream or of the current depends greatly on the prevailing winds. The greatest velocity observed was $2\frac{1}{2}$ knots.

It is high water, full and change, at Bangkok river bar at 7h. 40m., but this is subject to a large correction, the greater part of which varies with the moon's declination.

Outside the bar and near the anchorage the flood sets to the westward, and the ebb to the eastward, altering its direction according to the strength of the river stream. Along the eastern shore of the gulf toward cape Liant the ebb sets to the southward and flood to the northward.

Tong-King Gulf: It is high water, full and change, at Fai Tsi Long archipelago at about 5 hours.

The tidal streams among the islands attain a rate of 2 knots an hour in places where confined; in the offing the streams run from one to $1\frac{1}{2}$ knots an hour; the flood coming from the south-west, and the ebb from the north-east; off Kebao the streams run nearly tide and half tide.

Hainan Strait: In North channel the flood sets S.W. by W. from one to 3 knots an hour, and the ebb N.E. by E. from one to $3\frac{1}{2}$ knots.

In Middle channel, at the position charted (12 miles N.E. by E. $\frac{3}{4}$ E. of Hainan point), the flood sets N.N.W. from $1\frac{1}{2}$ to 3 knots, and the ebb N.E. by E. one to 3 knots. (This is probably for only a portion of the time of flood and ebb.)

On full and change days in summer the E. set commences at	3 p. m.
" " " " " " " " W. " " "	11 p. m.
" " " " " " " " winter " E. " " "	3 a. m.
" " " " " " " " " W. " " "	11 a. m.

and occurs about one hour later every day.

It is high water, full and change, at West or Fort point, Nau chau, at 10h. 20m.; springs rise $12\frac{1}{2}$ feet, neaps 8 feet.

At Nau chau the stream runs $2\frac{1}{2}$ knots at springs, changing about one hour after high and low water, the flood setting to the southward, and the ebb to the northward.

It is high water, full and change, at Breaker point at 10h. 0m. approximately; springs rise 8 feet.

From January to May, between Hongkong and Breaker point, the ebb tidal stream ran eastward, but generally speaking it was weak. Eastward of Breaker point the flood stream sets eastward.

Port Swatau: The flood stream is said to continue for one to $1\frac{1}{2}$ hours after high water on the bar.

Namoa Island: The flood stream comes in both northward and southward of the island.

The ebb tidal stream off Jokako point has been observed to set south-westward $4\frac{1}{2}$ knots in one tide.

Amoy: In the Inner harbour the duration of the flood tide is about $7\frac{1}{2}$ hours, and of the ebb $5\frac{1}{2}$ hours. The rate of the ebb stream during the first three hours is 4 to 5 knots at springs, and during its latter part 2 to 3 knots; the average rate of the flood stream is 2 to 3 knots at springs. The flood stream runs from three-quarters of an hour before low water to a quarter of an hour after high water.

Kwing Bay: In March, off Tau point, the flood stream sets south-westward and the ebb north-eastward, 2 to $2\frac{1}{2}$ knots an hour.

The flood stream enters Hai tan strait by both the northern and southern entrances; these streams meet between Rocky and Middle islands, in which vicinity, and more especially between Hill and Middle islands, there are, with strong winds, heavy overfalls, dangerous for boats.

Pescadores Islands: Off the Rover group, the north-going or flood stream makes at 4 hours after high water, and the south-going or ebb stream at 2 hours before high water. The rate of the north-going stream in Shôgun suidô sometimes exceeds 4 knots during the strength of the south-west monsoon, while the south-going stream rarely reaches 3 knots, but these rates may be reversed during the north-east monsoon.

At about 3 miles off the coast, in the vicinity of Tamsui harbour, the ebb tidal stream sets north-eastward at $2\frac{1}{2}$ to 3 knots an hour. This stream runs round the northern end of Formosa, and causes a turbulent ripple off Syau ki and Foki kaku. The flood stream runs south-westward at 2 knots an hour.

Chusan Archipelago: The tidal streams around and between the islands are very rapid, sometimes attaining a rate of 7 and 8 knots; and the tide rippings are numerous and dangerous for boats when there is much wind. As a rule, the sea does not run high, but the day before the approach of a typhoon, and during its continuance, a heavy swell rolls in upon the rock-bound coast. The direction of the tidal streams eastward of Chang tau is rotary, turning with the hands of a watch; but in the straits between the islands, in the mouth of Hangchau bay, and close to the land, it follows the conformation of the coast. Clear of local influences, the following is a broad guide:

The first half of the flood runs in directions from South to West; the last half from West to North.

The first half of the ebb runs from North to East, the last half from East to South.

Usually, as the moon crosses the meridian, the bore passes Haining, where it is nearly a straight line across the river, 9 cables wide, 8 to 11 feet high, and traveling 12 to 13 knots an hour; its front being a uniform sloping cascade of bubbling foam, falling forward and pounding on itself and on the river before it at an angle of between 40° and 70° . The highest and steepest part is over the deep channel of the river.

A quarter of an hour after the bore has passed Haining, the water has risen 13 feet; at 2h. 0m. it has risen 18 feet; it is high water at 3h. 0m. when the tide has reached a height of 19 feet, and the stream at once commences to run out swiftly. At 5h. 0m. it is at the mean level; at 8h. 0m. it is nearly low water. The out-going stream, however, continues to run rapidly eastward until the arrival of the next bore. The water is at its lowest for the 2 hours preceding the bore.

It is high water at Hangchau fu about the same time as at Haining, but the rise and fall does not exceed 6 or 7 feet.

At Haining the flood lasts for 3 hours; the ebb for 9 hours. At Hangchau fu the flood continues for $1\frac{1}{2}$ hours, and is nearly all in the bore.

The tidal streams off the mouth of the Yangtse kiang are rotary and turn in a direction with the hands of a watch; the first half of the flood runs in directions from South to West, the last half from West to North; the first half of the ebb runs in directions from North to East, the last half from East to South.

The rate of the stream varies with the age of the moon between one and 4 knots an hour.

The streams on the south-eastern coast of Shantung, eastward to Staunton island, appear to follow the general direction of the coast, the flood stream setting west-south-westward, and the ebb east-north-eastward, at an average rate of $1\frac{1}{2}$ knots an hour.

The times of the high water at various parts of the promontory, from Tsing hai bay to Wei hai wei, alter considerably at short intervals of distance, whilst the tidal streams change almost simultaneously at short intervals of distance.

Pe Chili Strait: In Charybdis harbour it is high water, full and change, at 10h. 30m.; springs rise 9 feet. The tidal stream sets northward in Hope sound and Charybdis harbour during the flood, and southward during the ebb. For some distance eastward of Miao tau strait the flood stream sets westward, and the ebb eastward; but within the strait, a few miles westward of Teng chau, the flood sets eastward and the ebb westward.

Northward of the Li tsin ho, the flood sets north-westward along the shore, and the ebb south-eastward, turning, but not regularly, at high and low water. At Lan mun sha banks, near the shore, the flood sets southward and the ebb northward.

Liantung Gulf (Sand Point): The flood stream sets northward along the shore, the ebb south-eastward; the streams turn earlier near the shore than in the offing.

On the east coasts of Kamchatka, Yezo, and Nipon the tidal streams are weak, and no exact observations are available; probably the streams set to the southward with a rising tide, and to the northward with a falling tide. Along the south coast of Japan, the flood stream sets to the westward and the ebb to the eastward.

The flood sets northward up the Kii and Bungo channels, the stream from the first channel setting westward in the Seto Uchi, and that from the Bungo channel dividing into two parts, one stream setting westward toward Simonoseki strait, and the other eastward, meeting the Kii channel stream at the east end of Bingo Nada. On the west coast of Kiusiu and in Korea strait the flood stream runs to the northward; to the westward and north-west through the Korean archipelago, and to the northward along the west coast of Korea.

In the Japan sea the tidal streams are weak and irregular. In the gulf of Tartary the flood stream sets to the northward.

Throughout the above coasts the streams overrun the rise and fall of tide by about one hour on the open coast, to 2 to 3 hours and even more in the Seto Uchi, among the inner islands, and in confined straits.

Amongst the islands off the south coast of Korea the flood stream sets to the westward, the ebb to the eastward, turning about 2 hours after high and low water by the shore.

The flood stream sets north-eastward along the western shore of the strait of Tartary at the rate of 2 miles an hour.

Southwest Japan: The tides near Kusakaki sima appear regular, flood flowing to the northward and ebb southward.

The tides near Mikomoto are regular, the flood setting W.S.W., and the ebb E.N.E. from $1\frac{1}{2}$ to 3 miles an hour.

It is high water, full and change, in Yokohama bay at 5h. 45m. Springs rise 5 feet; neaps $3\frac{1}{2}$ feet. The tidal stream is scarcely perceptible in Yokohama bay.

Near the extreme of Futsu saki the tidal streams sweep round at a rate of more than 3 knots an hour at springs.

The rise in Ofunato harbour is about 5 feet; there is no perceptible tidal stream there.

It is high water, full and change, in Fuk ura, eastward of Naruto passage, at 6h. 14m.; springs rise $6\frac{1}{2}$ feet, neaps $4\frac{1}{2}$ feet. At Anaga ura, northward of the passage, the time of high water is variable,

and the rise is 2 to 4 feet. The stream sweeps through the passage with great velocity, and the roar of its breakers can be heard for several miles. The south-going stream begins, at springs, 3 hours and 25 minutes after the moon's meridian passage, and 2 hours and 8 minutes after the moon's meridian passage at neaps.

Me sima Group: Strong tidal streams set through the channels between the islands, the flood to the north-west, and the ebb to the south-east; but the general direction of the tidal streams is more to the northward and southward.

West coast of Kiusiu: Between Me saki and Noma no hana the flood stream sets to the northward along the coast, and the ebb tide to the southward, attaining at spring tides a velocity of from $2\frac{1}{2}$ to 3 knots an hour. The stream sweeps round the bays, causing tide-rips off the prominent points.

It is high water, full and change, at Nagasaki at 8h. 11m.; springs rise $10\frac{1}{2}$ feet, and neaps about 7 feet, but they are variable.

La P rouse Strait: The tides set east and west through the strait, the east-going stream attaining at spring tides a velocity of from 4 to 5 knots an hour.

Eastern Archipelago.

Arru Islands: The flood stream in Dobbo harbour comes in from the westward, and the ebb stream from the eastward. In the south-east monsoon the flood is weak, but the ebb runs from one to $1\frac{1}{2}$ knots an hour.

In the offing the flood stream sets to the S.S.E. and the ebb to the N.N.W.

Dampier Strait: The flood stream sets to W.S.W. and the ebb to E.N.E., but the streams appear to be greatly affected by the prevailing monsoons.

95. Table of slack waters and mean maximum velocities.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	h. m.	Time	h. m.	Time	h. m.	Time	h. m.
BAY OF FUNDY.											
From Cape Roseway, S. 51° E., 11 mi.*		W. Bell Dawson.	1904								
Cape Sable, S. 22° W., 3½ mi.*		do.	1904	LW. †-3 15				HW. -2 30			
S. 22° W., 12½ mi.*		do.	1904	LW. †-1 17				HW. -1 08			
Seal Island Light, S. 8° W., 8 mi.*		do.	1904	LW. †-1 02				HW. -1 13			
S. 70° W., 13 mi.*		do.	1904	LW. †-13				HW. + 25			
Lurcher Shoal, S. 82° E., 6 mi.*		do.	1904	LW. †+ 10				HW. + 20			
S. 80° W., 10 mi.*		do.	1904	LW. †+ 35				HW. + 22			
Brier Island Light, S. 84° W., 5½ mi.*		do.	1904	LW. †+ 41				HW. + 44			
Petit Passage, N. 28° W., 9½ mi.*		do.	1904	LW. †+ 43				HW. + 1 00			
Brier Island Light, N. 63° W., 15 mi.*		do.	1904	LW. †- 49				HW. + 04			
Gannet Rock, S. 48° E., 5 mi.*		do.	1904	LW. †+ 35				HW. - 05			
Big Duck Island, N. 87° E., 3½ mi.*		do.	1904	LW. †+ 1 40				HW. - 55			
W. Quoddy Light, S. 17° W., 4½ mi.*		do.	1904	LW. †+ 10				HW. + 30			
Moose Peak Light, S. 48° E., 6 mi.*		do.	1904	LW. †+ 1 05				HW. + 05			

Directions true unless otherwise noted.

* Directions and bearings magnetic; variation 18° W.

† Tides at St. John, N. B.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	h. m.	Time	h. m.	Time	h. m.	Time	h. m.
ST. LAWRENCE RIVER.	0 11										
Quebec Harbor		W. F. Maxwell	1885-9	L.W.*+1 10	Duration, 5 00			HW.+1 05	Duration, 7 30		
St. Laurent		do	1885-9	L.W.*+1 31	Duration, 5 00			HW.+1 54	Duration, 7 25		
Berthier		do	1885-9	L.W.*+1 10	Duration, 5 05			HW.+1 25	Duration, 7 20		
Grosse Isle		do	1885-9	L.W.*-1 19	Duration, 5 10			HW.+1 08	Duration, 7 10		
L'Islet		R. Pelletier	1900	L.W.*-1 19	Duration, 5 30			HW.-1 57	Duration, 6 50		
In Upper Traverse		A. Fournier	1900	L.W.+1 3 52	Duration, 5 25			HW.+3 13	Duration, 7 00		
In Lower Traverse		E. Lebel	1900	L.W.+1 3 57	Duration, 5 45			HW.+3 35	Duration, 6 45		
Orignaux Point		W. F. Maxwell	1885-9	L.W.+1 2 18	Duration, 5 55			HW.+2 45	Duration, 6 30		
In Brandy Pot Channel		do	1885-9	L.W.+1 2 04	Duration, 6 05			HW.+1 46	Duration, 6 20		
Tadoussac		do	1885-9		Duration, 6 08				Duration, 6 15		
Green Island		do	1885-9		Duration, 6 00				Duration, 6 24		
Bic Island		do	1885-9		Duration, 5 50				Duration, 6 34		
GULF OF MAINE.											
Off Massachusetts coast.	41 35 23	J. E. Pillsbury	July 27-28, 1885		HW.†-05		S. 25 W.		L.W.-30		N. 22 E. 1.27
Do	69 57 52	do	July 28-29, 1885		HW.†-1 20		S. 64 W.		L.W.+18		N. 20 E. 1.36
CAPE COD BAY.	41 48 11										
Off Scusset	70 31 25	H. Mitchell	July 23-24, 1860		T.‡-2 43		N. 58 W.		T.+2 58		N. 46 E. 0.10
Off Manomet Point	41 50 46	do	July 22-23, 1860		T.‡-5 26		S. 8 E.		T.+1 09		Northerly 0.58
GULF OF MAINE.	70 30 24										
Cape Cod, near Race Point.	42 04 37	Robert Platt	Aug. 20-21, 1877		HW.†-2 53		S. 46 W.		HW.-20		N. 61 E. 0.91
Off Massachusetts coast.	42 52 25	J. E. Pillsbury	July 30-31, 1885		HW.†-3 12		N. 46 W.		L.W.-4 14		N. 86 E. 0.56

* Tides at Quebec. † Tides at Father Point. ‡ Tides at Boston. § Local transits.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	h. m.	Time	h. m.	Time	h. m.	Time	h. m.
FORE RIVER.	0 11										
Portland, Me.	43 39 13	J. B. Weir.	July 14, 1873								
Do	43 38 59	do	July 12, 1873								
Do	43 38 42	do	July 5, 1873								
Do	43 38 51	do	July 11, 1873								
Do	43 38 34	do	July 10, 1873								
Do	43 38 32	do	July 9, 1873								
Do	43 38 29	do	July 8, 1873								
Do	43 38 26	do	July 7, 1873								
Do	43 38 25	do	July 1, 1873								
Do	43 38 32	do	July 2, 1872								
NANTUCKET SHOALS.											
Nantucket Shoals	41 00 ..	Lieut. C. H. Mc-	July 9-11,								
	69 27 ..	Blair	1852								
Do	41 02 ..	do	Aug. 14-16,								
Do	69 34 ..	do	1852								
Do	41 05 ..	do	Aug. 16-18,								
Do	69 37 ..	do	1852								
Do	41 12 30	do	July 5-7,								
	69 43 24		1852								

Directions true unless otherwise noted.

* Tides at Boston.

† Tides at Governors Island.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	Velocity	Time	Direction	Time	Velocity	Time	Direction
NANTUCKET SHOALS—continued.	0 1										
Nantucket Shoals	41 11 ..	Lieut. C. H. Mc-Blair	July 7-8, 1852	A. M. L.W. * - 2 30	Knots 1.83	A. M. H.W. - 0 31	N. 46 E.	A. M. H.W. + 2 03	Knots 1.66	A. M. L.W. - 1 01	S. 16 W.
Do	69 51 ..	do	Aug. 23-25, 1852	H.W. * - 2 44	1.94	H.W. + 0 15	N. 35 E.	H.W. + 3 59	1.66	L.W. + 0 32	S. 36 W.
East coast, Nantucket	41 17 26	H. P. Ritter	Sept. 6, 19, 1890	L.W. † - 0 36	0.91	L.W. + 1 34	N. 11 W.	H.W. - 1 50	0.99	H.W. + 1 40	S. 38 E.
NANTUCKET SOUND.	69 55 39										
Shovel Lt. Ship	41 32 42	H. L. Marindin	Sept. 18-27, 1887	L.W. † - 1 52				H.W. - 1 36			
Nantucket Sound	69 59 17	H. Mitchell	Aug. 27-31, 1857		0.30	L.W. † + 0 40	N. 6 E.		0.22	H.W. - 0 11	S. 42 E.
Do	41 32 13	do	Aug. 9-10, 1857		1.78	L.W. † + 0 58	S. 34 E.		1.75	H.W. + 1 00	N. 71 W.
Do	70 01 06	do	Aug. 9-10, 1857		1.13	L.W. † + 1 19	S. 6 E.		0.63	H.W. + 1 18	N. 14 W.
Do	41 34 20	do	Aug. 24-25, 1857		0.19	L.W. † + 0 03	S. 38 E.		0.94	H.W. + 1 50	N. 44 W.
Do	41 22 32	do	Aug. 7-8, 1857		0.78	L.W. † + 2 58	N. 81 E.		0.57	H.W. + 2 45	N. 65½ W.
Do	70 02 30	do	July 12-14, 1857		1.28	L.W. † + 1 38	N. 4 E.		1.26	H.W. + 1 51	S. 21 W.
Do	41 33 34	do	Sept. 28-29, 1887	L.W. † - 1 18				H.W. - 1 14			
Handkerchief Lt. Ship	70 03 51	H. L. Marindin	July 18-19, 1857		1.55	L.W. † + 3 52	N. 86 E.		1.41	H.W. + 3 38	N. 76 W.
Nantucket Sound	41 24 35(?)	H. Mitchell	July 15-21, 1871	L.W. † + 0 44	0.47	L.W. † + 2 10	Westerly	H.W. + 1 04	0.51	L.W. - 2 20	N. 85 E.
Edgartown	41 27 21	do	July 24-25, 1857		2.79	L.W. † + 2 45	N. 59 E.		2.04	H.W. + 1 40	S. 68 W.
Nantucket Sound	70 26 49	do									
	41 23 23										
	70 30 20										
	41 28 52										
	70 37 10										

Directions true unless otherwise noted.

* Tides at Governors Island.

† Tides at Boston.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	Knots	Time	Direction	Time	Knots	Time	Direction
NANTUCKET SOUND— continued.	0 1 "										
Nantucket Sound	41 29 41 70 38 43	H. Mitchell	July 29-30, 1857								
Do	41 30 41 70 34 50	do	July 30-31, 1871	HW. *+0 03	1.94	HW. *+2 59	N. 66 W.	LW. -0 10	2.35	HW. +2 47	S. 84 E.
BUZZARDS BAY.											
Bet. Mashpee Island and Bennetts Neck.	41 37 53 70 37 41	do	July 14-15, 1860			T.†+5 52	N. 23 E.		0.38	T. -1 28	S. 43 W.
In Back River Harbor	41 43 23 70 37 15	do	July 16-17, 1860			T.†+7 02			0.13	T. -1 30	S. 24 W.
NARRAGANSETT BAY.											
Providence Harbor, R. I.	41 47 53 71 23 09	H. L. Marindin	Oct. 10, 1874							HW. *+0 02	S. 51 E.
Do.	41 48 03	do	Sept. 11-12, 1874			LW. *+0 06	N. 31 W.		0.41	HW. +0 32	S. 40 E.
Do	41 23 27	do	Sept. 10-11, 1874			LW. *+1 20	N. 44 W.		0.70	HW. +0 04	S. 33 E.
Do	41 23 37	do	Sept. 9-10, 1874			LW. *+2 10	N. 22 W.		0.65	HW. -1 00	S. 30 E.
Do	41 48 25	do	Sept. 8-9, 25, 1874			LW. *+0 22	N. 14 W.		0.74	HW. -0 02	S. 12 E.
Do	41 23 40	do	Sept. 10, 21, 1874			LW. *+2 20	N. 35 E.		0.39	HW. +0 30	S. 13 W.
Do	41 48 47	do	Sept. 14-15, 1874			LW. *+2 30	N. 35 E.		0.56	HW. +0 01	S. 41 W.
Do	41 23 50	do	Sept. 17, 1874			LW. *+2 51	N. 62 E.		1.02	HW. -0 11	S. 83 W.
Do	41 48 59	do	Sept. 15-16, 1874			LW. *+2 30	N. 45 E.		0.50	HW. -0 04	S. 63 W.
Do	41 23 24	do	Sept. 17, 1874			LW. *+2 42(?)	S. 81 E.		0.27	HW. -0 28	N. 74 W.

Directions true unless otherwise stated.

* Tides at Boston.

† Local tides.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb.	
				Time	Velocity	Time	Direction	Time	Velocity	Time	Direction
NARRAGANSETT BAY— continued.											
Providence Harbor, R. I.	41 49 03	H. L. Marindin...	Oct. 9, 1874	A. M.	Knots	A. M.	N. 23 E.	A. M.	Knots	A. M.	S. 30 W.
Do	71 23 17			L.W. * -2 10	0.69			HW. -0 03	1.79		
Do	41 49 11	do	Sept. 23, 1874				N. 8 W.		0.61		S. 10 E.
Do	71 23 16			L.W. * -2 11	0.36			HW. +0 30	0.75		S. 2 1/2 W.
Do	41 49 17	do	Sept. 23, 1874				N. 17 W.		0.87		S. 58 W.
Do	71 23 16	do	Sept. 23, 1874				N. 35 E.		0.96		S. 39 W.
Do	41 49 23	do	Sept. 24, 1874				N. 85 E.		1.32		S. 44 W.
Do	71 23 16	do	Sept. 24, 1874				N. 26 E.				
Do	41 49 25	do	Sept. 24, 1874				N. 21 1/2 E.				
Do	71 23 05	do		L.W. * -2 03	0.50						
Do	41 49 29	do		L.W. * -2 10	0.35						
Do	71 22 58	do		L.W. * -1 58	0.74						
Do	41 49 35	do									
Do	71 22 53	do									
EASTERN LONG ISLAND SOUND.											
Between Latimers Reef and Bel Grass Ground.	41 18 25	Lieutenant Blake	July 5, 6 and 14, 1845	T.† +5 05	1.38	T.† +8 10	S. 71 W.	T.† -1 14	1.35	T.† +1 45	N. 77 E.
Off Groton Point	71 56 43		July 15, 16 and 19, 20, 1845	T.† +4 49	1.05	T.† -4 31	S. 82 W.	T.† -1 22	1.16	T.† +1 14	S. 75 E.
	41 17 56	do									
	72 00 12										
Near Race Point	41 14 32	do	Aug. 9, 10, 1845	T.† -6 36	3.46	T.† -3 50	N. 68 W.	T.† -53	3.73	T.† +2 23	S. 9 E.
	72 02 53										
	41 06 13	Lieut. F. H. Cross- by	May 11, 12, 1887	HW.† -2 16	0.8	HW. 15	S. 61 W.	LW.† -1 52	0.8	LW. + 15	N. 58 E.
	72 45 43										

Directions true unless otherwise noted.

* Tides at Boston.

† Local transits.

‡ Tides at Governors Island.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	<i>h. m.</i>	Time	<i>h. m.</i>	Time	<i>h. m.</i>	Time	Direction
WESTERN LONG ISLAND SOUND.	41 05 55	Lieut. F. H. Cross by W. J. Sears, U. S. Navy	May 4, 5, 1887	HW.*-2 30	HW. + 21	HW. + 21	HW. + 21	HW. + 21	HW. + 21	HW. + 21	0
	72 56 00			HW.†-4 16	HW. -1 31	HW. -1 31	HW. -1 31	HW. -1 31	HW. -1 31	HW. -1 31	N. 62 E.
	2.5 mi. N 3° W. from Stamford Shoal Lt. S. E. from Bridgeport.	do	Nov. 21, 22, 1886	HW.†-4 16	HW. -1 31	HW. -1 31	HW. -1 31	HW. -1 31	HW. -1 31	HW. -1 31	N. 88 E.
	7.4 mi. N. 109 W. from Stamford Shoal Light.			HW.†-4 35	HW. -1 17	HW. -1 17	HW. -1 17	HW. -1 17	HW. -1 17	HW. -1 17	N. 83 E.
	41 01 15	Lieut. F. H. Cross by do	Nov. 11, 12, 1886	HW.*-2 05	HW. + 40	HW. + 40	HW. + 40	HW. + 40	HW. + 40	HW. + 40	N. 70 E.
NEW YORK EAST RIVER.	40 54 25			HW.*-1 16	HW. + 1 00	HW. + 1 00	HW. + 1 00	HW. + 1 00	HW. + 1 00	HW. + 1 00	N. 61 E.
	73 41 31	do	Nov. 10, 11, 1886	HW.*+1 12	HW. + 4 12	HW. + 4 12	HW. + 4 12	HW. + 4 12	HW. + 4 12	HW. + 4 12	S. 50 E.
	73 44 54			HW.*+1 40	HW. + 44	HW. + 44	HW. + 44	HW. + 44	HW. + 44	HW. + 44	N. 65 E.
	40 52 22	H. Mitchell	July 16-18, 1858	HW.*-1 08	HW. -2 12	HW. -2 12	HW. -2 12	HW. -2 12	HW. -2 12	HW. -2 12	S. 17 W.
	73 44 23			HW.*-0 12	HW. -1 24	HW. -1 24	HW. -1 24	HW. -1 24	HW. -1 24	HW. -1 24	S. 85 W.
Between Stepping Stones and City Island.	40 48 08	do	July 18-24, 1858	HW.*-0 06	HW. -1 53	HW. -1 53	HW. -1 53	HW. -1 53	HW. -1 53	HW. -1 53	S. 69 W.
	73 47 26			HW.*+1 20	LW. + 4 30	LW. + 4 30	LW. + 4 30	LW. + 4 30	LW. + 4 30	LW. + 4 30	S. 49 W.
	40 48 12(?)	do	July 11-20, 1858	LW.*+1 43	HW. -1 18	HW. -1 18	HW. -1 18	HW. -1 18	HW. -1 18	HW. -1 18	S. 35 W.
	73 49 50(?)			LW.*+1 24	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	S. 36 W.
	40 47 25	do	May 23-June 1, 1857	LW.*+1 24	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	S. 36 W.
Off Old Ferry Point.	73 56 38.1			LW.*+1 24	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	S. 36 W.
	40 47 08(?)	do	June 1-9, 1857	LW.*+1 43	HW. -1 18	HW. -1 18	HW. -1 18	HW. -1 18	HW. -1 18	HW. -1 18	S. 35 W.
	73 55 10(?)			LW.*+1 24	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	S. 36 W.
	40 45 58.9	do	May 23-June 1, 1857	LW.*+1 24	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	S. 36 W.
	73 56 38.1			LW.*+1 24	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	HW. -1 20	S. 36 W.

Directions true unless otherwise noted.

*Tides at Governors Island.

†Tides at New London.

Table of slack waters and mean maximum velocities—Continued

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	h. m.	Time	Direction	Time	h. m.	Time	Direction
NEW YORK EAST RIVER—cont'd.	Off Eighty-first street... 40 46 15 73 56 42	H. Mitchell	May 8-18, 1857	LW.*+1 45	HW. -1 17	HW. -1 17	N. 38 E.	HW. +1 47	HW. +1 47	LW. -1 38	S. 38 W.
	Between Fifty-second street and Blackwells Island. 40 45 12 73 57 43	do	July 9-12, 1858	LW.*+1 21	HW. -1 08	HW. -1 08	N. 25 E.	HW. +1 43	HW. +1 43	LW. -2 11	S. 30 W.
	Off Twenty-third street. 40 43 57 73 57 58	H. L. Marindin	Oct. 4-7, 1886	LW.*-1 38	HW. -1 19	HW. -1 19	N. 16 W.	HW. +1 57	HW. +1 57	LW. -1 56	S. 12 E.
	Northern end Wallabout Bay. 40 42 30 73 58 22	Lieut. M. Woodhull	July 8, 9, 1854	LW.*+1 07	HW. - 40	HW. - 40	N. 37 E.	HW. +2 07	HW. +2 07	LW. -2 12	S. 38 W.
	Off Atlantic Dock. 40 41 07½ 74 00 44	H. Mitchell	July 28-30, 1858	HW.*-1 10	HW.*-1 10	N. 58 E.	LW. -1 43	S. 57 W.
NEW YORK LOWER BAY.	Main Channel. 40 28 49 74 00 28	H. L. Marindin	Aug. 4-5, July 8-9, 1887	LW.†+1 06	LW. +3 03	LW. +3 03	Not observed	HW. -0 09	HW. -0 09	HW. +2 53	Not observed
	Do. 40 28 57 74 00 49	H. Mitchell	Aug. 8-10, 1858	HW.†-2 13	HW.†-2 13	S. 86 W.	LW. -2 15	N. 85 E.
	Junction Main and Swash channels. 40 29 15 73 58 53	do	Aug. 22-23, 1858	HW.†-2 01	HW.†-2 01	N. 80 W.	LW. -2 35	S. 74 E.
	Gedney Channel. 40 29 18 73 57 51	H. L. Marindin	July 28-30, Aug. 5 and 9-10, 1887	LW.†+1 26	LW. +4 24	LW. +4 24	Not observed	HW. + 41	HW. + 41	HW. +3 38	Not observed
	North of Flynn's Knoll. 40 29 40 74 01 06	H. Mitchell	Sept. 7, 1858	HW.†-2 13	HW.†-2 13	N. 80 W.	LW. -2 56	S. 69 E.
Blind Channel between Eastern and Gedney channels.	Swash Channel. 40 30 36 74 00 48	do	Aug. 24-26, 1858	HW.†-1 42	HW.†-1 42	N. 32 W.	LW. -2 22	S. 63 E.
	Do. 40 30 33 74 01 12	H. L. Marindin	July 17, 29, 30, 1887	LW.†+ 46	LW. +3 58	LW. +3 58	Not observed	HW. + 48	HW. + 48	HW. +4 06	Not observed
	Do. 40 30 33 74 01 12	H. Mitchell	Aug. 12-13, 1858	HW.†-2 08	HW.†-2 08	N. 75 W.	LW. -1 57	S. 67 E.

Directions true unless otherwise noted.

* Tides at Governors Island.

† Tides at Sandy Hook.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Fbb	
				Time	Velocity	Time	Direction	Time	Velocity	Time	Direction
NEW YORK LOWER BAY—Cont'd.											
East Channel	40 30 51 73 57 55	H. L. Marindin	July 18-19, Aug. 9-10, 27-28, 1887	A. M. L.W. *+1 07	Knots 1.70	A. M. L.W. +4 31	Not observed	A. M. H.W. +1 01	Not observed	A. M. H.W. +4 22	Not observed
Do	40 31 26 74 00 25	do	June 29-30, July 19 and 28, 1887	L.W. *+1 09	2.13	L.W. +4 00	Not observed	H.W. +4 47	2.18	H.W. +4 54	Not observed
West entrance Swash Channel	40 31 33 74 02 19	H. Mitchell	Aug. 20-22, 1888		0.79	H.W. *—2 54	S. 69 W.		0.67	L.W. — 36	S. 45 E.
East Channel	40 31 44 74 00 56	do	Aug. 15-16, 1888		0.82	H.W. *—1 39	N. 51 W.		0.87	L.W. — 13	S. 58 E.
Fourteen Foot Channel	40 31 49 73 59 39	H. L. Marindin	June 20-24, July 13- 15, 1887	L.W. *+1 00	1.39	L.W. +3 58	Not observed	H.W. +1 21	1.94	H.W. +4 15	Not observed
Do	40 31 58 74 00 09	H. Mitchell	Aug. 16-18, 1888		0.96	H.W. *—2 00	N. 49 W.		1.27	L.W. —1 31	S. 66 E.
Off Elm Tree Beacon	40 33 11 70 05 00	do	June 30, July 1, 1889	L.W. †+1 12	0.35	H.W. — 41	N 30 E	H.W. +3 11	0.28	L.W. + 28	S. 72 W.
	40 33 42 74 01 45	do	Aug. 4-13, 1888		1.17	H.W. *— 23	N. 8 W.		2.27	L.W. — 03	S. 9 E.
	40 33 46 74 01 53	do	Aug. 15-18, 1888		0.94	H.W. *+ 06	N. 12 E.		1.43	L.W. + 22	S. 10 E.
	40 33 56 74 01 53	do	Aug. 18-20, 1888		0.81	H.W. *+ 41	N. 20 E.		1.36	L.W. + 39	S. 9 E.
Gravesend Bay	40 34 53(?) 74 01 09(?)	do	Aug. 28-29, 1888		0.37	H.W. *—2 34	N. 34 E.		0.54	L.W. —1 56	S. 69 E.
		do	July 30, Aug. 3, 1888								
The Narrows	40 36 10 74 02 35	H. L. Marindin	Sept. 15-16, 1888	L.W. *+3 23	1.0		N. 39 W.	H.W. +1 40	1.9		S. 31 E.
		G. C. Hannus	Oct. 4-6, 1886								

Directions true unless otherwise noted.

* Tides at Sandy Hook.

† Tides at Governors Island.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	<i>h. m.</i>	Time	Direction	Time	Direction	Time	Direction
NEW YORK LOWER BAY—Continued.											
The Narrows	40 36 15	H. Mitchell.	July 30-Aug. 3, 1858			<i>h. m.</i>	°	<i>h. m.</i>	°	<i>h. m.</i>	°
	74 02 19					HW. * - 0 57	N. 37 W.	1. 19	LW. - 44	S. 32 E. 1.73	
NEW YORK UPPER BAY.											
The Narrows	40 36 58	(H. Mitchell	July 30-Aug. 3, 1858								
	74 03 13					LW. † + 3 36		1. 2	HW. + 2 31	S. 16 E. 1.5	
Off Erie Basin	40 40 00	G. C. Hanus	Oct. 4-6, 1886								
	74 01 28					LW. * + 1 33	N. 18 E.	1. 24	HW. + 1 55	N. 20 E. 1.09	
Between Governors Island and Bedloe Island.	40 41 16	H. L. Marindin	Aug. 1872								
	74 02 03					HW. * + 0 3	N. 29 E.	1. 05	LW. - 0 33	S. 28 W. 1.71	
Off Castle Garden	40 42 10	Lieut. R. Wainwright.	Oct. 1855								
	74 01 12					HW. * - 50	S. 73 E. (?)	0. 63(?)	LW. + 1 56 (?)	S. 14 E. (?) 2. 10(?)	
HUDSON RIVER.											
Off Canal Basin	40 42 35	H. L. Marindin	Aug. 22, 1873								
	74 01 53					HW. * - 2 11	N. 17 E.	0. 67	HW. + 2 36	S. 15 W. 1.22	
Off Barclay street	40 42 55	Schooner Madison.	June 27-28, 1854								
	74 01 17					LW. * + 3 45	N. 24 E.	1. 87	HW. + 2 53	S. 14 W. 2.52	
	40 43 11	H. L. Marindin	Aug. 21, 22, 23, 1873								
	74 01 44					HW. * - 3 03	N. 13 E.	0. 87	HW. + 2 10	S. 11 W. 1.38	
Off Pier 45	40 43 43	do	Aug. 29 and Sept. 1, 1873								
	74 01 04					HW. * - 2 31	N. 10 E.	2. 07	HW. + 3 37	S. 7 W. 1.44	
Off Charlton street	40 43 43	Schooner Madison.	June 29-30, 1854								
	74 01 07					HW. * - 2 22	N. 18 E.	1. 74	LW. - 2 45	S. 8 W. 2.31	

Directions true unless otherwise noted.

* Tides at Governors Island.

† Tides at Sandy Hook.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	h. m.	Time	h. m.	Time	h. m.	Time	h. m.
HUDSON RIVER—cont'd.											
Off Pier 9, Jersey City	40 43 44	H. L. Marindin	Aug. 23 and 25, 1873	HW. * -2 54	HW. + 00	N. 4 E.	0.80	HW. +2 02	HW. -5 40	S. 12 W.	1.42
.....	74 01 38	do	Aug. 25-27, 1873	HW. * -2 27	HW. + 17	N. 20 E.	0.93	HW. +2 02	HW. -6 01	S. 18 W.	1.78
Off Seventeenth street	40 44 48	Schooner Madison.	July 1-2, 1854	HW. * -2 24	HW. + 04	N. 21 E.	1.78	LW. -3 14	LW. - 31	S. 20 W.	2.32
Off Thirty-second street.	40 45 29	do	July 5-6, 1854	HW. * -1 12	HW. +1 08	N. 18 E.	1.94	LW. -3 21	LW. + 42	S. 48 W.	2.74
Off Forty-first street.	40 45 50	H. Mitchell	Sept. 4-5, 1858	HW. * + 00	N. 27 E.	1.75	LW. + 07	S. 32 W.	2.32
.....	74 00 24	H. L. Marindin	Sept. 13-14, 1872	HW. * -2 17	HW. + 03	N. 34 E.	1.04	HW. +2 42	LW. 18	S. 26 W.	2.49
Off Forty-second street.	40 45 52	F. F. Nes	Sept. 13-14, 1872	HW. * -2 26	HW. + 22	N. 38 E.	1.36	LW. -3 24	LW. + 01	S. 34 W.	2.34
Jersey shore, opposite Forty-first street.	40 45 58	H. Mitchell	Sept. 13, 1872	HW. * + 02	N. 30 E.	1.13	LW. - 32	S. 29 W.	1.50
Off One hundred thirty-first street.	40 49 11	Lieut. R. Wright.	Aug. 17-18, 1855	HW. * -0 51	HW. +2 10	N. 38 E.	0.57	HW. +3 11	LW. +1 15	S. 18 W.	2.19
DELAWARE BAY.											
Off Cape Henlopen	38 49 47	H. L. Marindin	Aug. 24-28, 1886	LW. +1 53	LW. +4 25	N. 59 W.	1.42	HW. +1 23	HW. +5 12	S. 34 E.	2.24
New Castle	39 39 20	do	Aug. 2-9, 1886	LW. +1 17	LW. +1 24	3.07	HW. - 03	HW. +2 42	2.20
Philadelphia	39 56 52	F. A. Kimmel	Jan. 25 to July 1, 1902	LW. +1 38	LW. +3 00	N. 27 E.	1.3	HW. +1 04	HW. +3 40	S. 5 W.	1.3
Petty Island	39 57 31	H. L. Marindin	July 20-31, 1886	LW. +1 39	LW. +2 59	1.86	HW. +1 15	HW. +4 54	1.86

Directions true unless otherwise noted.

* Tides at Governors Island.

† Tides at Sandy Hook.

‡ Tides at Philadelphia.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	Velocity	Time	Direction	Time	Velocity	Time	Direction
CHESAPEAKE BAY.	0 1 "										
Off Wolf Trap Light Boat.	37 23 39	Lieut. J. I. Almy.	Nov. 17-19, 1851	H. m. HW. $\frac{1}{2}$ +0 35	0	H. m. HW. +2 50	N. 1 E.	H. m. LW. -0 50	0.5	H. m. LW. +3 00	0
Baltimore Harbor	39 09 59										
	39 15 55	F. A. Kummell	Jan.-June, 1903	LW. $\frac{1}{2}$ +0 05	N. 24 W.	LW. +3 30	N. 24 W.	HW. +0 30	0.10	HW. +3 05	S. 24 E.
CAPE FEAR RIVER.	36 34 29										
Off mouth of Elizabeth River.	33 54 14	W. I. Vinal	Feb. 16, 1872	HW. $\frac{1}{2}$ - 2 41	N. 40 W.	HW. - 54	N. 40 W.	HW. +0 35	0.36	LW. -1 30	N. 89 E.
SAVANNAH RIVER.	32 02 19										
Near Fort Pulaski	32 02 11	H. L. Marindin	May 25-29, June 1, 1874		N. 77 W.	HW. $\frac{1}{2}$ -1 53	N. 77 W.		1.40	HW. +2 50	S. 85 E.
Do	32 02 11	do	May 6-7, 1874		S. 85 W.	HW. $\frac{1}{2}$ -2 04	S. 85 W.		1.06	HW. +2 37	S. 89 E.
Off northeast corner of Hutchinson Island.	32 07 33	J. N. Maffitt	Mar. 23-24, 1852		N. 30 W.	HW. $\frac{1}{2}$ -1 10	N. 30 W.		0.74	LW. -1 17	S. 58 E.
Northwest of Kings Island.	32 07 30	do	Mar. 23, 1852		S. 67 W.	HW. $\frac{1}{2}$ -1 14	S. 67 W.		0.42	LW. -1 38	N. 76 E.
FLORIDA.	31 07 58										
Fernandina Bar	30 42 33	F. D. Granger	Apr. 21-22, 1874	LW. $\frac{1}{2}$ +1 18	S. 88 W.	LW. +3 38	S. 88 W.	HW. +0 27	0.53	LW. -2 27	N. 54 E.
Do	31 24 39	do	May 5, 7, 1874	LW. $\frac{1}{2}$ +0 19	N. 16 W.	HW. -2 36	N. 16 W.	HW. +0 32	0.56	LW. -1 56	S. 52 E.
Do	31 23 20	do	Apr. 23-24, 1874	LW. $\frac{1}{2}$ +0 25	N. 28 W.	LW. +3 25	N. 28 W.	HW. +0 32	1.02	HW. +3 55	S. 44 E.
Do	30 41 48	do	Apr. 18, May 1-2, 1874	LW. $\frac{1}{2}$ +0 09	N. 66 W.	LW. +3 00	N. 66 W.	HW. +0 12	0.60	LW. -2 33	S. 80 E.

Directions true unless otherwise noted.

* Tides at Old Point Comfort.

† Tides at Baltimore.

‡ Tides at Smithville (Southport), N. C.

§ Tides at Fort Pulaski, Savannah River, Ga.

¶ Tides at Savannah, Ga.

** Tides at Old Fernandina, Fla.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	<i>h. m.</i>	Time	<i>h. m.</i>	Time	<i>h. m.</i>	Time	Direction
SAN FRANCISCO BAY.											
Off Presidio	37 48 28 122 27 37	G. Bradford	Jan. 6-8, 1875	T.*+6 39		T. + 8 07		T. - 1 22		T. + 2 47	N. 84 W.
Raccoon Straits	37 52 23 122 26 06	do	Nov. 21-23, 1871	LW.†+1 17 LW.†+1 22		HW. - 2 34 T. + 9 13		HW. + 0 30 HW. + 0 49		LW. - 1 51 T. + 3 23	S. 48 W. N. 81 W.
Between Black Point and Anita Rock Spindle.	37 48 25 122 27 04	do	Jan. 4-6, 1875	T.*+7 10		T. + 8 41		T. - 1 03		T. + 2 11	S. 81 W.
Off Black Point	37 48 33 122 26 02	do	Nov. 12-14, 1874	T.*+5 58							
North end of Southamp- ton Shoal.	37 54 35 122 25 13	do	Feb. 26-28, 1873	LW.†+1 53 T.*+6 59		T. + 10 30		HW. + 1 26 T. + 45		T. + 3 46	S. 14 E.
Between north entrance of Raccoon Strait and Southampton Shoal.	37 52 53 122 25 13	do	Nov. 16-18, 1871	LW.†+2 40 LW.†+2 22		HW. - 1 08		HW. + 1 59 HW. + 2 11		LW. - 38	S. 16 E.
Off Southampton Shoal.	37 53 49 122 25 08	do	May 29-31, 1873	LW.†+2 45 T.*+7 37				HW. + 1 44 T. + 1 00		T. + 3 54	S. 16 E.
Off southeast point of Angel Island.	37 50 57 122 25 06	do	Nov. 7-9, 1871	LW.†+1 39 LW.†+1 30		T. + 10 34		HW. + 33 HW. + 48			S. 40 W.
Between North Point and Angel Island.	37 50 20 122 24 54	do	Nov. 7-9, 1871	LW.†+1 26 LW.†+2 20				HW. + 48 HW. + 48			N. 60 W.
Off Southampton Shoal.	37 52 54 122 24 37	do	Apr. 17-19, 1873	LW.†+2 47 T.*+7 54		LW. + 4 42		HW. - 27 HW. + 1 41		HW. + 3 55	N. 79 W.
Off Shore between Black Point and Alcatraz.	37 48 41 122 25 34	do	Nov. 10-12, 1874	T.*+7 54 T.*+6 05		T. + 11 05 T. + 9 03		T. + 1 15 T. + 12 11		T. + 4 33* T. + 2 49	S. 3 W. West.
Off Southampton Shoal.	37 53 09 122 23 31	do	Feb. 20-22, 1873	LW.†+3 05 LW.†+3 05		HW. - 38		HW. + 1 52 HW. + 1 25		LW. - 22	S. 9 E.
Off North Point	37 48 30 122 24 25	do	Mar. 20-22, 1873	T.*+6 05		T. + 8 38		T. + 11 22		T. + 2 02	N. 40 W.

Directions true unless otherwise noted.

* Local transits.

† Tides at North Point, San Francisco Bay.

‡ Tides at Angel Island, San Francisco Bay.

§ Tides at Fort Point, San Francisco Bay.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack	Ebb	
				Time	<i>h. m.</i>	Time	Direction		Time	Direction
SAN FRANCISCO BAY—continued.	° ' "			<i>h. m.</i>	<i>h. m.</i>	°	<i>Knobs</i>	<i>h. m.</i>		<i>Knobs</i>
South of middle of Southampton.	37 53 27 122 24 28	G. Bradford	Feb. 12-14, 1873	L.W. * + 51	HW. - 1 38	N. 19 E.	0.66	HW. + 1 20	S. 9 W.	1.21
One-eighth mile north of North Point.	37 48 23 122 24 24	do	Nov. 2-7, 1871	L.W. + 24	L.W. + 22	S. 60 E.	1.35	HW. - 15 HW. - 12	N. 60 W.	1.55
East of Angel Island.	37 51 50 122 24 18	do	Jan. 20-22, 1875	L.W. * + 3 42	T. + 11 16	N. 12 W.	2.20	HW. + 2 21 T. + 1 47	S. 6 E.	2.46
Southampton Shoal, south end.	37 53 04 122 24 12	do	Feb. 10-12, 1873	L.W. * + 1 25	HW. - 1 08	N. 13 E.	1.14	HW. + 1 30 HW. + 2 10	S. 10 W.	1.14
Between Point Richmond and Southampton Shoal.	37 54 10 122 24 10	do	Feb. 24, 26, 1873	L.W. + 2 49 T. + 4 8 13	T. + 11 12	N. 32 W.	1.95	T. + 1 40	S. 28 E.	2.05
Between North Point and Alcatraz.	37 48 40 122 25 03	do	Oct. 29-31, 1874	T. + 5 55	T. + 9 05	S. 78 E.	1.78	T. + 11 58	N. 76 W.	2.02
Off Telegraph Hill	37 48 08 122 23 53	do	Oct. 27-29, 1874	T. + 5 43	T. + 8 01	S. 38 E.	2.32	T. + 10 51	N. 43 W.	2.40
Between City and Shoals north of Goat Island.	37 47 46 122 23 29	do	Oct. 15-17, 1874	T. + 5 21	T. + 7 59	S. 37 E.	1.50	T. + 10 28	N. 28 W.	1.45
Between Mission Rock and Rincon Point.	37 48 34 122 23 17	do	July 30-Aug. 1, 1874	L.W. * + 2 03 L.W. * + 2 01	L.W. + 4 36 T. + 8 58	S. 44 E.	1.4	HW. + 45 HW. + 40	N. 31 W.	1.8
Northwest of Mission Rock.	37 47 01 122 23 09	do	Sept. 9-11, 1874	T. + 6 07 L.W. * + 1 05	T. + 8 50 T. + 8 51	S. 5 E.	1.88	HW. - 06 HW. - 40	N. 1 W.	2.20
Between Mission Rock and Merrimac Street Wharf.	37 46 11 122 23 03	do	Aug. 24-27, 1874	T. + 6 08 L.W. * + 50	T. + 8 51 T. + 8 39	S. 11 E.	1.42	T. + 11 38 HW. - 39	N. 11 E.	1.58
			Sept. 22-24, 1874	T. + 6 11	T. + 8 39	S. 30 E.	1.88	T. + 11 24	N. 8 W.	1.85

Directions true unless otherwise noted.

*Tides at Fort Point, San Francisco Bay.

†Tides at North Point, San Francisco Bay.

Local transits.

||| Tides at current station.
||| Tides at Angel Island, San Francisco Bay.

Tides at Angel Island, San Francisco Bay.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack		Flood		Slack		Ebb	
				Time	Velocity	Time	Direction	Time	Velocity	Time	Direction
SAN FRANCISCO BAY— continued.	0 1 "	G. Bradford	Sept. 7-9, 1874	A. m.	Knots	A. m.	0	A. m.	0	A. m.	0
				LW. #+1 15	2.00	HW. + 05	S. 8 E.	HW. + 05	2.18	T. +2 57	N. 1 W.
				T. +6 33	1.40	HW. - 07	S. 24 E.	T. +11 21	1.80	T. +2 49	N. 7 W.
				LW. #+1 15	2.38	HW. + 02	S. 27 E.	HW. + 02	2.15	T. +1 57	N. 2 W.
				T. +5 56	1.98	HW. + 53	S. 45 E.	T. +12 28	2.08	T. +2 26	N. 8 W.
				LW. #+1 23	2.10	HW. + 38	S. 25 E.	HW. + 38	2.18	T. +2 35	N. 25 W.
				T. +5 35	1.58	HW. + 10	S. 21 E.	T. +12 46	1.40	T. +2 58	N. 42 W.
				LW. #+1 56	1.58	HW. + 38	S. 53 E.	HW. + 38	1.68	T. +3 55	N. 25 W.
				T. +6 15	1.80	HW. - 10	S. 25 E.	T. +11 18	2.12	T. +2 15	N. 8 W.
				LW. #+0 59	0.34	HW. + 23	S. 87 W.	HW. + 23	0.29	LW. -2 30	N. 43 E.
CALIFORNIA COAST		A. P. Osborn	Jan. 7-8, 1897	A. m.	Knots	A. m.	0	A. m.	0	A. m.	0
				LW. #+1 11	0.34	HW. - 3 32	S. 12 W.	HW. + 28	0.32	LW. -2 18	N. 64 E.
				T. +6 31	0.89	HW. -2 19	N. 70 E.	HW. - 13	0.79	LW. - 49	West.
				LW. #+1 01	2	HW. -2 22	N. 64 W.	HW. - 13	2	LW. - 49	S. 60 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				LW. #+1 12	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				LW. #+1 12	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				LW. #+1 12	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
ALASKA COAST		C. M. Thomas	July 24, Aug. 5-7, 1887.	A. m.	Knots	A. m.	0	A. m.	0	A. m.	0
				LW. #+1 12	0.89	HW. -2 22	N. 70 E.	HW. - 13	0.79	LW. - 49	West.
				T. +5 41	2	HW. -2 22	N. 64 W.	HW. - 13	2	LW. - 49	S. 60 E.
				LW. #+1 12	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				LW. #+1 12	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				LW. #+1 12	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				LW. #+1 12	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.
				T. +5 41	1 to 6	HW. -2 22	N. 34 W.	HW. - 13	1 to 6	LW. - 49	S. 26 E.

Directions true unless otherwise noted. * Tides at Fort Point. † Local Transits. ‡ Tides at Sausalito, California. § Tides at Astoria, Oregon.

Table of slack waters and mean maximum velocities—Continued.

Station	Latitude and longitude	Observing party	Date	Slack			Flood			Ebb		
				Time	h. m.	Direction	Time	h. m.	Direction	Time	h. m.	Direction
NEW ZEALAND Tory Channel	41 13 ..	Light keeper	May 15, 29,	A. m.	T. *+2 13	o	A. m.		o	A. m.		o
	174 20 ..		June 14, 28, July 13, 27, 1900									
French Pass	40 55 30	do	do	T. *+1 19						T. +7 13		
Stephens Island	40 40 ..	do	do	T. *+2 33						T. +8 07		
	174 01 ..											

Directions true unless otherwise noted.

* Local transits.

96. Table showing hourly values of duration (azimuth) and velocity of the current.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
GULF OF MAINE.											
Nantucket Shoals Light Ship.	40 37 00	R. L. Faris.....	Sept. 9-10, 1903	HW.*	198	204	205	220	242	308	38
	69 37 15			I.W.*	0.4	0.4	0.4	0.4	6.3	0.1	0.2
Georges Bank.....	41 10 04	Robert Platt, U. S. Navy	June 6-10, 1877	HW.*	181	181	185	206	284	340	353
	68 55 35			I.W.*	1.1	1.1	0.9	0.6	0.4	0.6	0.9
Do.....	41 20 46do.....	June 21-24, 1877	HW.*	171	188	205	250	290	322	341
	68 23 07			I.W.*	1.6	1.3	0.9	0.6	0.9	1.5	1.7
Do.....	41 31 00do.....	June 25-27, 1877	HW.*	179	196	226	266	309	334	3
	67 52 30			I.W.*	1.7	1.2	0.9	0.8	1.4	1.6	1.4
Do.....	41 36 38do.....	Aug. 28-29, 1877	HW.*	164	178	215	262	308	332	342
	67 24 12			I.W.*	1.2	0.9	0.7	0.8	1.2	1.6	1.6
Off Chatham Lights.	41 37 30	H. Mitchell.....	Aug. 19-20, 1857	HW.*	187	16	11	5	9	16
	69 51 50			I.W.*	192	191	190	189	188	187
Georges Bank, Georges Shoals.	41 37 57	J. A. Howell.....	Aug. 22-23, 1872	HW.*	182	208	235	260	290	330	0
	67 43 30			I.W.*	1.8	1.8	2.0	1.4	1.4	1.3	1.9
Off Chatham Lights.	41 40 40	H. Mitchell.....	Aug. 20, 1857	HW.*	150	40	35	30	20	6
	69 52 12			I.W.*	0.2	0.6	0.9	0.7	0.4	0.1	0.0
Do.....	41 40 45	Robert Platt, U. S. Navy.	Sept. 14-15, 1857	HW.*	176	30	17	10	9	13	15
	69 45 40			I.W.*	0.2	0.2	0.6	0.8	0.9	0.8	0.6
About 7 mi. E. of Nantsett Lights.	41 52 40do.....	Sept. 12-13, 1877	HW.*	270	315	315	325	330	330	340
	69 48 00			I.W.*	0.2	0.2	0.3	0.6	0.8	1.0	1.0
Georges Bank.....	41 56 39do.....	July 15-17, 1877	HW.*	154	181	204	238	272	307	336
	66 38 15			I.W.*	1.8	1.5	1.0	1.0	1.3	1.6	1.8
Do.....	41 58 00do.....	Aug. 30-31, 1878	HW.*	142	146	148	318	339	346	349
	69 26 00			I.W.*	0.8	0.5	0.2	0.1	0.3	0.5	0.6
					348	351	355	72	135	146	144
					0.6	0.4	0.3	0.2	0.5	0.8	0.8

* Tides at Boston.

Directions true.

Table showing hourly values of duration (azimuth) and velocity of the current. —Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
GULF OF MAINE—CON.					°kn	°kn	°kn	°kn	°kn	°kn	°kn
5½ mi. E. ½ N. of Cape Cod Light	42 04 00	Robert Platt, U.S. Navy	Aug. 23-24, 1877	HW.*	70	0	320	325	315	305	295
	LW.*			0.3	0.2	0.4	0.5	0.6	0.7	0.8	
3½ mi. N. ½ W. of Race Point	42 07 04	do	Aug. 24-25, 1877	HW.*	70	80			250	240	245
	LW.*			0.7	0.5			0.7	0.8	0.8	
Stellwagen Bank	42 17 15	do	Aug. 30-31, 1877	HW.*	80	100	100	200(?)	235	240	260
	LW.*			0.5	0.4	0.5	0.2(?)	0.3	0.6	0.6	
BOSTON HARBOR †					0.6	0.5	0.3	0.2	0.5	0.6	0.5
Nantasket Roads	42 19 09	C. H. Davis	Oct. 6, 12, 1848	HW.*	78	78	70	75	208	202	200
	LW.*			1.8	1.7	1.2	0.1	0.6	1.4	2.0	
Hypocrite Channel	42 41 09	do		HW.*	200	202	200	315	75	77	78
	LW.*			2.0	1.8	1.9	0.1	0.9	1.6	1.8	
South Channel	42 20 35	H. Mitchell	Aug. 30-31, 1860	HW.*	39	42	45	240	239	239	240
	LW.*			1.1	0.8	0.4	0.1	0.6	1.0	1.1	
Broad Sound	42 22 57	C. H. Davis	Oct. 11, 1848	HW.*	240	242	245	60	55	43	39
	LW.*			1.1	0.8	0.3	0.1	0.6	1.0	1.1	
President Roads	42 19 54	do	Oct. 16, 1848	HW.*	75	76	70	77	239	241	243
	LW.*			1.5	1.3	0.9	0.1	0.8	1.5	1.8	
Off East Boston	42 21 33	do	June 16, 1848	HW.*	243	244	244	245	70	75	75
	LW.*			1.8	1.8	1.4	0.1	0.9	1.0	1.5	
GULF OF MAINE					13	10	357		209	274	260
Georges Bank	42 24 46	Robert Platt, U.S. Navy	July 19, 1877	HW.*	0.4	0.4	0.3	0.0	0.2	0.3	0.4
	LW.*			258	252	278	344	356	6	13	
Do	42 27 09	do	Aug. 15-16, 1878	HW.*	0.4	0.2	0.1	0.1	0.2	0.3	0.4
	LW.*			71	75	82	86	254	250	255	
	67 37 45			HW.*	1.3	1.3	0.9	0.2	0.5	0.9	1.0
	LW.*			254	240	230	75	71	79	71	
				HW.*	1.0	0.7	0.2	0.3	0.7	0.9	1.2
	LW.*			105	152	139	130	205	301	307	
				HW.*	1.0	1.0	0.7	0.1	0.4	0.7	0.9
	LW.*			309	313	320	325	160	169	168	
GULF OF MAINE					1.0	1.0	0.8	0.2	0.5	0.8	1.0
Georges Bank	42 24 46	Robert Platt, U.S. Navy	July 19, 1877	HW.*	130	110	60	358	333	327	343
	LW.*			0.7	0.6	0.3	0.2	0.4	0.9	1.0	
Do	42 27 09	do	Aug. 15-16, 1878	HW.*	347	37	85	117	137	140	130
	LW.*			0.9	0.6	0.3	0.7	0.7	0.7	0.8	
	67 37 45			HW.*	161	182	227	266	317	330	349
	LW.*			0.8	0.5	0.4	0.4	1.0	1.4	1.5	
				HW.*	351	16	45	74	114	144	160
	LW.*			1.5	1.0	0.7	0.4	0.4	0.7	0.8	

Directions true unless otherwise noted.

*Tides at Boston.

† For 47 additional stations in Boston Harbor, see Coast and Geodetic Survey Tide Tables for 1903.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—				Solar hours after—			
					3	2	1	0	1	2	3	
GULF OF MAINE—CON.												
Georges Bank	42 28 39 66 03 30	Robert Platt, U.S. Navy.	Aug. 13-14, 1877.	HW.*	125	125	131	149	178	215	278	
				I.W.*	305	3	49	102	121	124	125	
Off Thatcher Island..	42 33 11 70 31 17			HW.*	60	50	30	310	290	280	280	
				I.W.*	290	300	330	10	40	50	60	
Georges Bank	42 50 00 65 56 30do	Aug. 8-9, 1877.	HW.*	132	156	214	277	298	304	308	
				I.W.*	309	313	327	8	72	106	130	
Do	43 04 00 65 40 40			HW.*	97	118	173	242	276	292	316	
				I.W.*	326	354	33	68	83	90	93	
Do	43 47 23 67 37 30do	Aug. 23, 1878.	HW.*	228	214	205	242	285	312	340	
				I.W.*	343	310	292	267	238	240	220	
Portsmouth Harbor (S. 77° W. of Whale-back light).	43 03 30 70 42 10			HW.†	175	176	177	178	179	180	12	
				I.W.†	12	14	15	16	17	18	175	
Portsmouth Harbor (S. 78° E. of Portsmouth light).	43 04 15 70 42 19do	Oct. 6, 24, 1898.	HW.†	152	160	168	174	178	173	0	
				I.W.†	358	349	342	343	7	3	148	
Portsmouth Harbor (N. 5° W. of Portsmouth light).	43 04 35 70 42 37			HW.†	90	101	117	127	135	288	
				I.W.†	290	295	294	286	275	263	87	
Portsmouth Harbor (N. 25° W. of Portsmouth light).	43 04 40 70 42 50do	Sept. 26, 29-30, Oct. 10, 1898.	HW.†	71	77	83	89	94	100	235	
				I.W.†	237	244	249	249	245	238	70	
Portsmouth Harbor (S. of Clark Island).	43 04 31 70 43 31			HW.†	88	86	84	83	81	79	261	
				I.W.†	262	264	264	263	259	253	89	
Portsmouth Harbor (off Goat Island Ledge buoy).	43 04 27 70 43 59do	Oct. 7, 1898.	HW.†	88	87	86	85	84	83	268	
				I.W.†	268	267	266	265	264	263	88	
Portsmouth Harbor (S. of Portsmouth Navy-Yard).	43 04 35 70 44 28			HW.†	137	135	132	125	128	125	305	
				I.W.†	306	311	315	317	316	315	138	

Directions true unless otherwise noted.

*Tides at Boston.

†Tides at Portland.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
NANTUCKET SHOALS.	0 1 "				0/kn	0/kn	0/kn	0/kn	0/kn	0/kn	0/kn
NW. side Fishing Rip.	41 02 (?) 69 28 (?)	Lieut. C. H. Mc- Blair.	July 8-9, 1852.	HW.*	188	204	221	222	227	259	300
				LW.*	0.7	1.2	1.4	1.0	0.8	0.8	1.0
					308	341	6	22	66	187
Nantucket Shoals	41 00 (?) 69 27 (?)do.....	July 9-11, 1852.	HW.*	1.0	1.4	1.6	1.5	1.0	0.5	0.6
				LW.*	78	197	210	229	240	256	265
					1.0	0.2	1.3	1.9	1.7	1.5	1.3
Do	41 01 (?) 69 27 (?)do.....	July 10-11 1852.	HW.*	279	313	5	38	43	40	56
				LW.*	1.3	0.9	1.5	1.8	1.6	1.3	0.8
Do	41 08 08 69 31 48do.....	Aug. 24-25, 1852.	HW.*	186	209	222	236	244	254	256
				LW.*	0.3	0.7	1.3	2.2	2.2	1.2	0.8
					281	326	13	25	23	42	146
					0.7	0.5	0.8	1.2	1.2	0.4	0.3
Do	41 02 (?) 69 34 (?)do.....	Aug. 14-16, 1852.	HW.*	126	166	176	196	216	222	234
				LW.*	0.4	1.7	2.4	2.8	2.5	2.1	1.2
					247	299	356	20	35	36	108
					1.2	1.0	1.6	2.4	2.2	1.3	0.4
Do	41 05 .. 69 37do.....	Aug. 16-18, 1852.	HW.*	213	204	217	220	238	292	320
				LW.*	1.1	1.6	1.8	1.7	1.3	0.9	0.9
					326	358	18	32	66	105	159
					1.0	1.3	1.4	1.3	0.9	0.7	1.0
Do	41 08 38 69 39 51	Lieut. Vreeland ..	Sept. 21-22, 1891.	HW.†	193	206	227	252	238	245	257
				LW.†	1.3	2.0	2.0	1.9	1.6	1.0	0.9
					326	10	34	45	68	95	189
					0.9	1.5	2.2	2.0	1.1	0.6	1.2
Do	41 12 00 69 40 00do.....	Sept. 10-12 and 15-16, 1891.	HW.†	202	210	255	5	20	20	30
				LW.†	2.6	1.9	0.8	0.4	2.0	2.2	2.0
					30	40	180	195	200	202
					2.0	1.3	0.0	1.0	1.7	2.1	2.6
Do	41 12 18 69 40 31do.....	Sept. 16-17, 1891.	HW.†	205	215	250	300	340	10	30
				LW.†	1.1	0.8	0.4	0.4	0.8	1.3	1.6
					35	45	180	190	200	205
					1.6	1.1	0.6	1.2	1.3	1.1
Off Sankaty Head	41 22 10 69 42 34do.....	Sept. 5-7, 1891.	HW.†	220	230	270(?)	10	25	25
				LW.†	1.4	1.1	0.4	1.1	1.8	1.6
					25	25	100(?)	200	200	210	220
					1.6	0.9	0.2	1.0	1.7	1.7	1.4
Nantucket Shoals	41 12 30 69 43 24	Lieut. C. H. Mc- Blair.	July 5-7, 1852.	HW.*	187	205	350	15	15	15	25
				LW.*	1.6	0.6	0.4	1.1	1.5	1.3	0.9
					25	20	150	160	180	185	187
					0.8	0.4	0.2	0.9	1.7	1.8	1.6
SW. end Great Rip	41 12 (?) 69 45 (?)	Lieut. C. H. Davis.	Aug. 30-31, 1848.	HW.†	198	212	218	222	235	251	308
				LW.†	0.5	1.2	1.6	1.5	1.2	0.7	0.7
					324	7	16	19	25	24	196
					0.9	1.6	1.8	1.5	1.0	0.3	0.4
					232	245	251	26	58	64	55
					2.1	1.5	0.4	0.6	1.4	1.8	1.2
					55	72	171	200	215	224	230
					1.2	0.8	0.4	0.7	1.6	2.1	2.2

Directions true unless otherwise noted.

* Tides at Governors Island.

† Tides at Boston.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
NANTUCKET SHOALS—continued.					° kn	° kn	° kn	° kn	° kn	° kn	° kn
NE. of Davis South Shoal.	41 00 (?) 69 50 (?)	Lieut. J. N. Maffitt	Aug. 15-17, 1849.	HW.*	293	303	308	325	10	52	64
				LW.*	2.7	2.3	2.0	1.7	1.3	1.8	2.3
					65	69	82	127	212	259	293
Nantucket Shoals	41 04 41 69 50 25	Lieut. C. H. Davis	Aug. 6-8, 1846.	HW.*	274	282	318	46	94	52	94
				LW.*	2.0	1.3	0.8	0.8	1.7	2.3	2.4
					95	100	136	214	252	250	274
Do	41 11 (?) 69 51 (?)	Lieut. C. H. McBlair	July 7-8, 1852.	HW.†	146	192	213	226	229	270	330
				LW.†	0.7	1.2	1.8	1.7	1.4	1.1	1.2
					344	12	14	38	57	125	143
Do	41 21 25 69 51 22	do	Aug. 25-26, 1852	HW.†	175	200	215	228	233	253	288
				LW.†	0.4	1.4	2.2	2.2	1.7	1.4	0.5
					310	16	40	44	42	76	144
					0.5	1.2	1.8	1.6	1.7	0.9	0.3
Do	41 01 18 69 51 26	Lieut. C. H. Davis	Aug. 12-14, 1846.	HW.*	277	288	287	305	35	89	96
				LW.*	1.9	1.7	1.3	1.0	1.1	1.3	1.5
					98	104	136	201	243	261	277
Do	41 05 16 69 56 00	do	Aug. 14-15, 1846.	HW.*	1.5	1.3	0.9	0.5	0.9	1.6	1.9
				LW.*	242	257	257	300	15	66	79
					1.5	1.6	1.4	1.0	0.7	0.8	1.2
				LW.*	84	103	121	148	200	226	242
					1.2	1.1	0.9	0.6	0.7	1.1	1.5
Do	41 17 26 69 55 39	Lieut. C. H. McBlair	Aug. 23-25, 1852.	HW.†	130	160	191	219	225	230	254
				LW.†	1.0	1.0	1.2	1.5	1.5	1.3	0.9
					278	312	356	37	54	64	125
					0.8	0.7	0.9	1.5	1.6	1.2	1.0
Do	41 03 57 69 54 07	Lieut. C. H. Davis	Aug. 6-7, 1846.	HW.*	272	284	306	5	40	67	80
				LW.*	1.7	1.4	1.2	1.5	1.7	1.9	1.7
					85	109	154	203	230	249	270
					1.5	1.0	0.8	1.2	1.6	1.8	1.7
Do	41 11 18(?) 69 58 16(?)	Lieut. J. N. Maffitt	Aug. 5-7, 1849.	HW.*	256	258	265	274	0	41	70
				LW.*	1.4	1.5	1.1	0.8	0.6	0.9	1.3
					73	91	121	147	209	233	256
					1.4	1.4	1.2	0.9	0.8	1.2	1.4
Near Nantucket	41 11 21 69 58 16	Lieut. C. H. Davis	Aug. 23, 1847	HW.*	242	247	270	28	38	41	47
				LW.*	1.7	1.2	0.3	0.4	1.0	1.6	1.5
					46	50	...	205	232	238	242
					1.3	0.8	0.0	0.7	1.3	1.8	1.7
Do	41 25 27 70 01 57	H. Mitchell	Aug. 4-5, 7-8, 1857.	HW.*	242	251	261	20	66	72	85
				LW.*	1.4	0.8	0.3	0.3	0.8	1.4	1.4
					85	85	123	225	225	225	242
					1.3	0.8	0.4	0.8	1.5	1.5	1.4

Directions true unless otherwise noted.

*Tides at Boston.

†Tides at Governors Island.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—				Solar hours after—			
					3	2	1	0	1	2	3	
NANTUCKET SOUND.	0 1 "				° kn	° kn	° kn	° kn	° kn	° kn	° kn	
Off Shovelful, S. of Powder Hole.	41 32 00 70 01 02	Lieut. M. Woodhull.	Aug. 28-29, 1852.	HW.*	330	30	100	100	90	85	85	
				L.W.*	0.9	0.6	1.2	2.3	2.6	2.2	1.3	
					85	0	294	310	310	320	330	
Off Great Point.	41 24 15 70 03 40	do	Aug. 7-8, 1852.	HW.*	1.2	0.1	1.2	1.7	1.7	1.0	0.9	
				L.W.*	240	240		65	65	70	75	
					0.9	0.5	0.0	0.2	0.6	1.2	0.9	
					75		215	230	240	240	240	
N. of Handkerchief Shoal.	41 33 52 70 03 43	do	Sept. 1-2, 1852.	HW.*	0.9	0.0	0.3	0.6	0.7	0.8	0.9	
				L.W.*	285	260	160	140	150	150	165	
					0.5	0.1	0.3	0.4	0.4	0.3	0.1	
					0.0	0.0	250	285	285	285	285	
Nantucket Sound.	41 27 00 70 05 27	H. Mitchell.	July 5-6, 1857.	HW.*	0.0	0.1	0.3	0.5	0.6	0.6	0.5	
				L.W.*	273	284		67	70	72	81	
					1.4	0.9	0.0	0.6	1.0	1.2	1.0	
Do	41 21 31 70 19 55	Lieut. M. Woodhull.	Aug. 17-18, 1852.	HW.*	81	87		235	262	273	273	
				L.W.*	0.9	0.6	0.0	0.5	1.0	1.3	1.4	
					237	240	80	70	70	70	75	
					0.9	0.6	0.3	1.1	1.3	1.1	0.9	
					75	75		237	240	238	238	
Between Horseshoe Shoal and Bishop and Clerks Lightship.	41 32 20 70 15 10	H. Mitchell	July 13-14, 1857.	HW.*	0.9	0.5	0.0	0.5	1.0	1.2	0.9	
				L.W.*	295	255	300	100	81	81	81	
					1.0	0.8	0.3	0.4	0.8	0.9	0.8	
					81	50	20		210	200	295	
Between Cross Rip Light and Horseshoe Shoal.	41 27 13(?) 70 17 28(?)	do	July 8-9, 1857.	HW.*	0.8	0.7	0.4	0.0	0.6	0.8	1.1	
				L.W.*	261	261	261		81	81	81	
					1.4	0.9	0.5	0.0	1.0	1.1	0.9	
					81	81	81		250	255	261	
					0.9	0.8	0.4	0.0	0.3	0.9	1.4	
Between Longs and Nortons Shoal.	41 25 49 70 21 46	do	July 17-18, 1857.	HW.*	239	245	290	36	50	60	70	
				L.W.*	1.1	0.6	0.2	0.5	0.9	1.3	1.4	
					70	70	90	150	216	216	230	
					1.4	1.1	0.7	0.4	0.8	1.0	1.1	
Nantucket Sound.	41 20 08(?) 70 25 40(?)	do	July 19-20, 1857.	HW.*	194	210	216	36	36	36	36	
				L.W.*	2.8	2.2	0.6	0.6	3.1	3.1	2.8	
					36	36	36	194	194	194	194	
					2.7	2.0	1.1	0.1	2.3	3.4	3.0	
Do	41 24 20 70 26 26	Lieut. M. Woodhull.	Aug. 14-15, 1852.	HW.*	190	200	345	350	345	345	350	
				L.W.*	0.6	0.2	0.2	0.4	0.7	0.7	0.5	
					350	345	190	190	193	193	190	
					0.5	0.3	0.3	0.9	0.9	0.7	0.5	
Off Cape Poge.	41 25 47 70 26 50	do	Aug. 13-14, 1852.	HW.*	260	265	265	270		120	125	
				L.W.*	0.9	0.7	0.5	0.1	0.0	0.4	0.6	
					125	170	170	150	235	250	260	
					0.6	0.5	0.4	0.2	0.4	0.7	0.9	

Directions true unless otherwise noted.

*Tides at Boston.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
VINEYARD SOUND.	0 1 "				0/kn	0/kn	0/kn	0/kn	0/kn	0/kn	0/kn
Off Edgartown Light	41 23 14 70 30 00	H. L. Marindin..	Sept. 11-25, 1891.	HW.*	278	277	280	85	90	90	90
				LW.*	0.7	0.5	0.1	0.8	0.9	0.6	0.4
					90	292	286	278	280	274
					0.4	0.3	0.7	0.6	0.4	0.7
Swimming Place, near Edgartown.	41 22 33 70 30 24do.....	Sept. 14-25, 1891.	HW.*	169	173	0	0	0	0
				LW.*	1.3	1.0	0.3	1.6	1.8	1.4	0.8
					0	0(?)	180(?)	180	180	180	174
					0.8	0.4	0.5	1.2	1.3	1.2	1.2
Off West Chop.....	41 29 17 70 35 52	Lieut. M. Wood- hull.	July 31, Aug. 1, 1852.	HW.*	262	262	245	100	100	108	105
				LW.*	2.9	2.3	0.4	1.0	2.5	3.4	3.6
					105	105	115	260	262	262
					3.6	2.8	1.5	0.0	2.6	3.1	3.0
S. entrance to Woods Hole.	41 30 47 70 39 42do.....	Sept. 21-22, 1850.	HW.*	284	280	280	112	105	105	105
				LW.*	1.1	0.7	0.2	0.6	1.3	1.3	1.2
					105	100	300(?)	245	280	284	285
					1.1	0.5	0.3	0.7	1.0	1.1	1.1
Do	41 30 49 70 39 43do.....	July 31-Aug. 1, 1851.	HW.*	295	310	75	80	95	85
				LW.*	0.4	0.2	0.0	0.4	0.5	0.4	0.2
					85	300	290	300	295	295
					0.2	0.0	0.2	0.4	0.6	0.5	0.4
Do	41 30 45 70 40 02do.....	Oct. 10-11, 1850.	HW.*	310	315	160	150	140
				LW.*	0.8	0.5	0.0	1.3	1.5	1.4
					140	135	135	0	320	310	310
					1.3	0.9	0.4	0.2	0.5	0.9	0.8
Do	41 30 37 70 40 11	Lieut. J. R. Golds- borough.	Nov. 5-6, 1849.	HW.*	295	285	260	170	100	90	90
				LW.*	1.0	1.0	0.3	0.7	1.0	1.1	1.0
					90	90	100	280	295	295
					1.0	0.8	0.3	0.9	1.0	1.0
Do	41 30 42 70 40 15	Lieut. M. Wood- hull.	July 5-6, 1851.	HW.*	295	295	160	145	140	145
				LW.*	0.4	0.2	0.1	0.3	0.7	0.7	0.6
					150	170	190	295	290	290	295
					0.6	0.4	0.1	0.3	0.4	0.4	0.4
Robinsons Hole, S. end.	41 26 34 70 48 16do.....	Sept. 29-30, 1850.	HW.*	310	305	310	330	150	115	140
				LW.*	2.4	2.6	2.3	1.2	0.5	1.0	1.0
					145	170	170	180	240	315	310
					1.0	0.7	0.6	0.4	0.5	1.0	2.3
Do	41 26 34 70 48 16do.....	July 18-19, Aug. 16-17, 1851.	HW.*	284	290	300	150	120	115	145
				LW.*	0.3	0.2	0.1	0.1	0.6	0.7	0.7
					150	200	250	260	250	250	280
					0.6	0.4	0.3	0.4	0.4	0.4	0.3
S. entrance to Quicks Hole.	41 26 06 70 50 37	Lieut. J. R. Golds- borough.	Nov. 12-13, 1849.	HW.*	312	320	325	0	118	122	120
				LW.*	2.3	2.2	1.4	0.3	1.1	1.4	1.2
					118	136	140	200(?)	255	305	310
					1.1	0.7	0.4	0.2	0.5	1.8	2.2

Directions true unless otherwise stated.

*Tides at Boston.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
VINEYARD SOUND—CON.	0 1 "				0 kn	0 kn	0 kn	0 kn	0 kn	0 kn	0 kn
S. entrance to Quicks Hole.	41 25 59 70 50 38	Lieut. M. Woodhull.	Sept. 30 and Oct. 1, 1850.	HW.*	300	300	310	130	110	115	120
				LW.*	2.1	1.5	0.8	0.2	1.5	1.7	1.8
Between Gay Head and Nawashena Island.	41 23 16(?) 70 51 08(?)	H. Mitchell	July 6-7, 1857.	HW.*	120	120	125	125(?)	240	270	300
				LW.*	1.8	1.2	0.7	0.1(?)	0.5	1.6	2.1
BUZZARDS BAY.											
N. entrance to Woods Hole.	41 31 32 70 41 35	Lieut. M. Woodhull.	July 1-2 and Aug. 6-7, 1851.	HW.*	230	235	250	60	85	90
				LW.*	1.4	1.1	0.7	0.0	0.2	1.4	1.4
Do	41 31 29 70 41 39do.....	Sept. 22-23, Oct. 31, and Nov. 1, 1850.	HW.*	90	80	70	200(?)	230	230	230
				LW.*	1.4	1.1	0.4	0.3	1.0	1.3	1.4
Do	41 31 39 70 41 53	Lieut. J. R. Goldsborough.	Nov. 10-11, 1849.	HW.*	330	340	175	165	165	165
				LW.*	0.5	0.4	0.0	0.4	0.8	0.8	0.7
Between Scanticut Neck and West Falmouth.	41 36 30 70 42 48	Lieut. G. C. Hanus.	Apr. 23-24, 1896.	HW.†	170	165	165(?)	330	335	335	335
				LW.†	0.6	0.4	0.1	0.2	0.4	0.6	0.6
Between Clark Point and Naushon Island.	41 31 00 70 49 30do.....	Apr. 21-23, 1896.	HW.†	328	332	182	180	180	180
				LW.†	0.8	0.5	0.0	0.6	1.1	1.2	1.0
N. entrance to Quicks Hole.	41 27 13 70 50 58	Lieut. M. Woodhull.	Aug. 9-10, 1851.	HW.*	180	190	260	295	305	320	325
				LW.*	1.0	0.4	0.2	0.5	0.7	1.0	0.9
Do	41 27 20 70 50 59do.....	July 21-22, 1851.	HW.*	340	350	350	165	168	170	175
				LW.*	0.7	0.4	0.1	0.2	0.7	0.9	0.7
Do	41 27 21 70 51 02do.....	Sept. 24, 26, 1850.	HW.*	178	180	200	320	325	335	340
				LW.*	0.7	0.7	0.2	0.3	0.7	0.7	0.7
Do	41 27 16 70 51 03	Lieut. J. R. Goldsborough.	Nov. 13-14, 1849.	HW.*	218	218	220	275	?	20
				LW.†	0.1	0.2	0.1	0.0	0.1	0.2	0.3
				LW.†	30	30	30	30	195	210
				LW.†	0.2	0.2	0.2	0.1	0.0	0.1	0.1
				LW.†	220	200	210	210	90	30
				LW.†	0.2	0.2	0.3	0.2	0.0	0.1	0.2
				LW.†	40	70	70	90(?)	165	210
				LW.†	0.3	0.2	0.2	0.2	0.0	0.1	0.1
				LW.*	340	355	15	185	195	195
				LW.*	1.3	1.2	0.7	0.0	0.9	1.7	1.6
				LW.*	195	195	205	250	320	325	335
				LW.*	1.6	1.3	0.7	0.4	1.1	1.2	1.3
				LW.*	325	335	350	0	186	190	205
				LW.*	1.0	0.9	0.6	0.2	0.8	1.8	1.5
				LW.*	205	210	230	255	300	320	320
				LW.*	1.5	1.0	0.4	0.2	0.8	1.0	1.0
				LW.*	350	10	25	65	170	180	180
				LW.*	1.5	1.4	0.8	0.5	1.7	1.9	1.5
				LW.*	185	195	220	320	330	340	350
				LW.*	1.4	0.9	0.3	0.3	1.1	1.5	1.5
				LW.*	347	350	0	90(?)	187	204	215
				LW.*	1.4	0.9	0.5	0.2	1.2	2.1	1.8
				LW.*	215	214	215	270(?)	301	312	345
				LW.*	1.7	1.3	0.7	0.2	1.1	1.4	1.4

Directions true unless otherwise noted.

* Tides at Boston.

† Tides at Clark Point, near New Bedford.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—				Solar hours after—			
					3	2	1	0	1	2	3	
EASTERN LONG ISLAND SOUND.	0 1 "				°/kn	°/kn	°/kn	°/kn	°/kn	°/kn	°/kn	°/kn
South of New London.	41 13 32 72 04 45	Lieut. C. P. Perkins	June 17, 1887	HW.*	130	120	130	100	80?	265	280	
				L.W.*	1.7	2.4	2.2	1.1	0.5	0.9	2.4	
					250	280	280	285	285	285	100(?)	
					1.3	2.5	2.8	2.2	1.5	0.6	0.4	
Between Plum Island and Saybrook.	41 12 35 72 15 52do.....	June 16, 17, 1887	HW.†	96	98	110	160	253	260	268	
				L.W.†	1.5	1.6	1.2	0.4	1.2	1.9	2.1	
					275	278	273	278	276	0	86	
					2.2	2.5	1.9	1.6	0.6	0.1	1.1	
Off Terrys Point (Orient), Long Island.	41 10 11 72 18 53	E. E. Haskell....	July 19 and 21-22, 1890	HW.‡	350	70	80	88	110	(?)	220	
				L.W.‡	0.3	2.0	2.5	2.2	1.6	0.3	1.4	
					240	240	240	240	245	290(?)	350(?)	
					2.5	3.2	3.3	2.9	1.8	0.6(?)	0.3(?)	
Between Terrys Point, Long Island, and Saybrook.	41 12 14 72 19 31do.....	July 8, 11-12, 1890	HW.‡	350(?)	75	73	80	115	175	215	
				L.W.‡	0.7(?)	1.3	1.5	1.4	0.7	0.6	1.3	
					230	227	225	230	237	240	320(?)	
					1.4	2.1	2.4	2.4	2.0	1.1	0.7(?)	
Do	41 13 53 72 20 08do.....	July 7-8, 1890	HW.‡	105	105	105	105	105	125	228	
				L.W.‡	2.2	2.5	2.5	2.2	1.7	0.4	0.9	
					240	250	255	275	330	85	105	
					1.7	2.0	1.7	0.9	0.5	1.5	2.2	
Off Saybrook (Lynde Point) Light-house.	41 15 14 72 20 23do.....	July 5, 10-11, 1890	HW.‡	120	125	130	130	(?)	250	245	
				L.W.‡	1.4	1.2	0.9	0.3	(?)	1.2	1.7	
					265	270	275	275	305	104	120	
					1.8	1.7	1.2	0.7	0.3	1.1	1.3	
SW. from Saybrook ..	41 09 47 72 25 43	Lieut. C. P. Perkins	June 14-15, 1887	HW.†	65	75	75	85	150	195	220	
				L.W.†	2.1	2.3	1.7	1.2	0.4	1.2	2.1	
					220	230	260	275	300	30	65	
					2.1	2.3	1.9	1.2	0.4	0.7	2.1	
SW. from Saybrook ..	41 07 15 72 35 16do.....	June 9-10, 1887	HW.‡	73	73	73	110	232	260	250	
				L.W.‡	1.8	1.5	1.0	0.3	0.5	1.1	1.5	
					260	285	270	280	50	67	71	
					1.6	1.3	0.9	0.4	0.3	1.0	1.8	
Mid-Sound S. of The Thimbles.	41 07 00 72 44 58	E. E. Haskell....	July 28, 29, 1890	HW.‡	345	90	90	30	45	90	170	
				L.W.‡	0.2	0.6	0.8	0.8	0.7	0.5	0.2	
					250	240	240	240	235	195	(?)	
					0.6	1.1	1.2	1.1	0.5	0.3	(?)	
SE. from New Haven.	41 06 30 72 46 02	Lieut. C. P. Perkins	May 25, 26, 1887	HW.‡	80	75	70	70	250	240	240	
				L.W.‡	0.8	0.7	0.5	0.3	0.3	0.7	1.0	
					235	245	270	290	70	80	80	
					1.0	0.9	0.6	0.3	0.2	0.5	0.8	

Directions true unless otherwise noted.

* Tides at Little Gull Island.

† Tides at Saybrook.

‡ Tides at New London.

§ Tides at Falkner Island.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
WESTERN LONG ISLAND SOUND.	0 1 "				° kn	° kn	° kn	° kn	° kn	° kn	° kn
NW. of Eatons Point, Long Island.	40 58 00 73 27 36	Lieut. C. P. Perkins	May 18, 19, 1887	HW.*	27	33	50	0	285	265	260
				LW.*	0.5	0.6	0.3	0.1	0.4	0.5	0.8
					270	270	270	280	295	350	30
					0.8	0.9	0.9	0.5	0.3	0.3	0.5
Off Stamford Light.	40 59 16 73 31 03do.....	Apr. 26, 27, 1887	HW.*	67	55	50	80	300	265	240
				LW.*	0.5	0.6	0.4	0.1	0.1	0.3	0.4
					235	240	235	225	200	90	70
					0.5	0.5	0.4	0.2	0.1	0.2	0.4
Between Great Captain Light and Oyster Bay.	40 57 47 73 35 44	Lieut. Goldsborough	July 8, 9, 1847	HW.†	271	302	40	60	80	90	120
				LW.†	0.8	0.4	0.3	0.8	1.1	1.0	0.6
					130	167	210	233	247	257	268
					0.5	0.3	0.5	0.9	1.1	1.0	0.8
Off Matinicoek Point	40 55 02 73 38 38	E. E. Haskell....	Aug. 7, 8, 13, 14, 1890	HW. ‡	100	100	90	90?	210?	230?	230?
				LW. ‡	0.6	0.7	0.6	0.1?	0.1?	0.3?	0.6?
					230 (?)	215	208	208	180?	150?	100?
					0.6 (?)	0.6	0.4	0.2	0.2?	0.4?	0.6?
NEW YORK, LOWER BAY.											
Scotland Wreck Light Ship (old position).	40 26 24 73 55 56	Ensign J. M. Ellicott	Aug. 11-13, 1885	HW.†	125	135	135	145	170	300?	315
				LW.†	0.4	0.6	0.5	0.4	0.1	0.2	0.4
					315	330	320	330	340	103	125
					0.4	0.6	0.6	0.4	0.1	0.2	0.4
Inside Sandy Hook...	40 27 28 74 00 52	H. Mitchell	July 13 and 28, 1856	HW.†	338	212	174	172	176	181	181
				LW.†	0.12	0.17	0.39	0.49	0.56	0.44	0.34
					181	200	184	204	358	20	350
					0.18	0.27	0.22	0.05	0.14	0.26	0.25
Sandy Hook Bay	40 27 33 74 02 20do.....	July 13, 28, 29, 1856	HW.†	66	38	229	230	249	255	275
				LW.†	0.39	0.16	0.13	0.43	0.65	0.62	0.57
					271	285	242	290	353	25	61
					0.57	0.42	0.42	0.31	0.26	0.27	0.38
Do.....	40 27 39 73 55 25	Lieut. G. C. Hannus, U. S. Navy	Aug. 11-13, 1885	HW.†	125	133	140	143	200	280	291
				LW.†	0.6	0.7	0.6	0.4	0.1	0.2	0.6
					300	300	300	287	275	140	130
					0.6	0.8	0.7	0.4	0.1	0.2	0.6
Inside Sandy Hook: ..	40 27 50 74 00 56	H. Mitchell	July 10, 15, 26, 1856	HW.†	246	172	182	177	165	176	180
				LW.†	0.07	0.36	0.50	0.53	0.61	0.57	0.48
					175	179	178	139	346	341	280
					0.40	0.25	0.21	0.07	0.09	0.20	0.13
S. of SW. Spit, in Ship Channel.	40 28 27 74 01 54do.....	July 15-16, 1856	HW.†	52	69	277	358	208	247	223
				LW.†	0.31	0.24	0.24	0.28	0.15	0.27	0.68
					226	260	314	4	48	37	40
					0.69	0.81	0.40	0.51	0.62	0.43	0.32

Directions true unless otherwise noted.

* Tides at Stamford.

† Tides at Sandy Hook.

‡ Tides at Willets Point.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after			
					3	2	1	0	1	2	3
NEW YORK, LOWER BAY—continued.											
N. of Sandy Hook....	40 28 30	H. Mitchell	Aug. 2, July 10, 15, 16, 23, 1856	HW.*	86	89	126	128	209	251	256
	I.W.*			1.48	1.19	0.75	0.71	0.81	1.24	1.36	
Do	40 28 52	do	July 23, Aug. 1, 1856	HW.*	1.32	1.21	0.73	0.73	0.78	1.39	1.48
	I.W.*			111	108	126	184	244	272	267	
.....	40 29 17	Ensign E. F. Leiper, U. S. Navy.	Aug. 11-13, 1885	HW.*	1.68	1.45	1.02	0.94	1.16	1.64	1.53
	I.W.*			275	283	264	212	185	126	111	
.....	40 29 17	HW.*	1.41	0.94	0.48	0.52	0.69	1.20	1.66
	I.W.*			133	138	130	130	215	270	280	
.....	73 54 54	I.W.*	0.5	0.6	0.6	0.4	0.1	0.3	0.6
	I.W.*			290	290	290	270	255	170	135	
.....	40 30 12	Lieut. J. M. Hawley, U. S. Navy.	Aug. 28-30, 1885	HW.*	0.6	0.7	0.7	0.6	0.4	0.2	0.5
	I.W.*			82	110	106	130	217	254	268	
.....	74 02 50	I.W.*	0.7	0.5	0.3	0.1	0.1	0.4	0.6
	I.W.*			268	268	287	313	6	41	75	
HUDSON RIVER.											
Bull's Ferry, off Ninety-sixth street.	40 47 50	H. Mitchell	Sept. 26-27, 1871	HW.†	25	206	206	194	200	200
	I.W.†			0.1	0.5	0.9	1.2	1.4	0.8	0.0	
Off Verplanck's Point.	41 14[42]	do	Sept. 9, 11, and 23, 1871	HW.†	26	32	37	37	26	26
	I.W.†			0.5	1.2	1.5	1.6	1.2	0.9	0.0	
Off Cold Spring	73 58[16]	HW.†	326	326	184	184	184	184
	I.W.†			1.0	0.3	0.0	0.7	1.0	1.0	0.6	
Off New Windsor.....	41 24[56]	do	Sept. 13-15, 1871	HW.†	184	342	342	342	342	342
	I.W.†			0.4	0.0	0.4	0.9	0.9	1.0	1.0	
Off Carthage	73 57[51]	HW.†	353	340	330	150	150	160	160
	I.W.†			1.1	0.9	0.4	0.1	0.6	0.9	0.9	
Off New Windsor.....	41 28[22]	do	Sept. 14 and 20, 1871	HW.†	160	150	353	353	353	353
	I.W.†			0.9	0.5	0.0	0.5	0.9	1.1	1.1	
Off Carthage	70 00[20]	HW.†	40	40	20	165	160	173	165
	I.W.†			1.2	0.9	0.4	0.2	0.6	0.8	1.0	
Off Carthage	41 33[23]	do	Sept. 19-20, 1871	HW.†	170	170	170	15	44	41	40
	I.W.†			1.0	0.6	0.1	0.3	0.8	1.3	1.3	
Off Rossville.....	73 58[32]	HW.†	45	45	37	200	230	230	217
	I.W.†			0.9	0.7	0.2	0.2	0.7	0.9	1.0	
ARTHUR KILL.											
Off Tottenville.	40 31 05	H. Mitchell	Aug. 15, 20, 1856	HW.*	205	205	205	40	45	45
	I.W.*			0.9	0.8	0.2	0.0	0.8	0.9	0.9	
Off Rossville.....	74 15 05	HW.*	0.9	1.0	1.1	0.9	0.4	0.7	1.2
	I.W.*			45	45	45	45	225	225	225	
Off Rossville.....	40 33 28	do	Aug. 16, 1856	HW.*	1.2	1.1	0.9	0.5	0.3	0.5	0.7
	I.W.*			225	225	225	225	45	45	
Off Rossville.....	74 13 15	HW.*	0	0.5	0.4	0.2	0.0	0.2	0.5
	I.W.*			45	45	45	45	225	225	225	
.....											

Directions true unless otherwise noted.

* Tides at Sandy Hook.

† Tides at Governors Island.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3 .	2	1	0	1	2	3
HUDSON RIVER—CON.											
Off Island View	40 34 56 74 12 28	H. Mitchell	Aug. 23, 1856	HW *	200 0.8	200 0.8	200 0.8	200 0.6	200 0.2	20 0.1	20 0.6
				L.W.*	20 0.7	20 0.9	20 0.9	20 0.5	200 0.3	200 0.7
About 0.4 mi. N. 5° W. from Pralls Island.	40 37 12 74 12 08do.....	Aug. 23, 1856	HW.*	170 1.0	170 1.6	170 1.5	170 1.3	170 0.9	170 0.2	350 0.5
				L.W.*	350 0.5	350 0.8	350 1.0	350 1.1	350 0.9	350 0.5	170 1.0
NEWARK BAY.											
Off the mouth of Elizabethport Creek.	40 38 37 74 11 15do.....	Sept. 9, 1856.	HW.†	216 1.1	216 1.5	216 1.7	216 1.5	216 0.7	36 0.2	36 1.0
				L.W.†	36 1.0	36 1.3	36 1.3	36 1.1	36 0.7	216 0.7
About 0.2 mi. W. from Corner Stake Light.	40 38 50 74 10 34do.....	Aug. 16, Sept. 10, 1856	HW.†	265 1.1	265 1.2	265 1.2	265 0.8	265 0.4	85 0.2	85 0.8
				L.W.†	85 0.8	85 1.0	85 0.9	85 0.7	85 0.3	265 0.4	265 1.0
Off Newark, N. J., on Passaic River, at outlet of Morris Canal.	40 44 16 74 09 42do.....	Aug. 28, 1856.	HW.†	135 0.6	135 0.8	135 0.8	135 0.6	135 0.1	315 0.2	315 0.6
				L.W.†	315 0.6	315 0.8	315 0.8	315 0.7	315 0.5	315 0.1	135 0.5
KILL VAN KULL.											
About 0.1 mi. S. from Bergen Point Light.	40 38 27 74 09 03do.....	Aug. 21, Sept. 9, 1856	HW.†	105 1.8	105 1.7	105 1.1	105 0.6	285 0.0	285 0.7
				L.W.†	285 1.5	285 2.0	285 1.7	285 1.0	285 0.2	105 0.8	105 1.7
Off Port Richmond. .	40 38 39 74 07 49do.....	Aug. 15-16, 1856	HW.†	80 1.8	80 1.8	80 1.5	80 0.8	260 0.3	260 1.6	260 2.1
				L.W.†	260 2.1	260 2.2	260 1.6	260 0.9	80 1.2	80 1.7
Off New Brighton....	40 38 49 74 05 55do.....	Aug. 15, 1856.	HW.†	90 0.6	90 0.4	90 0.2	270 0.2	270 0.6	270 0.9	270 1.0
				L.W.†	270 1.0	270 0.9	270 0.6	270 0.2	270 0.2	90 0.5	90 0.6
CHESAPEAKE BAY.											
Elizabeth River, Nor- folk Harbor.	36 49 41 76 17 36	J. B. Weir	May 20, 21, 1876	HW.‡	350 0.7	350 0.8	350 0.7	350 0.4	170 0.4	170 0.8	170 0.8
				L.W.‡	170 0.8	170 0.7	170 0.8	170	170 0.6	350 0.5	350 0.2
Do	36 50 26 76 16 41do.....	May 15, 29, 1876	HW.‡	285 0.5	285 0.6	285 0.4	285 0.3	105 0.0	105 0.2
				L.W.‡	105 0.4	105 0.4	105 0.4	105 0.1	285 0.1	285 0.4	285 0.3

Directions true unless otherwise noted.

* Tides at Sandy Hook.

† Tides at Governors Island.

‡ Tides at Old Point Comfort.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
CHESAPEAKE BAY— continued.	° ' "				°/kn	°/kn	°/kn	°/kn	°/kn	°/kn	°/kn
Elizabeth River, Norfolk Harbor.	36 50 31 76 17 14	J. B. Weir.....	May 20, 21, 1876	HW.*	280	280	280	280	100	100	100
				LW.*	0.4	0.6	0.5	0.2	0.2	0.5	0.6
Do	36 50 37 76 17 45	do	May 15, 16, 29, 1876	HW.*	100	100	100	100	280	280	280
				LW.*	0.6	0.6	0.4	0.2	0.0	0.3	0.5
Do	36 51 41 76 19 05	do	May 24, 27, 1876	HW.*	330	330	330	330	150	150	150
				LW.*	0.5	0.5	0.4	0.3	0.0	0.4	0.6
Do	36 51 54 76 19 32	do	May 25, 26, 1876	HW.*	150	150	150	150	330	330	330
				LW.*	0.6	0.7	0.6	0.4	0.1	0.2	0.5
Approaches to Chesapeake Bay.	37 01 15 75 56 00	Lieut. B. F. Sands.	Sept. 11-12, 1851	HW.*	310	310	310	310	130	130	130
				LW.*	0.6	0.8	0.6	0.2	0.3	0.8	1.0
Do	37 03 37 75 54 50	do	Sept. 9, 10, 1851	HW.*	130	130	130	130	310	310	310
				LW.*	0.9	0.9	0.8	0.4	0.1	0.5	0.7
Grove Wharf, James River.	37 11 .. 76 39 ..	Lieut. J. N. Maffitt.	July 31-Aug. 2, 1855	HW.*	310	310	310	310	130	130	130
				LW.*	0.7	0.8	0.6	0.4	0.1	0.4	0.8
Off Wolf Trap Light Boat.	37 23 39 76 09 59	Lieut. J. I. Almy...	Nov. 16-19, 1851	HW.*	130	130	130	130	310	310	310
				LW.*	0.8	0.9	0.7	0.1	0.0	0.3	0.6
.....	37 36 49 76 10 29	Lieut. S. P. Lee ...	Nov. 5-6, 1850	HW.*	95	105	125	130	160	260	260
				LW.*	0.0	0.7	0.9	0.9	0.6	0.2	0.4
Near Watts Island Light.	37 42 25 75 55 57	G. Bradford	May 27-31, 1881	HW.*	260	280	280	285	285	300
				LW.*	0.4	0.9	1.5	1.7	1.1	0.4	0.0
Near Watts Island Light.	37 44 15 75 52 01do	Mar. 3-Apr. 23, May 19-20, 1881	HW.*	70	70	70	75	80	260
				LW.*	0.4	0.6	0.9	0.7	0.4	0.0	0.4
				LW.*	260	280	285	285	285	70
				LW.*	0.4	0.6	0.7	0.7	0.4	0.0	0.3
				LW.*	0.6	0.3	0.1	0.2	0.4	0.5	0.4
				LW.*	0.4	0.1	0.1	0.4	0.6	0.7	0.6
				LW.*	21	29	37	160	168	176	184
				LW.*	1.0	0.8	0.3	0.5	0.8	1.0	1.0
				LW.*	184	192	200	355	3	11	19
				LW.*	1.0	0.7	0.1	0.6	0.9	1.1	1.1
				LW.*	335	335	0	170?	170	165	170
				LW.*	0.7	0.6	0.4	0.3	0.4	0.5	0.5
				LW.*	170	170	170	340	340	335
				LW.*	0.5	0.4	0.4	0.2	0.4	0.6
				LW.*	26	32	240	250	242	228	217
				LW.*	0.6	0.2	0.3	0.4	0.5	0.5	0.3
				LW.*	216	194	42	47	49	48	44
				LW.*	0.3	0.3	0.5	1.1	1.3	1.2	0.9
				LW.*	36	272	275	266	245	258
				LW.*	0.3	0.2	0.3	0.5	0.5	0.3	0.0
				LW.*	50	51	46	51	54	43
				LW.*	0.0	0.4	0.4	0.5	0.5	0.4	0.3

Directions true unless otherwise noted.

* Tides at Old Point Comfort.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
CHESAPEAKE BAY— continued.	° ' ''				°/kn	°/kn	°/kn	°/kn	°/kn	°/kn	°/kn
West of Smiths Island	37 59 25 76 09 00	Lieut. S. P. Lee	Aug. 30, 31, 1849	HW.*	308	311	238	234	196	192	
				LW.*	182	176	176	94	94	277	
Near Point Lookout, Md.	38 00 30 76 17 37do.....	June 25, 26, 1849	HW.*	314	324	337	347	252	234	219
				LW.*	219	201	263	276	285	298	310
Near Point No Point	38 09 54 76 16 16do.....	Aug. 26, 27, 1849	HW.*	28	0	330	297	268	240	211
				LW.*	211	184	153	125	70	38
In Hooper Straits	38 13 14 76 05 50do.....	Sept. 30, Oct. 1, 1849	HW.*	36	29	21	200	209	218	226
				LW.*	226	235	72	64	55	47	39
Off Cedar Point, Md.	38 16 41 76 18 13do.....	Oct. 9, 10, 1849	HW.*	168	357	347	336	324	171	171
				LW.*	171	170	170	169	169	168	168
Mouth of Patuxent River.	38 19 19 76 23 28do.....	Oct. 4, 5, 1849	HW.*	247	252	256	261	265	50	68
				LW.*	68	87	108	239	242	246
Off Cove Point, Md.	38 25 47 76 22 50do.....	Oct. 12, 13, 1849	HW.*	6	4	2	359	357	355
				LW.*	355	164	167	173	178	183	188
Mouth of Great Chop- tank River.	38 36 15 76 19 49	Lieut. McArthur	July 25, 1848	HW.*	45	42	42	42	200	211
				LW.*	211	211	200	200	53	48
Great Choptank River.	38 39 40 76 15 00do.....	July 18, 19, 1848	HW.*	90	93	90	90	83	80
				LW.*	230	255	265	267	270	270
Off Holland Point	38 42 28 76 27 29	Lieut. S. P. Lee	Oct. 20, 21, 1849	HW.*	182	198	15	15	14	14	13
				LW.*	13	13	12	124	140	156	175
Off Herring Bay, Md.	38 47 44 76 26 39do.....	Oct. 18, 19, 1849	HW.*	247	241	33	27	22	16
				LW.*	16	11	6	274	265	258	249
					0.4	0.3	0.1	0.1	0.3	0.4	0.4

Directions true unless otherwise noted.

* Tides at Old Point Comfort.

Table showing hourly values of duration (azimuth) and velocity of the current—Con.

Station	Latitude and longitude	Observing party	Date	Tides	Solar hours before—			Solar hours after—			
					3	2	1	0	1	2	3
CHESAPEAKE BAY—continued.	0 1 "				°/kn	°/kn	°/kn	°/kn	°/kn	°/kn	°/kn
Between Tilghmans Point and Mouth of Wye River.	38 50 47 76 13 41	Lieut. McArthur.	June 17, 18. 1847	HW.*	157	166	180	177	180	3
				LW.*	0.2	0.3	0.4	0.3	0.1	0.0	0.1
					3	5	10	5	5	0
					0.1	0.2	0.3	0.4	0.3	0.2	0.0
In Eastern Bay, Md..	38 52 24 76 17 36	Lieut. S. P. Lee ..	Oct. 15, 16, 1849	HW.*	18	27	36	46	55	65	201
				LW.*	0.1	0.2	0.3	0.3	0.2	0.1	0.2
					201	202	204	205	206	207
					0.2	0.4	0.5	0.5	0.4	0.1	0.0
Off Thomas Point Light.	38 53 04 76 24 53do.....	Oct. 17, 18, 1849	HW.*	223	3	10	11	19	33
				LW.*	0.1	0.2	0.5	0.6	0.8	0.9
					18	14	8	197	212	206
					0.7	0.5	0.2	0.2	0.3	0.4
Baltimore Harbor....	39 10 08 76 23 46	F. P. Webber	June 7-25, 1867	HW.†	170	170	170	170	170	170	345
				LW.†	0.4	0.5	0.6	0.5	0.3	0.1	0.2
					345	345	345	345	345	170	170
					0.2	0.4	0.4	0.3	0.1	0.2	0.4
Do	39 10 43 76 28 14do.....	May 31- June 4, 1867	HW.†	150	150	150	285	285	285	285
				LW.†	0.3	0.2	0.1	0.2	0.3	0.4	0.4
					285	285	285	150	150	150	150
					0.4	0.3	0.2	0.1	0.3	0.3	0.3
Do	39 10 49 76 26 30do.....	June 5, 1867	HW.†	210	210	210	210	210	10	10
				LW.†	0.5	0.8	0.8	0.5	0.2	0.3	0.8
					10	10	10	10	10	210
					0.8	1.0	0.9	0.7	0.4	0.0	0.4
Do	39 11 06 76 27 47do.....	May 30, 1867	HW.†	110	110	110	290	290	290	290
				LW.†	0.6	0.5	0.1	0.2	0.3	0.4	0.4
					290	290	290	110	110	110
					0.4	0.3	0.2	0.0	0.3	0.5	0.6
Do	39 11 19 76 24 00do.....	June 4, 1867	HW.†	190	190	190	190	190	25
				LW.†	0.5	0.6	0.5	0.4	0.2	0.0	0.6
					25	25	25	25	25	190	190
					0.6	1.0	0.9	0.5	0.2	0.2	0.5
Do	39 11 30 76 27 08do.....	May 28, 29, 1867	HW.†	140	140	140	280	280	280
				LW.†	0.2	0.2	0.1	0.0	0.1	0.2	0.3
					280	280	280	280	140	140
					0.3	0.3	0.2	0.1	0.0	0.2	0.2
Do	39 11 32 76 25 15do.....	May 27, 1867	HW.†	70	70	205	205	205	205
				LW.†	0.2	0.1	0.0	0.1	0.2	0.3	0.3
					205	205	205	70	70	70
					0.3	0.3	0.2	0.0	0.2	0.3	0.5

Directions true unless otherwise noted.

* Tides at Old Point Comfort.

† Tides at Baltimore.

97. Table of harmonic constants.

Station	Latitude and longitude	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch	Amplitude	True azimuth
GULF OF MAINE. Cashes Ledge*	0 1 "				Knots	o	Knots	o	Knots	o	Knots	o
	42 54 55	Capt. C. D. Sigsbee...	Sept. 1-4, 1875	M ₂	0.365	236.4	0.288	319.1	0.369	246.1	0.282	269.9
	68 54 29			C	-0.140		-0.078		0.156			15.1
NANTUCKET SHOALS. South end of Great Rip	41 12 09	Lieut. C. H. Davis...	Aug. 2-3, 1848	M ₂	1.805	207.0	0.656	245.8	1.880	210.6	0.395	286.6
	69 41 02			C	-0.238		0.113		0.263			334.6
	41 11 50	do	July 20-21, 1848	M ₂	1.454	202.3	1.054	228.9	1.752	211.2	0.391	295.5
	69 48 25			C	-0.248		0.098		0.267			328.9
NANTUCKET SOUND. Cross Rip light vessel*	41 26 46	Lightkeeper	July 23-Aug. 12, 1857	M ₂	0.049	139.1	0.920	245.5	0.920	246.0	0.047	349.8
	70 17 26			M ₄			0.045	116.7				
				M ₆			0.070	29.9				
				S ₂	0.035	101.9	0.154	260.2	0.157	261.1	0.013	1.0
ATLANTIC COAST. Off Martha's Vineyard*	40 43 ..	Lieut. Ackley	Aug. 6-7, 1879	M ₂	0.402	193.5	0.368	259.8	0.458	221.0	0.296	297.8
	70 28 ..			C	-0.110		-0.076		0.134			24.6
	40 38 30	Lieut. J. E. Pillsbury	Sept. 28-29, 1877	M ₂	0.243	236.0	0.364	281.2	0.410	89.1	0.153	139.2
	70 38 00			C	-0.221		-0.362		0.423			47.5
Do.*	40 53 50	do	Sept. 27-28, 1887	M ₂	0.104	239.2	0.259	283.7	0.270	99.2	0.070	151.8
	70 44 30			C	-0.206		-0.048		0.212			2.1
	41 07 40	do	Aug. 30-Sept. 3, 1887	M ₂	0.157	213.7	0.277	312.4	0.279	136.4	0.155	176.1
	70 54 10			C	0.070		-0.169		0.183			101.5
Off Block Island	40 37 00	do	Sept. 14, 18-20, 1887	M ₂	0.349	202.4	0.255	313.1	0.369	187.2	0.226	236.0
	71 09 30			C	0.079		-0.104		0.131			117.2
	41 04 30	do	Sept. 6, 20-22, 1887	M ₂	0.165	189.2	0.073	310.7	0.170	183.9	0.060	245.2
	71 18 30			C	-0.054		0.051		0.074			306.6
Off Block Island*	40 17 56	Lieut. Ackley	Aug. 21, 22, 1879	M ₂	0.368	177.6	0.356	255.3	0.399	212.1	0.321	301.5
	71 41 30			C	0.049		0.081		0.095			229.8

* For stations marked thus (*) results to the left of the double line refer to magnetic direction; all other results refer to true direction.

Table of harmonic constants—Continued.

Station	Latitude and longitude	Observing party	Date	Component	North and south		East and west		Resultant flood			Minimum after flood	
					Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch	True azimuth	Amplitude	True azimuth
ATLANTIC COAST—continued.	° ' "				<i>K</i> knots	°	<i>K</i> knots	°	<i>K</i> knots	°	°	<i>K</i> knots	°
Off Montauk Point *	40 49 45	Lieut. Pillsbury.....	Sept. 5-6, 15-16, 1887	M ₂	0.246	235.9	0.193	310.7	0.256	252.8	193.5	0.178	283.5
Do. *	71 44 00do.....	Sept. 8-9, 1887	C	-0.016	264.8	-0.058	311.8	0.060	118.6	49.5	0.244	139.5
Off New Jersey coast *	40 59 40			M ₂	0.378	264.8	0.546	311.8	0.617	118.6	49.5	0.244	139.5
Do. *	71 45 00			C	-0.007	267.6	-0.249	357.3	0.249	268.3	172.8	0.120	262.8
Off New Jersey coast *	39 33 41	Lieut. Ackley.....	Sept. 1-2, 1879	M ₂	0.145	267.6	0.120	357.3	0.145	268.3	172.8	0.120	262.8
Do. *	72 12 20do.....	Aug. 31-Sept. 2, 1879	C	-0.228	183.6	-0.238	289.3	0.330	38.2	38.2	0.075	242.7
Off Fire Island *	39 56 16			M ₂	0.112	183.6	0.080	289.3	0.116	170.9	152.7	0.075	242.7
Do. *	72 50 00			C	-0.169	220.0	-0.128	309.9	0.212	29.1	29.1	0.095	172.0
Off Fire Island *	40 32 30	Lieut. Pillsbury.....	Oct. 8-12, 1887	M ₂	0.095	220.0	0.251	309.9	0.251	130.0	82.0	0.095	172.0
Approaches to New York *	73 09 40do.....	Oct. 25-26, 29-30, 1887	M ₂	-0.024	123.7	0.077	322.9	0.081	130.0	136.9	0.045	46.9
Off New Jersey coast *	40 09 00	Lieut. Ackley.....	Sept. 10-11, 1879	C	0.236	123.7	0.166	322.9	0.285	158.0	117.3	0.053	210.1
Do. *	73 20 45			M ₂	0.031	172.9	-0.043	329.3	0.053	158.0	120.1	0.052	210.1
Off Barnegat Light *	38 57 36	Lieut. Pillsbury.....	Oct. 12-14, 1887	C	-0.224	88.3	-0.210	324.0	0.307	135.6	103.3	0.040	13.3
Do. *	73 33 00			M ₂	0.051	88.3	0.094	324.0	0.100	135.6	103.3	0.040	13.3
Off Ocean City, Md. *	39 53 00			C	0.096	223.8	0.049	323.9	0.108	200.0	200.0	0.039	264.0
Coast of Virginia.....	73 55 45	Lieut. Lee.....	Aug. 11-13, 1849	M ₂	0.274	223.8	0.040	323.9	0.274	223.6	174.0	0.039	264.0
Do. *	38 18 11			C	-0.094	265.0	0.034	291.1	0.100	339.1	339.1	0.026	145.9
Off coast of North Carolina *	74 56 01	Lieut. Almy.....	Aug. 9-11, 1851	M ₂	0.070	265.0	0.099	291.1	0.119	102.7	55.9	0.026	145.9
Do. *	37 54 39do.....	Aug. 12-14, 1851	C	-0.024	194.9	0.054	257.0	0.059	203.4	198.2	0.043	288.2
Off coast of North Carolina *	75 15 36	Lieut. Bankhead.....	Aug. 17-18, 1859	M ₂	0.092	125.6	0.050	282.9	0.095	115.3	138.6	0.097	228.6
Do. *	33 43 24			C	0.051	154.2	-0.012	272.8	0.052	42.7	24.8	0.060	236.4
Do. *	78 14 ..			M ₂	0.364	154.2	0.336	272.8	0.485	42.7	146.4	0.258	236.4
Do. *	33 23 15			C	0.023	154.2	0.055	272.8	0.060	42.7	146.4	0.258	236.4
Do. *	77 57 30			C	0.398	154.2	0.331	272.8	0.449	42.7	146.4	0.258	236.4
					0.163		0.434		0.464				

* For stations marked thus (*) the results to the left of the double line refer to magnetic direction; all other results refer to true direction.

Table of harmonic constants—Continued.

Station	Latitude and longitude	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch	Amplitude	True azimuth
CHESAPEAKE BAY. Off Rappahannock Light-boat*	37 34 40	Lieut. Almy.....	Nov. 20-23, 1851	M ₂	K ₁ m/s	327.4	0.094	110.6	0.508	326.4	0.056	259.9
	70 11 10			C	-0.246		0.209		0.320			318.8
	38 19 08	R. L. Faris.....	Oct. 28-Nov. 11, 1897.	M ₂	0.383	565	0.078	192.6	0.387	55.3	0.054	261.5
	76 20 23			S ₂	0.089	117.2	0.021	241.0	0.060	113.7	0.017	257.8
Sharps Island Light-house	38 31 04	do.....	Oct. 4-19, 1897	C	0.027		0.057		0.063			244.7
	70 25 50			M ₂	0.290	72.3	0.092	273.3	0.303	74.1	0.032	73.3
				S ₂	0.046	90.3	0.017	328.6	0.046	94.0	0.014	77.9
				C	-0.065		0.018		0.067			34.5
Bloody Point Bar Light.....	38 44 47	F. H. Tillman.....	Aug. 16-28, 1897	M ₂	0.315	120.9	0.033	154.7	0.316	121.2	0.018	275.0
	76 28 02			C	-0.017		0.012		0.021			35.2
	38 52 29	do.....	Sept. 3-Oct. 1, 1897.	M ₂	0.555	131.0	0.211	136.6	0.593	131.7	0.019	290.8
	76 25 10			C	-0.217		-0.040		0.221			10.4
Greenbury Point	38 56 26	do.....	July 29-Aug. 12, 1897.	M ₂	0.539	132.4	0.181	125.0	0.568	131.7	0.022	108.4
	76 25 48			S ₂	0.093	155.8	0.043	129.3	0.101	151.5	0.018	113.2
				C	-0.153		-0.070		0.168			24.6
	39 00 49	do.....	June 23-July 21, 1897.	K ₁	0.220	211.4	0.041	213.2	0.224	211.5	0.001	100.6
Off Sandy Point Light-house	70 22 02			M ₂	0.644	146.8	0.140	134.3	0.658	146.3	0.030	282.0
				O ₁	0.166	231.3	0.019	255.3	0.117	231.9	0.008	98.6
				S ₂	0.069	181.2	0.049	99.7	0.070	171.9	0.048	282.7
				C	-0.002		0.012		0.013			281.3
3½ mi. E. of Seven Foot Knoll	39 09 17	do.....	May 18-June 16, 1897.	M ₂	0.513	160.7	0.222	206.3	0.538	166.0	0.151	288.4
	70 19 49			M ₄	0.008	111.3	0.005	190.6	0.008	116.0	0.005	278.6
				M ₆	0.008	75.0	0.012	269.6	0.015	265.3	0.002	32.8
				S ₂	0.060	188.3	0.015	314.1	0.060	186.6	0.012	261.4
Baltimore Entrance	39 12 29	do.....	Apr. 28-May 7, 1897.	C	-0.099		-0.076		0.125			37.5
	70 24 23			M ₂	0.149	142.2	0.128	163.5	0.193	151.2	0.037	310.3
				C	-0.018		-0.040		0.044			56.0
									0.130 (?)	14.0 (?)		
FLORIDA STRAITS. Off Fowey Rocks	25 34 15	Lt. J. E. Pillsbury.....	May 7-June 1, 1885.	K ₁					0.216	91.5		
	79 56 43		Feb. 28-May 29, 1886.	M ₂					0.096 (?)	290.1 (?)		
				O ₁								
				C					2.573			

* For stations marked thus (*) the results to the left of the double line refer to magnetic direction; all other results refer to true direction.

Table of harmonic constants—Continued.

Station	Latitude and longitude	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch	Amplitude	True azimuth
FLORIDA STRAITS—continued. Off Fowey Rocks *.....	0° 1' N				<i>K</i> knots	°	<i>K</i> knots	°	<i>K</i> knots	°	<i>K</i> knots	°
	25 33 33	Lt. J. E. Pillsbury.	May 7-14, 1885.	<i>M</i> ₂	0.165	87.8	0.078	233.1	0.178	82.3	0.178	160.5
	79 55 25			<i>C</i>	2.130		-0.032		2.133		2.133	182.1
	23 37 48	do	Feb. 16-17, 1887.	<i>K</i> ₁					0.132	349.8	0.132	285.2
Between Rebecca Shoals and Cuba. 82 33 56			May 7-8, 1887.	<i>M</i> ₂					0.078	106.2	0.078	285.2
				<i>O</i> ₁					0.132	347.8	0.132	285.2
				<i>C</i> (34 fth.)					1.818		1.818	285.2
				<i>C</i> (15 fth.)					2.095		2.095	285.2
GULF OF MEXICO. Yucatan Channel.....				<i>C</i> (30 fth.)					1.919		1.919	285.2
				<i>C</i> (65 fth.)					1.756		1.756	285.2
	21 36 00*	Lt. J. E. Pillsbury.	Apr. 13-17, 1887.	<i>K</i> ₁					0.251	248.6	0.251	[185]
	86 16 10			<i>M</i> ₂					0.331	95.9	0.331	
Do *.....				<i>O</i> ₁					0.213	243.6	0.213	
				<i>C</i>					3.125		3.125	
	21 36 20	do	Mar. 25-26, 1887.	<i>M</i> ₂	0.158	277.4	0.200	161.0	0.230	318.0	0.230	129.6
	86 31 55			<i>C</i>	0.490		-0.080				0.119	39.6
WEST INDIES. Windward Passage *.....				<i>M</i> ₂	0.106	136.9	0.061	128.4	0.122	314.8	0.122	30.4
	20 02 ..	Lt. J. E. Pillsbury.	Apr. 1888.	<i>C</i>	-0.109		0.045		0.118		0.118	338.1
	73 43 ..	do	Mar. 1889.	<i>M</i> ₂	0.159	208.4	0.045	165.0	0.163	206.2	0.163	192.0
	18 01 15	do	Apr. 8, 1888.	<i>C</i>	-0.037		-0.058		0.069		0.069	57.5
Mona Passage.....	67 39 00		Feb. 28-Mar. 1, 1889.									
				<i>M</i> ₂	0.161	121.2	0.075	0.1	0.166	127.3	0.166	162.6
	16 19 09	do	Mar. 12-14, 1888.	<i>C</i>	0.244		0.014		0.244		0.244	181.8
	60 50 26	do	Mar. 19-20, 1888.	<i>M</i> ₂	0.126	123.8	0.307	74.4	0.319	259.2	0.319	72.7
East of Désiderade Island *.....				<i>C</i> (0 fth.)	0.396		-0.947		1.026		1.026	111.7
				<i>C</i> (15 fth.)	-0.308		-0.733		0.795		0.795	66.2
	13 33 45	do		<i>C</i> (30 fth.)	-0.391		-1.095		1.163		1.163	69.3
	60 46 45	do		<i>C</i> (65 fth.)	-0.094		-0.438		0.455		0.455	76.6
East of St. Lucia *.....				<i>C</i> (130 fth.)	0.026		0.152		0.154		0.154	259.3

* For stations marked thus (*) the results to the left of the double line refer to magnetic direction; all other results refer to true direction.

Table of harmonic constants—Continued.

Station	Latitude and longitude	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Ampli- tude	Epoch	Ampli- tude	Epoch	Ampli- tude	Epoch	Ampli- tude	True azi- muth
WEST INDIES—continued. Between St. Vincent and St. Lucia *	0° 13' 28" S	Lt. J. E. Pillsbury.	Feb. 22-25, 28-29, 1888.	M ₂	K _h 0.158	207.6	K _h 0.192	353.8	K _h 0.238	187.4	K _h 0.072	217.4
	61° 03' 45" N		Jan. 25, 26, 1889...	C(0 fth.)	0.329		-0.858		0.919			
				C(38 fth.)	0.352		-1.082		1.138			110.0
				C(15 fth.)	0.313		-1.040		1.086			107.0
Between Barbados and Tobago ...				C(30 fth.)	0.037		-1.080		1.081			105.8
				C(65 fth.)	0.139		-0.795		0.719			90.9
				C(130 fth.)	-0.160		+0.094		0.186			100.2
	12° 13' 00" Ndo.....	Jan. 26-28, 1888.	M ₂	0.055	298.4	0.120	48.3	0.121	232.8	0.051	190.7
Off Tobago Island	60° 02' 13" N			C	0.214		-0.769		0.798			105.6
	11° 31' 49" Ndo.....	Jan. 31-Feb. 12, 1888.	M ₂	0.140	148.0	0.177	308.6	0.222	136.0	0.037	218.1
				C(0 fth.)	1.514		-0.940		1.782			148.2
	60° 23' 40" N			C(15 fth.)	1.062		-0.322		1.110			163.1
				C(30 fth.)	0.856		-0.295		0.995			161.0
				C(65 fth.)	0.704		-0.283		0.759			158.1
				C(130 fth.)	1.030		-0.271		1.065			165.3

* For stations marked thus (*) the results to left of double line refer to magnetic direction; all other results refer to true direction.

Table of harmonic constants—Continued.

GULF STREAM PERMANENT CURRENT

[Observations by Lieut. J. H. Pillsbury and Lieut. C. E. Vreeland.]

Station	Latitude North	Longi- tude West	Date	Number of ob- serva- tions	Velocity	True azimuth
	° ' "	° ' "			<i>Knots.</i>	°
North of Bahama Islands.....	30 56 15	76 16 10	Apr. 21-22, 1889.....	56	0.67	205
Florida Strait.....	26 55 00	79 11 40	Nov. 24-26, 1889.....	39	0.54	160
Do.....	25 37 28	79 47 24	May 7-16, 1886.....	68	2.92	188
Do.....	25 34 43	79 25 24	Apr. 6-7, May 3-29, 1885; Mar. 19, 29, Apr. 19, 1886.	107	1.76	171
Do.....	25 34 27	79 53 34	May 19-20, 1885; Apr. 7-8, May 16-30, 1886.	164	3.28	176
Do.....	25 34 15	79 56 43	May 7-June 1, 1885; Feb. 28-May 20, 1886; Apr. 29, 1887.	568	2.57	179
Do.....	25 34 00	79 33 39	May 5-6, 1885; Apr. 22, 1886.	22	1.88	171
Do.....	25 32 41	79 41 13	Apr. 18, May 1-2, 1885; May 4-10, 1886.	70	2.64	179
Old Bahama Channel.....	22 35 00	78 06 30	Apr. 24-25, 1888; Mar. 27-29, 1889.	161	0.28	77
Santaren Channel.....	23 45 00	79 25 45	Apr. 8-9, 1889.....	53	0.08	225
Nicholas Channel.....	23 24 40	80 22 10	Mar. 30-31, Apr. 10-11, 1889.	65	0.52	292
Florida Strait.....	24 01 00	82 28 45	May 9-10, 1887.....	34	1.63	309
Do.....	23 10 35	82 29 00	May 6-7, 1887.....	65	1.40	246
Do.....	24 16 00	82 29 27	Mar. 2-3, 1887.....	29	1.11	267
Do.....	23 44 20	82 30 00	Feb. 12-13, 1887.....	19	3.37	264
Do.....	23 18 15	82 31 54	Feb. 26-27, 1887.....	37	0.95	247
Do.....	23 27 10	82 32 10	Feb. 17, 1887.....	16	1.96	259
Do.....	24 02 00	82 35 55	May 4-5, 1887.....	43	0.27	327
Do.....	23 44 30	82 36 00	May 10, 1887.....	16	1.73	297
Do.....	24 16 00	82 39 27	Feb. 15-16, 1887.....	22	0.30	119
Do.....	24 17 00	82 41 25	Feb. 9-10, 1887.....	7	0.13	229
Do.....	23 27 50	82 41 37	Apr. 27-28, 1887.....	48	2.52	265
SW. of Tortugas.....	24 38 10	84 00 00	Dec. 30-31, 1889.....	44	0.35	1
Do.....	24 11 50	84 05 05	Dec. 5-6, 19-21, 1889.....	111	1.79	341
North of Cuba.....	22 47 00	84 11 50	Dec. 13-14, 1889.....	52	0.99	51
Do.....	23 12 30	84 24 20	Dec. 15-16, 1889.....	50	0.27	355
Between Tortugas and Cuba.....	23 32 00	84 31 00	Dec. 16-17, 1889.....	57	0.82	326
Do.....	23 56 40	84 32 15	Dec. 18-19, 1889.....	51	1.41	329

Table of harmonic constants—Continued.

MISSISSIPPI RIVER PERMANENT CURRENT

[Observations by H. L. Marindin.]

Station	Latitude North	Longitude West	Date	Number of observations	Velocity	True azimuth
	° ' "	° ' "			<i>Knots.</i>	°
South West Pass	29 02 08	89 19 43	Feb. 23-24, Mar. 4, Apr. 4, 1876.	90	2.72	42
Do	29 04 02	89 18 18	Apr. 4, 1876	12	2.92	45
Do	29 06 32	89 16 19	Feb. 23-24, Mar. 4, Apr. 4, 17-18, 1876.	167	2.91	26
Above Cubits Gap	29 12 14	89 16 36	Feb. 14, 1876	33	2.94	335

YUCATAN CHANNEL PERMANENT CURRENTS

[Observations by Lieut. J. E. Pillsbury in 1887.]

Latitude		Longitude		Surface			3½ fathoms			15 fathoms		
North	West	North	West	Number of observations	Velocity	True azimuth	Number of observations	Velocity	True azimuth	Number of observations	Velocity	True azimuth
° ' "	° ' "	° ' "	° ' "		<i>Knots.</i>	°		<i>Knots.</i>	°		<i>Knots.</i>	°
21 54 00	85 11 37			47	0.48	238	25	0.63	6	0.85(?)
21 47 40	85 11 22			65	0.38	309	32	0.47	8	0.80(?)
21 44 40	86 21 10			38	3.01	177	17	3.12	5	2.80	182
21 43 00	85 29 40			109	1.16	173	52	1.19	12	1.16	185
21 42 15	85 46 25			33	1.64	179	15	1.63	2	1.50	163
21 42 00	86 01 50			91	2.77	185	45	2.78	11	2.36	185

Latitude		Longitude		30 fathoms			65 fathoms			130 fathoms		
North	West	North	West	Number of observations	Velocity	True azimuth	Number of observations	Velocity	True azimuth	Number of observations	Velocity	True azimuth
° ' "	° ' "	° ' "	° ' "		<i>Knots.</i>	°		<i>Knots.</i>	°		<i>Knots.</i>	°
21 54 00	85 11 37			5	0.70(?)	6	0.60(?)	5	0.40(?)
21 47 40	85 11 22			7	0.71	202	7	0.64	309	7	0.40(?)
21 44 40	86 21 10			4	2.80	182	4	2.35	182	4	2.12	193
21 43 00	85 29 40			12	1.22	191	11	1.30	179	11	1.02	179
21 42 15	85 46 25			3	1.55	151	4	1.56	179	3	1.13	174
21 42 00	86 01 50			8	2.40	193	10	2.20(?)	10	1.55	185

Table of harmonic constants—Continued.

Station	Latitude and longitude—magnetic variation	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch	Amplitude	True azimuth
SAN FRANCISCO BAY. Between Lime Point and Presidio*	0 1 "				<i>K</i> nodes	°	<i>K</i> nodes	°	<i>K</i> nodes	°	<i>K</i> nodes	°
	37 49 21	Edw. Cordell	Dec. 5, 6, 1866	<i>K</i> ₁	0.396	43.2	0.375	41.1	0.512	42.4	0.009	145.9
	122 27 37			<i>M</i> ₂	1.179	284.0	1.561	287.7	1.955+	286.4	0.061	339.5
	16° 5 E			<i>C</i>	-0.033		0.246		0.248			294.1
Off Presidio	37 48 28	G. Bradford	Jan. 6-8, 1875	<i>K</i> ₁					0.563	45.8		
	122 27 37			<i>M</i> ₂					0.803	252.3		
				<i>M</i> ₄					0.312	260.9		
				<i>O</i> ₁ †					0.354	28.0		
Off Yellow Point *	37 50 05	Edw. Cordell	Dec. 14-15, 1866	<i>S</i> ₂ †					0.181	256.9		
	122 27 00			<i>K</i> ₁	0.484	37.7	0.493	133.8	0.484	36.6	0.192	283.6
	16° 5 E			<i>M</i> ₂	1.825	266.9	0.812	309.6	1.928	272.1	0.525	306.0
				<i>C</i>	-0.075		-0.425		0.432			96.5
Off shore W. of Black Point *	37 48 25	G. Bradford	Dec. 21-23, 1874	<i>K</i> ₁	0.052	14.2	0.336	34.3	0.340	33.9	0.018	351.8
	122 26 37			<i>M</i> ₂	0.101	395.9	1.046	263.1	1.048	263.4	0.068	175.9
				<i>C</i>	-0.072		-0.266		0.276			74.9
				<i>K</i> ₁					0.377	46.6		
Between Black Point and Anita Rock Spindle.	37 48 25	do.	Jan. 4-6, 1875	<i>M</i> ₂					1.079	260.1		
	122 27 04			<i>M</i> ₄					0.095	55.4		
				<i>O</i> ₁ †					0.237	28.8		
				<i>S</i> ₂ †					0.243	264.7		
Off Black Point	37 48 33	do.	Nov. 12-14, 1874	<i>K</i> ₁					0.332	29.5		
	122 26 02			<i>M</i> ₂					1.451	269.4		
				<i>M</i> ₄					0.189	275.0		
				<i>O</i> ₁ †					0.209	11.7		
Off Black Point *	37 48 58	Edw. Cordell	Dec. 6-7, 1866	<i>S</i> ₂ †					0.326	274.0		
	122 26 02			<i>K</i> ₁	0.304	74.7	0.603	51.9	0.667	56.3	0.106	170.9
	16° 5 E			<i>M</i> ₂	0.733	298.5	2.143	280.1	2.254	281.9	0.220	178.3
				<i>C</i>	0.079		-0.212		0.226			126.9
Between Blossom Rock and Alcatraz*.	37 48 50	do.	Dec. 8-11, 1866	<i>K</i> ₁	0.031	61.4	0.791	32.3	0.792	32.3	0.015+	294.5
	122 24 42			<i>M</i> ₂	0.172	40.4	2.255+	276.7	2.257	276.5	0.143	298.9
	16° 5 E			<i>M</i> ₄	0.036	246.2	0.069	300.7	0.073	292.6	0.028	356.2

* For stations marked thus (*) the results to the left of the double line refer to magnetic directions; all other results refer to true direction.

† By inference from Fort Point, Cal., tides.

Table of harmonic constants—Continued.

Station	Latitude and longitude, magnetic variation	Observing party	Date	Component	North and south			East and west			Resultant flood			Minimum after flood	
					Ampli- tude	Epoch	K'nols	Ampli- tude	Epoch	K'nols	Ampli- tude	Epoch	True azi- muth	Ampli- tude	True azi- muth
SAN FRANCISCO BAY—Continued. Between Blossom Rock and Alcatraz*—Continued. Off Meigg's Wharf *	37 48 41 122 24 36 16° 5' E	Edw. Cordell..... do	Dec. 8-11, 1866. Mar. 22-26, 1866.	O ₁ †	0.190	43.6	0.498	0.498	14.5	0.498	0.498	14.5	284.5	0.009	249.5
				C	0.218	-0.040	0.222	0.222	186.1
				K ₁	0.173	233.1	0.690	0.690	23.5	0.706	0.706	25.0	299.0	0.084	29.0
				M ₂	0.605	49.0	1.377	265.9	1.418	1.418	260.6	260.6	307.8	0.340	217.8
Off shore Between Black Point and Alcatraz.	37 48 41 122 25 34	G. Bradford.....	Nov. 10-12, 1874.	M ₄	0.442	260.6	0.291	165.6	0.443	0.443	84.4	10.8	0.290	0.290	280.8
				O ₁ †	0.109	215.8	0.434	5.7	0.440	0.440	7.2	299.0	0.053	0.053	29.0
				C	0.490	-0.473	0.681	0.681	42.1	152.5
				K ₁	0.485	0.485	266.7
Off Blossom Rock *	37 49 00 122 24 24 16° 5' E	Edw. Cordell.....	Jan. 18, Feb. 1, 1867	M ₂	1.671	1.671	144.0
				M ₄	0.210	0.210	24.3
				O ₁ †	0.305	0.305	271.3
				S ₂ †	0.376	0.376	13.2	301.2	0.138	211.2
Between North Point and Alcatraz.	37 48 40 122 25 03	G. Bradford.....	Oct. 29-31, 1874..	K ₁	0.933	52.2	1.804	269.7	1.962	1.962	262.9	262.9	310.6	0.523	220.6
				M ₂	0.221	266.8	0.083	310.2	0.225	0.225	269.1	269.1	208.6	0.042	208.7
				M ₄	0.040	151.4	0.042	266.6	0.051	0.051	181.4	181.4	244.5	0.027	334.5
				O ₁	0.309	0.309	14.8	301.0	0.095	211.0
				S ₂	0.053	79.6	0.366	306.0	0.368	0.368	305.4	305.4	292.3	0.038	202.3
				C	0.192	-0.641	0.669	0.669	123.2
				K ₁	0.384	0.384	26.7
				M ₂	1.882	1.882	268.5
				M ₄	0.058	0.058	310.9
				O ₁ †	0.423	0.423	8.9
				S ₂ †	0.242	0.242	273.1

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† By inference from Fort Point, Cal., tides.

Table of harmonic constants—Continued.

Station	Latitude and longitude, magnetic variation	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Ampli- tude	Epoch	Ampli- tude	Epoch	Ampli- tude	Epoch	Ampli- tude	True azi- muth
SAN FRANCISCO BAY—continued. Off Telegraph Hill.....	37 48 08 122 23 53	G. Bradford	Oct. 27-29, 1874	K ₁	K'nols	°	K'nols	°	K'nols	°	K'nols	°
				M ₂			0.439	40.6				
				M ₄			1.610	249.4				
				O ₁ †			0.101	66.6				
				S ₂ †			0.270	22.8				
Between Pacific Mail Steamship Co. wharf and Yerba Buena.*	37 47 46 122 23 20	do	Oct. 15-17, 1874	K ₁			0.362	254.0				
				M ₂			0.266	11.1				
				M ₄			1.536	250.9				
				O ₁ †			0.259	230.1				
				S ₂ †			0.167	353.3				
Off Kincon Rock.*	37 48 03 122 23 14	Edw. Cordell	Mar. 26-31, 1866	K ₁	0.400	200.6	0.477	311.9	0.520	334.2	0.341	80.7
				M ₂	1.505	83.9	0.987	266.2	1.800	264.6	0.033	253.3
				M ₄	0.054	323.7	0.077	96.9	0.087	110.2	0.035	47.5
				O ₁ †	0.252	182.8	0.300	294.1	0.377	316.4	0.214	80.7
				C	0.908		-0.271		0.948			
Off Long Bridge wharf, Mission B.*	37 47 14 122 23 07	do	Nov. 3-6, 1865	K ₁	0.242	209.9	0.039	48.0	0.245	30.3	0.012	277.7
				M ₂	1.429	79.4	0.402	262.2	1.485	259.6	0.019	270.8
				M ₄	0.081	272.9	0.145	150.8	0.153	141.9	0.065	216.7
				O ₁ †	0.152	192.1	0.095	30.2	0.154	12.5	0.008	277.7
				C	0.418		0.057		0.422			
Between Pacific Mail Steamship Co. wharf and Oakland.*	37 46 07 122 23 06	do	Nov. 22-24, 1866	K ₁	0.152	205.6	0.126	42.3	0.195	32.3	0.028	247.1
				M ₂	1.136	77.2	0.387	260.5	1.200	257.5	0.021	267.7
				M ₄	0.101	257.1	0.098	91.1	0.140	83.9	0.017	242.5
				O ₁ †	0.096	187.8	0.079	24.5	0.123	14.5	0.018	247.1
				C	0.229		-0.027		0.231			
Between Pacific Mail Steamship Co. wharf and Oakland.*	37 47 16 122 22 27	do	Apr. 1-6, 1866	K ₁	0.218	210.7	0.260	45.9	0.352	40.2	0.045	234.1
				M ₂	1.795	85.5	1.281	272.0	2.202	267.7	0.118	251.0
				M ₄	0.107	327.0	0.054	190.4	0.115	154.1	0.035	264.1
				O ₁ †	0.137	292.9	0.176	26.1	0.221	22.4	0.028	234.1
				C	0.959		0.131		0.181			

* For stations marked thus (*) results to the left of the double line refer to magnetic direction; all other results refer to true direction.
† By inference from Fort Point, Cal., tides.

Table of harmonic constants—Continued.

Station	Latitude and longitude, magnetic variation	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Ampli- tude	Epoch	Ampli- tude	Epoch	Ampli- tude	Epoch	Ampli- tude	True azi- muth
SAN FRANCISCO BAY—Continued. Northward of Point Avisadero*	37 44 30 122 21 53	G. Bradford.	Dec. 7-9, 1874	K ₁	Knols	°	Knols	°	0.342	15.4	°	°
				M ₂					1.526	273.7		
				M ₄					1.217 (?)	147.4 (?)		
				O ₁ †					0.215	357.6		
				S ₂ †					0.343	278.3		
Northward from Point Avisadero*	37 44 04 122 21 39	do.	Dec. 10-12, 1874	K ₁	0.289	185.1	0.154	4.8	0.327	5.0	0.001	62.0
				M ₂	1.450	89.6	0.759	263.4	1.635	268.3	0.073	62.4
				C	-0.134		-0.042		0.140			
				K ₁	0.730	173.9	0.289	319.7	0.769	350.0	0.154	87.6
				M ₂	1.125	93.5	1.160	268.1	1.614	270.7	0.076	60.6
Southeast of Yerba Buena* ALASKA, BERING SEA.	37 48 06 122 20 57 160° 5 E.	A. F. Rodgers.	Oct. 16-18, 1862	C	0.125		0.104		0.163			
				M ₂	0.287	95.5	0.325	271.6	0.434	93.4	0.015	221.5
				C	0.268		-0.054		0.273			
				K ₁	0.367	48.7	0.390	55.9	0.535	52.5	0.034	336.2
				M ₂	0.608	273.6	0.698	264.9	0.923	268.7	0.070	198.5
Off Cape Romanzof* ALASKA, BERING SEA.	61 48 48 166 06 38	do.	Aug. 30-Sept. 2, 1899.	O ₁	0.200	23.7	0.218	30.9	0.300	27.5	0.019	336.2
				C	0.135		-0.003		0.135			
				M ₂	1.545	101.9	0.397	104.5	1.595	102.1	0.018	284.4
				C	0.182		0.043		0.187			
				M ₂	0.040	208.9	0.133	246.0	0.137	244.3	0.023	367.7
Off Northeast Cape West of Cape Nome*	63 16 04 168 41 03 64 26 .. 165 02 ..	do.	Aug. 9-13, 1902.	C	0.063		0.243		0.251			
				K ₁	0.174	252.6	0.029	186.4	0.174	252.0	0.026	115.9
				M ₂	0.100	315.6	0.020	74.4	0.100	314.6	0.017	286.3
				M ₄	0.011	192.1	0.008	148.0				
				M ₆	0.010	271.6	0.019	82.7				
Golofnin Bay* ALASKA, BERING SEA.	64 28 14 162 52 59	do.	Sept. 11-24, 1899	M ₂	0.007	115.4	0.004	8.6				
				O ₁	0.154	216.6	0.046	106.0	0.155	218.4	0.043	105.5
				S ₂	0.028	27.6	0.011	176.4	0.030	24.0	0.005	272.8
				C	-0.215		+0.018		0.216			
												17.2

* For stations marked thus (*) results to the left of the double line refer to magnetic direction; all other results refer to true direction.

† By inference from Fort Point, Cal., tides.

Table of harmonic constants—Continued.

Station	Latitude and longitude, magnetic variation	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood	
					Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch
ALASKA, BERING SEA—Cont'd.					<i>K</i> in ft.	°	<i>K</i> in ft.	°	<i>K</i> in ft.	°	<i>K</i> in ft.	°
Off Port Safety *	64 28 30 164 40 ..	Capt. J. F. Pratt	July-Aug., 1900	M ₂ C	0.056 -0.064	177.1 ..	0.096 -0.282	181.9 ..	0.111 0.284	180.7 98.0	0.004 0.004	350.4 ..
Off Nome Army Post *	64 28 35 165 18 30	do.	July 28-Oct. 8, 1900.	M ₂ C	0.057 0.048	111.8 ..	0.129 -0.063	250.3 ..	0.136 0.079	255.6 163.2	0.035 0.054	40.1 46.9
Off Nome City *	64 29 30 165 25 00	do.	Aug. 3, 4, 15, 16, 21-23, 1900.	M ₂ C	0.074 0.053	109.9 ..	0.115 -0.153	236.9 ..	0.125 0.157	248.9 122.7	0.054 0.062	46.9 20.2
Off Penny River *	64 31 30 165 44 ..	do.	Aug. 8-24, 1900.	M ₂ C	0.062 0.051	202.8 ..	0.191 0.058	352.8 ..	0.191 0.077	352.8 248.9	0.062 0.045	20.2 54.8
East of Tapkok Bay *	64 32 .. 163 51 ..	do.	July 21-Aug. 1, 1900.	M ₂ C	0.060 -0.002	300.8 ..	0.126 0.011	93.4 ..	0.148 0.014	104.8 300.0	0.045 0.045	54.8 ..
Off Solomon City *	64 32 20 164 24 30	do.	July 26-28, 1900.	M ₂ C	0.075 -0.209	189.8 ..	0.159 -0.182	179.8 ..	0.075 0.277	179.8 202.2	0.075 0.061	179.8 359.5
Port Clarence *	65 15 00 166 22 10	do.	Aug. 30-Sept. 20, 1900.	K ₁ M ₂ M ₄ M ₆ M ₈ O ₁ S ₂ C	0.066 0.060 0.016 0.021 0.006 0.016 0.026 -0.110	44.6 137.7 174.6 336.8 125.9 200.6 227.5 ..	0.084 0.044 0.008 0.011 0.006 0.048 0.021 -0.094	118.2 203.5 168.9 259.1 327.2 284.9 27.4 ..	0.093 0.064 0.008 0.011 0.006 0.048 0.021 0.148	103.7 153.7 168.9 259.1 327.2 284.9 27.4 63.0	0.061 0.037 0.007 0.007 0.006 0.016 0.006 0.031	359.5 317.2 18.9 342.5 63.0
Inside Cape Spencer *	65 16 28 166 45 10	do.	Sept. 22-Oct. 5, 1900.	K ₁ M ₂ O ₁ S ₂ C	0.047 0.081 0.054 0.034 0.140	75.6 285.0 246.1 61.3 ..	0.021 0.007 0.118 0.069 0.074	342.2 107.2 1.1 291.0 ..	0.047 0.112 0.121 0.073 0.158	256.4 106.1 6.4 284.0 172.1	0.031 0.002 0.048 0.025 0.031	268.1 243.0 33.0 220.0 ..

* For stations marked thus (*) the results to the left of the double line refer to magnetic direction; all other results refer to true direction.

Table of harmonic constants—Continued.

Station	Latitude and longitude, magnetic variation	Observing party	Date	Component	North and south		East and west		Resultant flood		Minimum after flood
					Amplitude	Epoch	Amplitude	Epoch	Amplitude	Epoch	
NORTH SEA	52 27 00 4 51 36	Discussion by J. P. Van der Stok Études des Phénomènes de Marée, etc.	1898-1899	M ₂	Knots	o	Knots	o	Knots	o	o
				M ₄	0.839	169.5	1.145	181.0	1.00	182.3	0.229
				S ₂	0.035	294.6	0.034	343.9	0.046	315.6	0.018
				C	0.186	230.6	0.296	251.0	0.311	244.2	0.053
									0.119		233
Haaks	52 57 48 4 18 18	do	1898-1899	M ₂	1.309	123.7	0.319	155.7	1.337	125.2	0.166
				M ₄	0.083	261.6	0.016	69.0	0.084	261.6	0.008
				S ₂	0.256	183.5	0.062	213.8	0.265	184.9	0.031
				C					0.138		178
									1.049	79.4	0.007
Maas	52 01 18 3 53 54	do	1898-1899	M ₂	0.893	75.8	0.648	86.4	1.049	79.4	0.031
				M ₄	0.054	115.2	0.052	166.9	0.069	91.9	0.031
				S ₂	0.208	137.1	0.163	146.8	0.261	140.7	0.021
				C					0.119		190
					0.036	75.0	0.762	55.2	1.190	64.7	0.202
Schomvenbank	51 47 18 3 27 18	do	1898-1899	M ₂	0.075	71.8	0.005	60.8	0.121	65.9	0.011
				M ₄	0.259	134.6	0.233	115.4	0.344	136.1	0.058
				S ₂					0.092		212
				C					1.226	52.5	0.237
					1.164	56.5	0.455	22.8	1.226	52.5	0.237
Noord Hinder	51 55 24 2 36 36	do	1898-1899	M ₂	0.107	6.9	0.050	1.4	0.118	5.8	0.005
				M ₄	0.293	107.4	0.209	81.7	0.352	99.0	0.075
				S ₂					0.042		201
				C							

Results refer to true direction.

CHAPTER VII.

TIDAL CURRENTS IN RELATION TO MARINE ENGINEERING.

98. Upon comparing rivers which discharge into inland seas with those discharging into tidal waters, it will at once be noticed that all estuaries belong to the latter class. Bars resulting from the deposit of alluvial matter are frequently found at the mouths of rivers of both classes, but deltas are more characteristic of rivers discharging into bodies of water having small tides.

It will be noticed that narrow straits connecting the ocean with bays, harbors, ports, or so-called lakes are frequently much deeper than the near-by waters off either end. Where the water is shallow and the current strong, as at the mouth of an estuary, the material easily moved will arrange itself in ridges extending in the direction of the flood and ebb. For, an obstacle upon the bottom will cause a deposition on either side of it—on the one side during the flood and upon the other during the ebb. In a somewhat similar manner a cape, around which the current flows, may be extended into a hook, the hook extending toward the inward side protected from the action of the waves; or it may protect, as it were, a narrow shoal. Again, if rocks are scattered over a bottom easily eroded, the eddying effect of the impinging streams will produce an excavation upon either side of the rocks.

99. *Tidal rivers.**

The scouring action of the tide goes on more or less at all parts of a river; the eroded matter is, upon the whole, driven seaward because the ebb stream is somewhat stronger than the flood stream owing to its smaller cross section and to the natural discharge of the river. The portion of the river meeting the ocean is often more favorably situated for scour than the other portions owing to the facility with which the eroded matter, much of which is held in suspension, can be dispersed through the action of the waves and currents; hence, the greater depth between the capes. Where the ebb stream loses a large portion of its velocity, matter driven or carried by the water will be deposited. As a rule, a bar will thus be formed off the mouth of the river. The action of the waves will take away the soluble matter, leaving the bar composed mainly of sand and shingle.

Tidal currents whose direction some distance offshore is that of the general shore line, are of great importance in preventing the formation of a bar. For example, the rivers discharging into Chesapeake Bay have no bars, because the tidal streams of the bay, although not strong are yet sufficient to drive away any accumulation off the river mouths. Again, the tidal streams of the North Sea prevent the formation of bars off the estuary of the Humber and the Wash. On the other hand, stationary tidal waves in which the particles move to and from the shore permit the formation of bars

*For the law governing the forms of tidal rivers under certain conditions, see section 33.

and deltas. For example, the Atlantic coast of the Southern States, northwestern coast of Brazil, the coasts of India. (See Figs. 14, 22, 16, Part IV B.)

As a rule, bars do not occur as frequently off broad estuaries and bays as off rivers and estuaries of moderate width. The broader the estuary the greater is the dispersive effect of wind waves and littoral currents. On the other hand, an island or shoal lying a short distance offshore and to the right or left of the estuary might cause a bar to be formed, even where the currents along the shore are considerable, because the effect of these currents is much reduced by the intervention of the island or shoal. Similarly for a cape or headland extending outward from the shore.

Sand will be driven along the bottom if the velocity there be 0.4 knot; fine gravel, if about 1 knot; shingle, about 1 inch in diameter, if 2.5 knots; angular stones, about 1½ inches in diameter, if 3.5 knots.

The wind waves play an important part by disintegrating the rocks along the shore and driving about beds of sand and gravel lying beneath the shallow waters. Such effects may even close up the mouths of small streams and divert the lower part of their courses.

100. *On the training of tidal rivers.*

In training a tidal river so as to aid in the production and maintenance of a good depth, two things are of prime importance: 1° The elimination of irregularities of shore line and of depth; 2° the conserving of the tidal volume.

(1) Irregularities of shore line may be gradually effaced by means of groins extending outward from the banks toward the channel. These may be extended from time to time as the material deposited through their agency accumulates. Finally, when the channel becomes sufficiently regular a permanent wall along either bank, and connecting the extremities of groins, should be constructed. This greatly reduces all minor irregularities which the groins alone would occasion.

The river thus brought to a channel having regular banks will exert a scouring effect upon irregularities of depth. However, on account of the hardness or compactness of the river bed some dredging is generally required. With the river properly trained laterally, the work of dredging is greatly facilitated by the increased scour of the stream.

It is evident that a training wall or pier should extend seaward to deep water, otherwise it will act as a groin in impeding the littoral currents and causing a deposit to be formed in the channel near its mouth.

Unless the piers are to form a harbor of refuge and a protection for the entrance to the river, they should not converge but rather should diverge in a manner somewhat analogous to the banks of an estuary; for, besides causing swift currents at the outer ends, a too narrow opening greatly impedes the entrance of the tidal wave. This remark does not apply to a basin-like harbor, which is in no sense a tidal estuary. In this case the motion through the entrance to the harbor will be chiefly hydraulic in character. Converging piers are then advantageous, because the area between them constitutes an addition to the impounded water, and because the narrowness of the entrance enables the tide to there exert a scouring effect in the channel.

In some instances one sea wall may be a sufficient protection for the channel. This should be situated upon the side exposed to drift whether resulting from wind

or tide or other current, and should generally be concave toward the protected channel. The outer end of the wall, which should reach deep water, therefore trends in approximately the direction of the movement of the littoral current. Some engineers do not follow this rule of making the jetty concave toward the channel. See a paper by T. W. Symons and discussions thereon, in the Transactions of the American Society of Civil Engineers, Vol. 36 (1896), pp. 109-138.

Where the tide is of little consequence in comparison with the current proper of the stream, parallel walls or jetties are commonly used for deepening the channel by scour and with good results. Such are the jetties constructed by James B. Eads at the outer end of South Pass of the Mississippi Delta. The channel thus narrowed quickly increased in depth. The jetties cause the deposits of the river to be made so far out from the Delta that littoral currents are highly effective in transporting the material elsewhere.

In some cases no sea wall need be constructed if advantage is taken of the most natural course of the streams through the shallow bay or estuary. The main channel of the stream having been selected, all shallow portions of its bed are deepened by means of dredging, and the material taken out used as advantageously as possible in building up shoals or closing up undesirable passages. The improvement of the entrance to New York Harbor on a plan proposed by Major (now General) Gillespie is a conspicuous example of this method.

(2) The deepening of the channel permits the tide to flow with less resistance, and so the range may be increased in the upper portion of the river. An increased range means an increased tidal volume available for scouring. This often means greater velocity for the water particles. In fact, by (142),

$$v = \zeta \sqrt{\frac{g}{h}},$$

where ζ is the height of the tide above the undisturbed surface—that is, the velocity of the current is directly proportional to the amplitude of the tide and inversely proportional to the square root of the depth. Hence, whether or not the velocity at any section will be increased, depends upon how the amplitude of the tide is affected by the increase in depth.

If the river and estuary are so short and so formed that the tide wave is largely stationary, the velocities in some portions may become greatly reduced. The training, deepening, and extension of the tidal river will cause the tidal movement to be propagated with less irregularity and the current to become generally stronger.

If, however, it is not possible to extend the tidal river, as would be the case where rocky formations occur near the head of tide water, it becomes especially important to preserve broad flats as tidal reservoirs. Being usually covered with vegetable growth they offer no serious menace to the near-by channels.

In some instances artificial tidal reservoirs are used for flushing out channels or harbors.* By means of gates a large volume of water from the reservoir can be made to flow out in a comparatively short time, thereby making the scouring effect considerable.

*Thomas Stevenson: The Design and Construction of Harbors, 3d ed., pp. 301-305.

A lake-like broadening of a tidal river is generally serviceable in maintaining the depths at the portions of the river below it. However, the broadening and subsequent contraction should, if possible, be gradual, in order that no great amount of energy be lost by the change of cross section.

The following tidal rivers or estuaries along the coasts of the United States have bars off their mouths: Connecticut River; Hudson River; Winyah Bay; Charleston Harbor; Stone Inlet; North Edisto River; St. Helena Sound; Port Royal Sound; Tybee Roads; Savannah River; Ossabaw Sound; St. Catherine Sound; Sapelo Sound; Doboy Sound; Altamaha Sound; St. Simon Sound; St. Andrews Sound; Cumberland Sound; Nassau Sound; St. Johns River; Mississippi River; Brazos River; Coos Bay; Columbia River; Willapa Bay; Grays Harbor. Nearly all of these bars have been improved by dredging and jetties.

The following are a few references to papers and books relating to the improvement of tidal rivers and harbors:

D. Stevenson: Canal and River Engineering.

E. L. Corthell: A History of the Jetties at the Mouth of the Mississippi River, New York, 1881.

Thos. Stevenson: The Design and Construction of Harbors, Edinburgh, 1886.

L. F. Vernon-Harcourt: The principles of training rivers through tidal estuaries, as illustrated by investigation into the methods of improving the navigation channels of the Estuaries of the Seine, Proc. Roy. Soc. of London, Vol. 45 (1888-89), pp. 504-524.

W. H. Wheeler: Tidal Rivers, London, 1893.

Maj. C. E. Gillette: Seacoast Harbors in the United States, Trans. Am. Soc. of Civil Engineers, Vol. 54 (1904), Part A, pp. 297-324, also papers and discussions by others, Ibid., pp. 325-451.

J. N. Schoolbred: The tidal régime of the River Mersey, as affected by recent dredging at the Bar in Liverpool Bay, Proc. Roy. Soc., Vol. 78 (1906), pp. 161-166.

The reports of harbor commissioners of various cities and states contain many details bearing upon this subject.

101. *Harbors, bays, or lakes connected with the sea by means of a narrow strait.*

Knowing the rise and fall of tide off the outer end of the strait, also the dimensions and depths of the strait and inner body, it is not difficult to ascertain the velocity in the strait at any time, provided that the inner body is sufficiently deep in comparison with its horizontal dimensions for remaining sensibly level and the strait is but a small part of a wave-length long. For then the motion will be nearly steady and Bernoulli's theorem with resistance will apply. See sections 104-106, Part IV A, and sections 9, 15-17, Part V.

If ζ_o , ζ_i , denote the heights of the surfaces of the outer and inner bodies, then

$$v = \sqrt{2g} \times \sqrt{\zeta_o - \zeta_i} \times \sqrt{\frac{1}{1 + \zeta' \frac{lP}{\Omega}}}$$

The average value of $\sqrt{2g}$ is 8.0215. The empirical coefficient ζ' is, according to Eytelwein, 0.007565; other values are given in section 8. $\frac{\Omega}{P}$, the hydraulic mean

depth, is, for most straits, nearly the average depth, or $\frac{\Omega}{b}$, b denoting the breadth at the surface. If the value of $\zeta' \frac{1P}{\Omega}$ is many times unity, as might be the case in a strait of considerable length, the above reduces to Chézy's formula

$$v = c \sqrt{\text{mean depth} \times \text{slope of surface}}$$

wherein $c = 92\frac{1}{2}$ if ζ' has Eytelwein's value.

When the dimensions and depths are such that at any given time the flow through the strait is not practically steady, the problem becomes one of great difficulty, and will not be considered here.

In soil which is easily eroded, the swift currents produce and maintain a deep tideway between the outer and inner bodies. Off the outer end of the strait a bar is generally formed, and in some instances off the inner end also. Dredging is usually required at the bar, and the channel thus constructed should be protected by jetties against detritus from the neighboring banks and shoals. The jetties should be parallel or slightly convergent and should extend outward into as deep water as may be practicable, in order that the subsequent dredging may be reduced to a minimum.

The action of the waves upon the beach may cause fine matter to become suspended in the outside water. The flood stream thus discolored will, upon passing the strait, deposit this fine material upon the bottom and shores of the quiet inner body. The ebb stream will be comparatively clear. Thus it is seen that lagoons along the coast may receive sedimentary deposits from the waters outside as well as from the fresh-water streams which may discharge into them.

Examples of erosion in straits.—Lake Pontchartrain and the Rigolets passes. The larger pass at one point reaches the extraordinary depth of 95 feet, and the smaller one (Chef Menteur Pass) the depth of 90 feet. A bar covered by from 1 to 6 feet of water lies off the inner ends of the passes. Rockaway Inlet, leading to Jamaica Bay, New York, is at one point 57 feet deep, while the depth across the bar ranges from nothing to 16 feet. The average depth of San Francisco Bay is less than 10 fathoms. At its narrowest portion the depth of the Golden Gate reaches 60 fathoms. A nearly continuous bar, covered by from 4 to 6 fathoms of water and having a semicircular form, lies to the west of the Golden Gate. The center of the circle is 4.8 miles from Fort Point, and the radius is 2.7 miles.

Other examples along the eastern coast of the United States are Robinsons Hole, Massachusetts; Hatteras Inlet; Ocracoke Inlet; inlet opposite Beaufort, N. C.; New River Entrance; Cape Fear River Entrance; Boca Grande; Charlotte Harbor; West Pass, Apalachicola Bay; entrance to Pensacola Bay; entrance to Mobile Bay; Grand Pass, Barataria Bay; South West Pass, Vermilion Bay; entrance to Galveston Harbor; Pass Cavallo; Aransas Pass; Corpus Christi Pass. All of these inlets have bars outside the capes; in numerous instances they have been improved by dredging and the construction of jetties.

102. *Destructive effects due chiefly to wind waves.*

The destructive effects of the waves during severe storms upon an exposed coast line are frequently so great as to cause much alarm in the locality affected, and to

justify the expenditure of large sums of money in preventing them. The power of waves to tear down land is made far more effective where a littoral current, tidal or otherwise, is sufficiently strong for carrying away much of the matter thus brought into the reach of the sea. Where no such current exists, the tendency to form a protecting shoal along the exposed coast is greatly increased.

According to an estimate of Prof. W. M. Davis, all of the mainland of Cape Cod Peninsula north of the bend will be consumed by the waves in eight or ten thousand years.

According to Edward A. Martin, F. G. S., the coast denudation for England has amounted to 41378 acres in thirty-three years (1867-1900).

103. *The formation or arrangement of shoals.*

Through the encroachment of the sea upon the land, particularly noticeable after heavy storms, the near-by waters become discolored by the soluble ingredients of the soil, while the heavier matter remains on the bottom, comparatively near to the scene of the erosion. In this manner beds of sand and shingle are formed.

Immense quantities of alluvial matter are brought to the shallow waters of the sea through the agency of rivers. Besides forming shoals and bars off the mouths of these streams, as was mentioned in section 98, this material, through the action of the waves and currents, is scattered and transported to near-by localities favorable to the formation of shoals, islands, and shore extensions. It is, however, difficult to say how much of the material composing the shallow bed of the ocean adjacent to the shore is transported from river mouths and how much is due to the degradation of the coast line. Maps of soundings constitute almost the only guide in this matter. It will be noticed that the alluvium in the littoral waters, which is continually forming shoals and lowlands, is especially abundant in the vicinity of river mouths.

The effect of currents becomes conspicuous only where their velocity at the bottom is in excess of 0.3 knot. Shoals thus formed, or at least modified, often appear as ridges whose direction coincides with the lines of flow of the maximum current. Any sunken object may serve as the nucleus of a detached shoal. The sand driven along the even bottom will be arrested if it come in contact with an object constituting an irregularity in the bottom. Both flood and ebb currents may bring up sand, and from both directions. Such shoals occur in the following localities: In the North Sea, especially off Lincoln, Norfolk; in the Thames Estuary; off Belgium and between Holland and Norfolk; southeast of Nantucket Island, Massachusetts; south of Cornfield Point, Connecticut; eastern end of Vineyard Sound; Lower New York Bay; Delaware Bay; off Chincoteague Island; Chesapeake Bay Entrance; Essequibo River; and the Gulf of Cambay.

As time goes on, shoals of this kind may rise to the surface and become low, flat islands. But even before they reach the surface the ordinary action of the wind waves may be to drive the sand higher and higher upon the shoals, and so to facilitate their growth, just as heavy matter is being continually washed ashore.

A cape or point sometimes serves to check the motion of the water, and thus aid in the formation of a shoal,—e. g., shoal northeasterly from Great Point, Nantucket Island; Hen and Chickens Shoal, Cape Henlopen; and Hampton Bar, Old Point Comfort.

104. *Littoral drift, deposition, and beach formation.*

In driving material along the foreshore, the influence of the flood stream is much greater than that of the ebb, and so, as a rule, determines the prevailing direction of the drift; for, the material available for transportation results from the disintegration of rocks and soil, which process goes on above high-water mark, and is by the action of the destroying waves brought more within reach of the flood stream than within that of the ebb. Littoral drift is frequently due chiefly to the repeated impacts of wind waves. In fact, stones more than an inch or two in diameter could seldom be moved by tidal currents alone. Moreover there is abundant evidence of such drift in tideless lakes and seas. Wind waves deposit sand and stones upon the shore because the material driven along the bottom beneath the crest of the wave continues to advance as long as the water immediately surrounding it moves shoreward.

In this way sand and stones are driven high upon a shelving beach, the kinetic energy possessed by the moving material and surrounding water being consumed or converted into potential energy in the process. The receding wave can not move all of the stony material thus brought in, because energy must be consumed in moving and imparting velocity to it; the returning current is too feeble at and near the highest point reached by the wave to produce the necessary impact.

Whether matter is held in suspension or driven along the bottom, deposition will take place whenever the velocity of the water becomes sufficiently reduced. Therefore, if any current follows the shoreline and if groins or piers be extended outward, comparatively still water will be found between the groins; and in the course of time solid matter will there be deposited. In this way the lines of high and low water may be carried seaward.

If a straight sandy coast turns suddenly away from the sea, a sharp point or narrow arm may spring from the angle and take the original direction of the coast, although its extreme tip, forming a hook, may be continually directed inland, receiving its direction from the flood tide or incoming waves.

The streams along the coast following the general direction of a growing arm can not turn aside immediately upon arriving at its extremity. There comparatively slow streams and even eddies favor the growth of the arm. A hook results when the end of the arm is so rounded off that the flood stream can follow it well and so drive matter inward before losing too much of its velocity. The effect of the ebb is to turn the hook in the opposite direction or outward. Hence, when the rise and fall of tide is great, the effect of the flood (where the tide is progressive) upon the foreshore will exceed that of the ebb and there will result a hook turning inward. But where the rise and fall of the tide is not great (or where the tide is stationary), a slender arm may be extended through shallow water and form a nearly straight beach, although the advancing end will often be turned slightly inward; e. g., Rockaway Beach and Coney Island. When a hook of considerable extent is formed at the end of the arm, the effect will be that of a receding shoreline, and under some suitable circumstances another and much smaller arm will be formed following the direction of the outer shore of the main arm. This will grow, and finally become hooked. Another slender arm may form an extension of the outer shoreline; and so on. The result will be an arm whose outer shore is nearly straight while the inner shore is indented with bays. Sandy Hook is an example of this mode

of growth. The deep water east of this peninsula indicates that the tidal streams in conjunction with the winds are responsible for its origin and growth. But Mitchell says (p. 108, Coast Survey Report, 1873):

The material forming Sandy Hook is swept up from Long Branch coast by the diagonal wash of the sea. This was placed beyond dispute by my observations of 1857. Materials of the same specific weight as the sand were placed in the sea at many different points down the outside shore, and at different distances off shore. Those within the action of the waves breaking near the shore were swept along to the northward, and finally collected at the point of the Hook. Those placed far off shore never came to land, so that I concluded that the tidal currents took very little part in the transaction.

In these cases of shore extension it is almost certain that the wind waves play an important part both by facilitating littoral drift and by building slender strips or beaches in shallow waters, as will be presently described. That the extremities of beaches hook or turn inland does not prove that their extensions are due to the flood tide; for, similar forms occur around the Great Lakes and the Black Sea. Moreover, large waves, which chiefly cause the drifting of material, can only arise when the "fetch" is considerable, which implies an on-shore wind.

Generally speaking, beaches are formed by the action of the waves in shallow water upon the detritus there occurring. The result is a slender strip of sandy beach remarkable for its straightness, particularly upon its outer side. The axis of such a beach generally follows what probably was a contour line before the existence of the beach. For the ocean, this contour line probably lay 4 or 5 fathoms below ordinary low water; for the Gulf of Mexico 3 or 4 fathoms, and for shallow bays 2 or 3 fathoms. Why a shoal should originate in waters of these depths is a question difficult to answer with certainty; but the following is probably a partial explanation:

Owing to the shelving character of the sea bottom along the coast, an on-shore wind will cause the surface (troughs and crests of waves being averaged) to assume a slope. This will cause the water at the bottom to flow seaward.* This seaward current becomes feebler as the water becomes deeper. At some depth it will fail to drive sand before it, deposition will take place, and a bar be formed along a certain contour line. As the bar grows in height, the current may be somewhat stronger than before immediately over the bar, but the bar itself would serve to intercept the detritus while being driven seaward. Finally, when the shoal approaches the surface of the water, the waves become more like waves of translation and throw up sand and other material as if breaking upon the original shoreline. Such waves produce an evening and compacting effect, thus explaining why the outer side of a beach is more regular than the inner. If separate islands are formed, currents will aid the wind in joining them together through process of beach extension.

The beaches in the following localities are examples of those probably formed chiefly by the waves, but usually modified or cut by the tidal currents. At the mouths of rivers, bays, and harbors, beaches extend from the land. Besides the deposition received from the recoil or wind waves, another usually occurs as a result of wind waves and current driving material along the margins of the narrow strips of land, causing extension:

Islands of Nantucket, Chappaquiddick, and Marthas Vineyard; southern coast of Long Island; the coast of New Jersey; the coast of North Carolina; the coast of

* Cf. Thomas Stevenson: *The Design and Construction of Harbors*. 3d Ed., pp. 300-301.

Louisiana; the coast of Texas; Bolinas Lagoon; Humboldt Bay; Coos Bay; Tillamook Bay; mouth of the Columbia River; Willapa Bay; Grays Harbor; Drayton Harbor; Port Clarence; southern coast of England.

If but one cape at the entrance to a bay or river receives an extension, this arm may crowd the channel up close to the opposite bank or even move the mouth of a river some distance along the coast. E. g. Beach at Yarmouth; Oxfordness Beach; beaches at the mouths of several rivers in Oregon. Great Point, Nantucket Island, is a beach extension. It is probable that sand and other material is not driven along the eastern foreshore of the island by tidal currents alone, but that the impact of the wind waves continually drives loose objects northward. Lieut. Charles H. Davis, U. S. Navy, has mentioned several wrecks on the southern shore of the island, and called attention to the fact that coal and even bricks from the wreckage were found inside of Great Point.* These could not have been driven along by tidal currents.

In the following localities are beaches (bars) formed almost wholly by wave action, including beach-extension processes: Sea of Azov; northern portion of the Black Sea; Mediterranean shore east of Alexandria; Prince Edward County, Sodus Bays, Toronto Harbor, and Burlington Bay, on Lake Ontario; Erie Harbor, Long Point Bay, Point Pelee, Sandusky Bay, and Maumee Bay, on Lake Erie; Tawas Harbor, Lake Huron; eastern and southern shores of Lake Michigan; Chequamegon Bay and Duluth Harbor, Lake Superior.

105. *The formation of spits or submerged capes.*

A sandy cape or point upon an alluvial shore is generally supplemented by a shoal or spit extending outward to a considerable distance from the land. The littoral tidal currents have their velocities suddenly diminished in passing the cape, because they are there largely deflected and turned into deeper water. By virtue of both flood and ebb, the spit generally takes a direction nearly normal to the coast line at the cape, thus differing from a beach extension. But these two classes of points are not always distinct, because a shore extension originates at an angle in the coast line. As time goes on more sand is deposited upon the point and shoals, and in this manner the point continues to grow until other agencies or altered conditions cause the growth to cease.

Shoals of this character extend outward from Capes Hatteras, Lookout, Fear, Romain, and Canaveral, the character of the coast favoring the formation of detritus necessary in the building of shoals.

Examples of smaller shoals off capes and even off gentle curves in the shore line which may deflect the streams outward may be found along the northern shore of Long Island. Examples of slender capes, formed like beaches chiefly by wave action, occur around the Peconic Bays and Gardiners Bay, Long Island.

If a spit occurs at the junction of two tidal rivers, it may be regarded as the only portion of a bar off the mouth of the smaller river which the larger river will permit to remain owing to its own considerable currents.

Examples of such spits are: York Spit; Rappahannock Spit; off Cape Virgenes, Argentina.

* A memoir upon the geological action of the tidal and other currents of the ocean. Memoirs of the American Academy of Arts and Sciences, Vol. IV, 1849.

Nearly all matter deposited along rocky coasts is to be found in bays where the velocity of currents is diminished.

The following are a few papers relating to changes in shore line and the formation of beaches:

H. Mitchell: U. S. Coast and Geodetic Survey Reports: 1871, Appendix 9; 1873, Appendices 9, 10; 1876, Appendix 9; 1886, Appendix 8; 1887, Appendix 6.

H. L. Marindin: Ibid., 1889, Appendices 12 and 13; 1891, Appendix 8; 1892, Appendix 6; 1896, Appendix 8.

G. K. Gilbert: U. S. Geological Survey Report, 1883-84.

W. M. Davis: The outline of Cape Cod, Proceedings of the American Academy of Arts and Sciences, Vol. 23 (1896), pp. 303-332.

E. A. Martin: Coast denudation in England, Knowledge, Vol. 3 (1906), pp. 348-350.

106. *Why deposition takes place near the inner shore of a bend:*

If we take, by way of experiment, a circular vessel partially filled with water, we can, by moving a paddle round and round, soon set up a circular motion or vortex. If finely divided material like corn meal or fine sand be scattered upon the moving liquid, it will before long be found to be collecting at the center of the bottom. An inspection of the paths of these particles will show that they are driven along the bottom spirally toward the center. The explanation of this is that because of the friction of the bottom on the liquid, the motion is there somewhat reduced in amount. If there were no resistance in the vessel, the surface would be in equilibrium with the force of gravity and the centrifugal force. Since resistance exists, particularly at the bottom, the centrifugal force is there less than at the surface. The surface adjacent to the vessel is lowered, because of the decreased motion of the underlying strata. Hence it is no longer in equilibrium, but its particles tend outward. Since the surface along the vessel is elevated too much to correspond with the centrifugal force due to the smaller velocity near the bottom, an inward pressure gradient must exist at the bottom. Hence the inward velocity.

If the velocity set up is ascertained by observing particles on the surface of the liquid, it will be found that the theoretical height, section 12, Part IV A, is not realized. This discrepancy is due to the fact that the velocities below the surface are considerably reduced.

Now, the outer shore of a bend in a river corresponds to the edge of the vessel of water, while the inner shore corresponds to an imaginary boundary of the central area.

107. *Power contained in the tide.*

If a natural or artificial reservoir be connected with a tidal body by a narrow channel or sluice, a considerable difference in level between the surfaces of the two bodies will generally exist. This difference reduces to zero once on each rise or fall of tide. If the surface of the reservoir rise or fall 1 foot, it will absorb or give up

$$(5280)^2 \times 64 = 1,784,217,600 \quad (302)$$

foot-pounds of work for each square mile of impounded areas, or

$$(6080)^2 \times 64 = 2,365,849,600 \quad (303)$$

foot-pounds for each square nautical mile, 64 pounds being the assumed weight of a cubic foot of sea water.

The average available fall is generally much less than the range of tide in the reservoir; for, even with sluice gates making it possible to fill the reservoir at nearly the time of outside high tide and to empty it at nearly the time of outside low tide, the average fall for any considerable period of filling or emptying will be much less than the range of tide. When no gates are employed, the difference in level between the two water surfaces at any given phase of tide can be computed by means of section 9, Part IV A. If the inner body is so large and so shallow that its surface does not remain practically level, then the question of available height difference becomes more complicated. A large natural body of water used as a tidal reservoir would generally present difficulties of this kind.

One horsepower requires 550 foot-pounds of work per second, or 1,980,000 foot-pounds per hour or 24,592,790 foot-pounds per half tidal day.

Hence, 1 foot of available fall between reservoir and sea, occurring four times daily, has a maximum possible yield of 145.100 horsepower for each square statute mile, or 192.402 for each square nautical mile.

Some natural bodies of water suitable for tide mills are St. John River, New Brunswick; Great Bay and Piscataqua River, New Hampshire; Vancouver, British Columbia; Burrard Inlet and Narrows, British Columbia.

In estuaries, broad rivers, and shallow bays, artificial bodies of water can generally be formed by means of piers or dikes built across the tidal flats and more or less extended, according to the shape of the coast line.

The following are a few references to the subject of tide mills:

Lord Kelvin: Popular Lectures and Addresses, Vol. II, pp. 437-440.

W. H. Wheeler: A Practical Manual of Tides and Waves (1906), pp. 170-173.

CHAPTER VIII.

CIRCULATION OF THE SEA, AND ANNUAL INEQUALITY IN THE TIDES.

108. *General causes of the winds.*

The heat of the sun causes expansion in the lower strata of air, especially in the Torrid Zone. These cause all superincumbent strata to be elevated above their equilibrium levels. By considering a surface of equal pressure in the higher regions of the atmosphere, it will be seen that the surface must dip poleward, and so the fluid particles at high altitudes must move away from the equator. (See Figs. 18 and 19.)

In section 11, Part IV B, it is shown that a body moving in the Northern Hemisphere is deflected to the right, while a body moving in the Southern Hemisphere is deflected to the left. Consequently, a body moving from the equator toward either pole is deflected to the east. According to this reasoning, the winds in the upper strata of the atmosphere of the Northern Hemisphere should blow from the south in the equatorial regions and southwest and west in higher latitudes; for the Southern Hemisphere the winds should blow from the north, northwest, and west. This is somewhat at variance with experience, especially in the lower latitudes.

The outflow of the air from high altitudes of the equatorial regions tends to diminish the pressure observed there upon the earth's surface. This is seen very near the equator, where the eastward motion is theoretically small, and so does not tend to crowd the matter toward the equator as much as does such motion a little farther north or south.

At the equator and at the poles the meridional motion of particles must be in general comparatively small, since these places mark the limits of the excursions of the particles. Now, the deflecting force due to the earth's rotation varies with the velocity of the particles and the sine of the latitude conjointly. The effect of this force upon the general circulation of the atmosphere is to so divert the pole-seeking particles in the various latitudes that the attainment of velocities exceeding certain values becomes impossible, and consequently to hold a quantity of the upper atmosphere near the equator which would otherwise have gone toward the poles. The velocities of the general atmosphere diminish as the equator is approached, and so in the tropical regions the deflecting force in the higher strata must be very small. From observation it is known that in the Northern Hemisphere a belt of high pressure exists having, over the oceans, its axis along approximately the thirty-fifth parallel of latitude, while the similar belt in the Southern Hemisphere follows approximately the thirtieth parallel. To restore the air carried poleward in high or tolerably high altitudes, return currents of less altitude are necessitated. Between the ridges of high pressure the countercurrents extend to the surface of the earth, and being deflected westward by the earth's rotation, produce the trade winds.

The belts of high pressure cause the lowest layers of the return current in the near-by regions of higher latitude to reverse, and for some distance, to move poleward. It is thus seen that, in the tropics the winds experienced at the earth's surface are counter currents of the movements in high altitudes, while just outside the sub-tropical high-pressure areas a portion of the countercurrent is reversed.

The connection between wind and pressure at the earth's surface is generally such that the air is flowing away from a high area and toward a low area, but the directions of the movements are greatly influenced by the deflecting force of the earth's rotation acting upon the moving surface air and upon the air in higher altitudes. In fact, the movements of air are often the cause rather than the results of pressure gradients. Near a region of low pressure the directions of the motions may nearly coincide with the directions of the isobars, but farther away directions become approximately normal to each other.

The alternate heating and cooling of large continental areas involves a falling and rising in the atmospheric pressure at that part of the earth's surface. The lower air flows toward such an area during the summer season, and out of it during the winter season. This is the origin of the monsoons.

109. *The prevailing winds over the surface of the oceans.*

The prevailing winds in the Atlantic Ocean between parallels 35° and 60° north come from the southwest or west. In a zone extending for 30° on either side of the equator, the easterly winds are remarkably constant, but not strong, and are known as trade winds. North of the thermal equator they come from northeast, and south of it from southeast. Between parallels 35° and 60° south the winds are generally from the west or a little north of west.

In the Torrid Zone of the Pacific Ocean the prevailing winds are easterly. South of 40° south latitude they come from the west or a little north of west; north of 40° north latitude they come from the southwest.

The winds of the Indian Ocean north of the equator are northeasterly during the winter season and southwesterly during the summer. In the winter a high-pressure area exists over eastern Asia, and in the summer a low-pressure area over southern Asia. South of 40° south latitude they generally blow from the west or northwest.

In all oceans the wind velocity is small near the equator.

Along the coast of Norway the winds are generally southwesterly. In that portion of the Arctic traversed by the *Fram* the winds blow from near the New Siberian Islands toward southeastern Greenland, where a low-pressure area exists. In the part of the Arctic Archipelago just north of Lancaster and Melville Sounds, the wind is from the north. At Point Barrow it is east-northeast. North of Greenland and Grant Land it is westerly or northwesterly during the summer season.

It will be seen upon comparing the isobaric chart with the charts of the winds that the air particles approach and swirl round areas of low pressure and recede, in a similar manner but with less velocity, from areas of high pressure. Rotations against the sun indicate low areas in the northern hemisphere and high areas in the southern, and vice versa for rotation with the sun.

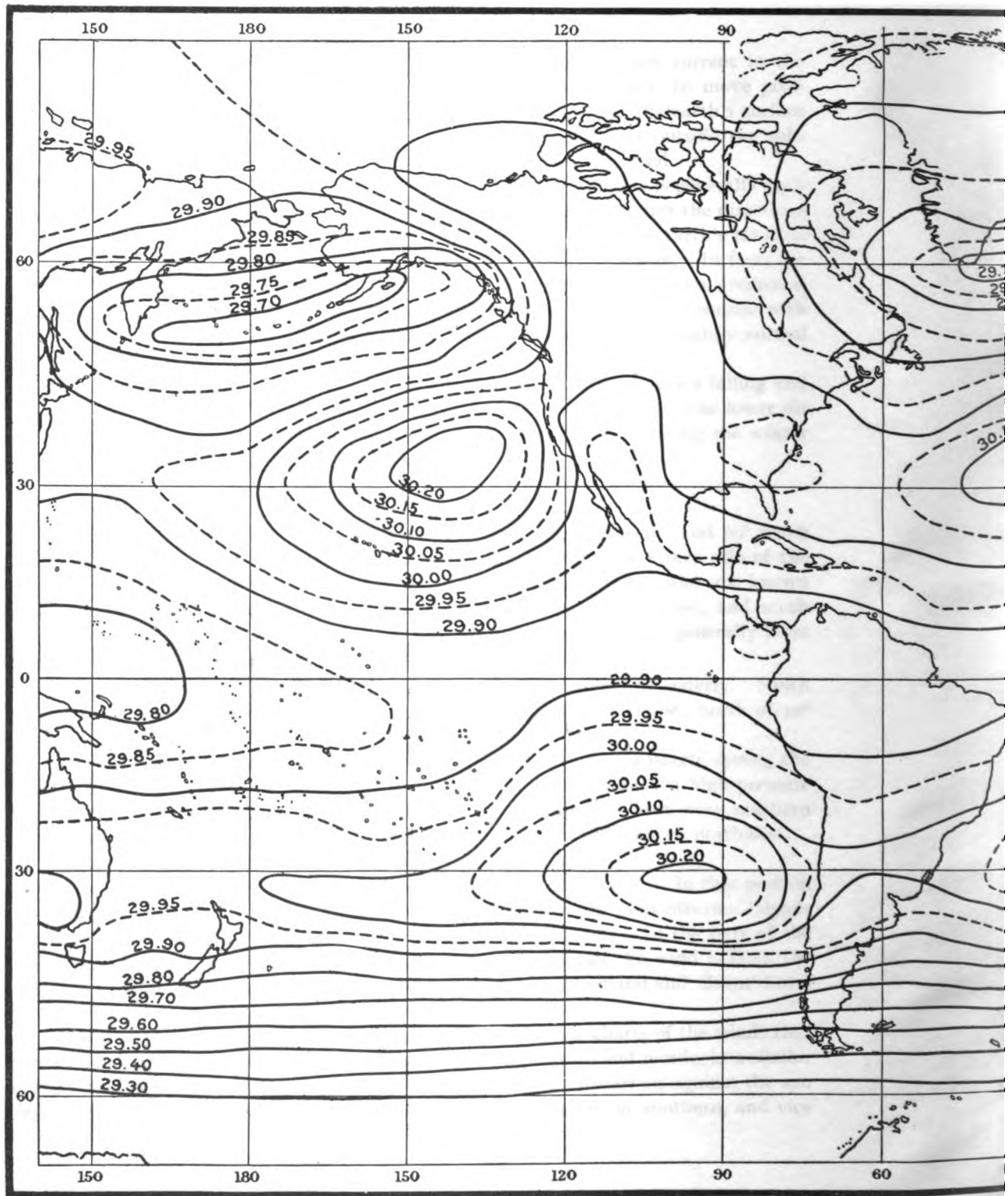
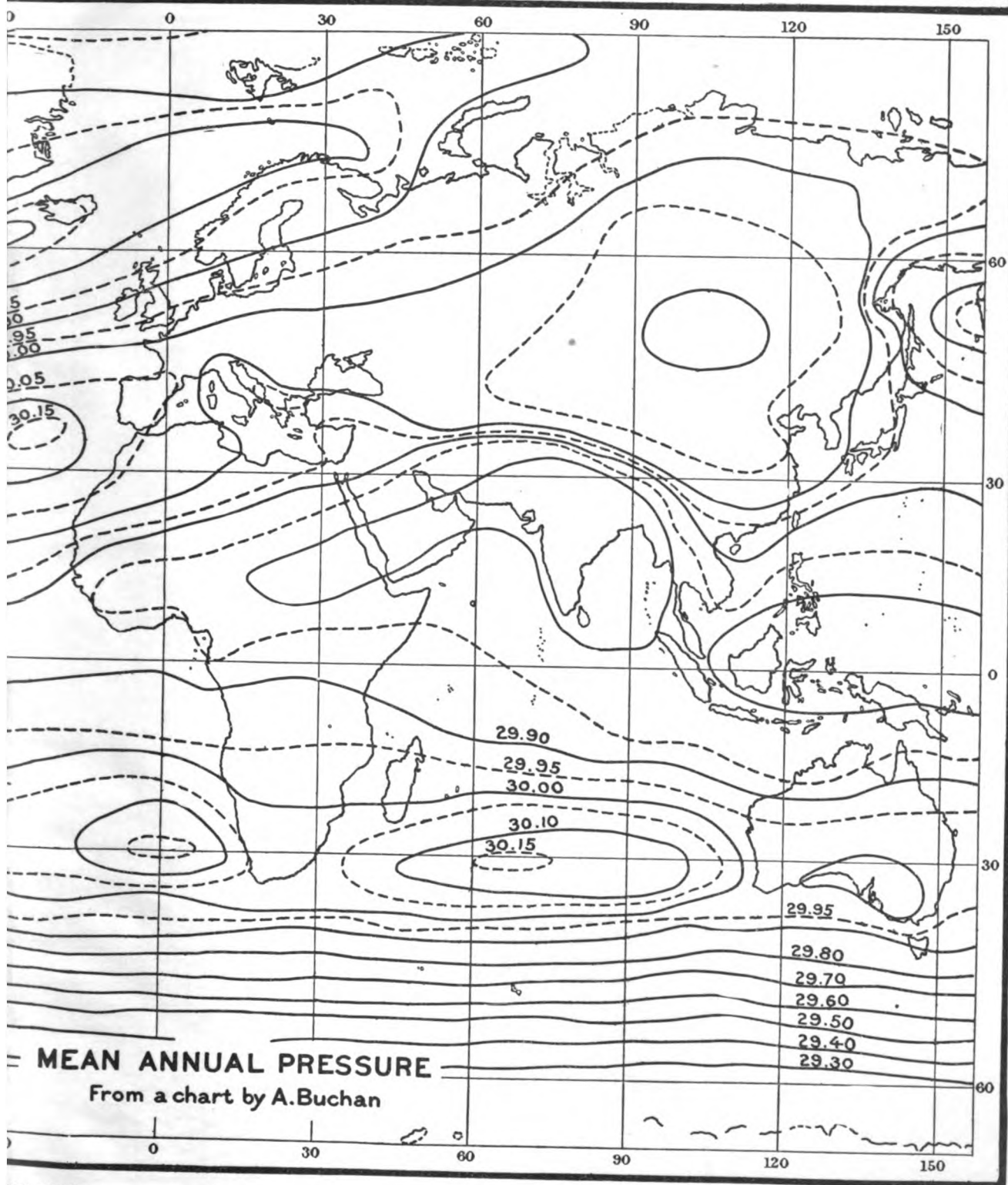


CHART OF ISOBARIC LINES. (FROM BA



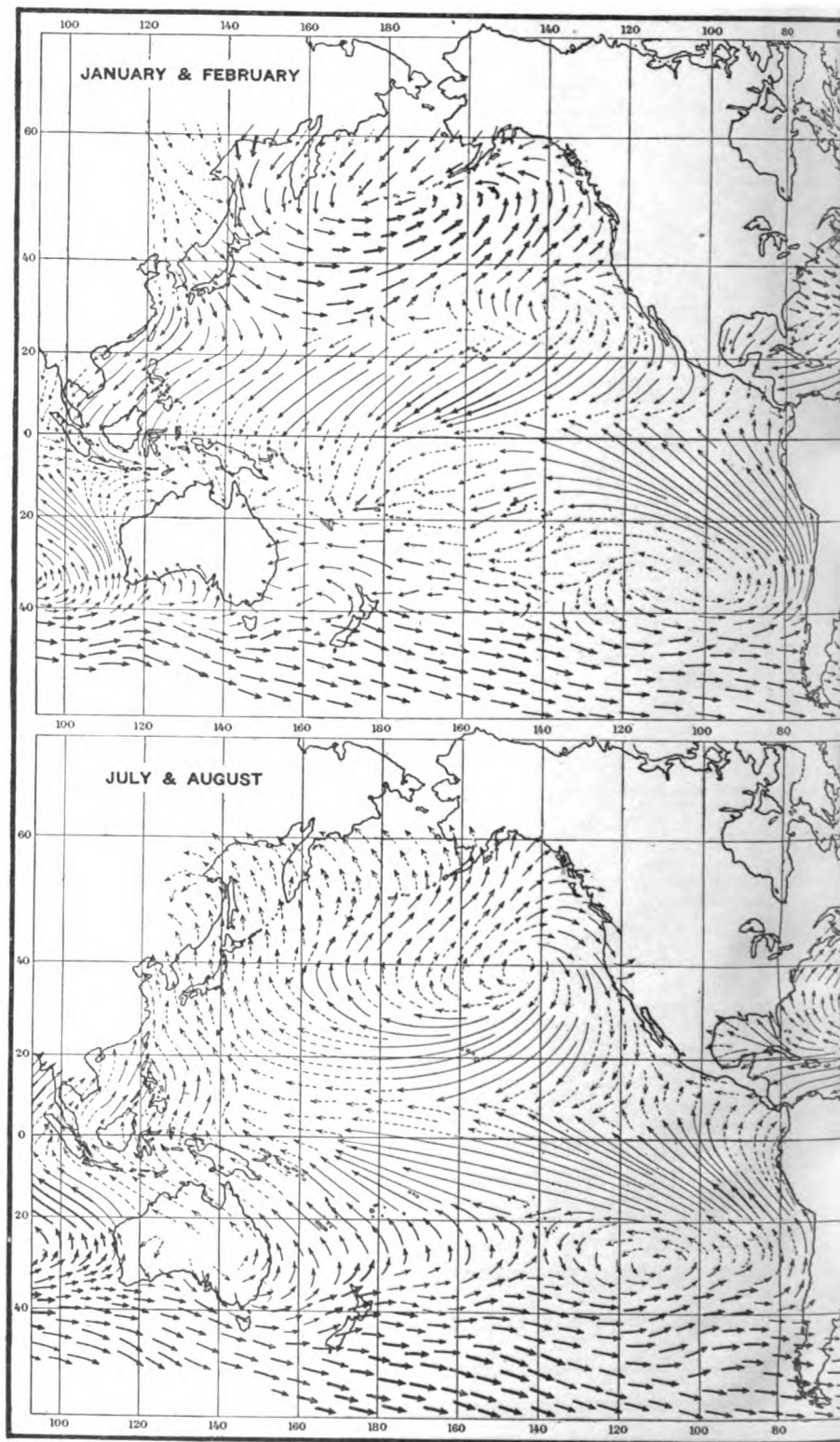
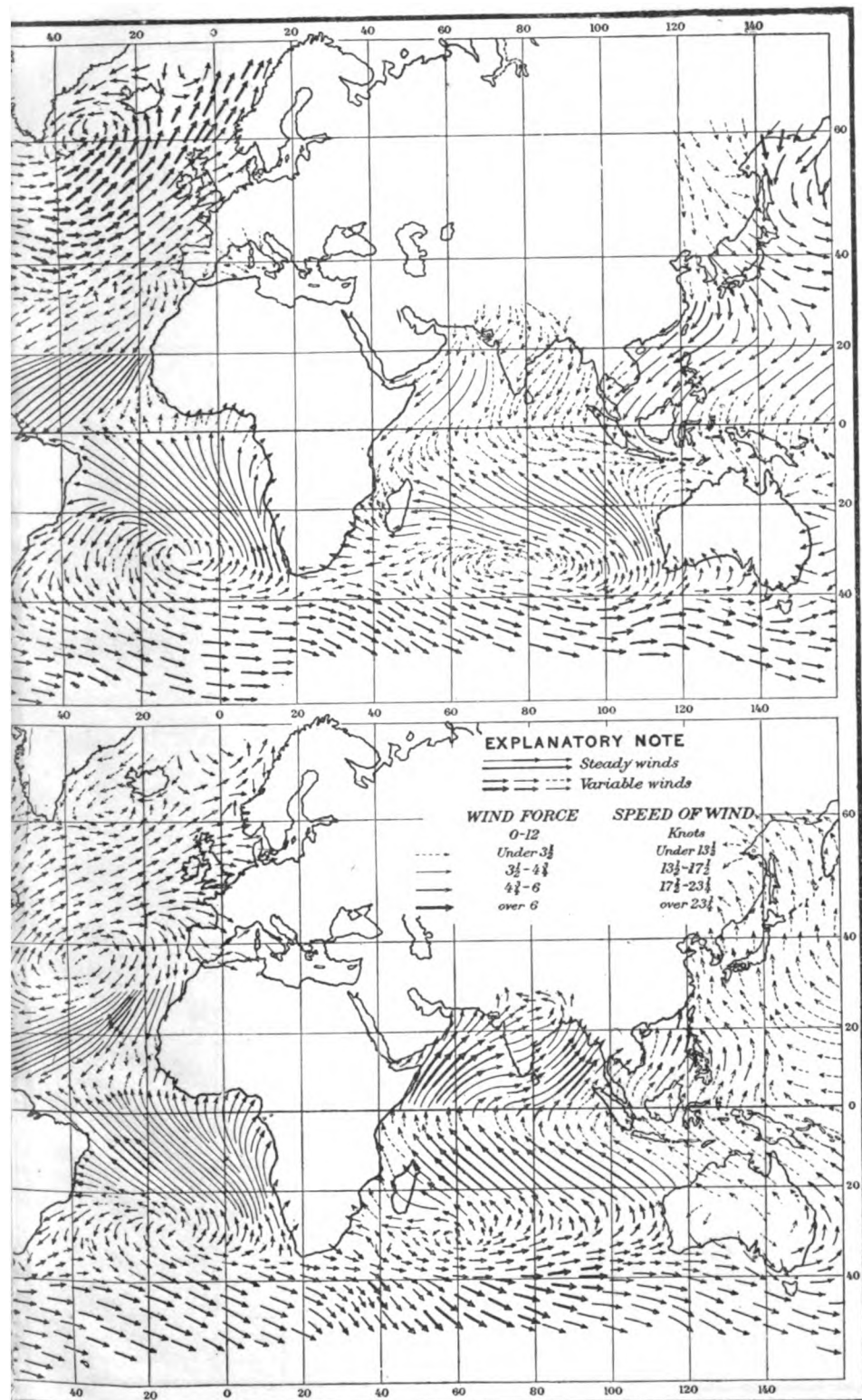


CHART OF PREVAILING WINDS OVER THE OCEANS



(DR. W. KÖPPEN IN BARTHOLOMEW'S PHYSICAL ATLAS.)

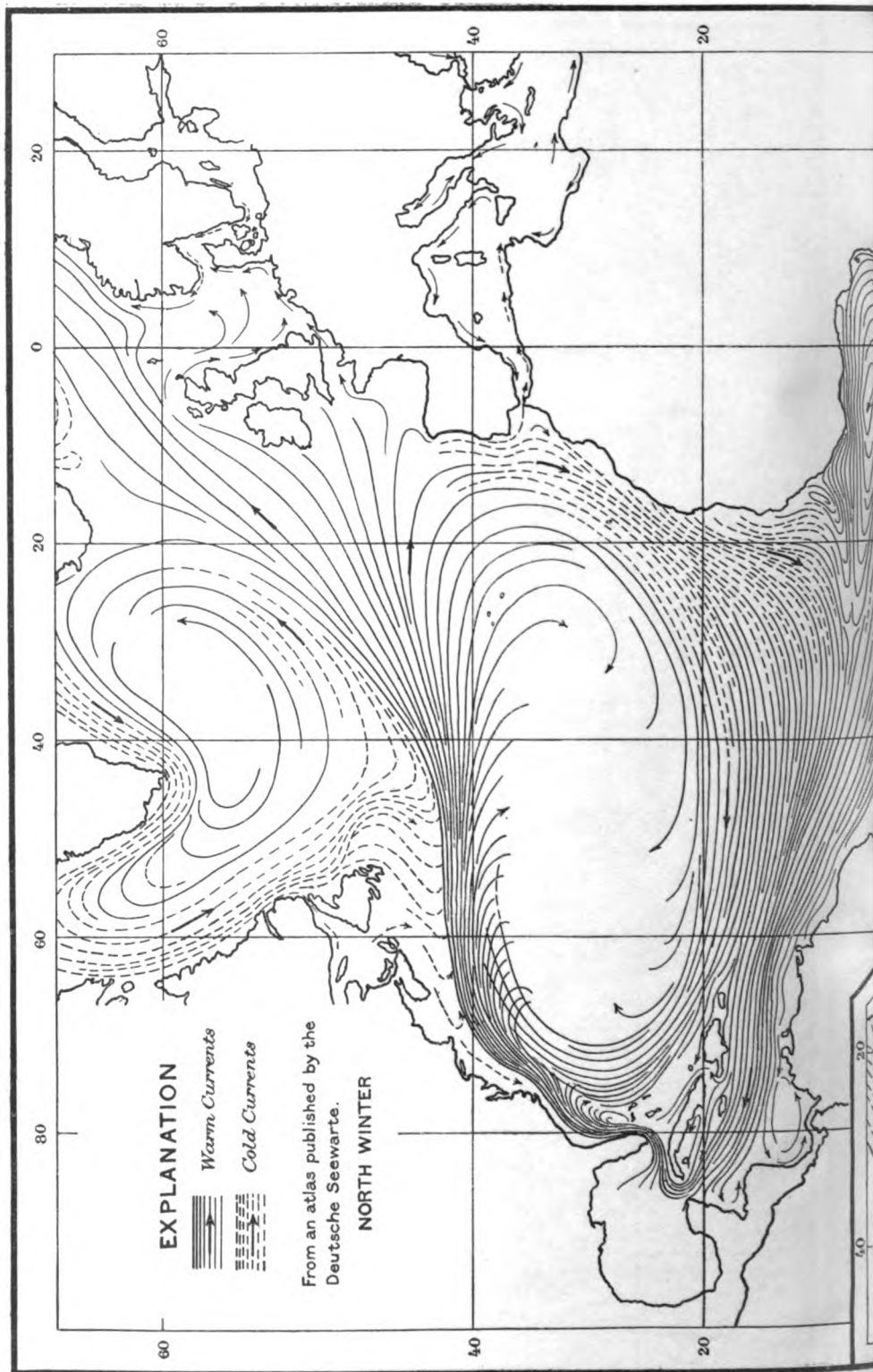
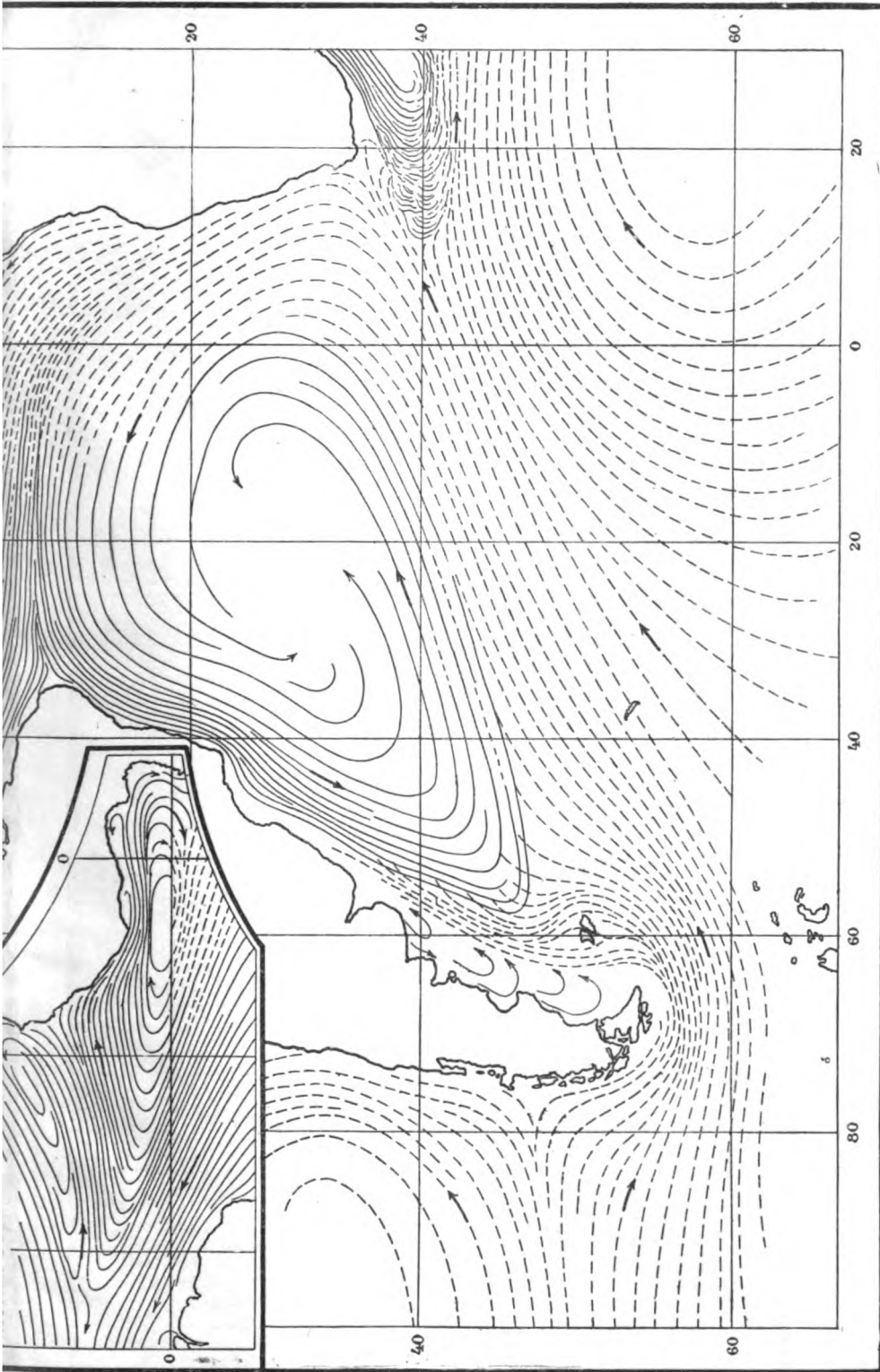


CHART OF NONTIDAL CURRENTS FOR THE ATLANTIC OCEAN, NORTH



WINTER. (PROF. O. KRÜMMEL IN ATLAS BY DEUTSCHE SEEWARTE.)

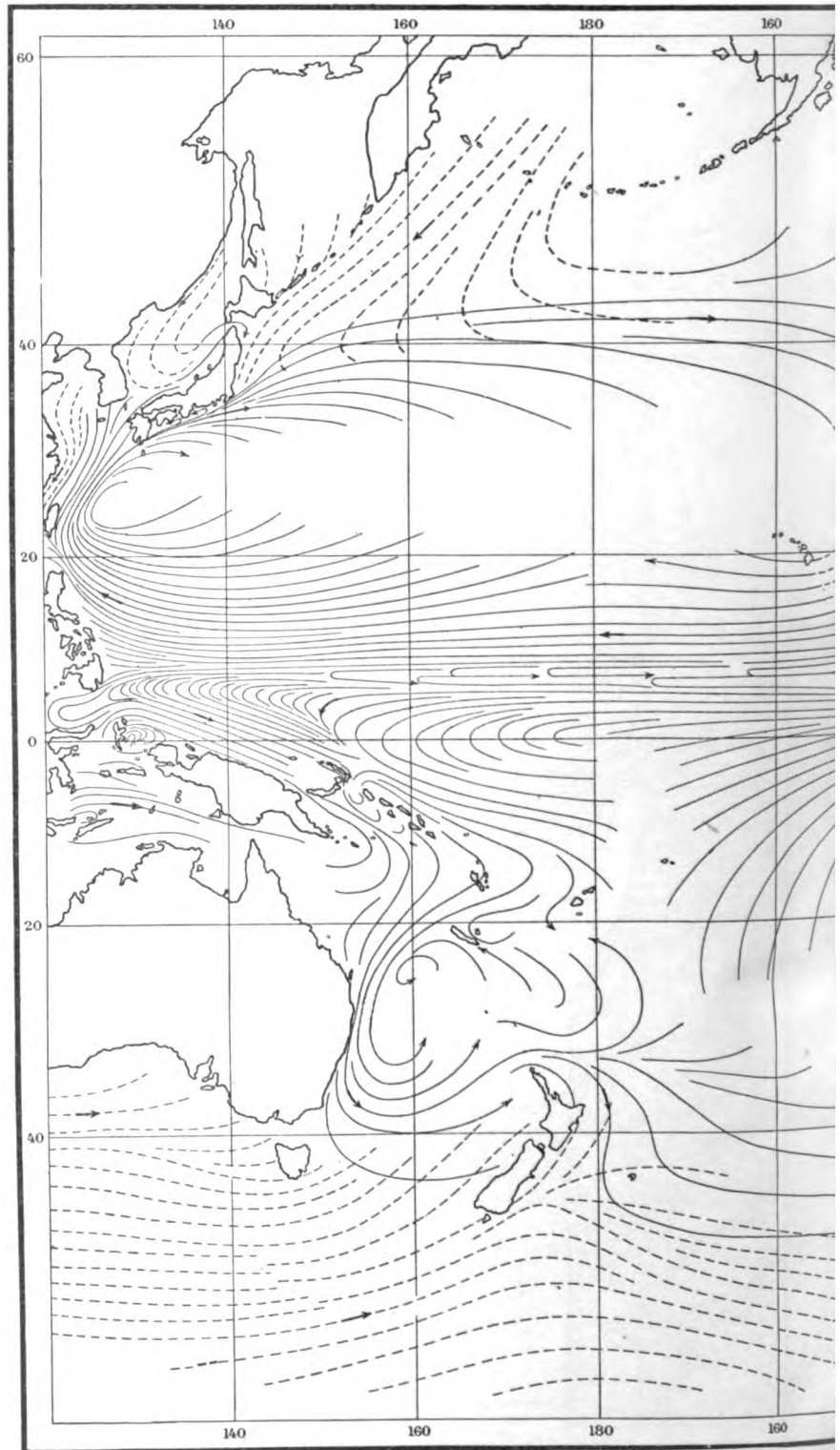
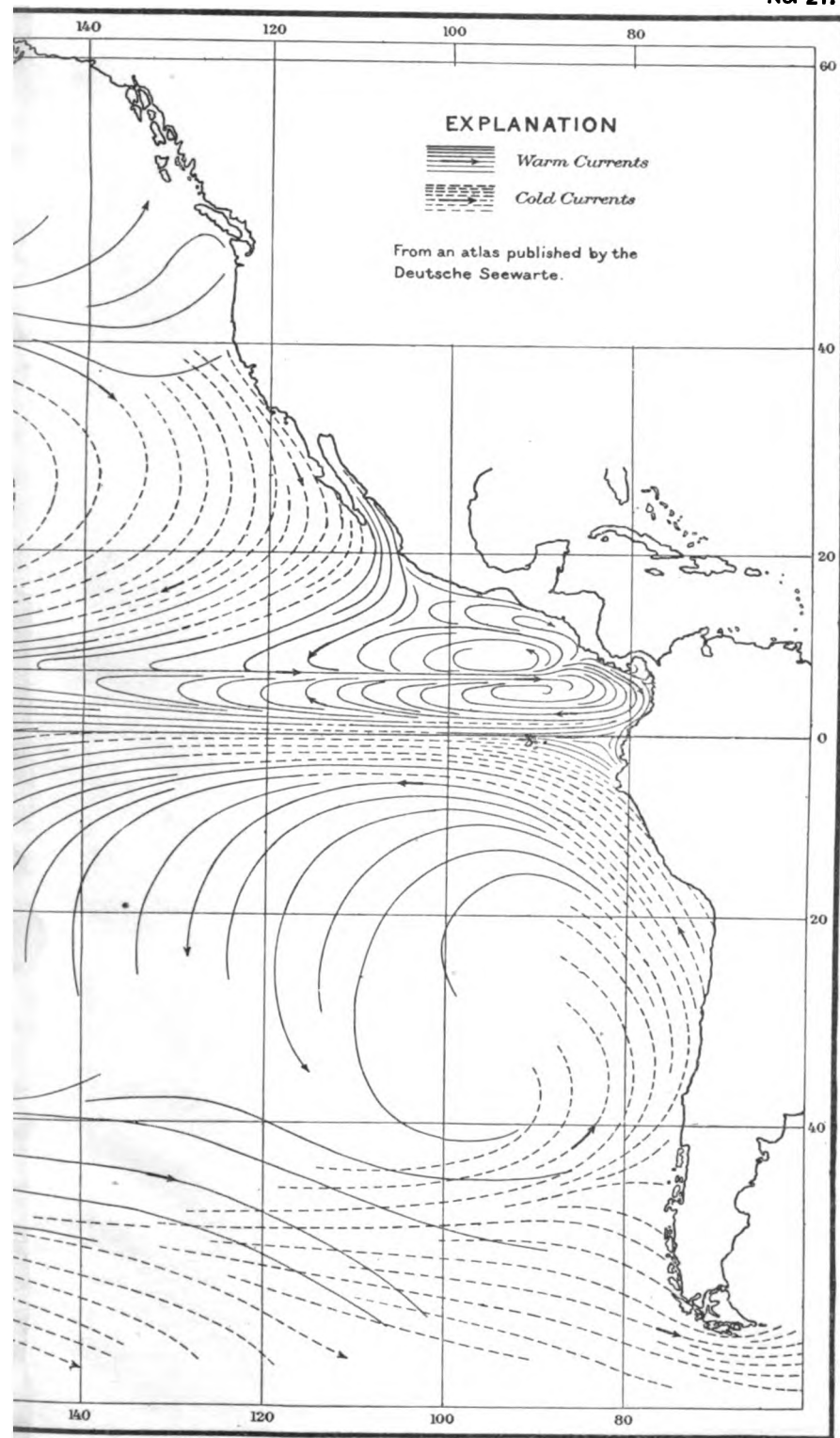


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PACIFIC OCEAN, JANUARY TO MARCH.

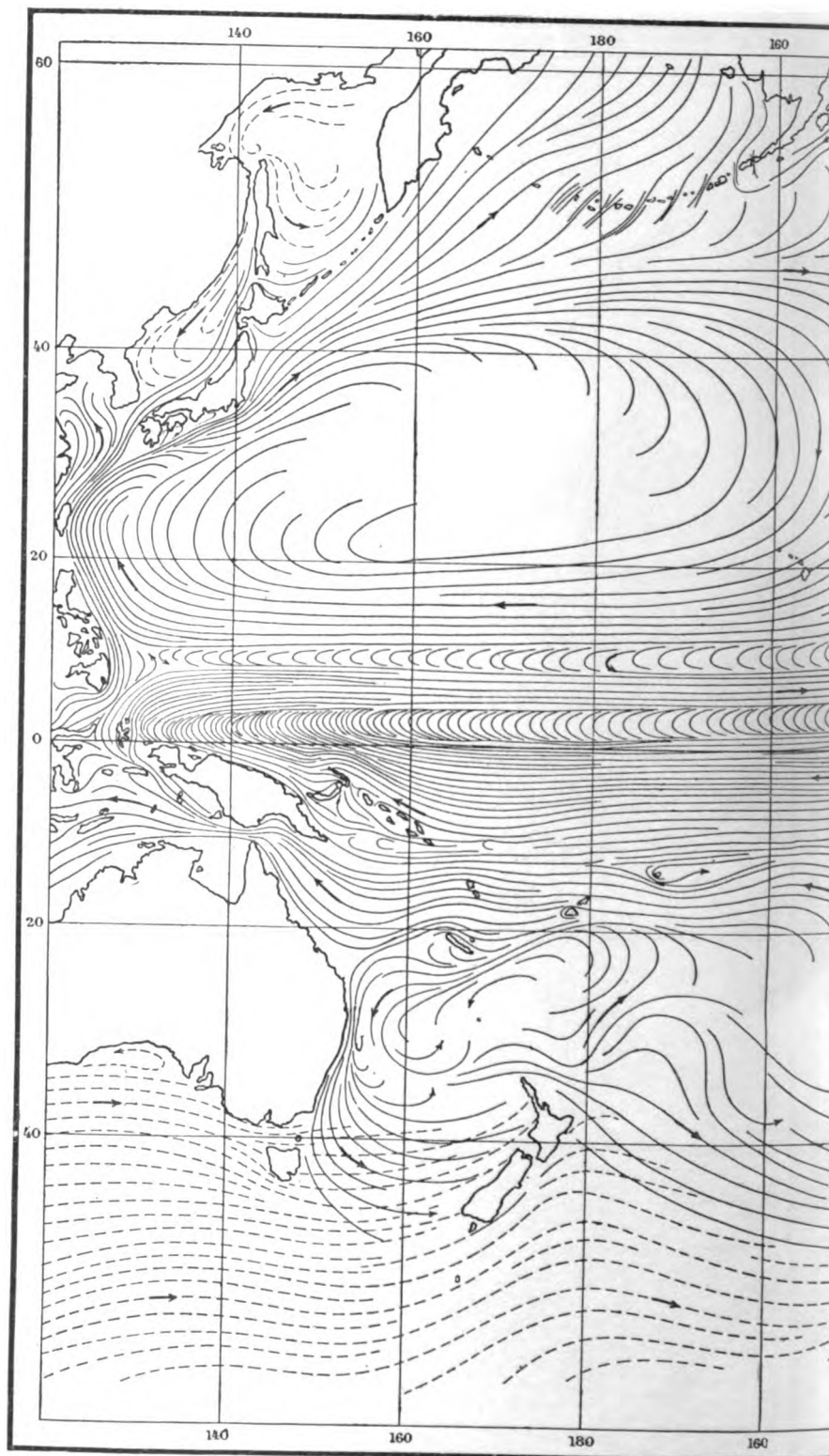
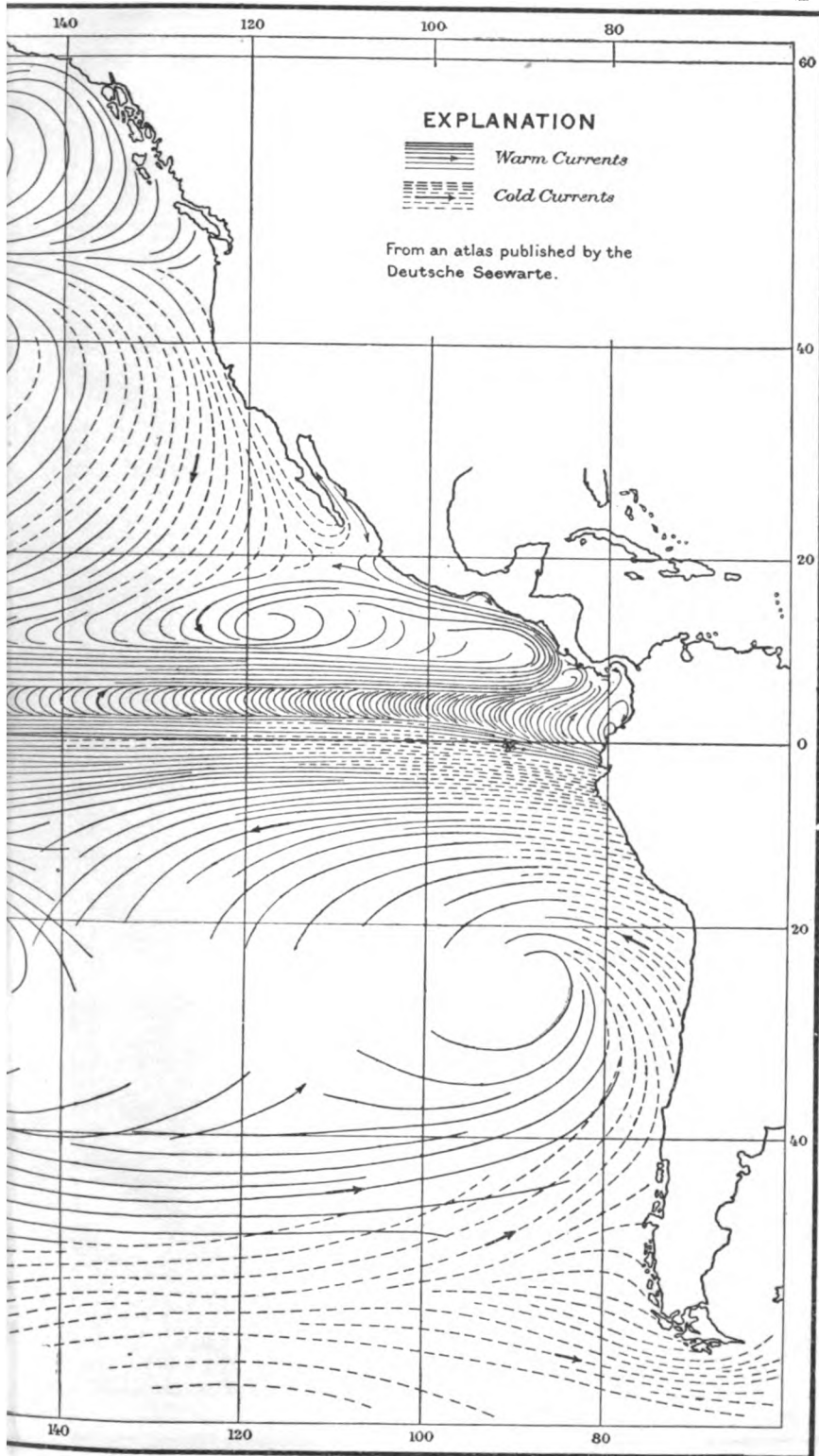


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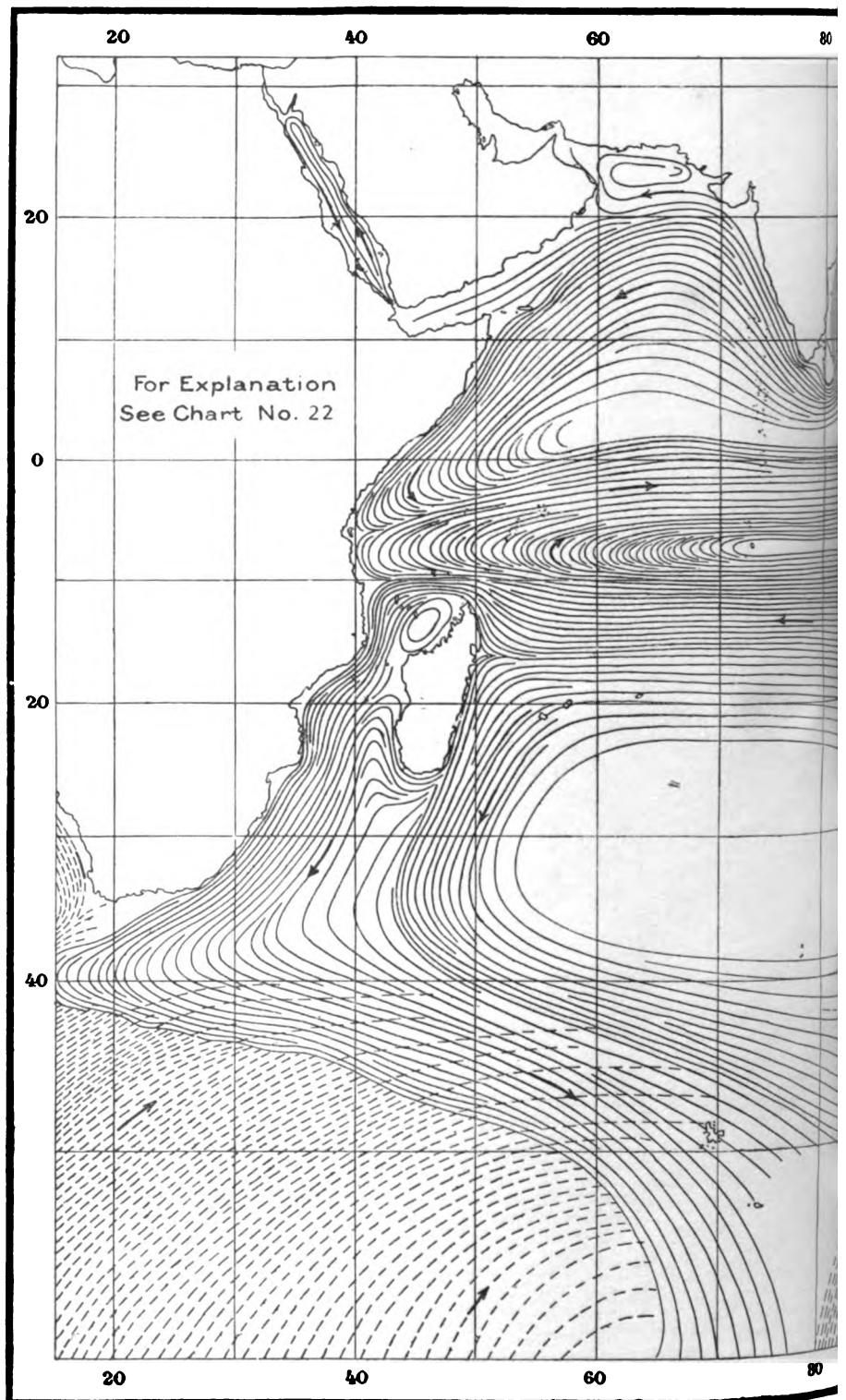
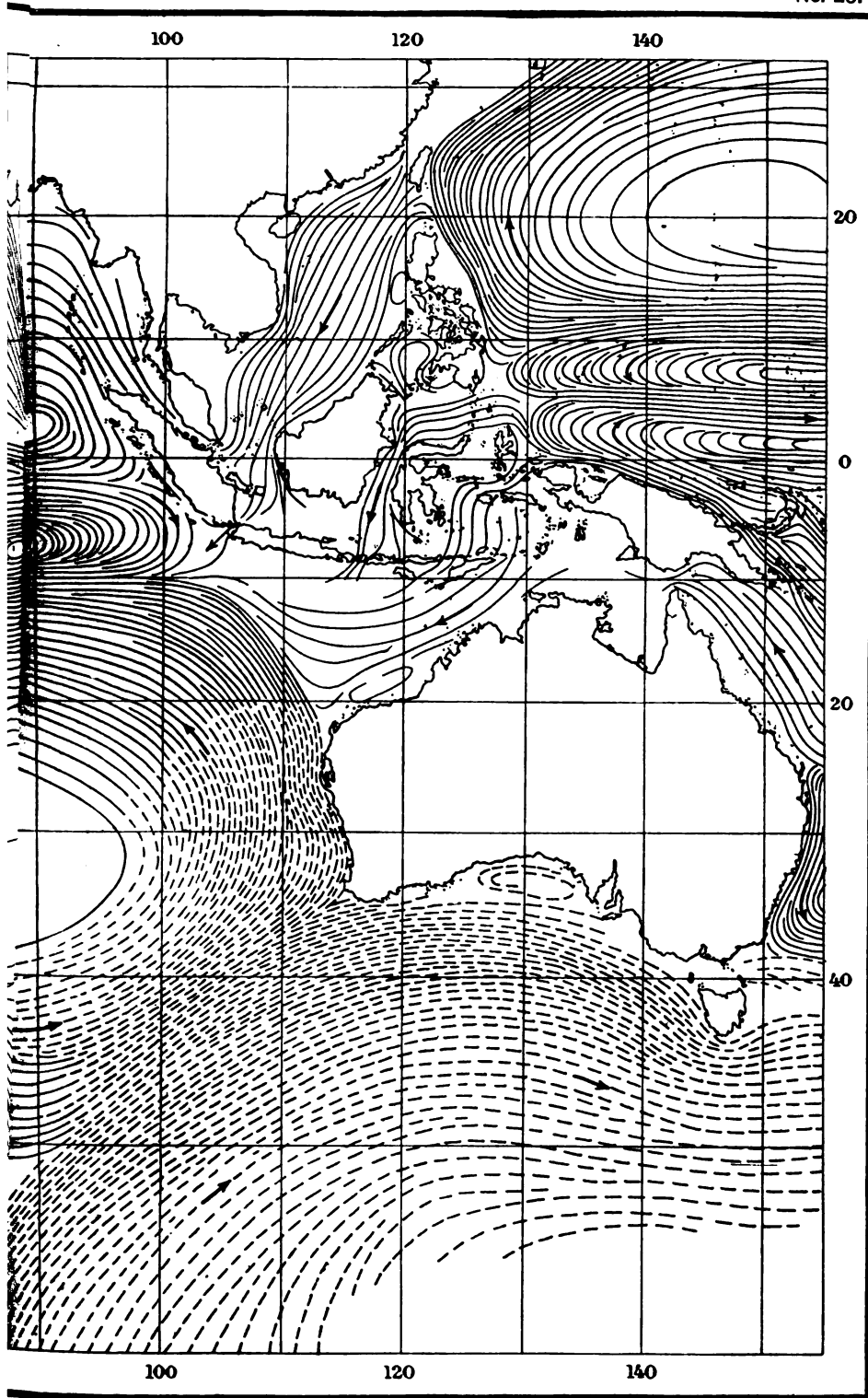


CHART OF NONTIDAL CURRENTS FOR THE INDIAN OCEAN.



SEPTEMBER TO FEBRUARY. (ATLAS BY DEUTSCHE SEEWARTE.)

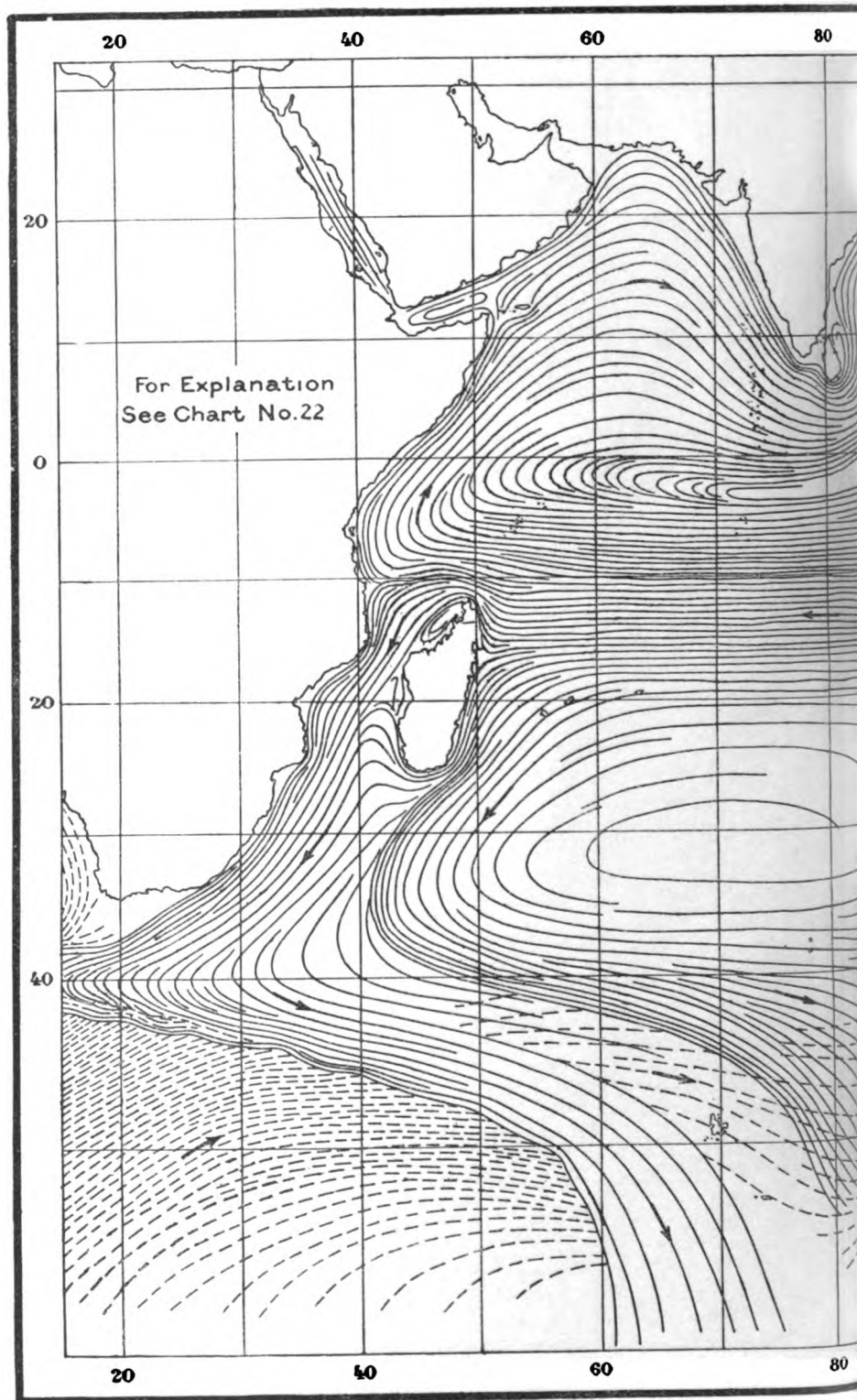
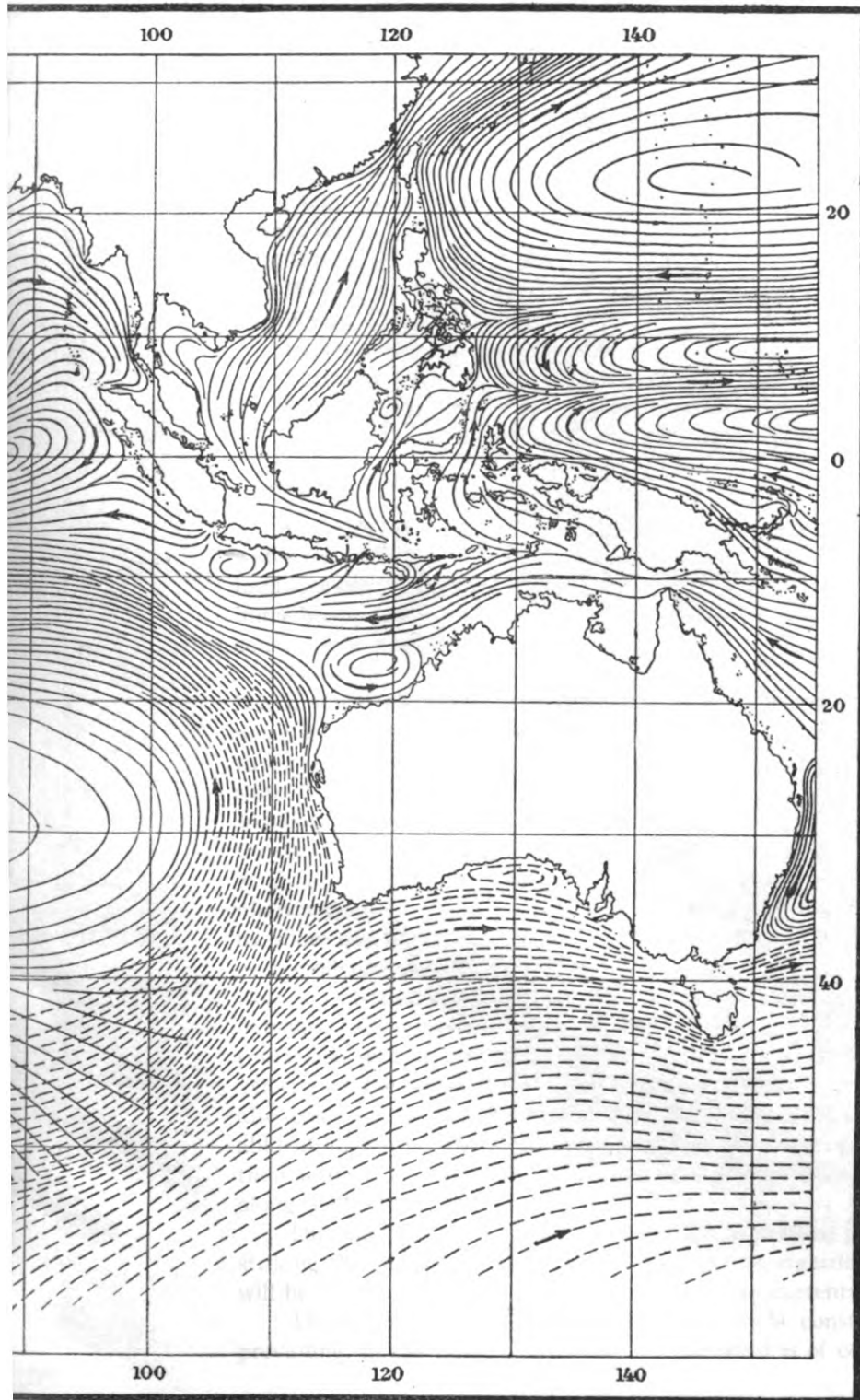


CHART OF NONTIDAL CURRENTS FOR THE INDIAN OCEAN



E TO AUGUST. (ATLAS BY DEUTSCHE SEEWARTE.)

The following are a few works containing matter relating to the movements of the atmosphere:

- Buchan: The Isobars and Prevailing Winds of the Globe, 1868.
 Ferrel (Wm.): A Popular Treatise on the Winds, New York, 1889.
 Deutsche Seewarte: Indischer Ozean, Hamburg, 1891.
 Abbe (Cleveland): The Mechanics of the Earth's Atmosphere, A Collection of Translations, Washington, 1891.
 Abbe (Cleveland): Article "Meteorology," New Volumes, Encyclopædia Britannica.
 Hann (Julius): Berghaus' Physikalischer Atlas, Gotha, 1892.
 Waldo (Frank): Elementary Meteorology, New York, 1896.
 Deutsche Seewarte: Stiller Ozean, Hamburg, 1896.
 Bartholomew's Physical Atlas: Vol. III, Atlas of Meteorology, 1899.
 Deutsche Seewarte: Atlantischer Ozean, Hamburg, 1902.
 Supan (Alex.): Grundzüge der Physischen Erdkunde, Leipzig, 1903.

110. *The direct effect of the wind.*

If a wind blows for a considerable time in one direction over an inclosed body of water the surface particles are carried or drifted from their original position through the impingement of the air upon them. These particles drag with them those situated immediately below the surface, and in time this dragging influence will be felt down to considerable depths.

The effect of these horizontal forces on the waters of a closed body is to increase the height of water level on the lee shore and to diminish it upon the opposite shore, although not generally by the same amount. In shallow bodies, or along the shelving shores of the ocean, the amount of this elevation may be considerable, as will be seen upon consulting sections 123, 124. In deep bodies with abrupt shores, the piling up is very small although there may be a good surface drift maintained by the wind. The reason for this is that the horizontal forces due to the wind do not act alike upon the particles at all depths, as do the tidal forces; for, they are considerable at the surface and insignificant near the bottom. Consequently the pressure due to the increased depth on the lee shore quickly gives rise to an acceleration in the reverse direction which exceeds, at even moderate depths, the acceleration imparted to the liquid elements by the moving elements situated nearer the surface. Hence the retrograde movement of the water not only near the bottom but for a considerable distance upward. Because of its much greater transverse section, the returning stream is as a rule scarcely perceptible, although the velocity of the surface stream may be considerable. Of course this counter movement also exists in the shallow bodies just referred to, because the wind's action can not be alike at all depths (like tidal forces), and so the body can not be in equilibrium under their action. Consequently there must be a drifting before the wind and a return current along the bottom.

This may be regarded as the circulation in vertical planes due directly to the wind striking the surface of the water. What may be regarded as the horizontal circulation will be briefly considered in describing the ocean currents, sections 111-115.

All the above remarks suppose the wind to be constant for some time, or at least prevailing, in the case of an ocean. If the wind is of comparatively short duration it

may give rise to seiche oscillations, which have been described in section 16, Part I, and will be further considered in Chapter IX, Part V.

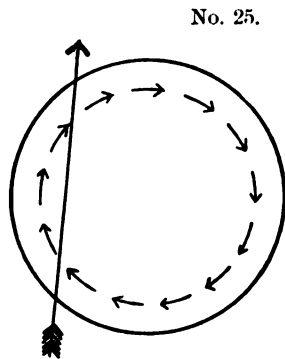
Circulation in horizontal orbits constitute a very general and obvious effect of the direct action of the winds upon the surface and are considered in several of the immediately following sections.

III. *Currents of the Atlantic Ocean.*

The effect which a constantly blowing wind may have in creating and maintaining surface currents or drifts in a given body of water depends largely upon the size, depth, and shape of the body. One of the most simple cases is a circular, elliptical, or oval body across one edge or limb of which the wind acts in approximately the direction of the boundary at this locality. (See Fig. 25.)

If at other edges of the basin the winds follow the direction of the near-by boundary still greater circulation will be imparted to the water, and it will reach to greater depths below the surface. The central portion will be comparatively quiet, forming a sargasso sea.

Examples of this are; the North Atlantic up to about the forty-fifth parallel; the South Atlantic to a line extending from Cape of Good Hope to the Falkland Islands. In either of these cases the body considered is not entirely surrounded by land, but an inspection of the wind directions will readily convince one that the results could hardly have been essentially different had there been a rigid wall extending across the North Atlantic along the forty-fifth parallel, another extending from Cape of Good Hope to the Falkland Islands, and even another along the Equator. In fact, were these walls in existence, then it would be possible for an east wind in the Torrid Zone to alone maintain the circulation in both basins. This is nearly the case which Krümmel* illustrated and established by experiment. (See Fig. 20.)



The north equatorial trade winds of the Atlantic blow from the northeast and east, giving rise to the resulting westerly drift, which was noticed by Columbus in his first voyage to the West Indies. He referred the cause of these motions to the westward motion of the primum mobile.†

The Gulf Stream was encountered by Ponce de Leon in 1513, as he skirted the coast of the peninsula of Florida, from a point north of Cape Canaveral to the Tortugas Islands. Franklin was the first to make a scientific study of the subject.

Its origin in the Straits of Florida is briefly explained in section 116; but its continuance across the Atlantic is due to the prevailing westerly winds, sometimes known as anti-trades. One portion of the stream impinges against Great Britain and Ireland, and probably extends as far to the northeast as Nova Zembla; the other branch continues easterly until it turns southward along the coasts of Portugal, Spain, and Africa.

Along the African coast this branch is called the Canary Stream, and it joins the North Equatorial Drift in the vicinity of the Cape Verde Islands. From here the drift is rapid, averaging about 10 miles a day. Through the Lesser Antilles the surface

* Handbuch der Ozeanographie, p. 358.

† See under Hakluyt and Bacon, this manual, secs. 74-76, Part I.

velocity exceeds 1 knot per hour. A good drift covers the Caribbean Sea and passes, mainly by force of gravity, through the Yucatan Channel. The larger portion of this water is spread out in the Gulf, but the smaller portion at once makes for the Straits of Florida in its effort to seek a lower level.* A portion of the western drift passes to the north of the Greater Antilles, and as a connecting current forms the western boundary of the Sargasso Sea.

According to the maps of Lieutenant Soley, one part of the Gulf Stream upon emerging from the Yucatan Channel turns westward and passes around or across the Campeche Bank—across if north winds prevail. From near Tampico, at the head of the Gulf, the main stream goes northeasterly to a point not far from the Mississippi Delta. An important northwestern branch goes from Yucatan Channel to join the branch just described near the Mississippi Delta. From this locality a stream flows southeasterly to the western end of Florida Strait.

The winds of the Gulf are northerly from October to February, inclusive, and southeasterly during the remainder of the year. The result of the southeasterly wind is to strengthen in the summer time the branch of the Gulf Stream extending from Yucatan Channel nearly to the Mississippi Delta. An eastern branch goes directly from Yucatan Channel to Florida Strait.

A counter current occupies the shallow northwestern corner of the Gulf. In a somewhat similar manner the shallow eastern edge of the Gulf is probably occupied by a weak northerly counter current. A current flows along the northern coast of Cuba and extends around Cape San Antonio, a little beyond the Isle of Pines.

The velocities of the principal streams within the Gulf generally lie between 0.5 knot and 2 knots.

It may be of interest to note here that as long ago as 1856 a bottle was picked up on Loggerhead Key, Florida Reef, by a Coast Survey party, which had been set adrift nearly two years before at a point south of the Mississippi Delta (Coast Survey Report, 1856, p. 279). This indicated the existence of a branch of the Gulf current mentioned above, which Lieutenant Soley has fully established.

The south equatorial trade winds blowing from the south and east, together with the resulting westerly drift, carried the navigator Cabral from his track, projected for rounding Africa, upon the coast of Brazil, thus giving to him the honor of its discovery. Off Cape Roque the drift divides, one branch, known as the Guiana Current, flows northwesterly, joining the north equatorial drift; the other, known as the Brazilian, flows southwesterly toward the Falkland Islands. The east-going stream across the Atlantic in the latitude of Tristan da Cunha, greatly increased in volume by cold water from the south, and the north-going stream west of South Africa, complete the circuit for this South Atlantic area.

112. *Currents around the Antarctic Continent.*

The prevailing winds off the coasts of the Antarctic Continent are westerly. These impart an eastward motion to the surrounding waters. This can be easily detected in the strait between Cape Horn and the South Shetland Islands, where the velocity is 11 miles per day. The general direction and amount of drift can be inferred from the fact that a bottle floated from Cape Horn to Port Phillip, on the south coast of Australia, a distance exceeding 8000 miles, in three and one-half years.

* Cf. A. Lindenkohl: U. S. Coast and Geodetic Survey Report for 1895, pp. 364, 365.

The immediate cause of the Cape Horn Stream is partly gravitational; for, the westerly winds force the waters against the Chilean coast, and the western coast of South Shetland Islands, and an escape to a slightly lower level is through the strait. Another portion of these accumulated waters flows northward, joining the Peruvian Current.

The cold streams from the Antarctic are felt as far north as off the Rio de la Plata, along the western coast of South Africa, and all along the western coast of Chile.*

113. *Currents in the Pacific Ocean.*

The Peruvian or Humboldt Current flows northerly, following the South American coast, sustained by the prevailing winds which here blow from the south; the velocity of this stream along the Peruvian coast lies between one-half and 1 knot. The current leaves the American coast near Cape Blanco, with a velocity perhaps as high as $1\frac{1}{2}$ knots and, much diminished in velocity and increased in width, flows westward under the influence of the south equatorial trade winds of the Pacific. Upon reaching the Pamotu Islands, a portion of this equatorial drift is deflected southerly and joins the general east-going drift referred to above. The remainder of the current has a general westerly direction.

The northern equatorial drift is mainly confined between parallels 10° and 25° north. Near the Hawaiian Islands the currents are variable but their general direction is probably westerly, with a velocity of 1 knot. Among the Marshall Islands the currents are irregular, with perhaps a prevailing westerly direction.

North of the fortieth parallel and across the Pacific Ocean the prevailing wind is from the southwest, and imparts an easterly motion to the waters of this region, and so accounts for the eastward continuation of the Kuro Siwo. This stream impinges upon the American coast, tempering the climate from Alaska to California. A portion of the drift known as the Californian Current flows southward off the coasts of Upper and Lower California, and finally joins the west-going drift.

The Kuro Siwo proper is a continuation and reflex of the north equatorial drift current, and so in part due to the easterly drift in the North Pacific; in other words, it is in part a connecting current. In part its immediate cause is gravitational, at least during the southwest monsoon. At that season the surface of the northern end of the China Sea must be on a higher level than the surface of the Pacific outside. This would account for such annual fluctuation of the intensity of the stream as exists southeast of Formosa.

East of Australia a counter-clockwise circulation exists, caused mainly by southeasterly winds acting upon the waters surrounding New Caledonia and the northwesterly wind acting upon the waters southeast from Tasmania.

114. *Currents in the Indian Ocean.*

North of the equator is the region of monsoons. Here the current has a northeasterly direction during the summer season of the northern hemisphere and a southwesterly direction during the winter season.

Between about 8° and 20° of south latitude is a west-going drift, due to the tolerably constant trade winds.

* The South American Pilot, Part II, ninth edition, under "Currents," pp. 22-25.

In the Indian Ocean south of the parallel of Cape of Good Hope the drift is in an easterly direction, agreeing with the general circulation around the Antarctic Continent. A branch of this drift turned northward by the coast of Australia, and known as the West-Australian Current, finally joins the west-going drift just referred to. The west-going drift is connected with the east-going by southwesterly currents along the coast of Madagascar.

115. *Currents in the Arctic Ocean.*

Some account of the movements of the surface currents of the Arctic from a point near Herald Island toward Cape Farewell, Greenland, has been given in section 46, Part IV B. Generally speaking, the direction of these currents in the open ocean approximately agrees with the direction toward which the winds blow. This was found to be the case by Nansen in the drifting of the *Fram*, and is probably true for most of the waters intersecting the Arctic Archipelago.

Along the northern coast of Greenland and north of Grant Land, Peary found the northwesterly winds and the ice setting easterly. Weyprecht and Payer, in the *Tegetthoff*, drifted from off Northern Nova Zembla northward and thus, in 1873, discovered Franz Josef Land. These indicate lateral movements toward the channel between Greenland and Spitzbergen and through which the ice escapes from the Arctic.

The southward current through Robeson and Kennedy channels has been well established by Hall, Greely, and others. The cold Labrador Current comes from Hudson Strait and the western side of Baffin Bay. It is felt as far south as Cape Hatteras, being sheltered from the west winds by the American coast, and being crowded against it by the deflecting force of the earth's rotation. A small portion of this stream traverses the Gulf of St. Lawrence, entering through the Strait of Belle Isle and leaving through Cabot Strait, although it is probably lost in the waters of the Gulf.

All of the above currents indicate a general surface movement from a region where the waters are of less density into one having warmer but denser waters. But by section 116 this implies that, save for the effect of the prevailing winds, the surface of the Arctic is slightly higher than the surface of the Atlantic. This seems reasonable because of the considerable precipitation and very small evaporation in cold regions. Because of its smaller density, the fresh water tends to remain upon the surface.

It may be noted here that the direction of the prevailing current at Point Barrow is supposed to be eastward, while a long series of observations at the Government station a few miles to the southwest of the point showed that the prevailing wind was from ENE.

As mentioned in section 116, the immediate cause of the reversible currents through Bering Strait is probably gravitational—similar to the flow between Lakes Michigan and Huron, when strong winds blow over their surfaces. When the northern part of Bering Sea is from any cause for some time higher than usual, the flow is northward; and when lower than usual, southward. The southerly winds in the summer bring about the first condition and the northerly winds during the remainder of the year, the second.

More detailed information concerning ocean currents can be obtained from the maps, Figs. 20-24.

116. *Currents whose immediate cause is a difference in surface-levels, in water-densities, or in both combined.*

Bodies of water, or two portions of the same body, may, through prevailing winds, evaporation, precipitation, or influx of water from the land, assume slightly different surface-levels or possess different densities.

These conditions give rise to currents, and may be divided into four cases, real or hypothetical, viz.: Like densities but different surface-levels, like surface-levels but different densities, the denser body having the higher surface-level, the lighter body having the higher surface-level.

Case 1.—Like densities and different surface-levels.

In attempting to restore equilibrium, the water will continually flow from the higher body (i. e., the body whose surface is the higher) into the lower. If the connecting strait is sufficiently small for enabling a sensible difference in surface-level to be maintained, the velocity may there be considerable. The velocity at the narrowest part of the strait and in the swiftest thread of the stream will be approximately equal to $\sqrt{2g(z_1 - z_2)}$ where z_1, z_2 , denote the heights of the surfaces of the two bodies above a fixed datum.

The effect of the north-equatorial trade winds is to elevate the water in the Caribbean Sea and the Gulf of Mexico above the general level of the Atlantic. Levels run across Florida between Cedar Keys and St. Augustine indicate a difference of level amounting to probably at least 0.8 foot. This implies a velocity of 7.2 feet per second, or 4.3 knots per hour, for the swiftest thread—an amount not greatly in excess of the observed value 3.4 knots (sec. 97). The above is in substantial agreement with Franklin's explanation. He says:

This stream is probably generated by the great accumulation of water on the eastern coast of America between the tropics by the trade winds which constantly blow there. It is known that a large piece of water, 10 miles broad and generally only 3 feet deep, has, by a strong wind, had its water driven to one side and sustained so as to become 6 feet deep, while the windward side was laid dry. This may give some idea of the quantity heaped upon the American coast, and the reason of its running down in a strong current through the islands into the Bay of Mexico and from thence proceeding along the coasts and banks of Newfoundland where it turns off toward and runs down through the Western Islands.*

Case 2.—Like surface-levels, different densities.

So long as this condition can be maintained, it is evident that the surfaces of equal pressure in the connecting strait (the free surface excepted) all slope downward toward the lighter body; hence any liquid element will be driven toward that body. The accelerating force will increase in going downward, but this does not mean that the velocity will be comparatively small at the surface; for, by the viscosity of water the under layers would finally impart their velocities to the waters above.

Case 3.—The denser body having the higher surface-level.

Here the flow at all depths is obviously from the higher and denser to the lower and less dense body.

An example of this is the current through Bering Strait from the Bering Sea to the Arctic during the summer season when the surface of the former stands at a higher level than the surface of the latter.

* Pillsbury: U. S. Coast and Geodetic Survey Report for 1890, p. 489.

Case 4.—The lighter body having the higher surface-level.

Since the surface of the less dense body or region is slightly higher than the surface of the one more dense, the water near the surface will flow toward the lower but denser body.

If at some depth below the surface the pressure due to depth be equal in the two bodies (that is, if there the surface of equal pressure be horizontal) there will be no tendency to flow in either direction. Below this surface, the surfaces of equal pressure will slope downward toward the lighter body; hence near the bottom the water will flow toward the lighter body notwithstanding the fact that its free surface is on a higher level.

The surface water of the Arctic Ocean moves toward the Atlantic through Denmark and Davis straits. The considerable precipitation, the influx from several large rivers, and especially the small evaporation, all go to maintaining a rather low density for Arctic waters as well as an increased, but of course very small, elevation of the surface.

The difference in the densities as actually existing is less than that indicated upon charts, because the chart values have been reduced to standard temperatures, and the polar seas, all depths considered, are in reality several degrees colder than the equatorial waters. But near the land, where the amount of fresh water greatly reduces the density, the water's surface is considerably elevated and may give rise to strong and cold currents, particularly in channels and on the right-hand side of the sea or body traversed. This crowding against the land, and prevention of the stream from dispersion, is due to the deflecting force of the earth's rotation. Doubtless a considerable amount of water passes as an undercurrent from the Atlantic into the Arctic through the straits east and west of Iceland. That such is the case can be inferred from the fact that the density of the Arctic, well below the surface, about equals the density found in these straits.

Another example of this case is the Gulf of St. Lawrence and the Atlantic Ocean. Strong surface currents exist in Cabot Strait, the velocity near Cape North being 2 knots.

The Atlantic Ocean and the Mediterranean Sea constitute another example. The surface current nearly always sets easterly in the axis of the strait, the velocity being about 3 knots; but below the surface this eastward flow is less marked. (See section 84, Part IV B.) It is certain that at the bottom the flow is westerly; for, the waters of the Mediterranean must be sufficiently dense through the evaporation of ocean water for causing the lines of equal pressure near the bottom of the strait to slope westerly, otherwise the Mediterranean would be continually deriving salt from the Atlantic.*

According to a report of M. Ch. Lallemand in the *Comptes-Rendus* of the twelfth general conference of the International Geodetic Association on the general leveling in France it appears that mean sea level for Biarritz, at the head of the Bay of Biscay, is 21 centimeters above that at Marseilles. It also appears that the mean sea level at the capes upon which Brest and Cherbourg are situated is somewhat lower than the level at Marseilles. These are conditions which might result from the action of the prevailing westerly winds.

By analogy it is reasonable to suppose that the sea level off the western end of the Strait of Gibraltar is elevated a few inches by the action of the prevailing winds upon the bay forming the approach to the strait; but this difference of level must be small,

* Cf. Boguslawski: *Handbuch der Ozeanographie*, Vol. 1, pp. 37, 38.

because the pressure at the depth of the bottom of the strait is known to be greater in the Mediterranean than in the Atlantic (in order to produce an outward flow), and the difference in density is not very great (section 120).

From page 21 of the Red Sea and Gulf of Aden Pilot, fifth edition, the following, concerning the currents in the strait of Bab el Mandeb, is quoted:

From about May to September, while the southwest monsoon is blowing in the Indian Ocean, the water runs out of the Red Sea; but, during the northeast monsoon, from October to March, it runs in; thus accounting for the difference of level before remarked upon, which has been observed to depend upon the season of the year.

In the strait of Bab el Mandeb these currents often have a rate of 30 or 40 miles a day, but their strength is much diminished a few miles up the sea; and in the strait it is somewhat confused through the irregular tidal influence there felt. At the change of the monsoon there is little or no current.

This is in accord with the general rule for the annual inequality in sea level in the western part of the Indian Ocean; for, upon referring to section 124 it will be seen that high water for this part of the ocean occurs in north winter while low water occurs in north summer.

In this strait the surface current has been observed to go in the direction toward which the wind blows, while near the bottom there was a contrary setting.

A fresh-water stream discharging into the ocean may cause counter currents at the bottom of the channel of the stream and for some distance along the bottom of the outer body.

Examples of this are the straits and sounds connecting the Baltic Sea with the North Sea, the Hudson River, and off the Mississippi River.

For the Baltic, see papers referred to below by F. L. Ekman, V. W. Ekman, and A. W. Cronander. For the Hudson, see a paper by Henry Mitchell, U. S. Coast and Geodetic Survey Report for 1887, pp. 301 311.

It has long been known that off the mouth of the Mississippi the colored surface water takes a different direction from that taken by the clear or blue water below. The existence of a counter current has not been established by observation. In fact observations made by R. Platt while engaged in work for this survey off the delta indicate chiefly that the under currents take a variety of directions with reference to the surface currents. In the river proper, the volume of fresh water is so great that salt water does not enter, and so counter currents do not there occur.

117. *Currents in lakes and bays.*

The Great Lakes of America have main currents moving toward their outlets. The position of such a current with reference to either shore of a lake is determined chiefly with reference to the directions of discharge. In case of Lakes Superior, Michigan, or Huron, the main current lies nearer to the right side than to the left. This suggests the deflecting force of the earth's rotation upon a freely moving body, as the influence governing the position of the axis of the stream. But as this tendency to crowding to the right is not conspicuous in either Lake Erie or Lake Ontario, where the velocity of the current proper must be much greater than that belonging to the upper lakes, it seems almost certain that the influence of this force is too small to be of importance in any of the other lakes.

Currents, especially at the surface, are caused by winds. In this region, the prevailing wind direction is westerly and so the currents produced directly by them, particularly in exposed places, take an easterly direction. On account of the limited size of any one of these lakes, it is not probable that in any forced current produced by the winds the paths of the water particles will show any general deviation to the right of the direction of the paths of the air particles.

The swiftest of all surface currents in the Great Lakes proper are probably those produced by the winds acting upon the waters of Lakes Michigan and Huron, causing the surfaces of the two bodies of water to differ in level by a considerable amount. This greatly increases for a time the discharge of water through the Strait of Mackinac. Observations off Manistee, Lake Michigan, showed a northerly current of from $1\frac{1}{2}$ to 4 statute miles per hour.

In protected places, and particularly along the more irregular shore, counter currents due chiefly to wind currents may exist.

The circulation of the Great Lakes is described in a paper by Mark W. Harrington, entitled "Currents of the Great Lakes, as deduced from the Movements of Bottle Papers during the seasons of 1892 and 1893," and in *Sailing Directions*, published by the U. S. Hydrographic Office (1900-1902), numbered 108 A, and 108 B, 108 C, and 108 D.

The winds which chiefly cause the currents in the Black Sea appear to take various directions over different parts of it.

A recent discussion of the currents of this sea is made by Walther Wissemann on pages 162-180 of the *Annalen der Hydrographie*, Volume 34, 1906.

A counterclockwise circulation takes place in the Mediterranean Sea, including the *Ægean* and the Adriatic seas. There is a dependent current or eddy in the gulf off Tripoli. The influx of water through the Strait of Gibraltar, the straightness of the African coast, and the prevailing westerly winds, account, in the main, for the eastward current very near the southern border of the sea; while the irregularity of the European coast favors a return current protected from the prevailing winds. The deflecting force of the earth's rotation probably causes the return current to enter the *Ægean* and Adriatic seas.

Currents dependent upon contact with others are of common occurrence in partially inclosed bodies of water. If the outer currents flow clockwise in their circuit, the dependent current will flow counterclockwise, and vice versa. The current in the southwestern corner of the Caribbean Sea and that in the Bay of Honduras are examples of this. Other examples are the Gulf of Guinea and near-by waters, the waters lying between Iceland and southern Greenland. The currents in the Japan Sea consist of a branch of the Kuro Siwo, and a counter current necessitated by it flowing southerly along the continental coast line.

The currents of the Greenland Sea and northern extremities of the Atlantic Ocean have recently been ascertained with considerable precision through the agency of bottles set adrift and afterwards recovered. The results of such observations are given in a paper by C. Ryder in the *Nautical Meteorological Annual* for 1904 (Copenhagen, 1905).

It will be noticed that the north-going streams hug the coast to their right or the east, while those going southward, the coast to their right or the west. This agrees with what might have been anticipated from a knowledge of the deflecting force due to the earth's rotation.

But it appears that north winds prevail down the eastern coast of Greenland and southerly ones between Norway and Iceland, due to the fact that a region of low barometer lies between these two coasts. Hence it is probable that the circulation between Iceland and Norway and between Iceland and Southern Greenland is chiefly to be explained by Krümmel's experiment; but that the deflecting force of the earth's rotation causes the streams, whether free or under the action of the sustaining winds, to crowd upon the shores of Norway and southern Greenland.

The openness of these bodies of water would preclude the explanation given for return streams against the winds in lakes and land-locked seas, from applying here.

The deflecting force of the earth's rotation accounts for the fact that the currents flow inward along the eastern shores of the Adriatic and Red seas and Baffin Bay, and outward along the western shores.

118. *Note on recent theoretical work.*

The influence of the earth's rotation upon ocean currents is mentioned by MacLaurin in his prize essay upon the tides.* The equations of motion for a liquid upon a rotating sphere lie at the foundation of Laplace's dynamical theory of tides. Ferrel makes constant use of the principle, following at once from Laplace's equations, that a moving particle is deflected to the right in the northern hemisphere and to the left in the southern. Before the work of V. Bjerknes the ocean currents were treated as free currents, i. e., as if they would move onward by their own inertia after the forces ceased to act. Nansen suspected from observation, and V. W. Ekman confirmed by computation, that forced or sustained currents take, if circumstances permit, a direction to the right of the sustaining force in the northern hemisphere. Moreover, as wind action is from the surface downward (each layer moving the one underneath it) the direction of the lower layer will likewise be to the right of the one imparting the motion.

Supposing all motions in an indefinitely large body of water acted upon by the winds to be steady and horizontal, then the equations of motion become very simple. The external forces for a given water element are the components of the earth's deflecting force, and the only other forces acting are the components of the resistance due to viscosity. Integrating these equations and determining the arbitrary constants to suit the assumed problem, it follows, that in the northern hemisphere, the surface currents take a direction 45° to the right of the direction of the wind, and that this angle increases with the depth.

Ekman also considers currents caused by pressure-gradient and the earth's rotation alone, and wind currents influenced by the continents, currents which arise from a difference of density, and the action of both wind and density variation. See references to Ekman below.

On account of the actual distribution of land and water, it is difficult to say at this time to what extent Ekman's theory of forced currents accounts for the existing ocean currents. The fact that there is a tendency for the water to flow to the right or left of the direction toward which the wind blows will doubtless be brought out for many regions.

*This Manual, Part I, sec. 95.

The hydrodynamical problems involving the motions of waters of different densities, treated by V. Bjerknes and V. W. Ekman, are of fundamental importance in the mathematical treatment of the motions of the sea and adjacent waters.

119. *A few references to observations and discussions of permanent currents.*

General and theoretical.

F. L. Ekman: On the general causes of ocean currents, *Nova Acta, Roy. Soc. Sc. Ups.*, Ser. III (1876).

K. Zöppritz: Zur Theorie der Meeresströmungen, *Annalen der Physik*, Vol. III (1878).

Boguslawski and Krümmel: *Handbuch der Ozeanographie*, Vol. II, Stuttgart, 1887.

M. F. Maury: *Physical Geography of the Sea*.

James Croll: *Climate and Time*, 1887.

H. Berghaus: *Physikalischer Atlas*, Gotha, 1892.

Atlases by the Deutsche Seewarte for the Indian, Pacific, and Atlantic Oceans.

V. W. Ekman: Ein Beitrag zur Erklärung und Berechnung des Stromverlaufs an Flussmündungen, *Kongl. Vetenskaps-Akademiens Förhandlingar*, 1899, pp. 479-507.

A. Supan: *Grundzüge der Physischen Erdkunde*, Leipsic, 1903. (See references there given on pp. 308-309).

A. Lindenkohl: *Encyclopedia Americana* (1904), Article Oceanography.

Charts published by the British Admiralty and by the U. S. Hydrographic Office.

V. W. Ekman: On the influence of the earth's rotation on ocean-currents, *Arkiv.-för Matematik, Astronomi och Fysik*, Vol. 2, No. 11 (1905).

O. Pettersson: On the influence of ice-melting upon oceanic circulation, *Svenska Hydrografisk Biologiska Kommissionens, Skrifter*.

J. W. Sandström: On icemelting in seawater and currents raised by it, *Ibidem*.

V. W. Ekman: Beiträge zur Theorie der Meeresströmungen, *Annalen der Hydrographie und Maritimen Meteorologie*, Vol. 34 (1906), pp. 423-430, 472-484.

Gulf Stream.

Coast and Geodetic Survey Reports: 1846, App. 4; 1853, p. 46; 1854, App. 52; 1856, App. 46; 1858, App. 32; 1860, App. 17; 1868, App. 11; 1884, App. 32; 1885, App. 14; 1886, App. 11; 1887, App. 8; 1889, App. 16; 1890, App. 10; 1891 (2), App. 10; 1895, App. 6.

A. Petermann: Der Golfström und Standpunkt der thermometrischen Kenntnis des Nordatlantischen Ozeans und Landgebiets in Jahre, 1870, *Petermann's Geographische Mitteilungen*, Vol. 16 (1870).

Der Golfström von 10. mai bis zum 10. Juni, 1904, *Annalen der Hydrographie u. Maritime Meteorologie*, Vol. 33, 1905, pp. 314-320.

J. C. Soley: *Annalen der Hydrographie u. Maritime Meteorologie*, 1907, pp. 84-87, and *Pilot Charts North Atlantic Ocean*, issued by U. S. Hydrographic Office, 1907.

Labrador Current.

The Newfoundland and Labrador Pilot, published by the Admiralty. Sailing Directions for Nova Scotia, Bay of Fundy, etc., U. S. Hydrographic Office. Dawson's Reports on Survey of Tides and Currents in Canadian Waters.

Indian Ocean.

Atlas by Deutsche Seewarte. Pilots for Africa, Red Sea, and Gulf of Aden, Hindustan, Bay of Bengal, and Islands in the Southern Indian Ocean, published by the Admiralty.

Arctic Ocean.

Bathymetrical Features of the North Polar Sea, F. Nansen, Christiania, 1904. The Norwegian North Polar Expedition, 1893-96, Scientific Results, London, 1902. F. Nansen.

Pacific Ocean.

Atlas by Deutsche Seewarte. Pacific Islands (Sailing Directions), by the Admiralty. Vols. I, II, III. A. Lindenkohl in Petermanns Geog. Mitteilungen. 1897, Heft XII. Papers by H. C. Russell and by H. A. Lenehan in Jour. and Proc. Roy. Soc. N. S. W.

Kuro Siwo (Japan Stream).

The China Sea Directory, by the Admiralty. Vol. IV.

Northern portion of Atlantic Ocean and North and Baltic Seas.

C. Ryder: Some investigations relating to the ocean currents in the sea between Norway, Scotland, and Greenland.

Nautical Meteorological Annual for 1904, Copenhagen, 1905.

Svenska Hydrografisk Biologiska Kommissionens Skrifter, Vols. I and II.

A. W. Cronander: On the Laws of Movement of Sea-Currents and Rivers, Norrköping, 1898.

120. *Note on density observations.*

Density observations are easily made to a fair degree of precision by means of a hydrometer. One form of this instrument consists of a glass bulb terminated by a slender graduated stem, and a cup or can into which the specimen of sea water is placed and within which is mounted a thermometer for ascertaining the water's temperature. If the range of density is considerable, several bulbs should be provided so that the range of each need not much exceed 1 per cent.

Recent observations have in many cases been reduced to the temperature 15° C. (= 59° F.) for the sea water and the standard temperature of the distilled water is taken as 4° C. (= 39° 2 F.). Results obtained by the U. S. Coast and Geodetic Survey prior to 1892, suppose both sea water and the standard distilled water to have the temperature 60° F. See report by Buchanan, Physics and Chemistry, Volume 1, Challenger Reports. In the Atlases of the Deutsche Seewarte both standard temperatures are 17° 5 C.

Tables for reducing the observed densities to standard temperatures are given on page 155, Coast and Geodetic Survey Report for 1874, and page 277, Report for 1891.

In making the observations care must be taken to secure the water at the assumed position and depth, to give the attached thermometer time to adjust itself to the temperature of the water, and to see that the bulb is not in contact with the walls of the cup, but floats freely.

In tidal rivers the stage of the tide should be noted.

Doctor Boguslawski, on page 147 of his *Handbuch der Ozeanographie*, gives the following densities of the surface waters, which are taken from British source:

Region.	Specific gravity reduced to 16°.7 C. = 62°F.
North Atlantic Ocean up to 50° north latitude.....	1.02664
South Atlantic Ocean up to 50° south latitude.....	1.02676
North Pacific Ocean up to 50° north latitude.....	1.02548
South Pacific Ocean up to 50° south latitude.....	1.02658
Indian Ocean between 0° and 50° south latitude.....	1.02630
Mediterranean Sea.....	1.02689
Black Sea.....	1.01430
North Sea.....	1.02610
Baltic Sea.....	1.00860
Red Sea.....	1.02860

Recent density maps are given in the Atlases of the Pacific, Indian, and Atlantic Oceans, published by the Deutsche Seewarte.

The principal sources of information used in the construction of these charts are given in the texts of the Atlases, and to these the reader is referred.

References to densities measured by the Coast and Geodetic Survey:

Hudson River, Report 1871, p. 130; 1887, p. 303.

Mississippi River (South Pass), Report 1875, opp. p. 190, and p. 192.

Chesapeake Bay, Report 1877, p. 189 (results not published).

Chesapeake Bay, Report 1881, p. 303, Figs. 56-62.

Atlantic Ocean, Report 1882, last figure.

Gulf of Mexico, Report 1895, App. 6.

Northeastern Pacific Ocean and Bering Sea, Report 1898, p. 239.

A considerable number of density observations, not published, have been made at Sandy Hook, New Jersey; Delaware Breakwater, Delaware; Tybee Island, Georgia; in the Gulf Stream, and in the Northern Pacific Ocean.

A few references not included in the texts of the Atlases of the Deutsche Seewarte are as follows:

A. Lindenkohl: Das spezifische Gewicht des Meerwassers in Nordöst-Pacifischen Ozean, etc., Petermanns Geogr. Mitteilungen, 1897, Heft XII.

A. J. Robertson: Temperature and salinity of the surface waters of the North Sea in the year 1903, North Sea Fisheries Investigation Committee, 1903.

Fridtjof Nansen: The Oceanography of the North Polar Basin, The Norwegian North Polar Expedition, 1893-6, Vol. III (No. IX), 1901-2. The Bathymetrical Features of the North Polar Seas, etc., The Norwegian North Polar Expedition, Vol. IV (No. XII), 1904.

N. M. Knipowitsch: Grundzüge der Hydrologie des Europäischen Eismeers, 1906, pp. 1439-1464.

Svenska Hydrografisk Biologiska Kommissionens Skrifter, Vols. I and II.

THE ANNUAL FLUCTUATION OF WATER LEVEL.

121. *River fluctuations.*

In most fresh-water streams the surface height and the velocity of flow are subject to variations through a yearly period, although great irregularities generally exist. Only the average, or somewhat regular, fluctuation will be here considered. The principal causes are the varying amounts of rain and evaporation, and the melting of snow in higher latitudes.

The annual rising of the Nile had an important bearing upon early civilization and its cause was a subject of much speculation.* The brief description of the phenomenon by Herodotus finds itself in perfect accord with observations made at the dam below Cairo during the years 1849-1878. Herodotus says that the river begins to rise at the summer solstice and reaches its maximum height in about one hundred days. Figure 121 in Supan's *Grundzüge der Physischen Erdkunde* shows this agreement.

From the discussion of observations covering the seven years 1872-1878, it appears that the fluctuation in the surface of the Mississippi River at New Orleans resembles a sine curve whose maximum height is near the middle of May and whose minimum is near the middle of November; also that the range of this regular curve is 10 feet.

The fluctuations in the upper portions of the river and its tributaries is less regular in its character, and the range of the rise and fall is usually much greater than at New Orleans.

The stages of the Mississippi River are shown in the hydrographs published in the Reports of the Mississippi River Commission, which forms a supplement to the recent Reports of the Chief of Engineers (United States Army). From these it will be seen that there are often several well-defined maxima and minima each year. The times of the principal maximum and minimum at Carrollton, La., may be given as about April 20 and November 25, respectively, the average range being about 16 feet. The lesser maxima perhaps occur about February 1 and June 15.

It will be noticed that, as a rule, the rivers of the Atlantic slope of the United States north of Georgia, and gauged above tidal influences, will attain their regular annual maximum height and discharge in February, March, or April, and their minimum in August, September, or October. See next section.

From the known geography and climatology of a given river basin some estimate can generally be made of the times when floods and freshets may be expected, also when the contributing streams should run unusually low. If these times are at variance with those given by the annual inequality in the water level as observed at some tidal station upon the river, one may be reasonably certain that the floods and freshets are, at that station, of secondary importance in producing this inequality.

For instance, at Port Eads, situated on South Pass of the mouth of the Mississippi, the regular annual maximum height (sec. 124) occurs in October, the range being 0.9 foot, whereas, as already stated, the same river at New Orleans reaches its maximum height in April or May, the range being 10 feet or more.

On the other hand, if the range of the annual fluctuation much exceed 1 foot, excepting certain regions of shallow water, and those parts of the earth where monsoons

* This manual, Part I, secs. 64, 67, *ad finem*.

prevail, it is probable that the upper-river stages control the annual fluctuations unless known meteorological facts are to the contrary. But what part is due to the efflux of river water and what to periodic winds is often difficult to determine. For example, at the river stations Calcutta and Rangoon, the annual high stage occurs about the first of September. But for southeastern Asia the rainy season occurs during the summer months and the dry season during the winter and spring. Consequently the heights of the rivers at these stations are governed by the efflux of the rivers. If this fluctuation were due to the monsoons only, it is probable, as will be noted in the next section, that the maximum height would occur not later than August.

The Great Lakes have an annual fluctuation of from 1 to 2 feet. They stand highest in August and lowest in February. Records extending over more than forty years are published in the Reports of the Chief of Engineers (United States Army).*

* E. g., Report for 1904, Part IV, p. 4057.

122. Table showing average discharge in cubic feet per second.*

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean.
St. Croix River at Sprague Falls, Me., 1902 (1 month), 1903, 1904 (9 months), 1905 (3 months).....	6 463	4 998	7 684	5 131	3 909	2 466	2 902	1 666	932	1 046	1 066	2 172	3 370
Penobscot River at the Sunk Haze Rips, Me., 1899 (4 months), 1900 (9 months).....	14 876	37 262	43 828	52 407	56 632	20 435	13 769	10 722	3 658	2 764	7 170	9 014	22 711
Kennebec River at Waterville, Me., 1893-1902, 1905.....	2 939	3 291	8 163	23 783	19 169	9 729	5 984	3 727	2 901	2 868	4 472	3 655	7 557
Androscoggin River at Rumford Falls, Me., 1892 (8 months), 1893, 1894-1895 (4 months), 1896-1902, 1905.....	2 242	1 927	2 530	9 342	9 895	4 992	3 240	2 514	2 217	2 408	2 942	2 438	3 891
Merrimac River at Lawrence Mass., 1890-1899.....	6 284	6 347	13 976	17 091	9 967	5 739	3 604	3 011	3 196	4 208	5 948	5 869	7 103
Connecticut River at Holyoke, Mass., 1896-1898.....	8 744	7 850	25 723	35 695	17 096	13 024	10 313	6 238	4 835	8 949	14 945	13 031	13 870
Housatonic River at Gaylordville, Conn., 1900 (3 months), 1901, 1902 (11 months), 1903, 1905 (11 months).....	2 502	3 455	6 504	4 434	2 208	2 045	1 328	1 346	1 389	1 620	1 231	2 668	2 561
Hudson River at Mechanicsville, N. Y., 1890-1901, 1902 (11 months), 1903, 1904-1905 (10 months).....	7 004	6 411	13 449	19 716	10 677	6 582	4 370	4 388	4 415	5 128	6 814	6 971	7 994
Mohawk River at Dunsbach Ferry Bridge, 1901, 1902, 1903 (8 months), 1904, 1905.....	3 314	1 670	15 800	10 592	4 990	6 142	3 792	2 899	3 872	6 401	3 854	6 746	5 839
Delaware River at Lambertville, N. J., 1897 (6 months), 1898-1901, 1902 (11 months).....	17 333	22 320	32 972	25 112	16 451	8 736	11 053	11 265	8 228	10 263	11 480	21 510	16 477
Schuylkill River at Fairmount Dam, Pa., 1898 (11 months), 1904, 1905.....	3 743	2 788	5 743	3 068	2 539	1 399	634	2 720	2 166	1 570	2 115	2 226	2 559
Susquehanna River at Harrisburg, Pa., 1891-1904, 1905 (11 months).....	43 610	58 266	98 340	75 024	41 914	27 182	18 396	16 672	13 965	19 372	21 084	37 877	39 308
Octoraro Creek at Rowlandsville, Md., 1896 (2 months), 1897, 1898, 1899 (9 months).....	426	723	436	400	411	253	203	261	214	183	380	461	363

* From measurements made by the U. S. Geological Survey.

Table showing average discharge in cubic feet per second—Continued.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean.
Patuxent River at Woodstock, Md., 1896 (5 months), 1897, 1898-1899 (11 months), 1901 (11 months), 1902-1904, 1905 (11 months).....	492	742	670	488	376	340	337	330	189	202	260	416	410
Potomac River at Point of Rocks, Md., 1895 (11 months), 1896 (10 months), 1897-1906.....	11 999	16 462	23 477	18 055	11 090	7 958	6 057	6 350	2 891	5 283	4 001	9 432	10 255
Monocacy River at Frederick, Md., 1897-1906.....	1 554	1 876	2 562	1 630	915	764	650	755	313	524	525	1 568	1 136
James River at Cartersville, Va., 1899-1904, 1905 (11 months).....	11 289	12 845	16 061	12 030	8 485	8 205	5 314	5 404	3 551	3 201	3 310	7 712	8 117
Appomattox River at Mattoax, Va., 1900 (4 months), 1901-1905.....	1 158	1 647	1 478	1 460	951	601	469	937	483	376	396	1 078	920
Roanoke River at Neal, N. C., 1896 (5 months), 1897-1902, 1903 (5 months).....	13 110	19 562	21 487	17 069	10 934	7 767	6 459	7 616	5 195	6 696	5 336	9 677	10 909
Tar River at Tarboro, N. C., 1896 (6 months), 1897-1900.....	2 124	5 941	5 867	4 005	2 009	1 393	1 337	1 034	796	670	1 022	2 137	2 361
Cape Fear River at Fayetteville, N. C., 1889-1902, 1903 (5 months).....	6 919	10 571	8 657	6 811	4 017	3 264	4 706	5 742	3 143	2 884	2 607	3 408	5 232
Yadkin River at Norwood, N. C., 1896 (4 months), 1897-1899.....	7 137	13 002	14 520	8 034	6 127	4 672	4 637	4 129	4 106	4 027	4 002	5 064	6 629
Waterlee River at Camden, S. C., 1904 (4 months), 1905.....	4 828	11 100	5 206	4 358	7 205	3 025	8 617	10 860	2 929	1 960	2 244	6 333	5 724
Broad River at Alstar, S. C., 1897-1903, 1905.....	7 175	15 233	14 260	12 041	6 809	8 830	5 242	8 502	5 917	4 455	4 110	8 308	8 407
Saluda River at Waterloo, S. C., 1897-1905.....	1 699	3 402	2 984	2 520	1 596	1 997	1 549	1 981	1 452	1 226	1 183	1 771	1 947
Savannah River at Augusta, Ga., 1894-1905.....	9 990	22 168	18 665	13 358	8 617	10 799	8 362	11 041	9 811	6 862	6 234	11 120	11 419
Ocmulgee River at Macon, Ga., 1893-1905, 1905.....	2 765	5 665	5 391	3 825	1 991	2 375	2 528	2 656	2 127	1 599	1 594	2 683	2 933
Oconee River at Dublin, Ga., 1898 (11 months), 1899-1905.....	5 852	10 344	8 735	7 605	3 599	4 015	3 252	3 586	3 828	2 654	2 699	4 860	5 086
Ochopee River at Reidsville, Ga., 1904, 1905.....	903	2 194	2 346	1 060	344	900	916	1 501	708	266	408	650	1 016
Flint River at Albany, Ga., 1902-1905.....	7 622	15 266	11 796	9 145	6 710	5 052	4 233	6 228	4 592	2 798	3 331	6 827	6 996

Table showing average discharge in cubic feet per second—Continued.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Mean.
Chattahoochee River at West Point, Ga., 1896 (5 months), 1897-1905.....	6 059	10 907	10 953	8 170	5 077	5 238	4 685	5 326	3 773	3 001	3 087	5 766	6 004
Alabama River at Selma, Ala., 1896-1905.....	34 920	58 699	53 109	47 041	22 163	19 584	15 898	17 093	10 821	8 947	9 095	24 114	20 790
Columbia River at Centerville, Ala., 1902-1905.....	1 626	3 696	2 822	1 459	1 266	533	544	675	349	393	344	1 034	1 228
Black Warrior River at Tuscaloosa, Ala., 1895-1902.....	14 392	15 679	27 602	16 831	3 849	5 373	2 594	1 787	936	1 226	1 351	5 992	8 134
Tombigbee River at Epes, Ala., 1901, 1905.....	19 384	29 164	17 194	14 456	9 190	3 840	2 804	13 876	2 602	2 214	1 954	10 102	10 570
Mississippi River at mouth of Red River, 1881 (4 months), 1882, 1883, 1884 (8 months)*.....	848 979	1 752 929	1 304 354	1 009 433	925 451	868 211	613 107	333 677	212 933	316 419	497 978	533 430	768 075
Sabine River, near Longview, Tex., 1905.....	395	1 251	3 134	5 511	16 640	4 470	9 777	1 934	195	461	1 332	6 388	4 290
Neches River at Evadale, Tex., 1904 (6 months), 1905.....	5 090	7 171	10 210	12 870	15 070	7 865	5 604	3 003	724	448	2 213	3 178	6 121
Trinity River at Riverside, Tex., 1903, 1904, 1905.....	3 063	6 828	12 239	10 848	14 043	10 767	10 013	5 227	821	2 436	2 035	4 100	6 868
Brazos River at Richmond, Tex., 1903, 1904, 1905.....	3 651	8 855	16 698	9 018	25 032	8 653	9 183	6 538	2 123	3 687	2 104	2 347	8 157
Colorado River at Columbus, Tex., 1904, 1905.....	797	942	2 351	3 910	8 701	4 910	2 898	2 066	2 036	1 862	1 504	1 520	2 792
Rio Grande at Eagle Pass, Tex., 1900 (8 months), 1902-1905.....	2 495	2 503	3 365	3 131	5 678	10 530	8 468	7 909	16 118	9 177	5 349	4 590	6 607
Colorado River at Yuma, Ariz., 1902-1905.....	4 645	9 806	16 884	16 598	34 891	53 988	25 841	11 074	7 186	8 115	6 934	7 408	16 947
Sacramento River at Collinsville, Cal., 1878-1884.....	30 500	38 167	60 833	93 867	93 867	62 667	23 833	10 250	7 083	7 917	8 700	15 067	37 729

* From measurements made by Mississippi River Commission.

123. *Direct action of winds in causing the annual height inequality.*

In shallow arms of the ocean the direct action of the wind in altering the height of the surface is very apparent.

Along the eastern coast of the United States the prevailing wind is northwesterly during the winter season and southwesterly, south, or southeasterly during the summer season. As a consequence the height of the sea is considerably lowered in the winter season and, at least in many localities, increased during the summer season. This explains the annual inequality at Governors Island, Baltimore, Old Point Comfort, Savannah River Entrance, and Fernandina, and in the main that for Philadelphia, Washington, and Wilmington (N. C.), although the upper-river stages are factors of some importance at these stations.

At Philadelphia the high water of the annual fluctuation occurs about September 1 and the low water in December or January. There seems to be, however, a secondary high water in March and low water in May. According to river gaugings made by the U. S. Geological Survey at Lambertville, N. J., the greatest monthly discharge of the Delaware River takes place during the month of March, the least during the month of September.

At Washington, D. C., the high water of the annual fluctuation occurs in June or July, the low water about March 1. According to river gaugings made at Point of Rocks, Md., the greatest monthly discharge takes place during the month of March, the least during the month of September.

At Wilmington, N. C., the high water of the annual fluctuation occurs in September or October, the low water about March 1. Gaugings at Fayetteville show that the months of greatest and least discharge nearly coincide with February and November.

At gauging stations situated above tidal influence, an increase of height in the water-level is accompanied by an increase of discharge and the empirical relation between the two constitutes a rating table for the stream with reference to which it has been constructed.

The northerly winds across the Gulf of Mexico produce low water at Galveston in February, while the southerly winds produce high water in October, the range being 1.5 feet. (The inundation and destruction of Galveston resulting from a West India hurricane occurred September 8, 1900.)

According to observations made by the Coast Survey in 1847 and 1848, the average effect of a north wind is to depress the waters of Albemarle Sound, North Carolina, between one and two tenths of a foot, and the effect of a south or southwest wind is to raise the level by about the same amount.* The fact that the result was the same for both sides of the Sound indicates clearly that the action of the winds upon Pamlico Sound is the principal cause of the fluctuation in level.

According to observations made in 1848 the average effect of southeasterly winds at Cat Island is to elevate the surface of the water 0.3 foot and that of north or north-west winds to depress the level 0.2 foot.†

Along the coast of New England, Canada, and Newfoundland the annual inequality is small and uncertain, the winds there having a less marked annual change than farther southward.

At Boston the prevailing winds at all seasons of the year are westerly, a little north of west in the winter, and somewhat southwesterly in the summer.‡

* Report for 1856, p. 271.

† Report for 1856, p. 278.

‡ See Appendix No. 6, U. S. Coast Survey Report for 1871. Here Ferrel shows the connection between the fluctuations of the sea and the variations of the barometer and the winds.

The winter winds along the Atlantic coast of the United States are the strongest of the year; consequently the water is most diminished in height during that season.

The prevailing summer winds of the southern and eastern coasts of Asia are nearly opposite in direction from the prevailing winter winds. (See Figs. 18, 19, and table in section 124.) The southwest monsoon draws the water away from Aden and the Red Sea in the summer season, but tends to pile it up on the coast of Baluchistan and the upper India; it diminishes the height in the neighborhood of Minikoi and Ceylon, but increases the height at Bengal and the southwestern coast of Farther India. This explains the semiannual reversal of currents observed in the Strait of Bab-el-Mandeb and mentioned in section 116. The southeasterly trend of the Malabar coast prevents the southwest monsoon from actually increasing much the height along the western coast, as the long series of observations at Karachi shows.

At Dublat, the mouth of the Hooghly River, the maximum occurs late in July with a range of 1.8 feet; farther up, at Diamond Harbor, it occurs early in August with a range of 2.1 feet; while at Calcutta (Kidderpore), where the effects of the river stages are apparent, the maximum occurs about September 1, the range being 6.1 feet.

The coast stations Chittagong and Akyab have annual inequalities of 3.2 and 1.9 feet, respectively, with maxima occurring in July, thus agreeing with the monsoon season.

For Elephant Point, a coast station at the mouth of the Rangoon River, the range of the annual inequality is 1.7 feet, while at Rangoon it is 2.7 feet. The maximum at the former place occurs about July 27 and at the latter place about August 26. Other coast stations are Port Blair (Andaman Islands), Mergui, and Amherst, with ranges 0.5, 1.2, 1.6 feet, respectively, and with maximum falling in or near August. The increase of range in going from the Andaman Islands to the main land is in accordance with what would necessarily result from wind effects.

For details concerning the tidal stations and tides of India, see Account of the Operations of the Great Trigonometrical Survey of India, Volume 16, Tidal Observations.

By the southwest monsoon the water level is elevated during the summer season along the western coast of Luzon and especially in the Yellow Sea. Water is, however, drawn away from the southern portion of the China Sea. Among the East Indies the effects are various, because water can drift in or out from either the Indian or the Pacific Ocean. Although it is difficult to infer the time of maximum height from the known monsoons, there is little doubt that the annual fluctuation along the coast is chiefly a wind effect.

The shallow and northern part of Bering Sea is elevated by southerly winds, which blow over Bering Sea during the summer season, and depressed by the northerly winds in winter.

The southwesterly winds of autumn produce high water along the shores of the Gulf of Alaska, while the easterly winds of spring and summer produce low waters.

The easterly wind is strongest for the eastern coast of Australia in March, and in consequence the sea level at Sydney, Ballina, and Newcastle is highest in the (north) spring season.

At Panama southwesterly winds prevail through the greater part of the year; but during the month of February, the winds are northeasterly. The annual fluctuation attains its maximum on about November 1 and its minimum on about February 20. The range of the oscillation is 2.0 feet.

By consulting the table given in section 124, it will be seen that for the Atlantic coast of Europe annual high water occurs from about September to December and low water from March to June. The range of the fluctuation is 1.0 foot at Greenock, 0.9 foot at Liverpool, but only 0.4 foot at Edinburgh, 0.5 at Sheerness, and 0.4 at London, and 0.3 at Ramsgate. Since greater ranges of annual tide occur on the western side of Great Britain and around Ireland than on the eastern side, the inference most readily drawn is that the winds are the controlling factor in its production. By consulting Pls. 21-24 of the Atlas of the Atlantic Ocean, by the German Seewarte, also Pl. 12 of Bartholomew's Physical Atlas (Meteorology), it will be seen that the west winds along the coasts of Europe are strongest in the autumn and weakest in the spring.

As pointed out by Lentz in his *Fluth and Ebbe*, Chapter III, the annual tide of the Baltic coast of Germany is governed by the winds. In September it has a maximum height and in May a minimum. The minimum coincides in time with the maximum amount of northeasterly winds, which blow the water out of the Baltic into the North Sea.

Levels run from Helder to Memel prove the existence of a permanent displacement of level produced by the winds, and comparable in order of magnitude with the annual fluctuation. It increases in height from Kiel to Memel.*

At Wilhelmshaven the maximum height occurs about October 1, and the minimum early in April.

The foregoing statements and the table in section 124 indicate that as a rule the water at most tidal stations stands highest in the summer or autumn and lowest in the winter or spring. This suggests that the annual fluctuation may be due to the alternate heating and cooling of the ocean waters, causing alternate expansion and contraction of the volumes. This possibility will be considered in the next section. Here it may be noted that as sea breezes at the earth's surface occur by day and land breezes by night, so the wind blows from the ocean in summer and from the land in winter. Without further assumptions we thus have an agency for producing the annual fluctuation and whose potency in many instances can not be doubted.

High-pressure areas are formed on the lands in winter and in the oceans in summer. Hence, the annual inequality can not be directly due to this cause. Moreover, the annual barometric fluctuations are too small to account for the annual inequality in sea level at most places. It seldom has a range of more than 0.3 of an inch, and this could give only about 0.3 of a foot for the range of the annual inequality.† In some instances it undoubtedly increases the annual inequality, as at St. Paul, Kodiak Island, Alaska, where the minimum pressure occurs in winter and the maximum in summer.

Since winds along the surface of the earth are in a general way setting from high-pressure areas toward low-pressure areas, it follows that indirectly these annual fluctuations in the barometer do in this sense greatly influence and even occasion the annual inequality in sea level. But as already intimated, the direct effect is often at variance with the indirect and far more important effect, viz., that due to resulting winds.

* Hugo Lentz: *Fluth und Ebbe, und die Wirkungen des Windes auf den Meeresspiegel*, Hamburg, 1879, pp. 120, 121.

Cf. P. G. Rosen: *Om hafsyttans höjdförhållande vid några punkter af Sveriges kuster under tiden 1887-1900*, Vol. 1, Sv. Hydro. Biol. Kom. Skrifter.

† Berghaus' *Physikalischer Atlas*, Charts Nos. 33, 34. Coast Survey Report 1875, Figs. 33, 34.

124. Long-period tides and

No.	Station	Latitude	Longi- tude	Mf	Mf°	Mm	Mm°	Sa
		° ' "	° ' "	Feet	°	Feet	°	Feet
EAST COAST OF AMERICA.								
		North	West					
28	St. Johns, Newfoundland	47 34	52 41					0.200 268
32	Quebec	47 49	71 12	0.122	76	0.303	34	.490 65
36	St. Paul Island	47 14	60 08					.073 164
40	Halifax	44 40	63 35	{ * .025 † .042	{ * 324 † 178	{ * .029 † .113	{ * 215 † 64	.150 252
42	St. John, New Brunswick	45 14	66 04	.056	184	.047	137	.115 93
44	Eastport, Me.	44 54	66 59					.105 338
45	Pulpit Harbor	44 09	68 53	.043	88	.053	22	.163 172
46	Portland, Me.	43 39	70 15					.200 178
50	Boston	42 22	71 03					.094 116
58	Newport	41 29	71 20					.144 153
59	Bristol	41 40	71 16					.194 151
60	Providence	41 49	71 24					.244 106
63	New London	41 21	72 05					.242 153
66	Willels Point	40 48	73 46					.153 110
67	New York, Governors Island	40 41	74 01					.244 127
70	Sandy Hook	40 27	74 00					.068 208
77	Philadelphia	39 57	75 09					.417 146
85	Old Point Comfort	37 00	76 19					.320 126
90	Washington Navy Yard	38 52	77 01					.272 128
95	Baltimore	39 17	76 35					.260 123
98	Wilmington	34 14	77 57					.302 173
101	Charleston, Custom-house Wharf	32 47	79 55					.288 186
103	Savannah Entrance, Tybee Island Light	32 02	80 51					.217 124
105	Fernandina, Fort Clinch	30 41	81 28					.237 105
106	Fernandina, Dade St.	30 41	81 28					.406 146
113	Key West, Fort Taylor	24 33	81 48					.377 216
125	Pensacola, Warrington Navy Yard	30 21	87 16					.244 92
131	Port Eads	29 01	89 10					.361 173
140	Galveston	29 19	94 47					.528 170
154	Nassau, Bahamas	25 05	77 21					.312 144
		South						
186	Buenos Ayres	34 36	58 22					.389 321
WEST COAST OF AMERICA.								
200	Cape Horn, Orange Bay	55 31	68 05					.156 92
205	Valparaiso, Chile	33 02	71 39					.151 351
		North						
210	Panama, Naos Island	8 55	79 32					.685 170
215	Mazatlan, Mexico (west coast)	23 11	106 27					.126 153
221	San Diego	32 42	117 14					.231 189
224	San Francisco, Entrance	37 49	122 29					.398 156
225	Sausalito	37 50	122 28					.150 244
226	San Francisco, Presidio, Cal.	37 48	122 27					.149 156
227	Astoria, Oreg.	46 11	123 50					.244 284
230	Port Townsend	48 07	122 45					.270 288
241	Sand Heads, Fraser River	49 05	123 16	.081	121	.101	354	.169 264
250	Sitka	57 03	135 20					.260 284
265	Kodiak (St. Paul), Alaska	57 48	152 21					.899 216

* Two years.

† One year.

computed annual fluctuation.

Ssa		No. of years observed	Principal				Secondary				No.
			Maximum		Minimum		Maximum		Minimum		
			Time	Height	Time	Height	Time	Height	Time	Height	
Feet	°			Feet		Feet		Feet		Feet	
0.071	217	18	Dec. 31	0.26	May 12	-0.19					28
.428	110	6	May 19	0.91	Jan. 30	-0.58	Nov. 12	-0.05	Sept. 3	-0.41	32
.143	273		Aug. 12	0.21	May 2	-0.18	Feb. 3	0.08	Nov. 14	-0.11	36
.158	146	5	Dec. 4	0.31	Mar. 19	-0.18	June 5	0.01	Aug. 21	-0.17	40
.160	125	8	May 30	0.26	Feb. 14	-0.22	Nov. 17	0.06	Sept. 3	-0.11	42
.044	309	1	Feb. 26	0.15	Oct. 21	-0.08	Aug. 25	-0.06	July 5	-0.07	44
.086	87	5	Oct. 18	0.21	Feb. 16	-0.22	June 11	0.02	July 4	0.02	45
.016	181	1	Sept. 18	0.18	Mar. 21	-0.22					46
.081	99	1	May 25	0.13	Feb. 4	-0.17	Oct. 23	0.06	Aug. 20	0.00	50
.067	145	1	July 3	0.12	Mar. 7	-0.21	Oct. 28	0.09	Sept. 14	0.07	58
.059	87	1	Sept. 28	0.17	Feb. 12	-0.25					59
.044	17	1	July 26	0.21	Jan. 4	-0.29					60
.124	100	1	Oct. 12	0.24	Feb. 11	-0.35	June 13	0.11	July 19	0.10	63
.113	111	2	May 31	0.22	Feb. 7	-0.24	Oct. 26	0.04	Sept. 2	0.00	66
.173	47	3	Sept. 26	0.27	Jan. 18	-0.41	May 8	0.16	July 8	0.07	67
			Oct. 19	0.07	Apr. 20	-0.07					70
.342	325		Aug. 31	0.74	Dec. 20	-0.51	Mar. 13	-0.05	May 16	-0.28	77
.106	161	2	July 3	0.37	Feb. 21	-0.37					85
.194	163	1	June 25	0.40	Mar. 2	-0.41	Nov. 22	0.04	Oct. 2	-0.02	90
.060	35	1	Aug. 23	0.23	Jan. 16	-0.31					95
.027	94	2	Sept. 25	0.30	Mar. 7	-0.32					98
.165	84	1	Oct. 23	0.41	Feb. 18	-0.37	May 29	-0.03	July 4	-0.05	101
.103	245	1	Sept. 10	0.22	Jan. 10	-0.31	May 16	0.09	June 8	0.08	103
.168	37	1									105
.308	207	1	July 24	0.42	Apr. 3	-0.71	Dec. 15	0.32	Oct. 9	0.09	106
.075	86	1	Oct. 30	0.45	Apr. 10	-0.30					113
.212	359	1	Sept. 4	0.26	Dec. 21	-0.46	Apr. 8	0.24	June 20	0.03	125
.180	87	1	Oct. 18	0.45	Feb. 17	-0.49	June 19	0.03	June 24	0.03	131
.332	44	1	Oct. 4	0.80	Jan. 29	-0.67	May 2	-0.07	June 17	-0.15	140
.101	33	1	Sept. 15	0.34	Jan. 26	-0.38					154
.166	336	1	Feb. 28	0.53	July 3	-0.42					186
.013	226	1	June 30	0.17	Dec. 15	-0.15					200
.091	228	1	Jan. 31	0.19	Oct. 6	-0.22	June 18	-0.04	May 7	0.02	205
.478	114	1	Nov. 1	0.85	Feb. 20	-1.12	June 11	0.33	Aug. 5	0.12	210
.133	248	1	July 31	0.25	Apr. 15	-0.20	Jan. 17	0.03	Nov. 7	-0.08	215
.114	280	3	Aug. 28	0.29	Apr. 28	-0.31					221
.184	221	4	July 29	0.50	Mar. 27	-0.51					224
.043	233	1	Dec. 23	0.15	May 5	-0.17					225
.076	277	4	Aug. 15	0.22	Apr. 16	-0.15	Jan. 22	-0.06	Dec. 14	-0.07	226
.267	151	2	Dec. 12	0.49	Aug. 26	-0.40	May 30	0.06	Mar. 21	-0.17	227
.131	225	3	Jan. 11	0.40	May 14	-0.22	July 19	-0.14	Sept. 10	-0.18	230
.204	217	2	Jan. 5	0.36	Apr. 20	-0.29	July 17	0.05	Sept. 28	-0.15	241
.055	336	1	Jan. 30	0.25	June 23	-0.31					250
.495	49	1	Oct. 19	1.38	June 22	-0.85	Apr. 6	-0.37	Feb. 13	-0.54	265

‡ Months.

Long-period tides and

No	Station	Latitude	Longi- tude	Mf	Mf°	Mm	Mm°	Sa
		° ' "	° ' "	Feet	°	Feet	°	Feet
EAST COAST OF ASIA.								
		<i>North</i>	<i>East</i>					
342	Yokohama, Japan.....	35 27	139 39					0.341 190
430	Shanghai.....	31 21	121 30					1.518 128
436	Swatow.....	23 23	116 39	0.069	120	0.050	298	.467 270
438	Whampoa.....	23 05	113 26					.484 171
440	Hongkong.....	22 18	114 10	.083	310	.073	101	.450 234
490	Singapore.....	1 17	103 51					.308 209
OCEANICA.								
		<i>South</i>						
506	Pulu Besar Strait.....	2 54	106 06					.394 262
507	Telok Betong.....	5 27	105 16					.184 263
512	Padang.....	1 00	100 18					.258 144
		<i>North</i>						
513	Ajerbangies.....	0 12	99 24					.256 122
515	Natal.....	0 36	99 06					.244 112
516	Gunung Sitoli.....	1 18	97 36					.381 156
517	Siboga.....	1 42	98 48					.180 88
518	Baros.....	2 00	98 24					.157 284
523	Oleh-leh.....	5 36	95 18					.289 65
524	Sabang Bay.....	5 54	95 20					.302 165
625	Segli.....	5 18	96 00					.472 153
527	Edi.....	4 54	97 48					.436 136
528	Belawan Deli.....	3 48	98 42					.364 116
529	Tandjong Tiram.....	3 18	99 30					.958 108
531	Bengkalis.....	1 30	102 06					1.194 201
		<i>South</i>						
533	Kuala Ladjan.....	1 24	103 36					.755 286
535	Tandjong Ruton.....	0 12	104 36					.384 273
541	Edam Island.....	5 58	106 51					.056 172
544	Boompjes Island.....	5 54	108 24					.112 103
547	Semarang.....	7 00	110 24					.374 77
552	Sembilangan.....	7 06	112 42					.364 103
553	Surabaya.....	7 12	112 36					.112 292
554	Gading.....	7 12	112 54					.049 79
555	Karang Kleta.....	7 18	112 48					.075 33
556	Pasuruan.....	7 38	112 54					.075 40
557	Zwaantjes-droogte.....	7 30	113 06					
559	Pulu Sapudi.....	7 06	114 18					.289 67
560	Meinderts-droogte.....	7 36	114 24					.272 10
565	Labuan, Java.....	6 24	105 12					.121 324
566	Java Fourth Point.....	6 06	105 54					.354 218
570	Pulu Langkuas.....	2 30	107 36					.676 283
571	Ondiepwater Island.....	3 18	107 12					.282 184
580	Sukadana.....	1 12	109 54					.842 283
582	Pontianak.....	0 00	109 18					.440 308
		<i>North</i>						
584	Pemangkat.....	1 12	109 00					.458 267
590	Boeloengan.....	2 50	117 22					.416 347
		<i>South</i>						
593	Moera Djawa.....	0 38	117 18					.267 317

computed annual fluctuation—Continued.

Ssa		No. of years observed	Principal				Secondary				No.
			Maximum		Minimum		Maximum		Minimum		
			Time	Height	Time	Height	Time	Height	Time	Height	
Feet	°			Feet		Feet		Feet		Feet	
.100	118	1	Oct. 27	0.38	Mar. 11	-0.40					34
.478	73	1	Sept. 7	1.11	Jan. 28	-1.99	June 24	1.05	July 21	1.03	430
.258	96	1	Nov. 18	0.64	July 26	-0.61	Apr. 10	-0.01	Mar. 12	-0.03	436
.135	92	1	Oct. 10	0.51	Feb. 23	-0.57					438
.190	94	2	Nov. 11	0.64	July 6	-0.37	Apr. 26	-0.25	Mar. 20	-0.28	440
.312	234	1	Jan. 4	0.36	Apr. 20	-0.62	Aug. 3	0.34	Oct. 14	-0.01	490
.079	180	5									506
.312	120	Nov. 24	0.48	Aug. 14	-0.39	May 19	0.14	Mar. 1	-0.25	507
.214	120	2	June 9	0.28	Feb. 19	-0.47	Nov. 3	0.23	Aug. 24	0.04	512
.151	106	4	June 5	0.28	Feb. 7	-0.40	Oct. 20	0.12	Aug. 30	0.08	513
.226	32	2	Sept. 22	0.28	Jan. 8	-0.47	Apr. 24	0.23	July 6	0.02	515
.131	108	1	Oct. 9	0.32	Feb. 19	-0.51					516
.190	166	1	June 16	0.37	Mar. 2	-0.23	Dec. 13	0.01	Sept. 28	-0.20	517
.082	261	3	Jan. 23	0.23	May 24	-0.18	Aug. 23	-0.04	Oct. 1	-0.05	518
.223	145	3	June 2	0.51	Sept. 22	0.30	Dec. 7	-0.06	Feb. 13	-0.23	523
.279	114	Nov. 4	0.41	Feb. 21	-0.58	June 5	0.22	Aug. 12	0.00	524
.226	147	1	July 3	0.42	Mar. 3	-0.69	Nov. 3	0.28	Sept. 14	0.23	525
.223	143	1	June 25	0.48	Feb. 23	-0.63	Oct. 26	0.15	Sept. 27	0.13	527
.161	305	1	Aug. 11	0.48	Dec. 13	-0.43					528
.161	71	1	June 21	0.82	Jan. 12	-1.10					529
.266	194	1	Nov. 11	1.03	Apr. 6	-1.43					531
.289	85	1	Dec. 1	0.78	July 24	-0.99					533
.052	53	1	Dec. 7	0.35	July 4	-0.42					535
.075	143	3	Nov. 22	0.09	Mar. 6	-0.13	June 14	0.07	Aug. 31	-0.02	541
.052	107	5	June 3	0.14	Dec. 31	-0.15					544
			June 9	0.37	Dec. 8	-0.37					547
.184	150	4	June 16	0.52	Feb. 15	-0.41	Nov. 11	-0.10	Oct. 15	-0.11	552
.217	165	1	Dec. 17	0.32	Sept. 8	-0.28	June 10	0.12	Mar. 22	-0.17	553
.128	154	1	June 9	0.18	Mar. 4	-0.13	Dec. 8	0.08	Sept. 14	-0.13	554
.105	126	1	May 21	0.17	Sept. 2	-0.15	Nov. 30	0.04	Feb. 14	-0.07	555
.098	142	1	May 29	0.16	Sept. 11	-0.14	Dec. 9	0.04	Feb. 21	-0.07	556
.115	106	2									557
.144	167	5	June 10	0.42	Oct. 10	-0.28	Jan. 4	-0.12	Feb. 8	-0.14	559
.180	136	May 15	0.36	Sept. 8	-0.42	Dec. 24	0.08	Feb. 11	0.03	560
.217	174	1	Dec. 25	0.29	Sept. 14	-0.32	June 12	0.16	Mar. 25	-0.12	565
.210	169	5	Dec. 2	0.48	Mar. 30	-0.49	July 10	0.02	Aug. 24	-0.01	566
.066	138	4	Dec. 26	0.71	July 17	-0.66					570
.098	86	5	Oct. 18	0.34	Feb. 26	-0.32					571
.210	135	2	Dec. 18	1.00	Aug. 1	-0.87					580
.210	155	3	Dec. 27	0.54	Aug. 26	-0.58					582
.079	104	2	Dec. 5	0.51	July 9	-0.46					584
.188	28	1	Mar. 27	0.57	July 29	-0.45					590
.101	73	1	Mar. 25	0.22	Aug. 3	-0.37	Dec. 27	0.15	Dec. 30	0.15	593

Long-period tides and

No.	Station	Latitude	Longitude	Mf.	Mf°	Mm	Mm°	Sa	
		° ' "	° ' "	Feet	°	Feet	°	Feet	°
OCEANICA—continued.									
		South	East						
594	Bay of Balik	1 16	116 48					.315	19
597	De Bril	6 06	118 54						
598	Makasser	5 08	119 24					.088	326
600	Donggala	0 40	119 44					.068	315
		North							
602	Tontoli	1 00	120 53					.367	242
603	Kema	1 24	125 06					.955	107
604	Gorontalo	0 30	123 06					.328	231
		South							
605	Posso	1 24	120 54					.341	16
607	Kadjang	5 24	120 18					1.552	359
608	Bonthain	5 36	119 54					.453	259
609	Saley	6 06	120 30					.226	224
612	Bima	8 24	118 42					.121	291
614	Kupang	10 12	123 36					.075	54
620	Banda	4 30	129 54					.489	351
		North							
626	Gamsungi	0 12	128 48					.198	256
627	Ternate	0 48	127 24					.226	357
628	Galela	1 48	127 48					.541	340
632	Taruna	3 42	125 30					.850	64
640	Manila, P. I.	14 36	120 57					.451	162
641	Olongapo, P. I.	14 50	120 16					.480	162
660	Honolulu	21 18	157 52					.215	197
		South							
680.2	Auckland	36 51	174 48					.357	88
680.5	Wellington, New Zealand	41 17	174 46					.241	54
681	Port Chalmers	45 50	172 30					.080	2
681.5	Port Darwin	12 23	130 37					.970	76
682	Cooktown	15 27	145 15					.346	320
682.5	Cairns Harbour	16 35	145 47					.202	9
683	Brisbane Bar	27 31	153 00					.109	8
683.5	Ballina	28 52	153 33	.097	314	.102	2	.413	7
684	Newcastle	32 57	151 44	.051	105	.082	198	.232	70
685	Sydney	33 52	151 12					.093	16
689	Port Adelaide	34 51	138 30	.06	181	.07	200	.305	126
689.5	Princess Royal Harbour	35 08	118 00	.064	175	.065	135	.328	111
690	Freemantle	32 03	115 45	.082	25	.079	147	.537	27
INDIAN OCEAN.									
		North							
710	Mergui (Bay of Bengal)	12 26	98 36	.043	19	.067	14	.589	146
720	Amherst, Moulmein River	16 05	97 34	.067	327	.071	2	.758	136
722	Moulmein, Moulmein River	16 29	97 37	.328	39	.366	12	2.330	149
725	Elephant Point, Rangoon River	16 30	96 18	.108	35	.096	355	.842	140
726	Rangoon, Rangoon River	16 46	96 10	.181	34	.206	12	1.315	147
730	Diamond Island	15 52	94 15						
735	Akyab	20 08	92 54	.064	190	.045	321	.944	145
740	Chittagong	22 20	91 50	.141	17	.192	356	1.567	134

computed annual fluctuation—Continued.

Ssa		No. of years observed	Principal				Secondary				No.
			Maximum		Minimum		Maximum		Minimum		
			Time	Height	Time	Height	Time	Height	Time	Height	
Feet	°			Feet		Feet		Feet		Feet	
.092	208	1	May 15	0.24	Oct. 8	-0.41					594
.344	354	4									597
.021	229	5	Feb. 2	0.11	Sept. 12	-0.10					598
.018	307	4	Feb. 15	0.08	July 6	-0.06					600
.331	215	4	Dec. 30	0.61	Apr. 19	-0.61	July 24	0.11	Sept. 25	-0.10	602
.085	289	1	July 18	0.99	Dec. 28	-0.95					603
.141	208	1	Dec. 16	0.38	Apr. 20	-0.43					604
.049	137	1	Apr. 25	0.34	Sept. 25	-0.37					605
.413	155	1	Apr. 24	1.30	Sept. 14	-1.96					607
.564	275	Jan. 29	0.83	May 14	-0.96	Aug. 19	0.37	Oct. 31	-0.19	608
.167	37	1	Oct. 17	0.38	June 25	-0.29	Mar. 29	-0.03	Jan. 31	-0.11	609
.157	226	1	Jan. 14	0.28	Apr. 26	-0.17	July 16	0.04	Oct. 3	-0.16	612
.128	188	1	June 21	0.19	Oct. 1	-0.18	Jan. 1	0.07	Mar. 20	-0.08	614
.161	4	1	Mar. 20	0.64	July 30	-0.41					620
.114	100	Nov. 19	0.30	July 24	-0.23	Apr. 28	0.05	Mar. 7	-0.07	626
.059	145	Apr. 25	0.20	Sept. 11	-0.28					627
.141	97	1	Apr. 2	0.49	Aug. 21	-0.65					628
.180	48	1	May 10	0.95	Dec. 16	-0.90					632
.102	58	1	Sept. 25	0.49	Feb. 13	-0.50					640
.136	31	1	Sept. 21	0.57	Feb. 4	-0.51					641
.090	331	1	Oct. 8	0.30	June 1	-0.16	Apr. 9	-0.12	Feb. 13	-0.15	650
.185	266	2	July 21	0.47	Nov. 19	-0.47	Mar. 13	0.00	Mar. 29	0.00	680.2
.035	240	1									680.5
.064	168	1									681
.540	58	1	May 6	1.30	Jan. 6	-1.31	Sept. 20	0.06	Aug. 20	0.01	681.5
.051	36	1	Feb. 28	0.34	July 31	-0.38					682
.050	157	1	May 1	0.19	Sept. 20	-0.25					682.5
.005	156	1	Apr. 4	0.10	Sept. 27	-0.11					683
.063	257	1	Mar. 13	0.40	Oct. 11	-0.46					683.5
.074	201	1	June 19	0.29	Oct. 30	-0.24					684
.008	97	*6	Apr. 17	0.10	Sept. 28	-0.09					685
.225	88	2	May 26	0.32	Feb. 3	-0.53	Oct. 15	0.24	Aug. 10	0.08	689
.235	97	1	May 26	0.43	Feb. 2	-0.54	Oct. 18	0.14	Aug. 24	0.04	689.5
.175	126	1	May 10	0.65	Sept. 19	-0.58					690
.158	113	5	Sept. 9	0.44	Feb. 16	-0.74	July 29	0.43	Aug. 13	0.43	710
.149	145	5	July 18	0.68	Feb. 17	-0.88					720
.616	286	6	Aug. 18	2.94	Mar. 21	-1.82					722
.129	150	5	July 27	0.93	Feb. 20	-0.77					725
.160	339	17	Aug. 26	1.45	Feb. 3	-1.21					726
											730
.194	160	5	July 13	0.83	Feb. 27	-1.11					735
.153	195	5	July 27	1.64	Feb. 17	-1.55					740

* Months.

Long-period tides and

No.	Station	Latitude	Longitude	Mf	Mf°	Mm	Mm°	Sa
		°	°	Feet	°	Feet	°	Feet
INDIAN OCEAN—continued.								
		North	East					
745	Dublat, Hoogly River	21 38	88 06	0.061	60	0.037	89	0.876 151
746	Diamond Harbor, Hoogly River	22 11	88 12	.153	42	.117	10	1.058 142
747	Calcutta (Kidderpore)	22 32	88 20	.263	35	.287	4	2.852 156
748	False Point	20 23	86 47	.075	29	.046	67	.829 166
755	Vizagapatam	17 41	83 17	.054	14	.043	21	.694 184
756	Cocanada	16 56	82 15	*.084	* 11	.063	321	.720 202
763	Madras	13 05	80 18	.043	22	.034	28	.394 216
765	Negapatam	10 46	79 51	.066	1	.049	335	.444 234
770	Pamban Pass	9 16	79 09	.043	355	.048	27	.149 302
772	Tuticorin	8 48	78 09	.041	357	.044	93	.299 310
773	Trincomalee, Ceylon	8 33	81 13	.054	14	.042	* 352	.251 263
775	Point de Galle, Ceylon	6 02	80 13	.038	19	.039	14	.350 309
776	Colombo, Ceylon	6 56	79 50	.043	358	.037	27	.313 308
780	Port Blair, Andaman Islands	11 41	92 45	.048	11	.025	7	.200 144
785	Cochin	9 58	76 15	.056	356	.041	92	.337 302
787	Beypore	11 10	75 48	.068	† 23	.081	50	.308 311
793	Kárwár	14 48	74 06	.042	5	.065	27	.352 310
795	Goa or Mormugoa	15 25	73 48	.059	* 359	.030	369	.269 313
800	Bombay (Apollo Bandar)	18 55	72 50	.049	5	.052	5	.110 350
801	Bombay (Prince's Dock)	18 57	72 50	.053	13	.054	322	.102 303
802	Bhávnagar	21 48	72 09	.063	356	.109	0	.246 108
805	Port Albert Victor	20 58	71 33	.032	26	.045	126	.162 243
807	Porbandar	21 37	69 37	.016				.078 289.8
809	Okha Point and Bet Harbor	22 28	69 05	.050	44	.066	312	.162 3
810	Navanar	22 44	69 42					
811	Hanstal	22 55	70 21	.101	37	.121	14	.024 195
815	Karachi	24 47	66 58	.036	1	.051	25	.130 68
820	Minikoi Light	8 16	73 01	.052	8	.038	56	.357 354
825	Bushire	29 00	50 52	.027	179	.073	47	.283 149
830	Muscat	23 37	58 35	.036	5	.047	28	.138 97
840	Aden	12 47	44 59	.042	16	.035	20	.378 356
845	Suez	29 58	32 32	‡ (.042	‡ 171	‡ .106	‡ 53)	‡ (.499 ‡ 312
850	Perim	12 38	43 24	.046	20	.030	197	.362 342
		South.						
870	Port Louis, Mauritius Island	20 08	57 29	.036	350	.047	297	.211 346
WEST COAST OF AFRICA AND EUROPE.								
900	Cape Town	33 54	18 25					.124 256
908	Duala (Kamerun)	4 03	9 40					.361 206
		North						
925	Toulon, France	43 07	5 56	.061	159	.057	196	.123 254
926	Marseilles, France	43 18	5 23	.019	229	.010	293	.151 185
			West					
938	Socoa, France	43 24	1 40	.015	166	.056	327	.180 217
942	Boyard, France	46 00	1 13	.028	148	.049	203	.240 192
943	Rochelle, France	46 09	1 09	.033	26	.110	239	.401 257
944	St. Nazaire, France	47 16	2 12	.061	268	.099	37	.423 256
946	Brest, France	48 23	4 29	.043	173	.085	315	.203 229

* Four years.

† Five years.

‡ Two years.

§ Three years.

computed annual fluctuation—Continued.

Sta	No. of years observed	Principal				Secondary.				No.	
		Maximum		Minimum		Maximum		Minimum			
		Time	Height	Time	Height	Time	Height	Time	Height		
Feet	°		Feet		Feet		Feet		Feet		
0.195	141	5	July 30	0.72	Feb. 26	-1.07				745	
.097	129	5	Aug. 7	0.97	Feb. 16	-1.15				746	
.930	331	16	Sept. 2	3.76	Jan. 17	-2.32				747	
.279	151	4	Oct. 19	0.60	Mar. 9	-1.11	July 28	0.58	Sept. 3	0.55	748
.340	118	6	Nov. 2	0.82	Mar. 3	-0.96				755	
.392	105	5	Nov. 4	1.04	Mar. 3	-0.85	June 9	-0.15	July 11	-0.19	756
.314	124	12	Nov. 17	0.68	Mar. 9	-0.53	June 5	-0.02	Aug. 5	-0.20	763
.344	128	5	Nov. 23	0.78	Mar. 14	-0.48	May 30	-0.09	Aug. 6	-0.34	765
.157	108	4	Nov. 27	0.22	Aug. 12	-0.30	May 2	0.12	Feb. 21	-0.02	770
.140	84	5	Dec. 5	0.23	Aug. 3	-0.43	Apr. 1	0.21	Feb. 5	0.16	772
.203	134	5	Dec. 2	0.45	Aug. 13	-0.31	May 22	-0.03	Mar. 19	-0.17	773
.127	116	6	Dec. 21	0.32	Aug. 12	-0.45				775	
.133	111	6	Dec. 15	0.30	Aug. 11	-0.44				776	
.111	182	17	July 10	0.26	Mar. 12	-0.28	Nov. 20	0.03	Oct. 20	0.02	780
.118	150	6	Dec. 27	0.39	Aug. 19	-0.40				785	
.166	205	6	Jan. 13	0.45	Sept. 14	-0.36	June 11	-0.08	May 9	-0.09	787
.068	228	5	Jan. 24	0.41	Aug. 24	-0.31				793	
.116	138	5	Dec. 22	0.27	Aug. 18	-0.37				795	
.134	201	19	Jan. 13	0.18	Sept. 30	-0.24	June 23	0.10	Apr. 9	-0.04	800
.140	181	9	Dec. 26	0.24	Sept. 13	-0.19	June 17	0.05	Apr. 3	-0.11	801
.153	149	8	June 16	0.37	Feb. 19	-0.32	Nov. 16	-0.03	Oct. 3	-0.06	802
.149	136	3	Nov. 28	0.31	Mar. 16	-0.18	June 1	0.01	Aug. 13	-0.16	805
.144	144.7	2	Dec. 8	0.21	Aug. 29	-0.19	May 30	0.08	Mar. 12	-0.10	807
.121	145	1	May 19	0.20	Sept. 9	-0.28	Dec. 25	0.09	Feb. 23	0.02	809
										810	
.090	156	1	Dec. 6	0.10	Mar. 13	-0.11	June 14	0.08	Sept. 7	-0.07	811
.152	149	25	June 5	0.28	Sept. 18	-0.18	Dec. 7	0.02	Feb. 22	-0.15	815
.138	244	4	Feb. 14	0.41	Oct. 8	-0.45				820	
.122	224	3	July 28	0.37	Mar. 25	-0.33				825	
.140	187	2	June 26	0.28	Mar. 12	-0.16	Dec. 24	0.00	Oct. 10	-0.15	830
.128	131	18	Apr. 25	0.36	Sept. 5	-0.49				840	
‡.157	‡110)		Dec. 26	0.42	Aug. 11	-0.65				845	
.166	111	2	Apr. 21	0.35	Aug. 23	-0.52	Dec. 29	0.17	Jan. 25	0.16	850
.118	118	1	Apr. 29	0.22	Aug. 26	-0.32	Dec. 21	0.11	Feb. 6	0.08	870
.111	76	1	Nov. 7	0.21	July 19	-0.20	Apr. 17	0.02	Feb. 14	-0.05	900
.154	58	3	Oct. 20	0.51	Feb. 24	-0.28	Apr. 25	-0.21	June 13	-0.25	908
.108	114	3	Nov. 22	0.23	Aug. 4	-0.16	May 7	-0.01	Mar. 7	-0.09	925
.170	118	1	Nov. 11	0.27	Feb. 26	-0.30	June 2	0.09	Aug. 11	-0.05	926
.167	80	1	Oct. 31	0.35	Feb. 16	-0.20	May 3	-0.01	July 16	-0.18	938
.063	129	1	Oct. 29	0.25	Mar. 16	-0.28					942
.080	123		Dec. 1	0.47	July 3	-0.35					943
.190	65		July 8	0.54	Nov. 9	-0.52					944
.086	154	3	Nov. 28	0.27	Apr. 8	-0.22					946

‡ Three years.

Long-period tides and

No.	Station	Latitude	Longi- tude	Mf	Mf°	Mm	Mm°	Sa
		° ' "	° ' "	Feet.	°	Feet	°	Feet °
WEST COAST OF AFRICA AND EUROPE—CON.								
		<i>North</i>	<i>West</i>					
948	St. Servan (St. Malo).....	48 39	2 02	.085	230	.064	154	0.320 222
950	Cherbourg.....	49 39	1 37	.010	102	.032	100	.169 200
			<i>East</i>					
952	Havre.....	49 29	0 06	.010	359	.101	273	.311 218
953	Thurso, Scotland.....	58 37	3 32					.265 219
954	Edinburgh, Scotland.....	55 59	3 10					.177 220
956	West Hartlepool, England.....	54 41	1 12	.046	205	.127	93	.265 219
958	Sheerness, England.....	51 27	0 45					.209 196
			<i>West</i>					
960	London Bridge, England.....	51 30	0 07					.124 112
			<i>East</i>					
962	Ramsgate, England.....	51 20	1 25	.044	288	.029	45	.127 181
			<i>West</i>					
968	Pembroke, Wales.....	51 41	4 56					.392 250
970	Helbre Island.....	53 24	3 00					
971	Liverpool, England.....	53 24	3 00	.041	282	.126	278	.362 238
978	Greenock, Scotland.....	55 57	4 45					.485 240
980.5	Queenstown, Ireland.....	51 51	8 16					.265 234
981	Galway, Ireland.....	53 14	9 04					.261 234
			<i>East</i>					
982	Ostende, Belgium.....	51 14	2 55					.161 222
983	Hook of Holland.....	51 59	4 09					.223 219
984	Ymuiden, Holland.....	52 28	4 34					.315 221
985	Helder, Holland.....	52 58	4 46					.341 213
1000	Christiania, Norway.....	59 55	10 44	.144	166	.154	235	.449 185
1001	Oscarsborg.....	59 41	10 37	.197	228	.177	182	.449 179
1002	Arendal.....	58 27	8 46	.092	223	.098	229	.322 195
1003	Stavanger.....	58 59	5 44	.075	205	.121	229	.246 221
1004	Bergen.....	60 24	5 08	.113		.134	166	.302 217
1005	Trondhjem.....	63 27	10 24					.446 252
1006	Bodø.....	67 17	14 23	.121	189	.089		.359 245
1007	Fineide.....	67 17	15 30	.010	206	.079	8	.348 213
1008	Kabelvaag.....	68 13	14 30	.125	78	.094	171	.374 264
1011	Vardø.....	70 20	31 06	.092	227	.061	193	.482 249

computed annual fluctuation—Continued.

Sta		No. of years observed	Principal				Secondary.				No.
			Maximum		Minimum		Maximum		Minimum		
			Time	Height	Time	Height	Time	Height	Time	Height	
Feet	°			Feet		Feet		Feet		Feet	
0.128	92	1	Nov. 5	0.45	Mar. 15	-0.25	May 16	-0.19	June 26	-0.21	948
.061	158	1	Nov. 15	0.18	Mar. 25	-0.22					950
.148	151		Nov. 24	0.42	Mar. 27	-0.38					952
			Oct. 31	0.26	May 1	-0.26					953
.082	113	3	Nov. 11	0.25	Mar. 15	-0.17	June 9	-0.08	July 8	-0.09	954
.097	223	3	Dec. 11	0.24	Apr. 17	-0.35					956
.046	155	1	Nov. 12	0.27	Mar. 24	-0.26					958
.131	197	3	July 3	0.25	Mar. 18	-0.17	Dec. 26	0.01	Oct. 14	-0.12	960
.075	288	1	Aug. 27	0.18	May 1	-0.16	Jan. 21	-0.01	Dec. 14	-0.02	962
			Dec. 1	0.39	June 2	-0.39					968
.142	189	4	Dec. 11	0.46	Apr. 18	-0.41					970
.058	183	1	Dec. 1	0.52	May 9	-0.47					971
			Nov. 15	0.26	May 16	-0.26					978
			Nov. 15	0.26	May 16	-0.26					980.5
		3	Nov. 3	0.16	May 4	-0.16					981
.079	236	1	Dec. 13	0.19	Apr. 24	-0.30					982
.098	210	1	Dec. 5	0.31	Apr. 19	-0.40					983
.135	205	4	Dec. 5	0.33	Apr. 12	-0.46					984
.177	218		Aug. 12	0.40	Apr. 5	-0.62					985
.115	223		Aug. 21	0.42	Apr. 2	-0.55					1 000
.121	180		Nov. 18	0.29	Mar. 29	-0.44					1 001
.089	198		Dec. 5	0.26	Apr. 14	-0.31					1 002
.102	143		Nov. 17	0.37	Mar. 29	-0.32					1 003
.220	174		Dec. 13	0.65	Apr. 15	-0.42	July 4	-0.19	Aug. 14	-0.23	1 004
.147	189		Dec. 14	0.48	Apr. 20	-0.39					1 005
.226	202		Dec. 14	0.41	Apr. 9	-0.56	July 28	0.16	Sept. 19	0.08	1 006
.187	218		Jan. 1	0.54	May 3	-0.41	Aug. 6	-0.12	Sept. 1	-0.13	1 007
.200	188		Dec. 16	0.65	Apr. 21	-0.50					1 008
											1 011

125. *On the possibility of the annual height inequality being due to fluctuations in the temperature of sea water.*

In an ocean or portion of an ocean lying upon one side of the equator, the heat directly radiated from the sun will be absorbed most rapidly at the time of the summer solstice. The maximum rate of deriving heat from the earth's atmosphere and from inflowing streams must generally occur somewhat later. From the same sources least is being absorbed by the ocean soon after the winter solstice. From these considerations one may perhaps infer that the assumed body of water, all depths being considered, would contain the greatest amount of heat not earlier than the autumnal equinoxes, and the least not earlier than the vernal. For portions of the ocean lying in low latitudes these times will be accelerated, and the range of the annual temperature fluctuations will be less.*

While it is undoubtedly true that the surface of the water in high latitudes stands on a slightly higher level than in low latitudes (as was pointed out in sec. 115), and is higher in the early autumn than in the early spring, it can be easily seen that this annual fluctuation can not be considerable, i. e., it can scarcely be a measurable quantity.

Suppose the surface of the water at a point in high latitude to be 0.1 foot above the surface at a point 8000 miles away in the opposite hemisphere. The instantaneous slope will be 2 one-billionths. And so the accelerating force per unit mass will be $2g$ divided by one billion (Eq. 91, Part IV A). But this acting through, say, three months will give rise to a velocity of 0.52 foot per second at the surface. The velocity will diminish in going downward and the flow near the bottom will generally be opposite in direction to that at the surface. If the assumed distance were less than 8000 miles, this velocity would be increased in proportion.

As no such alternation of surface flow from one hemisphere to the other has been observed, it is practically certain that the results due to annual temperature changes in the water can not cause an annual inequality in sea level with a range as great as 0.1 of a foot, and so this portion of it may be neglected.

In an inclosed body of water the annual temperature changes may easily produce an annual height inequality, but this would be concealed by the much greater changes due to the varying amounts of evaporation and of tributary waters. In going from 0° to 15° centigrade, pure water expands only about three-fourths of a one-thousandth part of the original bulk or depth, while sea water having a density of 1.028 at 0° , has a density of 1.026 at 15° .

* See temperature maps in the Atlases of the Oceans published by the Deutsche Seewarte.

CHAPTER IX.

SEICHES IN LAKES, BAYS, ETC.

126. The problem of seiches in variously-shaped bodies of water is one of peculiar interest to the mathematician because it presents analytical difficulties which have not been overcome save in a few simple cases. The problem is to find the free periods and natural modes of oscillation for given bodies of water. No attempt will be made here to go into an elaborate mathematical treatment of the subject.

Chapters III and IV, Part IV A, show how to ascertain oscillations in square, triangular, and circular areas whether the uniform depth be very small in comparison with a wave length or not. A few questions relating to long-wave motion in bodies of variable depth are briefly considered in Chapter IV. A number of experimental tests are described in Chapter V of the same part.

For a rectangular body $\frac{1}{2}\lambda$ in length, the period τ of the slowest oscillation is given by the equation

$$\tau^2 = \frac{2\pi\lambda}{g} \bigg/ \tanh \frac{2\pi h}{\lambda}; \quad (304)$$

$$\therefore \tau = \frac{\lambda}{\sqrt{gh}} \left[1 + \frac{1}{6} \left(\frac{2\pi h}{\lambda} \right)^2 - \frac{1}{40} \left(\frac{2\pi h}{\lambda} \right)^4 + \dots \right] \quad (305)$$

For long-wave motion,

$$\tau = \frac{\lambda}{\sqrt{gh}} = \frac{\text{twice length of lake}}{\sqrt{gh}} = \frac{4L}{\sqrt{gh}} \quad (306)$$

where $2L$ denotes the length of the lake.

Tables 46-52 facilitate the computation of τ . The theory of simple wave motion is given in Chapter II, Part I, and brief mention of the seiches is made in section 16, Part I.

127. *Long-wave motion in a canal of variable cross section.**

Let Ω denote the area of a cross section and b the breadth at the surface, then the equation of continuity is

$$\zeta = - \frac{1}{b} \frac{\partial}{\partial x} (\Omega \xi), \quad (307)$$

because the variation in the sectional volume whose length is ξ (i. e., its change for a small unit length of x) must be equal to the volume of a horizontal layer whose length

* Taken mainly from Lamb's Hydrodynamics.

is unity, whose breadth is b , and whose height is ζ . This becomes

$$\zeta = -\frac{1}{b} \frac{\partial}{\partial x} (hb\xi) \quad (308)$$

if $\Omega = hb$ and h denote the mean depth for the section. The dynamical equation is

$$\frac{\partial^2 \xi}{\partial t^2} = -g \frac{\partial \zeta}{\partial x}. \quad (309)$$

Upon differentiating (308) twice with respect to t and substituting for $\frac{\partial^2 \xi}{\partial t^2}$ its value from (309), we have

$$\frac{\partial^2 \zeta}{\partial t^2} = \frac{g}{b} \frac{\partial}{\partial x} \left(hb \frac{\partial \zeta}{\partial x} \right). \quad (310)$$

If

$$\zeta = F(x) \cos (at + \alpha),$$

(310) becomes

$$\frac{g}{b} \frac{\partial}{\partial x} \left(hb \frac{\partial \zeta}{\partial x} \right) + a^2 \zeta = 0. \quad (311)$$

The stationary wave in a canal communicating with a tided sea is found by writing b , h , or both, as functions of x . If b be proportional to x , and h remain constant, the amplitude will vary as $J_0(kx)$ where $k = a^2 (gh)$; if the h be proportional to x , and b remain constant, the amplitude will vary as $J_0(2\kappa^{\frac{1}{2}}x^{\frac{1}{2}})$ where $\kappa = a^2 L (gh_0)$. In both cases the origin is the head of the canal.

Consider a canal of uniform breadth and whose mean depth varies uniformly from zero at either end to h_0 at the center; i. e. suppose $h = ix = \frac{h_0}{L} x$ where L is the distance from either end to the center, the origin being taken at one edge.

Equation (311) now becomes

$$x \frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial \zeta}{\partial x} + \kappa \zeta = 0 \quad (312)$$

where

$$\kappa = \frac{a^2}{gi} = \frac{a^2 L}{gh_0}. \quad (313)$$

writing ζ in the form

$$\zeta = A + Bx + Cx^2 + Dx^3 + Ex^4 + \dots, \quad (314)$$

and substituting in (312) we have finally

$$\zeta = A \left(1 - \frac{\kappa x}{1^2} + \frac{\kappa^2 x^2}{1^2 2^2} - \frac{\kappa^3 x^3}{1^2 2^2 3^2} + \frac{\kappa^4 x^4}{1^2 2^2 3^2 4^2} - \dots \right) \quad (315)$$

$$= A J_0(2\kappa^{\frac{1}{2}}x^{\frac{1}{2}}) \quad (316)$$

since

$$J_0(y) = 1 - \frac{y^2}{2^2} + \frac{y^4}{2^2 \cdot 4^2} - \frac{y^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots \quad (317)$$

For the slowest mode of oscillation ζ must be zero at the center;

$$\therefore J_0(2\kappa^{\frac{1}{2}}L^{\frac{1}{2}}) = 0. \quad (318)$$

From any table of Bessel's functions the roots of J_0 are

$$0.7655\pi, 1.7571\pi, 2.7546\pi, \dots; \quad (319)$$

$$\therefore 2\kappa^{\frac{1}{2}}L^{\frac{1}{2}} = 0.7655\pi;$$

$$\therefore \tau_1 = \frac{2\pi}{a} = 1.306 \frac{4L}{(gh_0)^{\frac{1}{2}}} = 1.306 \frac{\text{twice length of body}}{(gh_0)^{\frac{1}{2}}} \quad (320)$$

where $1.306 = 1/0.7655$.

Hence the period is about 1.3 times as great as that of a rectangular basin of equal length and whose uniform depth is h_0 , or 0.9235 times as long as one having a uniform depth of $\frac{1}{2}h_0$.

For a binodal oscillation,

$$\frac{\partial \zeta}{\partial x} = 0$$

where $x = L$;

$$\therefore J'_0(2\kappa^{\frac{1}{2}}L^{\frac{1}{2}}) = -J_1(2\kappa^{\frac{1}{2}}L^{\frac{1}{2}}) = 0 \quad (321)$$

The roots of J'_0 are

$$1.2197\pi, 2.2330\pi, 3.2383\pi;$$

$$\therefore \tau_2 = \frac{2\pi}{a} = 0.820 \frac{4L}{(gh_0)^{\frac{1}{2}}} = 0.820 \frac{\text{twice length of body}}{(gh_0)^{\frac{1}{2}}} \quad (322)$$

This mode of oscillation evidently applies to either half of the given body.

$$\therefore \tau_2 = 1.640 \frac{2L}{(gh_0)^{\frac{1}{2}}} = 1.640 \frac{\text{twice length of half body}}{(gh_0)^{\frac{1}{2}}} \quad (323)$$

Hence the period of the half body is about 1.64 times as great as that of a rectangular basin of equal length whose uniform depth is h_0 or 1.16 times as great as one having a uniform depth of $\frac{1}{2}h_0$.

To calculate the position of the node in this case, find a from (322) and substitute this value in (313). The result will be

$$\kappa = \frac{(1.2197)^2 \pi^2}{4L}.$$

Since x must be such at the node that ζ vanishes,

$$J(2\kappa^{\frac{1}{2}}x^{\frac{1}{2}}) = 0.$$

$$2\kappa^{\frac{1}{2}}x^{\frac{1}{2}} = 0.7655\pi;$$

$$\therefore \frac{x}{L} = \left(\frac{0.7655}{1.2197}\right)^2 = 0.3940$$

Next suppose the origin to be taken at the center of a canal of uniform width whose length is $2L$ and whose depth varies from h_0 at the middle to zero at either end in accordance with the law

$$h = h_0 \left(1 - \frac{x^2}{L^2} \right). \quad (324)$$

This value of h substituted in (311) gives, since b is constant,

$$\frac{\partial}{\partial x} \left[\left(1 - \frac{x^2}{L^2} \right) \frac{\partial \zeta}{\partial x} \right] + \frac{a^2}{g h_0} \zeta = 0. \quad (325)$$

If we put

$$a^2 = n(n+1) \frac{g h_0}{L^2},$$

this equation becomes of the form

$$\frac{d}{d\mu} \left[(1 - \mu^2) \frac{dP_n}{d\mu} \right] + n(n+1) P_n = 0 \quad (326)$$

where $\mu = x/L$.

Since ζ must remain finite, the n must be integral in the solution

$$\zeta = C P_n \left(\frac{x}{L} \right) \cos(at + \alpha). \quad (327)$$

Now

$$P_0(\mu) = 1, P_1(\mu) = \mu, P_2(\mu) = \frac{3}{2}\mu^2 - \frac{1}{2}, \dots \quad (328)$$

For $n=1$, the profile of the surface is a straight line. The period

$$\tau_1 = \frac{2\pi}{a} = \frac{\pi}{2\sqrt{2}} \cdot \frac{4L}{(g h_0)^{1/4}} = 1.111 \frac{4L}{(g h_0)^{1/4}}. \quad (329)$$

128. An extensive mathematical treatment of bodies of variable depths has been recently given by Professor Chrystal* and his theories have been experimentally tested by Messrs. Peter White and William Watson,† and applied to lakes Earn and Treig in Scotland by himself and E. Maclagan-Wedderburn‡. The computed periods of Loch Earn are 14.50, 8.14, 5.74 minutes and for Loch Treig, 9.09, 5.07, 3.59, the corresponding observed quantities are 14.55, 8.10, ———, 9.18, 5.15, ———.

Professor Chrystal considers small longitudinal oscillations in a body of water whose depth, $h(x)$, cross section, $A(x)$, and surface breadth, $b(x)$, vary slowly from point to point. As is usually done in treatment of long-wave motion, the vertical acceleration is here neglected. If v denotes the surface area extending from $x=0$ to $x=x$, $\sigma(v)$, a function of v , known from the given body of water, viz. $A(x)$ $b(x)$; and n , the natural seiche frequency; then n is determined by the equation.

$$A(x)\xi = P \sin nt + Q \cos nt \quad (330)$$

* Proc. Roy. Soc. Edinburgh, Vol. 25, I (1904), pp. 328-337; Vol. 25, II (1905), pp. 637-647. Trans. Roy. Soc. Edinburgh, Vol. 41, III (1905), pp. 599-649.

† Proc. Roy. Soc. Edinburgh, Vol. 26, I (1906), pp. 142-156.

‡ Trans. Roy. Soc. Edinburgh, Vol. 41, III (1905), pp. 823-850.

where P and Q are solutions of

$$\frac{d^2 P}{dv^2} + \frac{n^2 P}{g\sigma(v)} = 0. \quad (331)$$

If $\sigma(v)$ be a linear function of v , the differential equation is found to depend upon Bessel's Functions; if $\sigma(v) = h(1 \mp v^2/a^2)$, the differential equation takes the form

$$(1 \mp w^2) \frac{d^2 P}{du^2} + cP = 0. \quad (332)$$

Integrating by series, two new transcendents for each sign are encountered. He calls those obtained when the upper sign is used *seiche-cosine* and *seiche-sine*, and when the lower, *hyperbolic seiche-cosine* and *hyperbolic seiche-sine*. If w is small, the above equation approximates

$$\frac{d^2 P}{du^2} + cP = 0. \quad (333)$$

If $c=1$, this is satisfied by $\cos w$, or $\sin w$; if $c=-1$, by $\cosh w$, or $\sinh w$. The seiche-cosine and seiche-sine functions apply to lakes whose longitudinal section is concave along the bottom, and the hyperbolic forms to convex bottoms. While the assumed form of $\sigma(v)$ may appear to be very special. Professor Chrystal shows how, by properly combining the simple geometrical bodies permitting of treatment, good approximations can frequently be made to lakes which at first sight would defy analysis.

If the breadth be constant and the depth $h(x)$ be a quartic function of x such that

$$h(x) = h(a^2 \mp x^2)^2, \quad (334)$$

the differential equation to be satisfied is

$$\frac{d^2 P}{dx^2} + \frac{cP}{(a^2 \mp x^2)^2} = 0. \quad (335)$$

the upper or lower sign to be used according as to whether the bottom of the longitudinal section is concave or convex.

In the case of a symmetric rectilinear lake shelving at both ends the periods of the secondary oscillations in terms of the period of the principal oscillation are found to be 0.6276, 0.4357, 0.3428, 0.2779, 0.2365, etc.; while the distances of the corresponding nodal lines from the center are the following fractions of the length of half of the body, viz 0.6057; 0.0, 0.8102; 0.3809, 0.8825; 0.0, 0.5930, 0.9228; 0.3763, 0.9441.

For a lake with one end vertical, and shelving to zero depth at the other end, the periods of the secondary oscillations in terms of the period of the principal, are found to be 0.5462, 0.3767, 0.2883, etc.; while the distances of the corresponding nodal lines from the vertical end are in terms of the lake length, 0.6057; 0.3809, 0.8825; 0.3763, 0.7056, 0.9441.

The corresponding period ratios for a parabolic lake are found to be 0.577, 0.408, 0.316, 0.258, 0.218, etc., and for a semiparabolic lake, 0.548, 0.378, 0.289, 0.234, 0.196, etc.

Professor Chrystal concludes his paper in the Transactions (Vol. LXI, III) with an important bibliography of the subject.

A review of Chrystal's work is given by Dr. W. Halbfass.*

Mr. D. Isitani ascertains "the change of period due to a slight alteration in the area and volume of the oscillating liquid."†

Mr. T. Terada shows analytically "that any contraction or expansion at the middle part of the canal prolongs or shortens its natural period respectively, and that at the end shortens or prolongs it respectively." He tests his work experimentally and also by application to Lake Hakone.‡

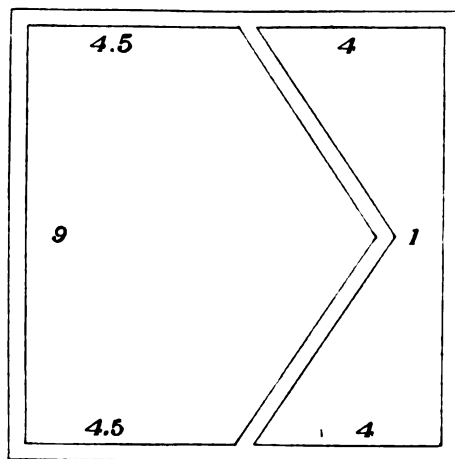
In order to show the relative periods of bodies having the same total lengths, I performed the following experiment:

A box 9 inches square on the inside was partitioned off by a portion in the manner indicated in the diagram. When the depth of water was 2 inches the observed number of periods a minute in the two parts were 91 and $63\frac{1}{2}$. The calculated number for a rectangular area 9 inches long is 86. The respective periods are, therefore, in seconds 0.66, 0.94 $\frac{1}{2}$, 0.70. Hence the period of the area narrow at the middle, is one-third greater than the period of a rectangular area of the same length.

129. Classification of seiches.

So far only completely surrounded bodies of water have been considered in reference

No. 26.



to seiche movements. But it is not difficult to see that bodies having incomplete boundaries may also possess free oscillations having periods suited to their dimensions (Cf. sec. 29, Part IV A). As a rule the more broken the boundary, the smaller will be the amplitude of the motion, *ceteribus paribus*. The existence of fractional oscillating areas is not so obvious; but it will be shown that the seiches due to them are common and important.

For convenience, seiche movements may be divided into the classes mentioned below. The names of these classes generally refer to the forms of the bodies or areas in which the seiche is produced, and, as a rule, scarcely require formal definition.

Name	Character of oscillations	General character of body of water
Lake seiche.....	Regular to irregular.....	Oscillating area.
Open-lake seiche.....	Fairly regular.....	Do.
Parallel-wall seiche.....	Fairly regular.....	Do.
Strait-and-harbor seiche.....	Irregular.....	Do.
Cul-de-sac seiche.....	Regular.....	Fractional area.
Shelving seiche.....	Irregular.....	Do.

* Zeitschrift der Gesellschaft für Erdkunde zu Berlin, 1907, pp. 5-24.

† Proceedings of the Tōkyō Mathematico-Physical Society, Vol. III (1906), pp. 170-173.

‡ Proceedings of the Tōkyō Mathematico-Physical Society, Vol. III (1906), pp. 174-181. See also paper in same journal, Vol. I, pp. 115-, by S. Nakamura and Y. Yoshida.

The most common cause of these periodic movements is the wind blowing over the bodies of water in which they occur. The sudden variations in barometric pressure may cause seiches in lakes and other nearly inclosed bodies of water. Earthquakes frequently produce seiches along the coast and in maritime harbors by means of the disturbance transmitted through the sea.

By "strait-and-harbor" seiche is meant an irregular oscillation caused by reflections of disturbances between irregular or nonparallel shores, lying generally only a few miles asunder. The markings on a tide curve resembles irregular saw teeth. An example of such a body is the Golden Gate, California.

The seiches belonging to the last two classes are generally found along open coasts. The tide curves from certain places contain at times quite regular oscillations, and from most places irregular saw-teeth markings whenever a considerable disturbance takes place, such as that produced by an earthquake or a strong wind. The regular oscillations are generally formed in a cul-de-sac or small bay suddenly terminated. The irregular ones are formed where the sea bottom is shelving and where the lengths, and so the periods, of the fractional areas are not as fixed as are those in the case just referred to. As will be presently pointed out, the oscillating strip of water in these two cases is about $\frac{1}{4} \lambda$ long; and so the more definite its boundary, the more perfect the oscillation.

A familiar illustration is a steam or air whistle closed at one end. The jet of steam or air forced across or into the mouth of the whistle causes the contained column of fluid, which is $\frac{1}{4} \lambda$ in length, to be thrown into a state of intense vibration in accordance with the well known principle of resonance. If the form of the interior of the resonator were not tolerably simple, the tones given out would be less certain, and would to a certain extent be selected in accordance with the intensity of the blast.

Suppose that a suspended pendulous body be exposed to the action of the wind. It will be found to oscillate almost incessantly, and in its own period, because the impinging current of air is not exactly uniform. So, under favorable conditions, a seiche of this kind may exist for days at a time, sustained by the variable action of the wind upon the oscillating arm of water and the larger body with which it is connected, the effect upon the latter being probably of greater importance.

Experiment No. 1.—Take a glass tube, open at both ends, about 20 inches long and one-half inch in diameter. Mark a point distant one-fourth of the length of a second's pendulum from the lower end of the tube. Immerse the tube up to this mark in a tank of water. By suction elevate a column of water, and then suddenly release it. The column will perform oscillations having a period of one second and a continually decreasing amplitude. The following are the elevations or amplitude in inches as found by experiment beginning with the first elevation occurring after letting go the column sustained by suction; the values in parentheses are the ratios of neighboring values: 4, (0.55), 2.20, (0.64), 1.40, (0.66), 0.93, (0.69), 0.64, (0.67), 0.43, (0.70), 0.30, (0.77), 0.23, (0.74), 0.17. From the nature of the case it is difficult to determine these quantities with precision; but they show that the amplitude decreases somewhat more slowly as it becomes smaller. If no attention is paid to the amplitude, it is easy to so time the sustaining suction impulses imparted from time to time that the natural period of oscillation shall be but little interfered with. In this way the number of oscillations per minute can be found with considerable accuracy (say to 1 per cent). The result will be 60 per minute very nearly.*

* Cf. Dr. Thomas Young: A Course of Lectures on Natural Philosophy (1845), p. 217.

Let L denote the length of the column of water, ζ the vertical displacement of the surface of the upper end; let Ω denote the area of the tube, and $\frac{\gamma}{g}$ the density of the liquid. Then by d'Alembert's principle the impressed force of gravity is equal to effective force;

$$\therefore -\zeta\Omega\gamma = M \frac{d^2\zeta}{dt^2} = L\Omega \frac{\gamma}{g} \frac{d^2\zeta}{dt^2}, \quad (336)$$

$$-\zeta g = L \frac{d^2\zeta}{dt^2}. \quad (337)$$

This equation is obviously satisfied by writing

$$\zeta = A \cos \left(\sqrt{\frac{g}{L}} t + \alpha \right). \quad (338)$$

The period of this oscillation is $2\pi \div \sqrt{\frac{g}{L}} = 2\pi \sqrt{\frac{L}{g}}$; the half-period of a pendulum of length l is $\pi \sqrt{\frac{l}{g}}$. If these two times are equal, L must be equal to $\frac{1}{4} l$.

Experiment No. 2.—Take a trough of rectangular cross section closed at one end, about 20 inches long, and containing a movable partition whereby various lengths can be readily secured. Place this in a tank of water so that the depth in the trough is from a fraction of an inch at the head to 2 inches at the mouth. By giving the water off the mouth of the trough an horizontal impulse, an oscillation will take place in the trough, which will continue for a few periods. These oscillations can be sustained by subsequent impulses, care being taken to not disturb the regularity of the motion. The period of the oscillation will be found to be approximately

$$4 \times \frac{\text{length of trough}}{\sqrt{gh}}, \quad (339)$$

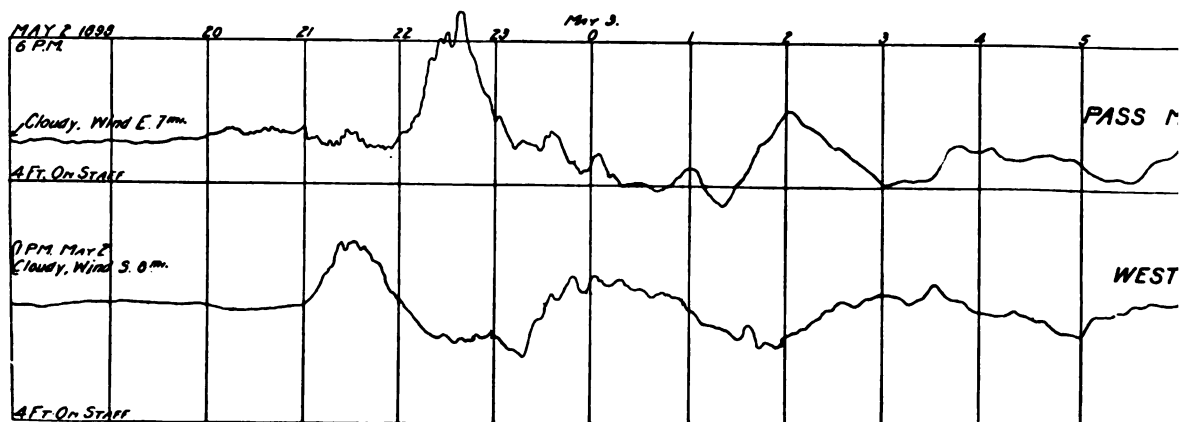
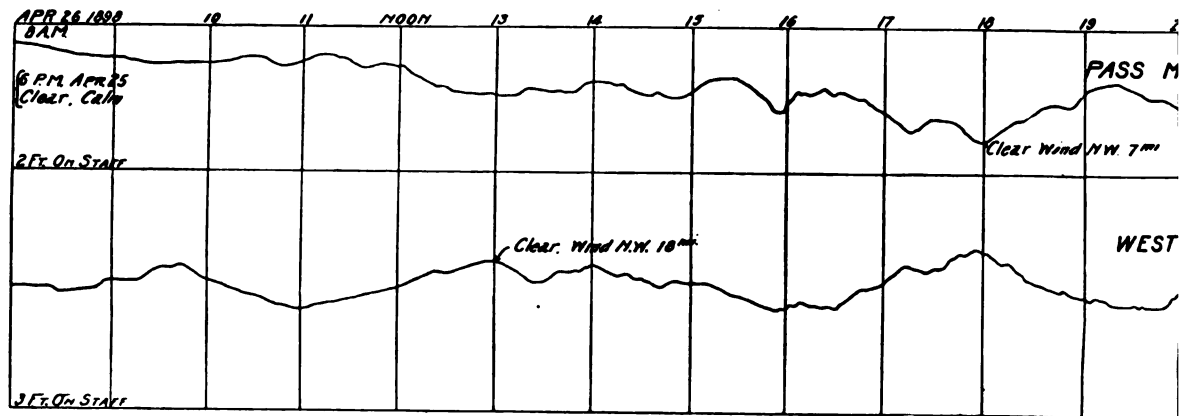
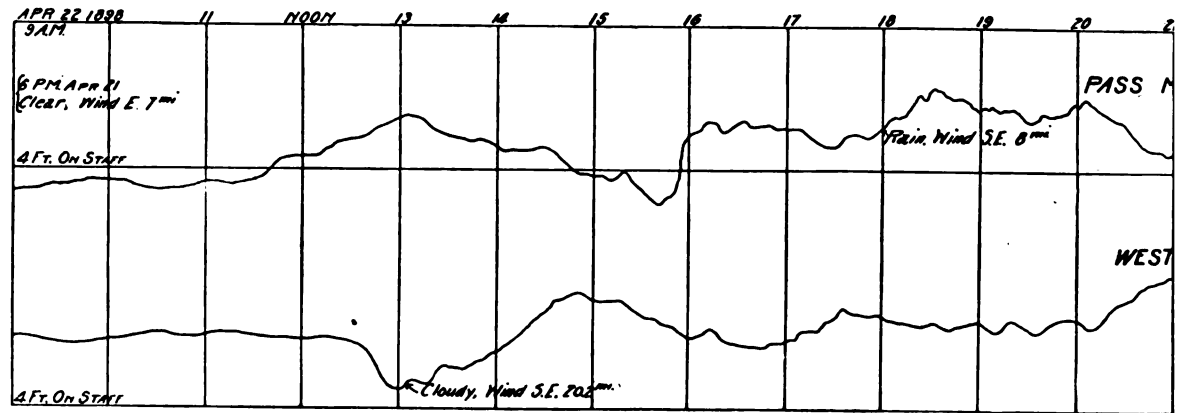
g being 386 inches (see Table 52).

Since the period of a rectangular lake whose depth decreases uniformly from the center to either end is nearly equal to what it would have been had the depth been constant and equal to its average depth, the period of an arm of water becoming shallow near the end must differ but little from the period calculated upon the assumption of a constant depth equal to its mean depth (see sec. 127).

130. Examples of lake seiches.

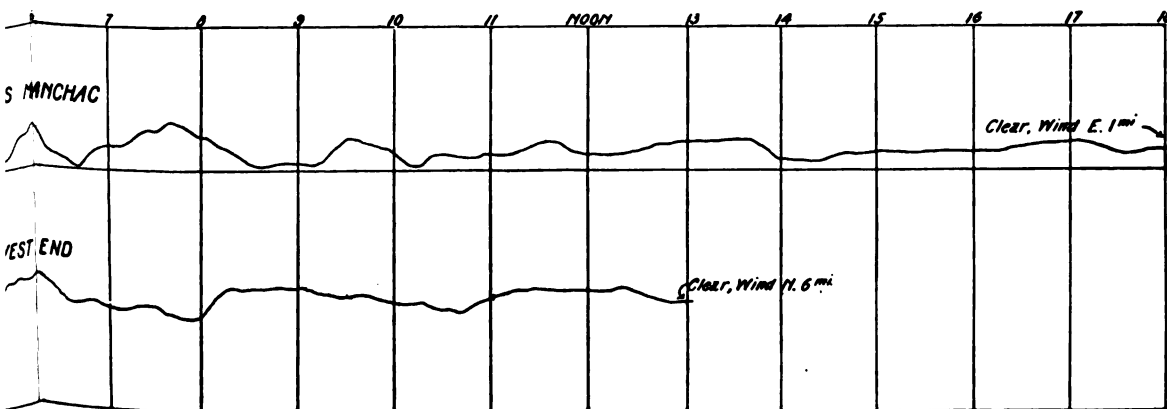
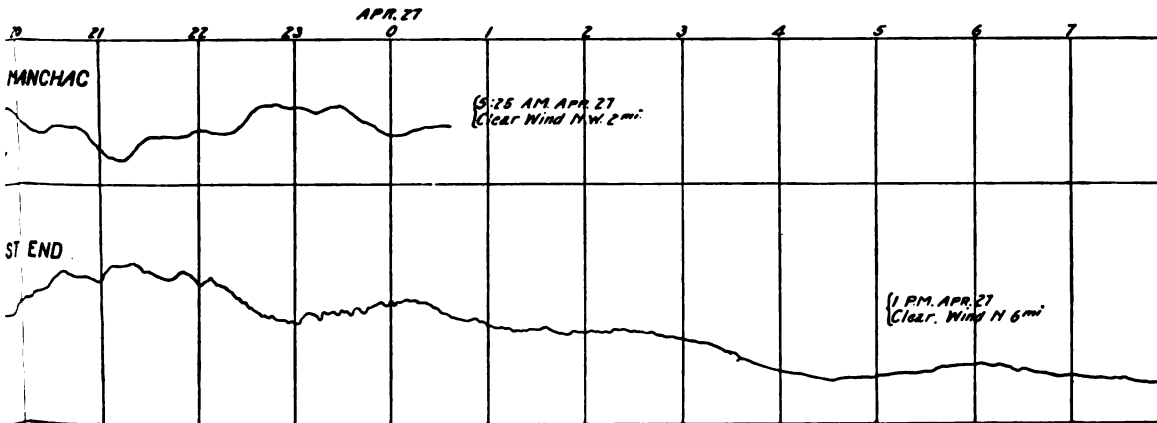
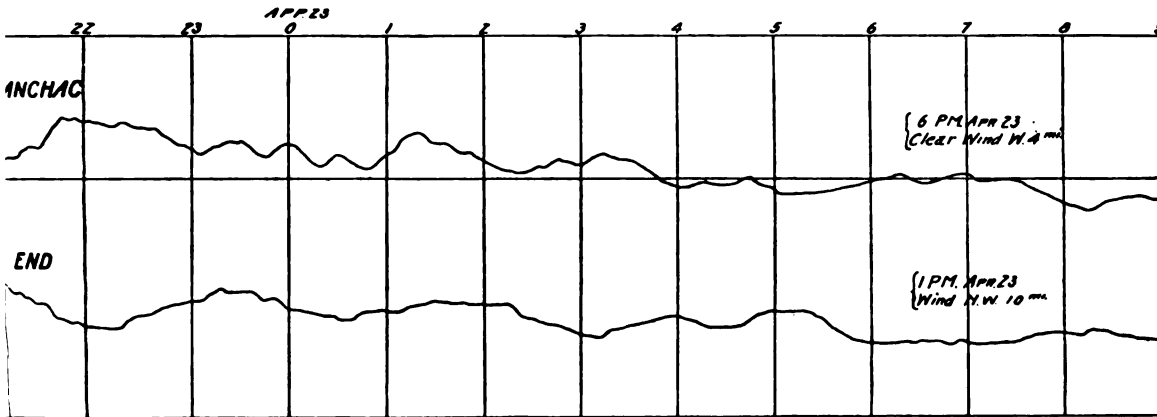
Dr. Forel was the first person to make an extensive and careful study of seiche phenomena (1876–1885). He found that Lake Geneva, upon whose northern shore he resided, had both longitudinal and transverse oscillations, and that the former often possessed more than a single nodal line; in fact, the record was far from being simple in character. The periods of the longitudinal seiches were found to be 73 and 35 minutes, and the period of the transverse seiche about 10 minutes. These periods agree fairly well with those computed by the simple formula

$$\tau = \text{twice length of lake} \div \sqrt{gh}.$$



IMPERFECT SEICHES IN

No. 27.



LES IN LAKE PONTCHARTRAIN.

He devised and used a delicate form of tide gauge which he styled "limnimeter" ($\lambda\acute{\iota}\mu\nu\eta$ =pool).

A list of Forel's papers on this subject is given by Darwin in his book entitled "The Tides and Kindred Phenomena in the Solar System," end of Chapter II; also by Chrystal, in the bibliography already alluded to.

Numerous modes of oscillation in the small lake Chiemsee, southern Bavaria, have been recently investigated and the corresponding periods carefully determined by A. Endrös.*

In a paper issued by the U. S. Weather Bureau entitled "Wind Velocity and Fluctuations of Water Level on Lake Erie," Prof. Alfred J. Henry finds the theoretical period of the lake to be 18 hours, while observations made at Buffalo and Amherstburg indicate an actual period of 14 hours or a little more.† In a note in the Monthly Weather Review‡ I have called attention to the shortening of the free period due to the narrowing ends of the lake and to the fact that actual depth is somewhat in excess of that used by Professor Henry. In fact, in a sharply-pointed convex lake, such as might be bounded by the lines of motion shown in Fig. 9, Part IV A, the period would be $1/\sqrt{2}=0.707$ times the period of a rectangular body having as length the extreme length of the lake. The mean between the period given on this assumption and that given by the assumption of a rectangular body is $14\frac{1}{4}$ hours, the length of the body in either case being 214 sea miles and the mean depth 10 fathoms.

The period of the longitudinal seiche of Lake Ontario is said to be four hours and forty-nine minutes.§

As will be noted in section 136, Lapham in 1852 observed oscillations at Milwaukee having a period of 2 hours, which is the theoretical period of an east-and-west oscillation across Lake Michigan.

The average depth of Lake Pontchartrain, La., is 14 feet. Calling the virtual length 30 miles and the width 20 miles, the two periods obtained for Table 50 are

$$\frac{30}{12.57} = 2.4 \text{ hours and } \frac{20}{12.57} = 1.6 \text{ hours.}$$

No regular seiches occur in this lake; Fig. 27 is given to show that a southeasterly wind may make the lake uncommonly low at New Orleans (West End), while it is uncommonly high at Pass Manchac, and vice versa; similarly for a northwest wind. Moreover, it shows a very rough oscillation at times with a period of about 2 hours.

In 1885 seiches were observed on Lake George, New South Wales, Australia, and described by H. C. Russell in his anniversary address.** They were found to have a period of 2 hours, very nearly. This is nearly the theoretical period for a longitudinal uninodal oscillation of a body having the length of the lake and a depth equal to the lake's mean depth.

* Zeitschrift für Instrumentenkunde, Vol. 24 (1904), pp. 180, 181.

† No. 262 (1902) Bulletin J. See also paper by same author in Monthly Weather Review, Vol. 28 (1900), pp. 203-205.

‡ Vol. 30 (1902), p. 312.

§ Monthly Weather Review, Vol. 26 (1898), pp. 261-262.

** Journal and Proceedings of the Royal Society of New South Wales, Vol. 19 (1885), pp. 13-19, and plates between pp. 82, 83.

One of the most regular seiches known occurs at times upon Lake Chiuzenji, a small lake in Japan. It was observed by Mr. K. Honda in 1906, by aid of two limnimeters of his own design.* He placed an instrument at each end of the lake and found low water at one end when it was high water at the other, and the period to be 7.70 minutes. The curves on the record are nearly perfect sine curves. By trial he found a point near the middle of the northern shore of the lake where the pencil of the limnimeter traced a straight line. The simple formula giving too short a period, Mr. Honda constructed a model of the lake and by means of it found the period of the lake to be 7.68 minutes. It seems probable that the period of the lake is slightly in excess of the period of a rectangular lake of the same length because of the somewhat dumb-bell shape of the lake.

131. *Examples of open-lake seiches.*

The celebrated tidal currents in the Euripus are chiefly due to the seiches in the Gulf of Talanta. The principal oscillation of the gulf has an observed period of $1\frac{1}{4}$ hours and a range of 6 inches or less. This period agrees fairly well with computation. The extreme length of the gulf is 54 miles, the average depth 52 fathoms. These dimensions imply a period of 1.8 hours.

The fact that at some points of this gulf the period of the oscillation is but little more than one hour indicates that a binodal seiche also exists.†

The oscillation in Narragansett Bay is remarkable in that the 10 miles extending from Providence to Bristol (the average depth being $2\frac{1}{2}$ fathoms) has a seiche of the binodal type, while the 10 miles extending from Bristol to Newport (the average depth being 11 fathoms) has one of the uninodal type. This is as it should be when depths are considered. By Table 50 the period of the binodal part is 0.77 hour and of the uninodal 0.74 hour.

Fig. 28 shows the period to be 0.75 hour. It shows that the oscillation is in approximately like phases at Providence and Bristol, while at Bristol and Newport the phases are nearly opposite.

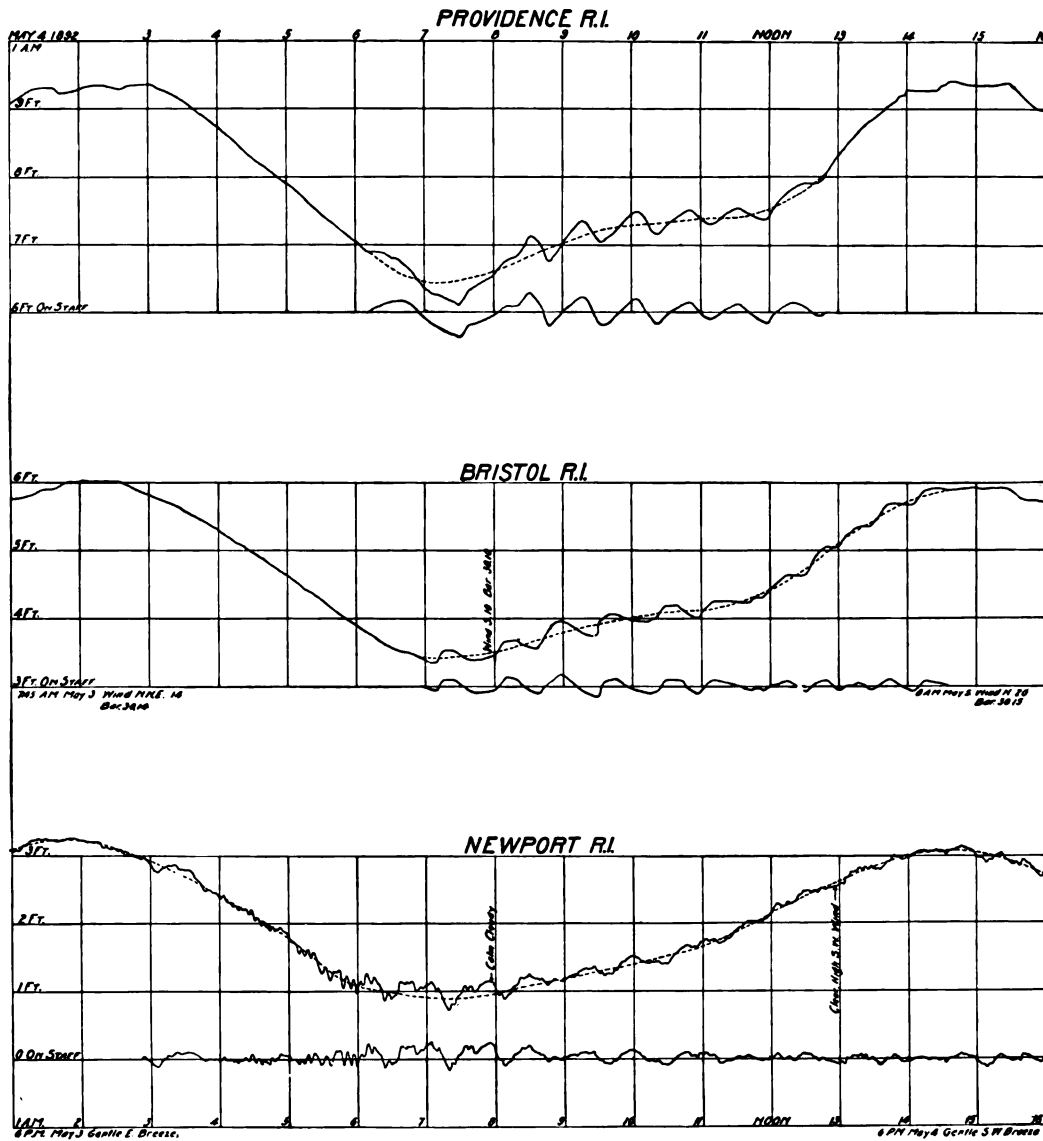
At Weeks, Louisiana, an imperfect seiche in Vermillion Bay and of considerable amplitude occurs during heavy winds. The observed period is about 3 hours; the period of two hours may be occasionally observed. The average depth of the bay is 8 feet. The virtual length is about 15 miles and virtual breadth about 10 miles. These dimensions give as periods

$$\frac{30}{9.5} = 3.2 \text{ hours, and } \frac{20}{9.5} = 2.1 \text{ hours.}$$

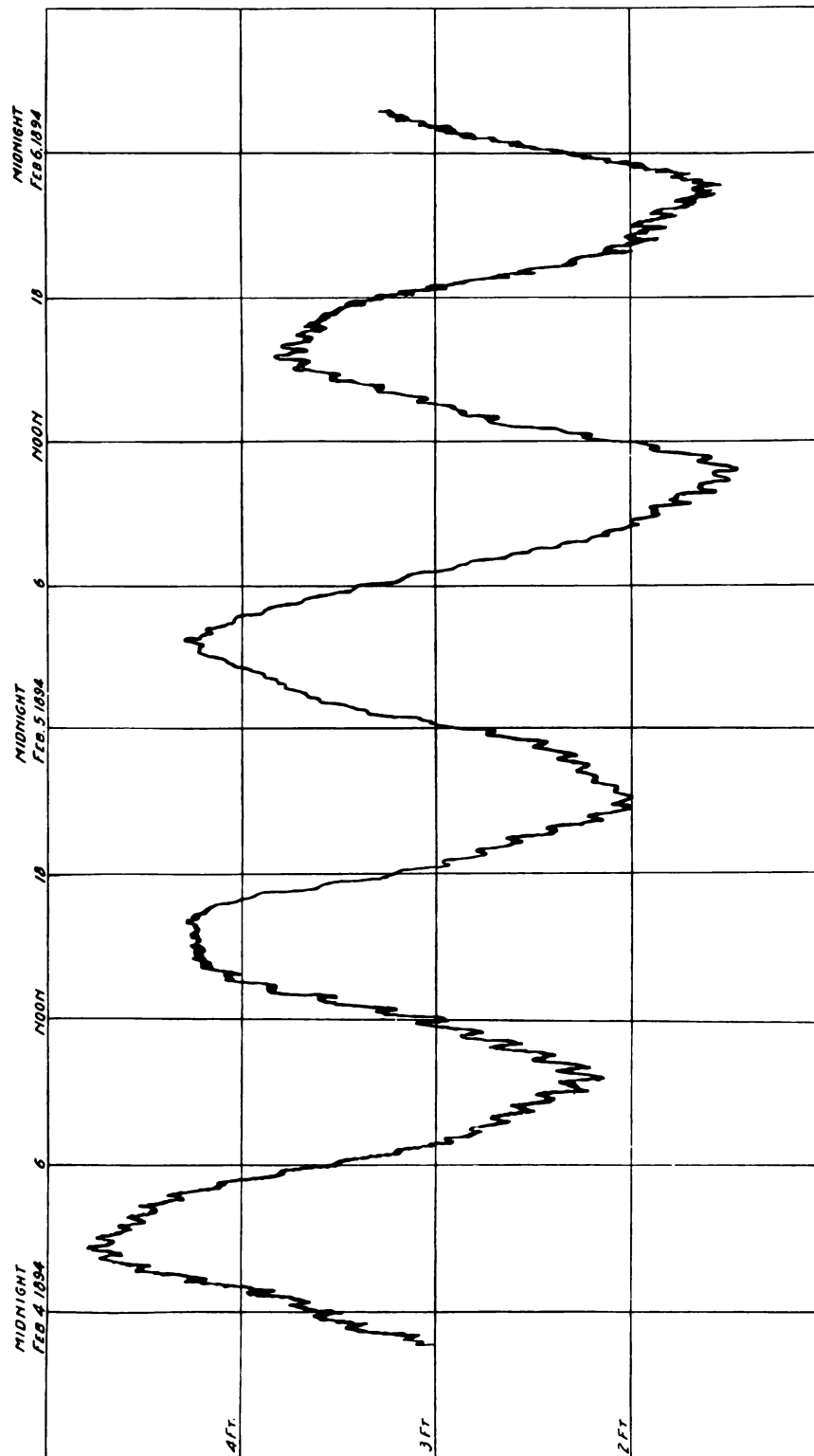
At Fort Point, Galveston, Tex., a small wave oscillation having a period of about $1\frac{1}{4}$ hours and an ordinary maximum range of about 0.2 foot has been observed. Now a partially enclosed body of water lies between Galveston, Pelican Island, and the mainland to the west. The virtual length of this body is about 5 miles, and the average

* Proceedings of the Tôkyô Mathemático-Physical Society, Vol. III (1907), pp. 220-223.

† The following are a few references to the seiches of the Euripus: F. J. P. Babin: Phil. Trans. 1671, Abr. Vol. I, p. 592. Barlow: An Exact Survey of the Tide, Ch. V, sections 4, 5. Lalande: Astronomie, Vol. IV, pp. 148-151. F. A. Forel: Comptes Rendus, Vol. 39 (1879), pp. 859-861. Nature, Vol. 21 (1879), p. 186. Krümmel: Handbuch der Ozeanographie, Vol. II, pp. 143-146. A. A. Μισοῦλης: Περὶ τῆς Παλλήροισας τοῦ Εὐρίπου (1882). See Fig. 37, Part IV A, and under Aristotle, Strabo and Pliny, secs. 64, 67, 69, Part I, this manual.



SEICHES IN NARRAGANSETT BAY.



SEICHES AT WELLINGTON, NEW ZEALAND.

depth at a mean stage of the tide is $5\frac{1}{2}$ feet; these dimensions give 1.27 hours for the free period of the body. By means of the channel along the city front the oscillation is propagated to Fort Point.

At Wellington, New Zealand, the observed period of the seiche oscillation is 28 minutes. The length of the harbor in a northeasterly and southwesterly direction is $6\frac{1}{2}$ miles and in a northwesterly and southeasterly direction 5 miles. The average depth for either direction is 10 fathoms. The computed period of the first is 30 minutes and of the second 23 minutes. Hence, it is probable that the seiche extends in a northeasterly and southwesterly direction; the period may be somewhat less than 30 minutes because the virtual length of an oval body is less than its extreme length and because the depth increases gradually for some distance from the shores.

A few small oscillations are shown upon the tide-record for Williamstown, Port Phillip, published in connection with the Krakatoa eruption. This body is 33 miles long in a north-and-south direction, its average depth being about 10 fathoms. The period of a binodal seiche in a rectangular body of these dimensions is 1.27 hours. On account of the oval shape of Port Phillip this value should be shortened somewhat. Observation seems to give about 1.20 hours for the period. An east-and-west seiche would have a slightly greater period, but it is probable that this could hardly be felt at Williamstown. However, the irregularity of the observed period indicates that either the mode of oscillation is not fixed or two or more motions coexist.

132. *Examples of parallel-wall seiches.*

On November 1, 1870, during a severe storm, a series of seiches were observed at Fiume, Austria.* Their average period was 2.7 hours and the maximum range about 1 foot. Now, the open portion of the Adriatic just below the bay upon which Fiume is situated is 67 sea miles in width; the depth is about 30 fathoms. This gives, by Table 50, 2.9 solar hours as the free period of a uninodal seiche.

Seiches have been observed at Ischia, by G. Grablovitz,† having an average period of $13\frac{3}{4}$ minutes and an average maximum range of perhaps 0.2 foot. The oscillating strip of water may be assumed to extend from the coast of Ischia to the middle of the opposing coast of Porcida. This distance is 2.6 miles, and the mean depth 9 fathoms. These dimensions give 0.2111 hour = 12.7 minutes as the period of the uninodal oscillation.

The eruption of Krakatoa, on August 27, 1883, produced a great disturbance in the waters of Sunda Strait, which was transmitted through the narrow part of the strait, and recorded on the tide gauge at Tandjong Priok, Batavia. Here the maximum range of the seiche was over 6 feet; the period was 2.20 hours.‡

The calculated period for uninodal seiche across the broad portions of Sunda Strait in a northwesterly and northeasterly direction is about 2.2 hours.

The tide curve at Tutticorn, Gulf of Manar, has at times an oscillation whose period is 3 hours and range about 0.5 foot.§ The distance across the gulf at this place is 92

* E. Stahlberger: Die Ebbe und Fluth in der Rhede von Fiume, Budapest, 1874

† Ricerche sulle mare d'Ischia, Rendiconti delle sedute della R. Accademia dei Lincei, Vol. 6 (1890), pp. 26-32.

‡ The Eruption of Krakatoa and Subsequent Phenomena, London, 1888

§ Account of the Operations of the Great Trigonometrical Survey of India, Vol. XVI (1901); Details of Tidal Observations, II, opp. page 57

miles, and the average depth of the upper end of the gulf may be taken as 55 fathoms. These values give a period of 3 hours.

133. *Examples of cul-de-sac seiches.*

The seiches at Malta are described by Airy in the Philosophical Transactions.* He finds the average observed period to be 21 minutes; the range to vary from nothing to a little more than 1 foot. The body of water responsible for this seiche he considers to be the deep arm of the sea lying between Sicily and Tunis, the shoals playing only a subordinate part.

From Figure 37, Part IV A, it is evident that the period of the transverse oscillation might vary from 1 to 2 or even more hours according to the amount of shoal water included in the estimate. Hence it is difficult to say how many nodal lines are present when the period of the oscillation is 21 minutes. Their number may be anywhere from 3 to 7.†

As Valetta, the place of observation, is situated upon the northeastern coast of the island, while the area in which the seiche arises in accordance with this hypothesis lies mainly to the west of the island, Airy says, "Such waves, once created, would be propagated to regions of the sea somewhat beyond those in which they are formed."

It seems probable that these oscillations are caused by the configuration of the harbor. For, the length of the harbor is 1.6 miles, and, if we call the average depth $6\frac{1}{2}$ fathoms, the period of the dependent oscillation would be $4 \times \sqrt{\frac{1}{2} \cdot \frac{1}{98}} = 0.31$ hour = 18 minutes.

Tidal observations made at St. Thomas Island, West Indies, show that oscillation is going on in the harbor most of the time. The most regular ones have a period of 0.45 hour and a range varying from 0.5 foot to nothing—it generally being 0.1 or 0.2 foot. At times there is an oscillation whose period is about 0.7 hour.

The harbor is about 1.3 miles long, measuring from the head to the capes at the mouth. The average depth is $3\frac{1}{2}$ fathoms. This gives $4 \times \sqrt{\frac{1}{2} \cdot \frac{1}{98}} = 0.34$ hour for the period of the oscillation. The broadening of the harbor near the head, and the contraction near the mouth due to Rupert Rock, must cause the period to be somewhat greater than the one just calculated. It is probable that the arm of water taken as $\frac{1}{2}\lambda$ does really extend outside of the mouth of the harbor proper.

The less perfect oscillation having a period of 0.7 hour may be caused by the strip of shallow water lying between St. Thomas Island, West Indies, and deep water to the south. The width of this strip is 8 miles and the average depth 20 fathoms. This gives $4 \times \sqrt{\frac{1}{2} \cdot \frac{1}{98}} = 0.9$ hour.

The seiches in St. John Harbor, an arm of the Bay of Fundy, has been briefly described by W. Bell Dawson ‡ and more fully by A. W. Duff, the point of observation of the latter being a short distance above the narrows.§

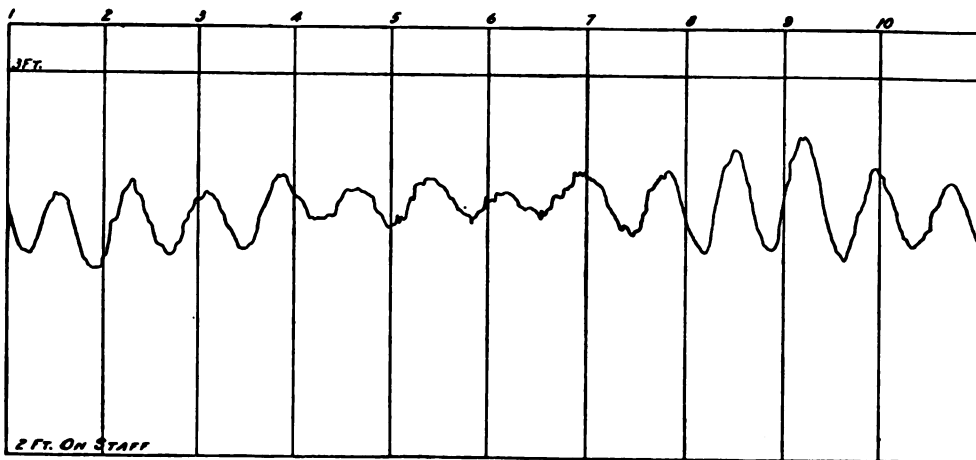
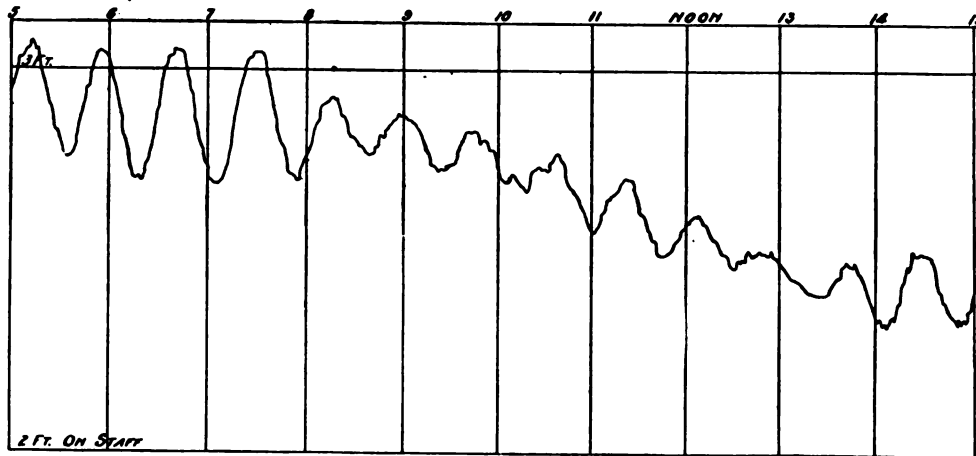
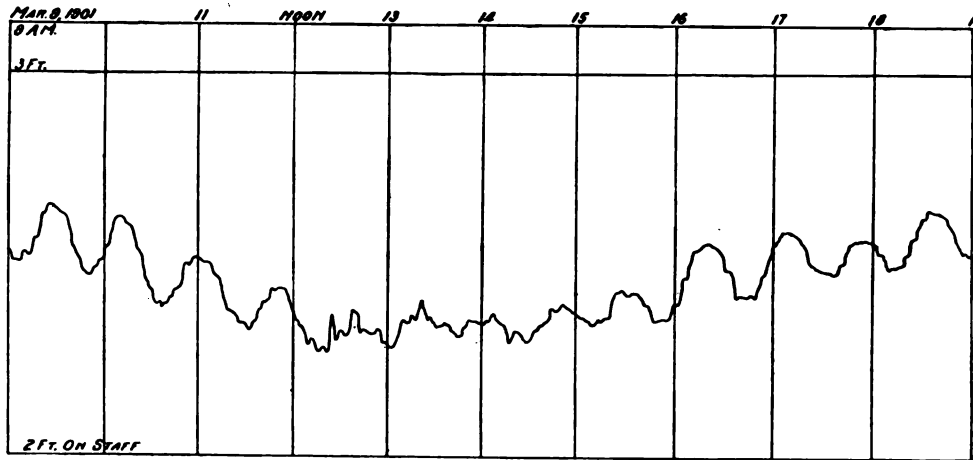
Duff gives 42.2 minutes as the length of the average period of the oscillations. Off St. John the bay is 32 miles wide. The average depth being 40 fathoms, it might be concluded from Table 50 that the period of the uninodal seiche is 1.23 hours.

* Vol. 169 (1878), pp. 136-138.

† Cf. Krümmel: Handbuch der Ozeanographie, Vol. II, p. 148.

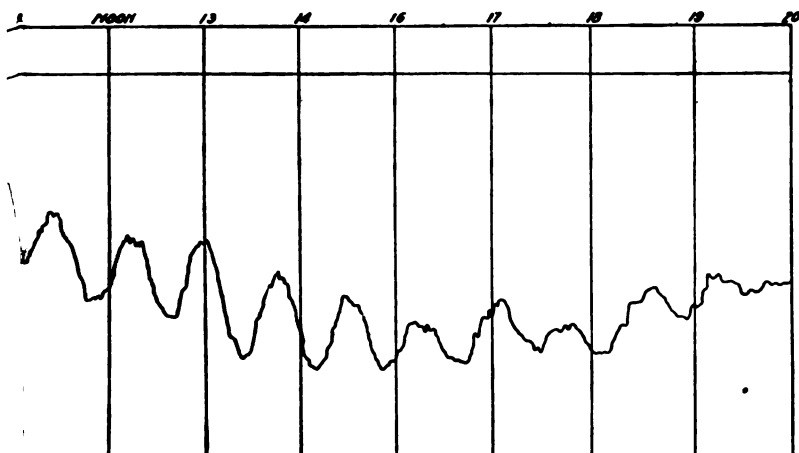
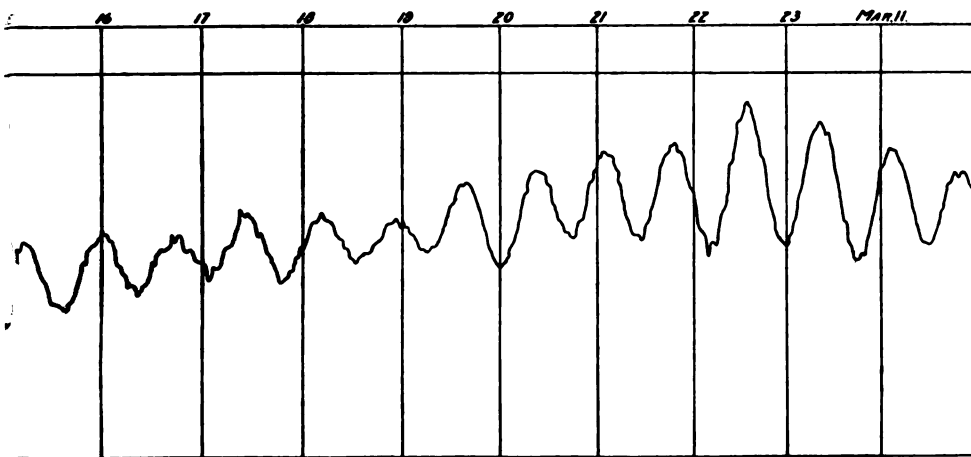
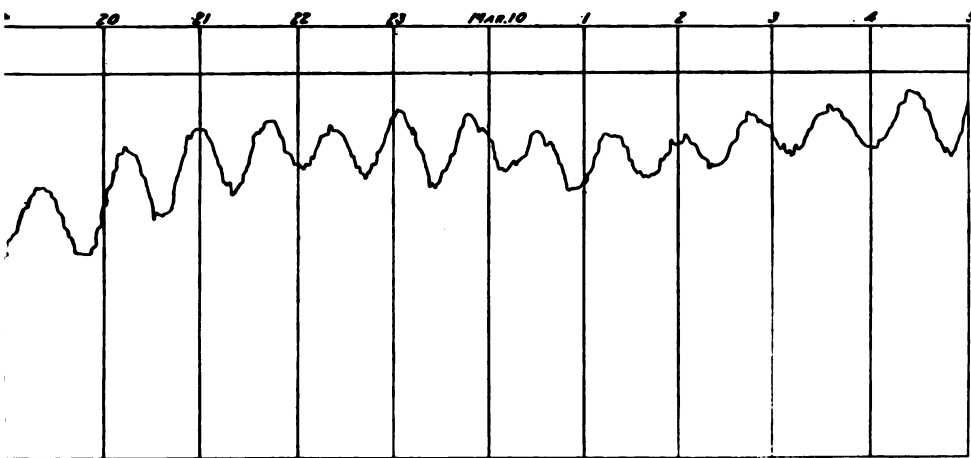
‡ Transactions of the Royal Society of Canada, Meeting of May, 1895, Section III, pp. 25-27.

§ Amer. Jour. Sci., Vol. 153 (1897), pp. 406-412.



SEICHES AT GUAI

No. 30.



U.S.A. PORTO RICO.

and the period of the binodal 0.62 hour. Now it seems in this case to be highly improbable that a binodal seiche should occur across the bay and not one of the uninodal type; for surely the winds sweeping over the surface of the bay would act most favorably across its axis and so incite uninodal motion. Again, Dawson says: "These minor undulations often continue for a week at a time; or even longer." This accords well with the facts witnessed at Guanica and described in the next paragraph. The harbor at St. John is 3.3 miles long from its head just south of the narrows to its mouth east of the northern part of Partridge Island. The average depth at half-tide level may be taken as 8 fathoms. These dimensions give as the period $4 \times \frac{3}{2} \cdot \frac{3}{2} = 0.567$ hour, or 34 minutes. At the bend in this arm depths of more than 20 fathoms occur, while near Partridge Island the depth of the channel is only 5 fathoms at half-tide level. These circumstances must increase the length of the period.

Remarkably regular seiches occur in Guanica Harbor. The observed period is 45 minutes, and the range varies from 1 to 4 inches. So persistent is this phenomenon that as many as 60 consecutive oscillations have been noted. (See Figure 30.)

That the phenomenon at Guanica does not depend upon a trinodal seiche across the Caribbean Sea is proved by the fact that no indication of a seiche having a period of 45 minutes occurs at Ponce or at Guayanilla.

The length of this harbor is 2 miles and the average depth 3 fathoms. The period of a rectangular dependent arm of these dimensions is, by Table 50, $4 \times \frac{2}{1} \cdot \frac{3}{2} = 0.55$ hour. This period must be increased by one-third part of itself in order to agree with the observed value. The average width of the harbor is at least one and one-half times its width at the narrowest part near the mouth. This fact increases the period somewhat, but it is partially offset by the increase in depth at the narrows. It seems probable that the oscillating arm of water extends some distance outside of the harbor proper, thus increasing the size and period of the body.

Port Real is a body of water 0.8 mile long and $1\frac{1}{2}$ fathoms deep. Here a fairly regular seiche occurs, having a period of 0.62 hour and a maximum range of generally 0.2 feet. A rectangular tongue of water would have for its critical period $4 \times \frac{1}{1} \cdot \frac{3}{2} = 0.32$ hour.

Near the entrance to this harbor the distance across the channel, counting from the 1-fathom line, is only one-twelfth of a mile, while the breadth within the capes, counting from this same contour, is two-thirds of a mile. It seems probable that the period for the actual body should be perhaps double that of the rectangular body just alluded to.

The oscillation does not persist as long as the one at Guanica; still as many as sixteen consecutive periods may at times be observed.

Edgartown Light-House, Massachusetts, is situated near the head of a bay bounded by land on one side and land and shoals on the other. This arm of water is 3 miles long and 4 fathoms deep on the average. These dimensions give for the critical period $4 \times \frac{3}{1} \cdot \frac{4}{2} = 0.73$ hour. Observation shows that when a strong northwest wind is blowing, a seiche may arise having a period of three-fourths of an hour and continuing for several hours.

The undulations in the tide curve at Colon on August 27-28, 1883, do not seem to have been connected with the Krakatoa eruption; for, they are remarkably regular

and occur too early for allowing a reasonable time for the disturbance to be propagated around South Africa. The observed period of this oscillation is 1.17 hours.

On September 6-7, 1882, the tide curve showed an earthquake disturbance. There were apparently two oscillations thus set up, the period of the first being 1.17 hours and of the second 0.40 hour; the maximum ranges being 1.3 feet and 0.5 foot, respectively.

On June 15 and again on June 17, 1883, a good oscillation, having a period of 1.17 hours and a range of 0.2 or 0.3 foot, is recorded on the tide curve.

The form of the western portion of Carribean Sea is not favorable to seiche oscillations, even if such were possible in a sea of such great depth. Turning to the harbor of Colon, we find its length to be 4 miles and its average depth $2\frac{1}{2}$ fathoms. The resulting period is therefore $4 \times \frac{4}{13.64} = 1.17$ hours.

At Dutch Harbor, Alaska, observations show a seiche having a period of about one-half hour. The extreme length of Iliuliuk Bay, of which Dutch Harbor is a branch, is $4\frac{1}{2}$ miles. The average depth is about 15 fathoms. These dimensions give, by Table 50, a period of $4 \times \frac{4\frac{1}{2}}{31.86} = 0.565$ hour = 34 minutes.

At Honolulu, Hawaiian Islands, the period of a regular sine-like fluctuation, groups of which appear from time to time, is $0.43\frac{1}{2}$ hour or 26 minutes. The larger ones are caused by earthquake sea waves, but the smaller ones may be due to meteorological disturbances. The following are some of the earthquakes whose observed effects had very nearly the above period: Krakatoa, August 27, 1883; Northern Japan, June 15, 1896; Ecuador, January 31, 1906; and Valparaiso, Chile, August 16, 1906.

The maximum range, depending upon the intensity of the disturbance at Honolulu, varies from 0.1 foot to 0.4 foot.

Assuming that the dependent body at Honolulu Harbor is $1\frac{3}{4}$ miles long and $2\frac{1}{2}$ fathoms deep on an average, the computed period is $4 \times \frac{1\frac{3}{4}}{13.01} = 0.42$ hour.

Following the Krakatoa eruption, irregular oscillations, having a period of 2.6 hours and a maximum range of $2\frac{1}{2}$ feet, were observed at Lyttleton, New Zealand. The length of Port Cooper is $8\frac{1}{4}$ miles, and average depth $2\frac{1}{2}$ fathoms. The resulting period is therefore $4 \times \frac{8\frac{1}{4}}{13.01} = 2.54$ hours.

At Olongapo, Subic Bay, Philippine Islands, a seiche, having a period of 1.3 hours and range of about 9 inches, has been observed; also smaller undulations having a period of from one-third to one-half an hour. The former can hardly be due to an oscillation of the greater part of China Sea lying between Luzon and Siam and Hainan Island; for, although a trinodal east-and-west seiche has a computed period of about $1\frac{1}{4}$ hours, no oscillation of such period is to be found at Manila.

Now the length of Subic Bay is 10 miles and its average depth 18 fathoms. Regarding this bay as a dependent arm, the period computed by Table 50 is $4 \times \frac{10}{36.8} = 1.09$ hours. By supposing the line marking the mouth of the bay to be convex, the length of the bay may be increased to 11 or 12 miles. The computed period of the east-and-west oscillation of Subic Bay at Olongapo is 0.3 hour and in a southeast-and-northwest direction 0.4 hour.

A seiche was observed at Aden, Arabia, as a result of the Krakatoa eruption, the period being 1.02 hours and the maximum range of about 0.8 foot. Small undulations of this period may sometimes be seen on the tide curve for this station.*

Aden Harbor is about 1.8 miles long, measuring from the mouth to the low-water line at the head. It is 0.4 mile broad at the mouth and $2\frac{1}{2}$ miles at its broadest part. The average depth is $1\frac{1}{2}$ fathoms. A rectangular arm 1.8 miles long and $1\frac{1}{2}$ fathoms deep has as its period $4 \times \frac{1.8}{10.08} = 0.71$ hour. For the actual harbor the period must be considerably greater than 0.71 hour and may approach the observed value.†

134. *Examples of shelving seiches.*

The larger earthquake oscillations at San Francisco must be due to the shelving shore outside and not to any oscillation of the bay, for, the disturbance resulting from the earthquake at Arica, Chile (May 9, 1877), and which reached California early on the following day, arrived at Fort Point about 7 minutes earlier than at Sausalito. This is about the difference which the depths between these two stations might imply. There is a tolerably close resemblance between the records upon the tide gauges at the two places. The maximum range of this disturbance is about 1 foot, and the irregular periods vary from 0.3 hour to an hour or more.

The saw-teeth-like disturbance having an average period of three or four minutes and which frequently occur during and after heavy westerly winds are, as already stated, due to irregular reflections across the Golden Gate and so belong to another class of seiches.

The disturbance caused by the Krakatoa eruption was well marked at Negapatam, Madras, False Point, Beypore, Port Elizabeth, and Table Bay. The resulting oscillations, although very irregular, may be said to have the following respective periods and maximum ranges:

1.5, 1.4, 2.8, 1.1, 1.1, 1.0 hours, and
1.5, 0.5, 1.6, 1.2, 4.5, 1.5 feet.

These periods are too short for permitting the assumption that all of the shallow water along the continental shelf forms a stationary wave $\frac{1}{4}\lambda$ in length. It seems probable that more or less of the water will partake of such motion at a given station according as the intensity of the disturbing force varies. The very uniformity of the slope of the bottom in these regions of shallow water must militate against the existence of definite oscillating arms of water; hence, the great variety of periods and amplitudes at a given place of observation.

Shelving seiches are by far the most common of all. Every considerable earthquake disturbance of the water of the ocean produces them in nearly all parts of a shelving coast. When caused by winds, their appearance is less striking, in that their

* Great Trigonometrical Survey of India, Vol. XVI, Details of the Tidal Observations, Part II, opposite p. 4.

[† After the MS. of Part V had gone to the printer, I learned that Messrs. K. Honda, T. Terada, and D. Isitana, had anticipated me in applying $\tau = 4L/\sqrt{gh}$ to such seiches as are described in sections 133 and 134. This oversight was due to the fact that while the MS. was being prepared, only three numbers of the Proceedings of the Tôkyô Mathematico-Physical Society were accessible to me.—R. A. H., Dec. 24, 1907.]

amplitude is generally less and the tendency to periodicity less evident. The record of these may then be likened to irregular saw-teeth.

In regular broad and open bays their size and regularity are increased, e. g., the Krakatoa disturbance is much greater and more seiche-like at Port Elizabeth than at Table Bay. At Galle, Ceylon, the tide curves contain at times minute oscillations less regular than a cul-de-sac seiche, but more regular than a shelving seiche.* The period is about 0.4 hour and the range about 0.1 foot. The bay or harbor is broad and open, but does not extend far inland. Hence, it is natural to expect a case lying between the two kinds just mentioned. The harbor helps in making the seiche definite, but because of its openness the outside water forms a large part of the fractional oscillating area.

135. *References to papers relating to the Causes of Seiches.*

Nature, Vol. 17 (1878), p. 234; Nature, Vol. 18 (1878), pp. 100, 101.

F. A. Forel: Seiches and earthquakes, Nature, Vol. 17 (1878), p. 281; Les seiches des lacs; leurs causes, Comptes Rendus, Vol. 86 (1878), pp. 1500-1503.

E. A. Perkins: The seiche in America, The American Meteorological Journal, Vol. 10 (1893), pp. 251-263.

J. R. H. MacFarlane: Occurrence of seiches in Lake Derravaragh, Co. Westmeath, 1893, 1894, Scientific Proceedings of the Royal Dublin Society, Vol. 8 (1895), pp. 288-296.

W. H. Wheeler: Undulations in lakes and inland seas due to wind and atmospheric pressure, Nature, Vol. 57 (1898), pp. 321, 322.

F. Napier Denison: The Great Lakes as a sensitive barometer, B. A. A. S. (1897), pp. 567, 568.

References to descriptions of Limnimeters or Limnographs.

G. Grablovitz: Descrizione d'un maregrafo portatile, Rendiconti delle sedute della R. Accademia dei Lincei, Vol. 6 (1890), pp. 359-362.

Dr. H. Ebert: Sarasin's neues selbstregistrirendes Limnimeter, Zeitschrift für Instrumentkunde, Vol. 21 (1901), pp. 193-201.

A. Endrös: Seichensforschungen am Chiemsee, Zeitschrift für Instrumentkunde, Vol. 24 (1904), pp. 180, 181.

* Details of Tidal Observations, *l. c.*, II, opp. p. 64.

CHAPTER X.

TIDES IN LAKES AND WELLS.

136. *Tides in lakes or inland seas.*

Passing over the tides of the eastern portion of the Mediterranean Sea which were familiar to the ancients and which have been already considered in this manual, it may be noted here that Daniel Bernoulli in Chapter XI of his "*Traité sur le Flux et Reflux de la Mer*" was the first to attempt to compute the tides on inland seas. (See sec. 94, Part I.)

E. B. Barlow in his book entitled "*An Exact Survey of the Tide*" (1717) mentions the existence of semidaily tides in Lake Huron, and thus speaks of the tides of Lake Superior:

Moreover, the French report of the *Upper Lake*, which lies to the N. West of this, and falls into the same River; that notwithstanding it is situated *five hundred Fathoms* above the Superficies of the Sea, and lies at *two hundred and seventy leagues* distant from it; yet it Floods to *two, three, and four Foot* in Height; which plainly shews, that these Tumours proceed absolutely from the *Moon's Pressure*, without the Intermediation of the *Ocean* to raise 'em by way of *Libration*, at so great a Distance from the Sea, and exalted so many Fathoms above it.

Early letters and discussions relating to the tides in the Great Lakes of North America are given by Gen. H. A. S. Dearborn upon pages 78 et seq., Volume XVI (1829), *American Journal of Sciences and Arts*. Dearborn says:

The phenomenon appears to have attracted the attention of Fra Marquette in 1673, of Baron Hontan in 1689, of Charlevoix in 1721.

Maj. Samuel A. Storrow observed in Green Bay in the year 1817 a fluctuation of the water surface having a range of from 5 to 8 inches and a half period of 11 or 12 hours. Schoolcraft (about 1820) concludes that there are no regular tides in the lakes, but that the observed fluctuations are meteorological in character. Dr. Joseph Lovell in 1827 expresses a similar view. Capt. Henry Whiting, U. S. A., observed the tides upon a graduated staff in Green Bay from June 1 to 6, 1819. He says:

The height of the rise and fall, was from twelve to eighteen inches. Both the ebb and flow were very sudden, and in that respect deviate from the general character of the tides. It was seldom more than an hour, in attaining its height, and was generally as rapid in making the descent, though several hours would often intervene between the changes.

In his letter dated September 11, 1827, he expresses his conviction that the Green Bay tides are due to the winds.

Capt. Greenleaf Dearborn, U. S. A., mentions that while stationed at the outlet of Lake Superior, tides having a range of about .18 inches and a time of rising or falling of $2\frac{1}{2}$ to $2\frac{1}{2}$ hours were observed.

In a paper entitled "Remarks on the supposed tides, and periodical rise and fall of the North America Lakes," * Maj. Henry Whiting expresses (p. 212) a belief that an astronomical tide must exist, but that it must be very small. He gives Governor Cass's observations made in 1828 (July 15 to August 29, generally 4 times daily) on Fox River, Green Bay. Governor L. Cass thus comments upon his observations:

The slightest inspection will satisfy you, that the changes in the elevation of the water are entirely too variable to be traced to any regular permanent cause; and that consequently there is no perceptible tide at Green Bay, which is the result of observation.

Lieut. D. Ruggles, in an article entitled "Tides in the North American Lakes," † gives some history of the subject and expresses the belief that the tide can not well be detected from observations made, say, four times daily (as was the case with those made in 1828), and so he causes hourly observations to be made for one week at Green Bay in 1836. It seems doubtful whether or not he really detected any tide, for his observations indicate oscillations of a foot or two.

Upon page 130 of his book entitled "Wisconsin: Its geography and topography, history, geology, and mineralogy" (2d ed. Milwaukee, 1846), Mr. Increase A. Lapham says:

The question whether there is a regular tide on the lakes, still remains undecided. That there are strong and variable currents in Lake Michigan has been known ever since the days of Hennepin.

In 1849 Lapham kept a meteorological journal, in which he gives the height of Lake Michigan five times daily and referred his readings to the city datum. Whether or not he really detected an astronomical tide from these observations is uncertain. He, however, believed that he did, as can be seen from his statements in an appendix to Charles Whittlesey's paper, page 447, *American Journal of Sciences and Arts*, Volume XXVII (1859). However this may be, he subsequently obtained hourly readings day and night from September 14 to November 14, 1852. This record is in the office of the Coast and Geodetic Survey. From these it is possible to determine the tide, as has been done in section 137; but no work of Lapham's appears to be extant which might prove that he detected the true tide. A plotting of the hourly readings clearly shows the transverse seiche at Milwaukee, with a period of almost exactly two hours—which is the theoretical time of such oscillation—and a range of 0.2 foot.

A paper by Major Lachlan entitled "On the periodical rise and fall of the lakes" appears in the *American Journal of Sciences and Arts*, Volume XIX (1855), pages 60-71, 164-175, and Volume XX (1855), pages 45-53. On pages 63 et seq., Volume XIX, is an historical account of the subject. On page 168 he quotes an extract from a report by Colonel Whittlesey for 1838-39, in which Whittlesey strongly contends that observations fail to show any astronomical tide in Lake Erie. Lachlan comes to the conclusion (Vol. XX., p. 51) that "a long, regular course of minute observations" is necessary to settle the question of tides in lakes. At the close of the paper (Vol. XX, p. 53) the editors refer to an important paper by Whittlesey (1851), which goes to show that the fluctuations are not periodical.

Major Lachlan also discusses the annual and other variations in lake level.

* *Am. Jour. Sci. & Arts*, Vol. XX (1831), pp. 205-219.

† *Am. Jour. Sci. & Arts*, Vol. XLV (1843), pp. 18-27.

Col. J. D. Graham, in an article entitled "Investigation of the problem regarding the existence of a lunar tidal wave on the great fresh-water lakes of North America,"* finds from six months of half hourly readings, the spring range on Lake Michigan at Chicago, to be 0.25 foot, the mean range 0.15 foot, and the mean lunital interval to be 30 minutes. He says:

This result was announced in my annual report to the Topographical Bureau of the War Department on the 15th of November, 1858, and also before the Chicago Historical Society at its annual meeting on the 30th of that month.

This discovery is referred to on page 127, Volume 37 (1859), *Journal Franklin Institute*.

The later work of Colonel Graham is discussed by Ferrel, "Tidal researches," pages 250-255, who finds for the values of approximately the same quantities—0.21 foot, 0.14 foot, and 32 minutes, respectively.

In the *American Journal of Sciences and Arts*, Volume XXVII (1859), pages 305-310, Charles Whittlesey reviews observations made by Mr. Underwood in 1858 at Green Bay, but he does not believe the tide on Lake Michigan yet discovered. On page 447 of the same volume is a note or appendix to Mr. Whittlesey's paper in which I. A. Lapham states that he detected (and announced in 1849) a tide from observations made every three hours during the month of August, 1849. Also, that subsequent hourly observations made day and night for two months fully confirmed this conclusion. In this note Whittlesey mentions Colonel Graham's discovery of the tide in 1858, as chronicled in the proceedings of the Chicago Historical Society for the meeting of November 30, 1858.

It will be noticed that the earlier attempt at finding a lake tide failed because fluctuations many times greater than the astronomical tide were invariably mistaken for it, and also because continuous hourly readings were seldom undertaken. The justice of Lapham's claim to the discovery of a lake tide must depend upon what values he may have obtained for its range and interval.

In the Annual Report of the Survey of the Northern and Northwestern Lakes for 1872, by Maj. C. B. Comstock, is given on pages 9-14 a discussion of the tides at Milwaukee. A self-registering gauge was there maintained for several years, and portions of the record from 1867 to 1871 are discussed. Judging from the diagrams, Plates I-III, $M_2 = 0.0395$, $S_2 = 0.018$, and $S_2 + M_2 = 0.045$ foot, and the lunital interval, 30 minutes. The tabular values on page 12 of this report, when harmonically analyzed give $M_2 = 0.0340$ ft., $M_2^0 = 19^\circ.46$; $S_2 = 0.0154$ ft., $S_2^0 = 26^\circ.43$. In discussing these tides, the writer ascertains the theoretical equilibrium tide at Chicago and Milwaukee. These applications of the equilibrium theory, especially to the results obtained from observations by Colonel Graham at Chicago, constitute the first instance in the history of tides where the forces and tides have been rationally connected. But in this case the writer obtains a result twice as great as the equilibrium value, from an erroneous assumption concerning the kinetic energy in the tides of the lake.

In the next year's report of the Lake Survey, pages 28 and 30, and Plates I and II, are given similar results from observations at Duluth, but no theoretical considerations are introduced. An analysis of the tabular values on page 30, gives $M_2 = 0.0643$ ft., $M_2^0 = 64^\circ.45$; $S_2 = 0.0360$ ft.; $S_2^0 = 85^\circ.75$.

* Report A. A. A. S., 1860, pp. 52-60.

137. *Harmonic constants for the Great Lakes.*

Components	Milwaukee, Lake Michigan, lat. $43^{\circ} 02' N.$, long. $87^{\circ} 54' W.$				Marquette, Lake Superior, lat. $46^{\circ} 32' N.$, long. $87^{\circ} 23' W.$				Duluth, Lake Superior, lat. $46^{\circ} 47' N.$, long. $92^{\circ} 05' W.$			
	Sept. 15, 1852–Oct. 13, 1852		Oct. 13, 1852–Nov. 10, 1852		Jan. 1, 1904–Jan. 3, 1905		Oct. 11, 1901–Oct. 14, 1902		May 9, 1902–May 12, 1903			
	<i>H</i> Ft.	κ °	<i>H</i> Ft.	κ °	<i>H</i> Ft.	κ °	<i>H</i> Ft.	κ °	<i>H</i> Ft.	κ °		
K_1					0.0066	263.03	0.0297	85.04	0.0345	99.37		
K_2					0.0030	333.62	0.0101	84.32	0.0079	77.04		
M_1					0.0019	213.27	0.0054	177.28	0.0052	170.68		
M_2	0.0247	37.4	0.0228	40.12	0.0130	298.84	0.0631	71.05	0.0658	72.22		
O_1					0.0040	233.42	0.0249	89.36	0.0213	92.54		
P_1					0.0026	261.28	0.0155	85.36	0.0168	59.50		
S_1							0.0159	80.93	0.0092	98.05		
S_2	0.0103	48.3	0.0208	70.55	0.0086	349.33	0.0336	87.91	0.0337	87.70		

The no-tide point of Lake Superior (center of gravity of the surface, section 49, Part II), is 9 statute miles north of Keweenaw Point. The latitude and longitude of the no-tide point are about $47^{\circ} 32' N.$, $87^{\circ} 43' W.$ By means of this section, or preferably by means of sections 1–4, Part IV A, it is readily seen that the values in this table agree well, as a rule, with the corrected equilibrium theory; but see section 21.

The observations at Milwaukee are hourly readings made by I. A. Laphan; those at Marquette and Duluth are taken from records of self-registering gauges maintained by the United States Army Engineers.

The large size of the tide at Chicago and Milwaukee indicates the stationary-wave motion of sec. 21.

The values of M_1 and M_1^0 given in section 124 agree tolerably well with the theoretical values given in Table 37. . . . So far as semidaily, daily, or fortnightly forces are concerned the earth behaves nearly as a rigid body.

But according to a publication of the Royal Prussian Geodetic Institute, New Series, No. 30, 1907, O. Hecker finds from observations upon a horizontal pendulum some yielding of the earth to the tidal forces.

138. *Tides in wells or springs.*

Pliny the Elder calls attention to wells in Spain that rise and fall with the tide of the ocean and to those which rise and fall contrary to it. (Sec. 69, Part I, this manual).

Barlow on pages 30–32, First Treatise, of his "Exact Survey of the Tides," places little or no credence in the possibility of intermittent springs being influenced by the rise and fall of the ocean tide.

On pages 308 and 309 of Volume IV of his *Astronomie*, Lalande makes mention of wells or springs which have periodic fluctuations. Some of these are simply intermittent springs; e. g., those described in *Philosophical Transactions* 1665, No. 7, and 1693, No. 204.

According to the Admiralty Sailing Directions for the Pacific Islands, Volume III, wells of fresh water in Fanning Island rise and fall with the tide.

A detailed account of a considerable number of tidal wells, is given by A. C. Veatch in Water-Supply and Irrigation Paper, No. 155, U. S. Geological Survey, 1906, entitled "Fluctuations of the Water Level in Wells, with Special Reference to Long Island, New York." He finds the tide in the wells of the locality considered to be due to the periodic deformation of a clay layer immediately above the water-bearing stratum, the deformation being caused by the varying weight of the ocean tide above. The effect is a maximum at the low-water line and decreases with the distance from the shore. The lag of the tide in the well upon the ocean tide varies from a few minutes at the shore to more than an hour at a few hundred feet inland. At the shore the range of the fluctuation may be a considerable fraction of the range of the ocean tide.

The tidal oscillations in shallow wells along the sea shore he concludes to be generally due to the action of sea water upon the outflowing fresh water in the porous material into which the well is dug. The tides of the well may be behind those upon the outside by an interval of several hours.

Upon page 67 is a bibliography of the subject.

CHAPTER XI.

MISCELLANEOUS REMARKS ON TIDES AND MODES OF REDUCTION.

*On the lengths of series.*139. *Effect of one or more components on observed mean sea level.*

A component produces no disturbance in mean sea level if the series of ordinates extend over exactly a whole number of its periods. Let τ denote the length of series actually used; let $\nu_1 \frac{360^\circ}{a}$ denote the positive integral number of periods of the component A which falls nearest the length τ ; and put

$$\tau = \nu_1 \frac{360^\circ}{a} + \epsilon_1$$

or

$$\epsilon_1 = \tau - \nu_1 \frac{360^\circ}{a}.$$

The displacement or disturbance of mean sea level is

$$\begin{aligned} \frac{A}{\tau} \int_0^\tau \cos (at + \alpha) dt &= \frac{A}{\tau} \int_{t=0}^{t=\tau-\epsilon_1} \cos (at + \alpha) dt \\ &= \frac{A}{a\tau} \left[\sin (a\epsilon_1 + \alpha) - \sin \alpha \right]. \end{aligned} \quad (340)$$

Thus we see that the amount of disturbance of mean sea level due to a component, depends in general upon its initial phase, as well as upon its speed and amplitude. Consequently, if we have several components A, B, C, \dots , a length τ such that the disturbance shall be zero for assumed values of the initial phases $\alpha, \beta, \gamma, \dots$, this length will not give a zero disturbance for other assumed values of $\alpha, \beta, \gamma, \dots$.

Special case $\alpha=0$, assuming τ nearly equal to a multiple of periods of A .—Since ϵ_1 is assumed to be small, the disturbance due to A becomes $A \frac{\epsilon_1}{\tau}$. This becomes $-A \frac{\epsilon_1}{\tau}$ if α be taken as 180° . Hence

The disturbance of mean sea level under such assumed conditions that an improper length has the greatest possible effect, is independent of the speed of the component and is directly proportional to the error in length.

If there are several components the expression for the disturbance is

$$\frac{A\epsilon_1}{\tau} + \frac{B\epsilon_2}{\tau} + \frac{C\epsilon_3}{\tau} + \dots$$

This becomes zero if

$$A\epsilon_1 + B\epsilon_2 + C\epsilon_3 + \dots = 0.$$

Now

$$\epsilon_1 = \tau - \nu_1 \frac{360}{a}, \quad \epsilon_2 = \tau - \nu_2 \frac{360}{b}, \quad \epsilon_3 = \tau - \nu_3 \frac{360}{c}, \dots$$

and so the value of τ satisfying the last equation is

$$\tau' = \frac{A\nu_1 \frac{360}{a} + B\nu_2 \frac{360}{b} + C\nu_3 \frac{360}{c} + \dots}{A + B + C + \dots} \quad (341)$$

Hence

The length of series giving the best value of sea level may be found by weighing the integral numbers of component periods (each multiple period being nearly equal to τ) according to the respective amplitudes.

Because ϵ_1 is supposed to be a small quantity, we may write the general expression for the disturbance in the form

$$\frac{1}{a\tau} \left[a\epsilon_1 \cos \alpha - \frac{a^2 \epsilon_1^2}{2} \sin \alpha \right]$$

If $\alpha = \mp 90^\circ$, this becomes $\pm \frac{Aa\epsilon_1^2}{\tau}$. In assigning limits to the integration it was assumed that ϵ_1 might have either a positive or a negative value. If we had supposed ϵ_1 to be an essentially positive quantity, the expression for the disturbance would have been either

$$\frac{A}{a\tau} [\sin (a\epsilon_1 + \alpha) - \sin \alpha] \text{ or } \frac{A}{a\tau} [\sin \alpha - \sin (a\epsilon_1 + \alpha)]$$

according as ϵ_1 is placed at the upper or lower limit of the integration. Hence we shall have as the required disturbance of sea level when $\alpha = -90^\circ$ the very small quantities,

$$\frac{Aa\epsilon_1^2}{\tau} \text{ or } -\frac{Aa\epsilon_1^2}{\tau} \quad (342)$$

according as ϵ_1 is positive or negative. Hence

The disturbance of mean sea level under such assumed conditions that an improper length has the least possible effect is proportional to the speed of the component and to the square of the error in length.

For several components the disturbance is

$$\pm \frac{Aa\epsilon_1^2}{\tau} \pm \frac{Bb\epsilon_2^2}{\tau} \pm \frac{Cc\epsilon_3^2}{\tau} \pm \dots \quad (343)$$

This equated to zero would give a time τ' slightly different from the τ' already found.

We shall suppose τ' to be determined by the foregoing rule, which weights the components according to their amplitudes.

The periods of the several components expressed in mean solar days are given in the column headed $\frac{S_1}{c}$, Table 1. Let such multiples of these periods be taken as shall

give in each case very nearly 29 solar days. The exact times thus determined are to be multiplied by the respective amplitudes of the components. If we use the theoretical amplitudes as given in Table 1, the above rule gives when M_2 , S_2 , N_2 , ν_2 , L_2 , T_2 , $2N$, μ_2 and K_1 , O_1 , P_1 , Q_1 , are substituted in the formula,

$$\tau' = 28.99 \text{ days.}$$

The result remains sensibly the same when we use semidiurnals alone or diurnals alone.

Thus we see that 29 days is almost an ideal period for the determination of mean sea level, so far as a period can be decided irrespective of the time and locality.

It may be remarked that although the above rule applies and gives $\tau' = 29$ days, it does not follow that multiples of 29 days are the most accurate lengths which can be selected. And this is so because the rule presupposes that, for any assumed length, the fractional portions of the component periods are very small, at least this should be the case with the larger components.

In attempting to find suitable lengths of series, it is to be noted that, since S_2 is a comparatively large component, such lengths should not fall from a whole number of days or of half days. This consideration, taken in connection with some of the uncertainties attending the determination of τ' , seems to justify one in always cutting the series off at a whole number of days or of half days.

Before attempting to actually find the lengths suitable for determining mean sea level, it is best to prepare a table showing the value, in mean solar days, of multiples of various component days (and half days in case of semidiurnals), covering a period of a little more than one year. This having been done, the approximate lengths can be seen at once; for, the multiple periods of any component of considerable size must fall not far from the required times.

Having found the approximate lengths, the formula for τ' will be of assistance in determining the more exact values, or in discriminating between two nearly equal assumed values.

The following are fairly satisfactory lengths expressed in solar days:

29, 58, 87; 104.5, 133.5, 162.5, 191.5, 220.5, 249.5, 278.5; 297, 326, 355, 384 or 282, 311, 340, 369.

Lengths approximately equal to 3 or 9 months are the most uncertain.

140. *Effect of one or more inequalities on the observed range of tide.*

An inequality in the tide can be represented by a curve resembling somewhat a curve of sines. The effect of the inequality will disappear if τ be an exact multiple of its period. When this is not the case, the disturbance in the mean quantity due to the error in the length of series used depends upon the initial phase of the inequality. A given error in length will have the greatest effect when the initial phase is 0° or 180° .

The inequality in height (range) and that in time (interval) should be separately considered. For, if the initial phase of the height inequality in a particular series were zero, the initial phase of the corresponding time inequality would be $\pm 90^\circ$. But for determining a suitable length of a series we can assume in our calculations that the initial phase of the time inequality is also zero; in other words, that the disturbance due to error in length is the greatest possible.

Now each subordinate component B, C, \dots can be connected with an inequality whose period is its synodic period with A . Hence we have as a suitable length of series for determining the mean range of tide or the mean lunital interval

$$\tau' = \frac{B\nu_1 \frac{360}{b-a} + C\nu_2 \frac{360}{c-a} + \dots}{B+C+\dots} \quad (344)$$

141. *Effect of one or more components on the observed values of another component.*

A series is cut off at a certain length τ . The A -summation gives the A -wave uncorrected for the effects of B, C, \dots . These effects are the same upon the component A as would be the direct superposition upon A of the small waves B', C', \dots . These latter denote, for the moment, waves of speed a whose amplitudes and phases depend upon the value of τ . The amplitudes and phases must have such values that the wave compounded of B', C', \dots shall have its amplitude zero.

To simplify matters, assume the phases of B', C', \dots relative to A to be zero. The correction to the amplitude of A becomes, section 59, Part II.

$$\delta A = -\delta \bar{A} = \frac{\sin \frac{1}{2}(b-a)\tau}{\frac{1}{2}(b-a)\tau} B + \frac{\sin \frac{1}{2}(c-a)\tau}{\frac{1}{2}(c-a)\tau} C + \dots \quad (345)$$

Since τ is assumed to be near the synodic periods of A and B, A and C , etc., we have

$$\sin \frac{1}{2}(b-a)\tau = \frac{1}{2}(b-a) \left(\tau - \nu_1 \frac{360}{b-a} \right),$$

$$\sin \frac{1}{2}(c-a)\tau = \frac{1}{2}(c-a) \left(\tau - \nu_2 \frac{360}{c-a} \right),$$

The disturbance in the amplitude of A due to the components B, C, \dots will be zero if $\tau = \tau'$, where

$$\tau' = \frac{B\nu_1 \frac{360}{b-a} + C\nu_2 \frac{360}{c-a} + \dots}{B+C+\dots} \quad (346)$$

Hence

A suitable length of series for the determination of any component can be found from its several synodic periods with the other components, by weighting these periods according to the amplitudes of the corresponding components.

By assuming other initial phases, slightly different results would be obtained.

142. *Rules for supplying missing portions of a tidal or tidal-current record.*

Since it is advisable to assume no particular knowledge of the tide at a given locality, the rules here laid down are designed to apply reasonably well to all stations, but somewhat better to those where the number of the tides is governed by the lunar semidiurnal constituent.

Short gaps can be filled by repeating the preceding record, starting the repeated portion at the same phase of tide (higher high, lower low, etc.) as that which imme-

diately precedes the gap. In a similar way the curve can be extended across the gap from the record which follows. A mean between the two sets of values can then be used.

In filling a gap more than a day or two in extent which may occur in a record several months long, advantage should be taken of the fact that the tidal forces, and so the tides, are after the lapse of certain intervals of time, nearly the same as before. Such intervals are 29, 191.5, 355, and 384 days. These are the times required for most of the inequalities to recur as nearly as may be; consequently, by making use of these intervals, all inequalities may be ignored and the missing tides inferred from the preceding (or following) record through the principal lunar constituent only. The length, 162.5 days, is less satisfactory.

Periods approximately equal to the above lengths, but consisting of a whole number of half lunar days, are given here:

	d	d	h	d	h	m
2 lunar half days =	1.03505010	=	1	0.8412	=	1 0 50.47
56	= 28.981403	=	28	23.5537	=	28 23 33.22
314	= 162.502866	=	162	12.0688	=	162 12 4.13
370	= 191.484268	=	191	11.6224	=	191 11 37.35
686	= 355.022184	=	355	0.5324	=	355 0 31.95
742	= 384.003587	=	384	0.0861	=	384 0 05.17

From these values there result the following rules, which apply to any form of tide or current record:

Tides 29 days after a given date are the same as those of the given date, but occur 27 minutes earlier in the day.

Tides 29 days before a given date are the same as those of the given date, but occur 27 minutes later in the day.

Tides 191.5 days after a given date are the same as those of the given date, but occur 23 minutes earlier in the half day.

Tides 191.5 days before a given date are the same as those of the given date, but occur 23 minutes later in the half day.

A. M. tides on the earlier date will generally be P. M. tides on the later, and P. M. tides on the earlier date will generally be A. M. tides on the next day following the later date.

Tides 355 days after a given date are the same as those on the given date, but occur 32 minutes later in the day.

Tides 355 days before a given date are the same as those on the given date, but occur 32 minutes earlier in the day.

Tides 384 days after a given date are the same as those on the given date, but occur 5 minutes later in the day.

Tides 384 days before a given date are the same as those on the given date, but occur 5 minutes earlier in the day.

Tides 162.5 days after a given date are the same as those of the given date, but occur 4 minutes later in the half day.

Tides 162.5 days before a given date are the same as those of the given date, but occur 4 minutes earlier in the half day.

A.M. tides on the earlier date will generally be P.M. tides on the later, and P.M. tides on the earlier date will generally be A. M. tides on the next day following the later date.

It is often convenient to take for the interpolated value the mean of the values occurring 29 days before and after the given date.

As a practical test, the first of these rules was applied to the predictions for San Francisco, Cal., as given in the Coast Survey Tide Tables for 1902. Comparison was made between the 112 tides there given for May 1-29 and those occurring 29 days later. The latter were found to occur on an average 26.62 minutes earlier in the day than the former, the individual difference ranging from 1 to 55 minutes. The greatest difference between corresponding heights is 0.6 foot.

To facilitate the application of these rules, Table 57 has been prepared showing dates upon which the tide should be almost the same in time and height.

In using these tables, care must be taken to properly allow for February 29 when this date occurs in the period considered. The table is for common years.

143. *General rule for inferring a component from larger components.*

Let A and B denote two large components, both diurnals or both semidiurnals. The age of the inequality in the A tide due to the B wave is

$$\frac{B^0 - A^0}{b - a}$$

The epoch of C , assuming the age of the C inequality is the same as of the B inequality, is

$$C^0 = A^0 + \frac{c - a}{b - a} (B^0 - A^0). \quad (347)$$

In particular

$$\begin{aligned} M_1^0 &= K_1^0 + \frac{m_1 - k_1}{o_1 - k_1} (O_1^0 - K_1^0) \\ &= K_1^0 + \frac{1}{2} O_1^0 - \frac{1}{2} K_1^0 = \frac{1}{2} (K_1^0 + O_1^0). \end{aligned}$$

The amplitude may be taken as the theoretical percentage of B or of A or of $A+B$. In particular, (see Table 1).

$$[M_1] = 0.079 O_1 \text{ or } 0.056 K_1 \text{ or } 0.033 (K_1 + O_1).$$

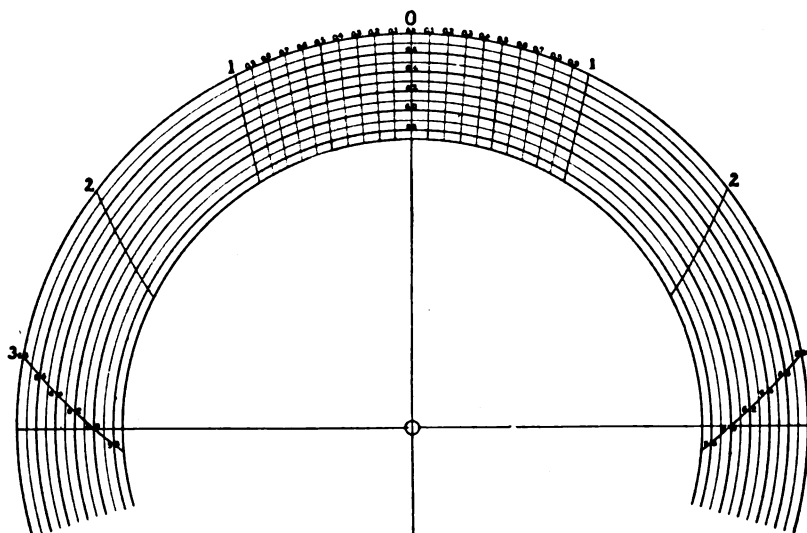
If c has very nearly the value a or b take C as the theoretical percentage of A or B (Table 1). If c has not very nearly the speed of either of these components, consider whether or not the period of the C inequality is equal to or commensurable with the period of the B inequality. If so, C can be taken as a fraction of B , even though c differs considerably from b . For example, the amplitude of L_2 can be better inferred as a fraction of the amplitude of M_2 or N_2 than of S_2 , although the speed of L_2 differs comparatively little from the speed of S_2 .

144. *Remarks on reductions and analyses.*

In section 74, Part II, it is explained how the harmonic components may be obtained from observed high and low waters. In the method as there laid down some error is incurred because the period of the semidaily tide is not constant and because its amplitude, as there determined, is affected by the residual effect of the diurnal tide. This means that the principal tide, M_2 , can not be well determined from the interpolated ordinates, and so must be inferred from a "first reduction." The smaller components, however, are well determined, as the example given in the section just referred to goes to show.

The more correct expression for the range of the semidaily wave is (498), Part II. A few extra columns on the reduction sheet will enable one to make the correction for the effect of the daily tide.

No. 31.



For interpolating hourly ordinates, when the period of the tide varies from day to day, various schemes might be suggested, such as tables or several sets of sine curves. A convenient apparatus may be described as follows:

Upon a sheet of drawing paper (tacked to a drawing board) draw a series of, say, 12 consecutive circles 0.2 inch apart, the radius of the inner circle being 6 inches. Let 3 solar hours represent a quadrant of the 4th (which is for a tidal day of just 24 solar hours) circle, counting the inmost circle as the first. Lay off from an initial diameter 1, 2, and 3 solar hours on either side. From the same initial diameter, but upon other circles, lay off distances such that the angles at the center shall not be 0° , 30° , 90° , as before, but these angles multiplied by $c_2 s_2$ where c_2 is the hourly speed of the tide and s_2 that of the component S_2 , or 30° per hour, c_2 can be assumed to decrease uniformly in going outward from the inner circle (see Fig. 31).

The values used for the duration of a tide, and which should be written upon the circles, may range from, say, 5.7 to 6.8 hours. Either way from the initial diameter the first hour should be divided into tenths to represent the tenths of hours in the times of occurrence of mean sea level of the semidaily tide. The hours are reckoned in either direction; they increase, say, from right to left for a rising tide and from left to right for a falling tide. Just within the circle is a disc, 12 inches in diameter, ruled with parallel lines, say, one-sixth of an inch apart, representing tenths of feet of amplitude, and numbered to make mean sea level any convenient reading. The disc bears a pointer extending across the system of circumferences, already described. After setting the pointer at the proper fraction of hour (the fraction of hour in the time of occurrence of mean sea level, column 5, p. 570, Part II) and across the proper circle (column 6), the disc is weighted down or otherwise held in position. A graduated scale revolving about the center of the circles is set with its edge upon the exact hours of this circle. The amplitude of the tide taken from this scale is then projected by means of the parallel lines upon the disc, upon a line perpendicular to them. This gives the required hourly heights.

In computing the effects of the diurnal components upon the amplitude (No. 498, Part II) it is convenient to make use of a system of parabolas whose ordinates are the ranges of the diurnal wave and whose abscissæ are twice the ranges of the semidaily wave. The diurnal components are found as described on page 571, Part II.

As a test of this process, ordinates have been inferred from high and low waters at Presidio, California, extending from January 1, 1898, over a series of 191½ days. The results as obtained from these ordinates generally compares well with results obtained from analyzing the observed ordinates for the same period. The results, not corrected for imperfect elimination, are as follows:

Component	From ordinates obtained from high and low waters		From observed hourly ordinates	
	<i>H</i>	κ	<i>H</i>	κ
	<i>Feet</i>	$^{\circ}$	<i>Feet</i>	$^{\circ}$
K_2 ---	0.0965	313.85	0.0912	330.28
M_2 ---	1.8165	332.27	1.7987	330.40
N_2 ---	0.3778	307.25	0.3833	305.58
S_2 ----	0.4100	341.47	0.4034	338.39
μ_2 ----	0.0327	175.26	0.0328	192.56
K_1 ---	1.1989	104.59	1.2494	105.84
O_1 ----	0.6525	83.59	0.7646	88.64
P_1 ----	0.4281	107.84	0.4131	105.34

145. In sections 30, 31, Part III, a plan is outlined for finding the spring and neap ranges of tide and the approximate value of S_2 from observed high and low waters. In practice it is generally best to assume that the age of the phase inequality is known from an harmonic analysis at the given station or at one not far away (see tables

under section 97, Part IV A, section 19, Part IV B). If harmonic constants are to be found, it seems best to use the method described in section 74, Part II, and slightly improved in section 144, Part V. The assuming of the age makes it possible to use comparatively short groups at springs and neaps.

In correcting for parallax it has been found best to make use of mean, rather than true, perigees and apogees. The times of the occurrence of these are given in Table 58, and the effect of the parallax wave upon the spring and neap ranges is given in Table 59.

146. In sections 36-39, Part III, a treatment of high and low waters for obtaining quantities connected with the diurnal part of the tide is given. In practice we can generally assume that the age of the diurnal wave is known from harmonic analyses at the given station or at one not very far away (see tables under section 97, Part IV A, and section 19, Part IV B). If harmonic constants are to be found, it seems best to use the method described in section 74, Part II, and slightly improved in section 144, Part V.

The object of giving the below computations is to show how the nonharmonic quantities can be obtained and how corrections for the time of year and for the longitude of the moon's node are to be applied. The results obtained from a one-month series and from one six months in length are seen to be in fair accord with each other. Before beginning these computations, it is assumed that if Greenwich transits have been used, the intervals have been so corrected as to refer to the local meridian. The effect of solar K_1 upon the interval can generally be ignored in computations relating to short series.

STATION: PRE .

Declinational tides, January 6 to December 26,

- (1) HWQ from reduction.
 (2) LWQ from reduction.
 (3) (1) × (group factor, Table 35) = HWQ corrected for group.
 (4) (2) × (group factor, Table 35) = LWQ corrected for group.
 (5) (Gc from reduction) + $\frac{1}{2}[(3) + (4) - (1) - (2)]$ = Gc corrected for group.
 (6) (Sc from reduction) - $\frac{1}{2}[(3) + (4) - (1) - (2)]$ = Sc corrected for group.
 (7) (Tropic LLW from reduction) - $\frac{1}{4}[(4) - (2)]$ = Tropic LLW corrected for group.
 (8) $(3)^2 + (4)^2 = (HWQ)^2 + (LWQ)^2$ = (approx. $2D_1$)².
 (9) (Mc from reduction) - (8) + $[(.06) \times 16 \text{ Mc}] = \text{Mc} - \frac{(2D_1)^2}{16\text{Mc}} = 2D_2$.
 (10) $(3) + (9) = \frac{HWQ}{2D_2}$ = argument Table 19.
 (11) $(4) + (9) = \frac{LWQ}{2D_2}$ = argument Table 19.
 (12) $2D_1$ Table 19. $2D_2$ = unity.
 (13) HW phase Table 19. See Part III, p. 161.
 (14) $1.02 F_1$, Table 14, or $\frac{1.02(K_1 + O_1)}{C_{11}K_1 + O'_1}$, when O_1/K_1 differs much from 0.7.
 *(15) $(9) \times (12) \times (14) = 2D_2 \times (2D_1 \text{ Table 19}) \times 1.02 F_1$ = corrected $2D_1$.
 *(16) $(3) \times (14) = HWQ \times 1.02 F_1$ = corrected HWQ.
 *(17) $(4) \times (14) = LWQ \times 1.02 F_1$ = corrected LWQ.
 (18) $37.6 = \frac{K_2}{M_2} \sin(K_1^0 - O_1^0 - K_2^0 - M_2^0)$ = increase in interval due to solar K_2 .
 (19) $(9) \times [F(\text{Mn}), \text{Table 14}, K_1 + O_1 = 0] = 2D_2$ corrected for moon's node.
 (20) $(14) + F(\text{Mn}) = [(D_2 \text{ uncorr.}) \times (D_1 \text{ corr.})] + [(D_2 \text{ corr.}) \times (D_1 \text{ uncorr.})]$ = factor for time inequalities.
 (21) $\text{HWI} + (18)$ = Mean tropic HWI.
 (22) $\text{LWI} + (18)$ = Mean tropic LWI.
 *(23) $[\text{HHWI} - (21)] \times (20) + (21)$ = corrected HHWI.
 *(24) $[\text{LHWI} - (21)] \times (20) + (21)$ = corrected LHWI.
 *(25) $[\text{HLWI} - (22)] \times (20) + (22)$ = corrected HLWI.
 *(26) $[\text{LLWI} - (22)] \times (20) + (22)$ = corrected LLWI.
 *(27) $[(5) - (9)] \times (14) + (19)$ = corrected Gc.
 *(28) $[(6) - (9)] \times (14) + (19)$ = corrected Sc.
 *(29) $[\text{Gt} - \text{Mn}] \times (14) + \text{Mn}$ = corrected Gt.
 *(30) $2\text{Mn} - \text{Gt}$ = corrected Sl.
 *(31) $[(7) - (\text{HTL} - \frac{1}{2}(9))] \times (14) + \text{HTL} - \frac{1}{2}(19) = [\text{Tropic LLW} - \text{uncorr. } D_2 \text{LW}] \times 1.02 F_1 + \text{corr. } D_2 \text{LW} = \text{corrected Tropic LLW}$.
 *(32) $[\text{Mean LLW} - \text{LW}] \times (14) + \text{LW}$ = corrected mean LLW.
 *(33) $\frac{1}{2}[\text{HWI} + \text{LWI} \pm 6^h 12^m 6^s] + (18) - 0^h 06^m \times (13) = (M_2^0 \text{ accel. by solar } K_2 - \text{HW phase}) + m_1 = D_1 \text{HWI}$.

An asterisk (*) indicates one of the quantities sought.

SIDIO, CAL., 1898.

(length of series a multiple of six months).

- (1) 1.670.
- (2) 3.714
- (3) $1.670 \times 1.012 = 1.690$
- (4) $3.714 \times 1.012 = 3.759$
- (5) $6.451 + \frac{1}{2} [0.065] = 6.483.$
- (6) $1.167 - \frac{1}{2} [0.065] = 1.135.$
- (7) $4.688 - \frac{1}{2} [0.045] = 4.666.$
- (8) $(1.690)^2 + (3.759)^2 = 2.856 + 14.130 = 16.986.$
- (9) $3.809 - 16.986 \div (16 \times 3.809) = 3.809 - 0.279 = 3.530.$
- (10) $1.690 \div 3.530 = 0.479.$
- (11) $3.759 \div 3.530 = 1.065.$
- (12) 1.178.
- (13) $65^\circ.$
- (14) $1.02 \times 0.944. [Table 14] = 0.963.$
- * (15) $3.530 \times 1.178 \times 0.963 = 4.004 = \text{corrected } 2D_1.$
- * (16) $1.690 \times 0.963 = 1.627 = \text{corrected HWQ.}$
- * (17) $3.759 \times 0.963 \quad 3.620 = \text{corrected LWQ.}$
- (18) $37^m.6 \times \frac{0.116}{1.696} \sin 21.^\circ 5' = 1^m.0.$
- (19) $3.530 \times 1.009 = 3.562.$
- (20) $0.963 + 1.009 = 0.954.$
- (21) $11^h 43^m.3 + 1^m.0 = 11^h 44^m.3.$
- (22) $5 \ 05.8 + 1.0 = 5 \ 06.8.$
- * (23) $(10^h 39^m.4 - 11 \ 44.3) \times 0.954 + 11^h 44^m.3 = 10^h 40^m.5 = \text{HHWI.}$
- * (24) $(13 \ 08.3 - 11 \ 44.3) \times 0.954 + 11 \ 44.3 = 13 \ 02.5 = \text{LHWI.}$
- * (25) $(4 \ 42.0 - 5 \ 06.8) \times 0.954 + 5 \ 06.8 = 4 \ 41.2 = \text{HLWI.}$
- * (26) $(5 \ 32.5 - 5 \ 06.8) \times 0.954 + 5 \ 06.8 = 5 \ 29.4 = \text{LIWI.}$
- * (27) $[6.483 - 3.530] \times 0.963 + 3.562 = 6.408 = \text{Gc.}$
- * (28) $[1.135 - 3.530] \times 0.963 + 3.562 = 1.256 = \text{Sc.}$
- * (29) $[5.696 - 3.897] \times 0.963 + 3.920 = 5.652 = \text{Gt.}$
- * (30) $2 \times 3.920 - 5.652 = 2.188 = \text{Sl.}$
- * (31) $[4.666 - 8.366 + 1.765] \times 0.963 + 8.366 - 1.781 = 4.722 = \text{corrected Tropic LLW.}$
- * (32) $[5.191 - 6.416] \times 0.963 + 6.416 = 5.236 = \text{corrected mean LLW.}$
- * (33) $\frac{1}{2} [11^h 43^m.3 + 5^h 05^m.8 + 6^h 12^m.6] + 1^m.0 - 4^h 48^m.5 = 7^h 02^m.7.$

Declinational tides, March 1 to April 16,

- (1) HWQ from reduction.
- (2) LWQ from reduction.
- (3) (1) × (group factor, Table 35) = HWQ corrected for group.
- (4) (2) × (group factor, Table 35) = LWQ corrected for group.
- (5) $\sqrt{(3)^2 + (4)^2} = \sqrt{(HWQ)^2 + (LWQ)^2} = \text{approx. } 2D_1$.
- (6) Mc from reduction or 0.88 Mn or $\text{Mn} - 0.54 \text{ S}_4$.
- (7) $(6) - (5)^2 + 16 \times (6) = \text{Mc} - \frac{(2D_1)^2}{16 \text{ Mc}} = 2D_2$.
- (8) $(3) \div (7) = \frac{HWQ}{2D_2} = \text{Arg. Table 19.}$
- (9) $(4) \div (7) = \frac{LWQ}{2D_2} = \text{Arg. Table 19.}$
- (10) $2 D_1$ from Table 19. $2D_2 = \text{unity.}$
- (11) HW phase from Table 19. See Part III, p. 161.
- (12) $1.02 F_1$, Table 32, or $\frac{102 (K_1 + O_1)}{c_{11} K'_1 + O'_1}$ Table 31, if O_1/K_1 differs much from 0.7.
- (13) $0.606 \times (\text{Table 31, col. 2}) \div (12) = \text{accel. of diurnal wave due to } P_1$.
- (14) $37^m.6 \frac{K_2}{M_2} \sin (K_2 + O_2 - K_1 - M_2) = \text{Increase in interval at tropic tides.}$
- (15) $\frac{1}{4} [2(HWI + LWI) + 2(14)] - (HHWI + LHWI + HLWI + LLWI) = \text{accel. of } D_2 \text{ due to solar effect in minutes and multiplied by } 0.483 \text{ in } M_2 \text{ degrees.}$
- (16) $[(7) - 2S_2 (\text{average phase corr. Table 24}) \times (\text{col. 9 Table 31})] [F(\text{Mn}) - \text{Table 14 } (K_1 + O_1) = 0] = 2D_3 \text{ corrected for solar effect and moon's node.}$
- *(17) $(7) \times (10) \times (12) = 2D_1 \text{ corrected.}$
- (18) $\frac{1}{4} [(15) - (13) + (11)] = \text{corrected HW phase. Arg. Tables 17 and 18.}$
- (19) $= \frac{(17)}{(16)} = \frac{D_1}{D_2} \text{ arg. Tables 17 and 18.}$
- (20) Table 17, arg. (18) and (19). Accelerations.
 - (i) HHW.
 - (ii) LHW.
 - (iii) HLW.
 - (iv) LLW.
- (21) Table 17, arg. (10) and (11). Accelerations.
 - (i) HHW.
 - (ii) LHW.
 - (iii) HLW.
 - (iv) LLW.
- *(22) (i) HHWI from reduction + $[(21i) - (20i)] \times 2^m.07 + (15) = \text{corrected HHWI.}$
 (ii) LHWI from reduction + $[(21ii) - (20ii)] \times 2.07 + (15) = \text{corrected LHWI.}$
 (iii) HLWI from reduction + $[(21iii) - (20iii)] \times 2.07 + (15) = \text{corrected HLWI.}$
 (iv) LLWI from reduction + $[(21iv) - (20iv)] \times 2.07 + (15) = \text{corrected LLWI.}$
- (23) Table (18) arg. 18 and 19.
 - (i) HHW.
 - (ii) LHW.
 - (iii) HLW.
 - (iv) LLW.
- *(24) $\begin{pmatrix} (23i) \\ (23ii) \\ (23iii) \\ (23iv) \end{pmatrix} \times \frac{1}{4} (16) = \begin{pmatrix} \text{HHW} \\ \text{LHW} \\ \text{HLW} \\ \text{LLW} \end{pmatrix} \text{ from HTL}$
- *(25) $(23i) - (23ii) = \text{HWQ.}$
- *(26) $(23iii) - (23iv) = \text{LWQ.}$
- *(27) $(23i) - (23iv) = \text{Gc.}$
- *(28) $(23ii) - (23iii) = \text{Sc.}$
- *(29) $(\text{Gt} - \text{Mn}') \times (12) + \text{Mn} = \text{corrected Gt.}$
- *(30) $\text{HTL} - (23i) = \text{corrected Tropic LLW on staff.}$
- *(31) $(\text{Mean LLW} - \text{LW}^1) \times (12) + \text{LW} = \text{corrected mean LLW.}$
- *(32) $\frac{1}{4} [HWI + LWI + 6^h 12^m 6] + (14) - 0^h.069 \times (18) = [M_2^2 \text{ accel. by solar } K_2 - \text{HW phase}] + m_1 = D_1 \text{ HWI.}$

An asterisk (*) indicates one of the quantities sought.

(length of series not a multiple of six months).

- (1) 1.42.
- (2) 3.38.
- (3) $1.42 \times 1.012 = 1.437$.
- (4) $3.38 \times 1.012 = 3.421$.
- (5) $\frac{1}{2} [(1.437)^2 + (3.421)^2] = \frac{1}{2} 2.065 + 11.703 = 3.711$.
- (6) 3.12.
- (7) $312 - \frac{13.768}{16 \times 3.12} = 2.844$.
- (8) $\frac{1.437}{2.844} = 0.505$.
- (9) $\frac{3.421}{2.844} = 1.203$.
- (10) 1.330.
- (11) 66.5.
- (12) $1.02 \times 1.081 = 1.103$.
- (13) $0.606 \times (13^{\circ}.3 \times 0.946) + 1.103 = 7^{\circ}.0$.
- (14) $37^m.6 \times \frac{0.116}{1.696} \sin 21^{\circ}.5 = 1^m.0$.
- (15) $\frac{1}{4} [2(11^h 40^m.5 + 5^h 01^m.9 + 2^m.0) - (10^h 14^m.1 + 12^h 56^m.6 + 4^h 21^m.5 + 4^h 58^m.3)] = 14^m.6 = 7^{\circ}.0$.
- (16) $2.844 - 0.80 \times \frac{-0.68 - 0.94}{2} = 3.492$. $3.492 \times 1.011 = 3.530$.
- (17) $2.844 \times 1.330 \times 1.103 = 4.172$.
- (18) $3^{\circ}.5 - 7^{\circ}.0 + 66^{\circ}.5 = 63^{\circ}.0$.
- (19) $\frac{4.172}{3.530} = 1.182$.
- (20) Accel. of HW and L.W. Arg. $63^{\circ}.0$ and 1.182 .
 - (i) HHW $+26^{\circ}.8$.
 - (ii) LHW -35.5 .
 - (iii) HLW $+21.0$.
 - (iv) LLW -12.2 .
- (21) Accel. of HW and L.W. Arg. $66^{\circ}.5$ and 1.330 .
 - (i) HHW $+31^m.0$.
 - (ii) LHW -41.7 .
 - (iii) HLW $+22.1$.
 - (iv) LLW -11.7 .
- *(22) (i) $10^h 14^m.1 + 8^m.7 + 14^m.6 = 10^h 37^m.4 = \text{HHWI}$
 (ii) $12^h 56^m.6 - 13^h 2 + 14^h .6 = 12^h 5^m.4 = \text{LHWI}$
 (iii) $4^h 21^m.5 + 2^h .3 + 14^h .6 = 4^h 3^m.4 = \text{HLWI}$
 (iv) $4^h 58^m.3 + 1^h .0 + 14^h .6 = 5^h 13^m.9 = \text{LLWI}$
- (23) Arg. $63^{\circ}.0$ and 1.182 .
 - (i) $+1.655$.
 - (ii) $+0.620$.
 - (iii) 0.000 .
 - (iv) -2.080 .
- *(24) (i) 1.655 $\left. \begin{array}{l} 2.921 = \text{HHW} \\ 1.094 = \text{LHW} \\ 0.000 = \text{LHW} \\ -3.671 = \text{LLW} \end{array} \right\} \times 1.705 = \left. \begin{array}{l} 2.921 = \text{HHW} \\ 1.094 = \text{LHW} \\ 0.000 = \text{LHW} \\ -3.671 = \text{LLW} \end{array} \right\} \text{from HTL.}$
- *(25) $2.921 - 1.094 = 1.827 = \text{HWQ}$.
- *(26) $0.000 + 3.671 = 3.671 = \text{LQW}$.
- *(27) $2.921 + 3.671 = 6.592 = \text{Gc}$.
- *(28) $1.094 + 0.000 = 1.094 = \text{Sc}$.
- *(29) $(5.356 - 3.884) \times 1.103 + 3.911 = 5.535 = \text{Gt}$.
- (30) $2 \times 3.911 - 5.535 + 2.287 = \text{Sl}$.
- (31) $(5.056 - 6.104) \times 1.103 + 6.090 = 4.933 = \text{mean LLW}$.
- (32) $\frac{1}{2} (11^h 40^m.5 + 5^h 01^m.9 + 6^h 12^m.6) + 1^m.0 - 0.069 \times 63.0 = 11^h 28^m.5 - 4^h 20^m.8 = 7^h 07^m.7$.

147. Where the tides are chiefly diurnal, it becomes necessary to make reduction of the equatorial tides in order to ascertain the semidaily wave. The semidaily wave being thus known, the diurnal can be ascertained at the time of the tropic tides by means of the great tropic range of tide and the times of occurrence of the tropic high and low waters. This procedure is preferable to that proposed in Part III, which makes use of Table 20, because mean sea level is subject to considerable variation. Given the displacements (accelerations) v and w of the semidaily wave due to the diurnal wave, then the HW-phase, β , can be found either by means of a table, like Table 17 extended, or by means of the formula:

$$\tan (\text{HW-phase}) = -\tan \frac{1}{2} v \cot \frac{1}{2} w \tan (45^\circ - \frac{v+w}{4}). \quad (348)$$

This equation is found by successively substituting for b t in the derivative of (27), Part III, equated to zero, $-\frac{1}{2} v$ and $90^\circ - \frac{1}{2} w$, and eliminating B and A .

A reduction of the tides for one year at Manila shows that, not only can the ranges and intervals be determined from observed high and low waters, but also that the principal harmonic constants determined by this means agree well with the constants obtained by analyzing hourly ordinates.

148. *Miscellaneous remarks and corrections.*

In section 24, Part I, the wave profile is shown to be trochoidal; but the approximation there used for y requires a correction when the amplitude of the wave is a considerable fraction of its length. The necessity for this correction is apparent in Fig. 12, Part IV A. Using the value of x given in (51) Part I and increasing the right-hand member of (52) by a quantity ϵ , the value of $\int_0^{2\pi} y' dx$, where $y' = y - h$, equated to zero gives

$$\epsilon = \frac{\lambda m^2}{4\pi}. \quad (349)$$

In other words, the condition of continuity where a whole wave length is considered prevents $y' = y - h$ from having the form (52).

In equation (1), Part I, for 50, substitute 48.8. The same substitution should be made in second footnote, section 93. Here also substitute for "daily retardation," "retardation per solar day," and for 36, 88, 51, and 49; 35, 87, 50, and 48, respectively.

In section 5, Part III, a graphic process is outlined for determining the times of maxima and minima of a wave composed of two simple waves, the period of one being twice that of the other. In construction of Tables 4 and 44 the following process of further approximation was used:

Let v' denote the approximate values of v , either directly from construction or from a second or higher approximation.

Then

$$\frac{1}{2} v = \frac{1}{2} v' + 57.3 y \quad (350)$$

where

$$y = \frac{B \sin \beta}{4 \cos \frac{1}{2} v' + B \cos \beta} - \tan \frac{1}{2} v'. \quad (351)$$

If this value of v does not satisfy equation (28), Part III, where $A=1$, a second approximation should be made in the same manner.

If we write

$$z = \cos^2 \beta, \quad (352)$$

then, in the limiting or discriminating case, B and β are connected by the relation

$$z^2 - z = \frac{1}{2B^4} \left(\frac{B^2 - 16}{6} \right)^3. \quad (353)$$

This equation, equated to zero, represents the discriminant of eq. (28), Part III, wherein x is written for

$$\cos \frac{1}{2} vt.$$

In section 20, Part III, for 1.006, put 1.012.

An harmonic analysis of the tide at Tamatave, made by the French, show that more of the cotidal lines should meet the eastern coast of Madagascar than are represented as doing so in Figs. 6, 7, Part IV B. In fact, under this station, page 350, Part IV B, the M_2 -cotidal hour is given as X.33, indicating that at least lines $X\frac{1}{2}$ — $XII\frac{1}{2}$ meet the coast. The values used in the construction of the cotidal lines for this region were taken chiefly from the Admiralty Tide Tables; the French values for Tamatave did not come to my notice until after the chart had been constructed.

Recent observations at Kiska Harbor, Aleutian Islands, show that the crowding up of the cotidal lines should occur in this vicinity instead of a little farther east, as shown in Figs. 34, 35, Part IV B. Observations give for mean high and low water intervals $2^h 07^m$ and $8^h 17^m$, and for mean range of tide 1.9 feet. This fact, together with the small range of tide found at Kiska, go to confirm the existence of the nodal line shown in Fig. 23, Part IV A.

In third line from bottom of page 545, Part IV A, for e read c .

A few corrections of errata are indicated in section 76, Part II; others have been indicated on errata sheets accompanying the Reports in which the parts of this manual have appeared.

For want of time, several matters whose treatment was contemplated must be omitted here. For example, the determination of the moon's mass from the ratio P_1/K_1 for ocean stations where the diurnal wave is large. That the assumed mass of the moon approximately agrees with the value inferred from the tides can be seen upon comparing the ratios P_1/K_1 for such stations in section 97, Part IV A, and section 19, Part IV B, with the assumed theoretical ratio. Other astronomical quantities can be estimated from the tides.

The slow rate of propagation in rivers, or other arms of the sea, having broad marginal strips of shallow water, or inequalities in shoreline like those possessed by Chesapeake Bay, is another matter passed over for want of time.

A few other matters are: The course of the diurnal wave, the K_1 -wave, for instance; the analysis and discussion of long-period gravitational tides; the latitude-variational tide; improvements in the methods of making and reducing observations; and the construction of additional auxiliary tables.

ADDITIONAL ERRATA, PARTS I TO IV B.

Part I. Fig. 19, opposite page 384, all numerals denoting heights should be multiplied by two.

Page 421, strike out lines 16-19.

Part II. Page 479, line 15, for "*bench mark*" read "*water*;" for "*water*" read "*bench mark*;" for "difference between" read "sum of."

Transfer "++" at end of line 35 to end of line 37.

Page 514, 10th line from bottom, delete "a small displacement of."

Page 517, 14th line from bottom, for "1" write " $1 + \frac{3}{2}e^2$."

Page 535, line 15, for " 87° " write " $87^\circ 43'$."

Page 553, 11th line from bottom, for "in local time" read "in standard time."

Page 556, 1st table, for " $\zeta(A)$ " write " $\zeta_c(A)$ "; for " $\zeta_c(B)$ " write " $\zeta(B)$ "; for " $\kappa' = +V_0 + u$ " write " $\kappa' = \zeta + V_0 + u$."

Page 573, 9th line from bottom, for "all multiplied by the same constant" read "each divided by a constant proportional to its speed;" 7th line from bottom, for "This shows that" read "In certain straits, see section 34, Part I."

Page 583, Table, after "All" add "except S."

Part IV A. Page 564, 5th line, for " t " read " x ;" 6th line after "have" add "for the flood stream;" 12th line, for "+" read " \pm ;" 13th line, for "+" read "-;" in eq. (74) and following expressions on page 564, for " α " read " $-\alpha$."

Page 579, in 2d, 4th, and 6th lines from bottom of page, for " Z " read " z ."

Page 616, near middle of page, for "three" read "these."

Page 620, annex " $\cos \alpha_{\nu \mu}$ " to each parenthesis of eq. (314).

Page 623, 4th line from bottom of page, for "(321)" read "(318)."

Part IV B. Page 332, 1st line before eq. (24), for " x " write " χ ."

Page 351, 3d column from right-hand side, for "5.87" read "11.87."

Page 351, 5th, 4th, and 2d columns from right-hand side for "224.5" read "324.5;" for "14.97," read "21.63;" for "6.27" read "12.93."

Page 371, 9th line from bottom, before "occur," insert "not."

Page 375, 13th line from bottom, delete "approach."

Page 386, 2d line from bottom, delete "scarcely."

Page 388, near middle of page, for "III" write " $III\frac{1}{2}$."

AUXILIARY TABLES
FOR THE
REDUCTION AND PREDICTION
OF
TIDES.

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TABLE 1.—The principal harmonic components,

Sym- bol.	Name of component, etc.	Speed per mean solar hour.		Speed ratios.				Synodic period.* 15/(m~c)
		Formula.	c	$\frac{c}{s_1}$	$\frac{s_1}{c}$	$\frac{c}{m_2}$	$\frac{c}{k_1}$	
			°					d.
J ₁	Smaller lunar elliptic diurnal	$\gamma + \sigma - \omega$	15° 58' 54.433	1° 03' 90.3	0° 96' 24.4	0° 53' 77.2	1° 03' 61.9	13° 7' 28.79
	Lunar elliptic diurnal, second order	$\gamma - 4\sigma + 2\omega$	12° 85' 42.862	0° 85' 69.5	1° 16' 69.3	0° 44' 34.9	0° 85' 46.1	9° 15' 88.2
	Larger lunar evectional diurnal	$\gamma - 3\sigma - \omega + 2\eta$	13° 47' 15.144	0° 89' 81.0	1° 11' 34.6	0° 46' 47.9	0° 89' 56.5	14° 6' 58.13
[K ₁]	Lunar diurnal	γ	15° 04' 10.686	1° 00' 27.4	0° 99' 72.7	0° 51' 89.4	1° 00' 00.0	27° 32' 15.8
[K ₁]	Solar diurnal	γ	15° 04' 10.686	1° 00' 27.4	0° 99' 72.7	0° 51' 89.4	1° 00' 00.0	27° 32' 15.8
K ₁	Luni-solar diurnal	γ	15° 04' 10.686	1° 00' 27.4	0° 99' 72.7	0° 51' 89.4	1° 00' 00.0	27° 32' 15.8
[K ₂]	Lunar semidiurnal	2 γ	30° 08' 21.372	2° 00' 54.8	0° 49' 86.3	1° 03' 78.8	2° 00' 00.0	13° 66' 07.9
[K ₂]	Solar semidiurnal	2 γ	30° 08' 21.372	2° 00' 54.8	0° 49' 86.3	1° 03' 78.8	2° 00' 00.0	13° 66' 07.9
K ₂	Luni-solar semidiurnal	2 γ	30° 08' 21.372	2° 00' 54.8	0° 49' 86.3	1° 03' 78.8	2° 00' 00.0	13° 66' 07.9
L ₂ {	Smaller lunar elliptic semidiurnal	{ 2 $\gamma - \sigma - \omega$ and }	29° 52' 84.788	1° 06' 85.7	0° 50' 79.8	1° 01' 87.8	1° 06' 31.9	27° 55' 45.6
[L ₂]	[Ferrel's L ₂]	{ 2 $\gamma - \sigma + \omega$ }	29° 52' 84.788	1° 06' 85.7	0° 50' 79.8	1° 01' 87.8	1° 06' 31.9	27° 55' 45.6
		2 $\gamma - \sigma - \omega$	29° 52' 84.788	1° 06' 85.7	0° 50' 79.8	1° 01' 87.8	1° 06' 31.9	27° 55' 45.6
		2 $\gamma + \sigma - \omega$	30° 62' 65.120	2° 04' 17.7	0° 48' 97.7	1° 05' 66.7	2° 03' 61.9	9° 13' 29.3
M ₁	Smaller lunar elliptic diurnal	{ $\gamma - \sigma - \omega$ and }	14° 49' 20.521	0° 66' 61.4	1° 03' 50.5	0° 50' 00.0	0° 66' 35.0	
[M ₁]	[Ferrel's m ₁ or Q ₁ ']	{ $\gamma - \sigma + \omega$ }	14° 49' 66.939	0° 66' 64.5	1° 03' 47.2	0° 50' 01.6	0° 66' 38.1	
		$\gamma - \sigma + \omega$	14° 49' 66.939	0° 66' 64.5	1° 03' 47.2	0° 50' 01.6	0° 66' 38.1	
M ₂		2 ($\gamma - \sigma$)	28° 98' 41.042	1° 93' 22.7	0° 51' 57.3	1° 00' 00.0	1° 92' 70.0	
M ₃		3 ($\gamma - \sigma$)	43° 47' 61.563	2° 89' 84.1	0° 34' 50.2	1° 50' 00.0	2° 89' 05.0	
M ₄	Principal lunar series	4 ($\gamma - \sigma$)	57° 96' 82.084	3° 86' 45.5	0° 25' 87.6	2° 00' 00.0	3° 85' 40.0	
M ₅		6 ($\gamma - \sigma$)	86° 95' 23.126	5° 79' 68.2	0° 17' 25.1	3° 00' 00.0	5° 78' 09.9	
M ₆		8 ($\gamma - \sigma$)	115° 93' 64.168	7° 72' 90.9	0° 12' 93.8	4° 00' 00.0	7° 70' 79.9	
N ₂	Larger lunar elliptic, semidiurnal	2 $\gamma - 3\sigma + \omega$	28° 43' 97.296	1° 89' 59.8	0° 52' 74.3	0° 98' 12.2	1° 89' 08.1	27° 55' 45.6
2 N	Lunar elliptic semidiurnal, second order	2 $\gamma - 4\sigma + 2\omega$	27° 89' 53.548	1° 85' 96.9	0° 53' 77.2	0° 96' 24.4	1° 85' 46.1	13° 77' 72.8
O ₁	Lunar diurnal	$\gamma - 2\sigma$	13° 94' 30.356	0° 92' 95.4	1° 07' 58.1	0° 48' 10.6	0° 92' 70.0	27° 32' 15.8
OO	Lunar diurnal, second order	$\gamma + 2\sigma$	16° 13' 91.016	1° 07' 59.4	0° 92' 94.2	0° 55' 68.3	1° 07' 30.0	9° 10' 71.9
P ₁	Solar diurnal	$\gamma - 2\eta$	14° 95' 89.314	0° 99' 72.6	1° 00' 27.5	0° 51' 61.1	0° 99' 45.4	32° 12' 28.2
Q ₁	Larger lunar elliptic diurnal	$\gamma - 3\sigma + \omega$	13° 39' 86.609	0° 89' 32.4	1° 11' 95.1	0° 46' 22.8	0° 89' 08.1	13° 72' 87.9
R ₂	Smaller solar elliptic	2 $\gamma - \eta$	30° 04' 10.686	2° 00' 27.4	0° 49' 93.2	1° 03' 64.7	1° 99' 72.7	14° 19' 15.8
S ₁		$\gamma - \eta$	15° 00' 00.000	1° 00' 00.0	1° 00' 00.0	0° 51' 57.3	0° 99' 72.7	29° 53' 05.9
S ₂		2 ($\gamma - \eta$)	30° 00' 00.000	2° 00' 00.0	0° 50' 00.0	1° 03' 50.5	1° 99' 45.4	14° 76' 52.9
S ₃	Principal solar series	3 ($\gamma - \eta$)	45° 00' 00.000	3° 00' 00.0	0° 33' 33.3	1° 55' 25.7	2° 99' 18.1	
S ₄		4 ($\gamma - \eta$)	60° 00' 00.000	4° 00' 00.0	0° 25' 00.0	2° 07' 01.0	3° 98' 90.8	
T ₂	Larger solar elliptic	2 $\gamma - 3\eta$	29° 95' 89.314	1° 99' 72.6	0° 50' 06.9	1° 03' 63.3	1° 99' 18.1	15° 38' 73.4
λ_2	Smaller lunar evectional	2 $\gamma - \sigma + \omega - 2\eta$	29° 45' 56.254	1° 06' 37.1	0° 50' 92.4	1° 01' 62.7	1° 95' 83.5	31° 8' 11.93
μ_2	Variational	2 $\gamma - 4\sigma + 2\eta$	27° 96' 82.084	1° 86' 45.5	0° 53' 63.2	0° 96' 49.5	1° 85' 94.6	14° 76' 52.9
ν_2	Larger lunar evectional	2 $\gamma - 3\sigma - \omega + 2\eta$	28° 51' 25.830	1° 90' 08.4	0° 52' 60.8	0° 98' 37.3	1° 89' 56.5	31° 8' 11.93

*The diurnal components are synodic with M₁, and the semidiurnal components with M₂.†The first of these components is Ferrel's n_1 ; the second his m_1 or Q₁', and is the same as [M₁]. U. S. C. and G. S. Report, 1878, App. 11.

with their speeds, coefficients, etc.

Sym- bol.	Coefficients.		Coeff. ratios.		Factors for reduction.	Equilibrium arguments.	
	Formula.	Mean value.	$\frac{C}{M_2}$	$\frac{C}{K_1}$	F	V	u
J_1	$\frac{1}{2} e \sin 2 I$ $\frac{1}{2} e^2 \sin I \cos^2 \frac{1}{2} I$ $\frac{1}{8} m e \sin I \cos^2 \frac{1}{2} I$.01485 .00487 .00512 .00708	.03269 .01072 .01127 .01559	.05600 .01836 .01930 .02669	$F(J_1) = \frac{0.72147}{\sin 2 I}$ $F = F(O_1)$ $F = F(O_1)$	$t+h+s-p+90^\circ$ $t+h-4s+2p-90^\circ$ $t+3h-3s-p-90^\circ$	$-v$ $2\xi-v$ $2\xi-v$
$[K_1]$	$(\frac{1}{2} + \frac{1}{2} e^2) \sin 2 I$.18115	.39878	.68302	$F([K_1]) = \frac{0.72147}{\sin 2 I} = F(J_1)$	$t+h+90^\circ$	$-v$
$[K_1]$	$(\frac{1}{2} + \frac{1}{2} e_1^2) \frac{T_1}{T^2} \sin 2 \omega$.08407	.18507	.31698	$F([K_1]) = \text{unity}$ $F(K_1) = \frac{1.05628}{(\sin^2 2 I + 0.66962 \cos 2 \nu \sin 2 I + 0.11210)}$	$t+h+90^\circ$	Zero
K_1	$(\frac{1}{2} + \frac{1}{2} e^2) \sin 2 I + (\frac{1}{2} + \frac{1}{2} e_1^2) \frac{T_1}{T^2} \sin 2 \omega$.26522	.58385	1.00000		$t+h+90^\circ$	$-v'$
$[K_2]$	$(\frac{1}{2} + \frac{1}{2} e^2) \sin^2 I$.03929	.08649	.14814	$F([K_2]) = \frac{0.15652}{\sin^2 I}$	$2t+2h$	$-2v$
$[K_2]$	$(\frac{1}{2} + \frac{1}{2} e_1^2) \frac{T_1}{T^2} \sin^2 \omega$.01823	.04013	.06874	$F([K_2]) = \text{unity.}$ $F(K_2) = \frac{0.22915}{(\sin^2 I + 0.14527 \cos 2 \nu \sin^2 I + 0.00528)}$	$2t+2h$	Zero
K_2	$(\frac{1}{2} + \frac{1}{2} e^2) \sin^2 I + (\frac{1}{2} + \frac{1}{2} e_1^2) \frac{T_1}{T^2} \sin^2 \omega$.05752	.12662	.21688		$2t+2h$	$-2v''$
L_2	$\frac{1}{2} e \left\{ 1 - 12 \tan^2 \frac{1}{2} I \cos 2 I' \right\}^{\frac{1}{2}} \cos^4 \frac{1}{2} I$.01257	.02767	.04739	$F(L_2) = F(M_2) \times R'$	$2t+2h-s-p+180^\circ$	$2\xi-2v-R$
$[L_2]$	$\frac{1}{2} e \cos^4 \frac{1}{2} I$ $\frac{1}{2} e \sin^2 I$.01257 .00323	.02767 .00711	.04739 .01218	$F([L_2]) = \frac{0.91538}{\cos^4 \frac{1}{2} I} = F(M_2)$ $F = \frac{0.15652}{\sin^2 I}$	$2t+2h-s-p+180^\circ$ $2t+2h+s-p$	$2\xi-2v$ $-2v$
M_1	$\frac{1}{2} e \left\{ \frac{1}{2} + \frac{1}{2} \cos 2 I' \right\}^{\frac{1}{2}} \sin I \cos^2 \frac{1}{2} I$.00522 .01649	.01149 .03630	.01968 .06217	$F(M_1) = F(O_1) \times Q'$	$t+h-s+90^\circ$	$\xi-v+Q$
$[M_1]$	$\frac{1}{2} e \sin 2 I$.01485	.03269	.05599	$F([M_1]) = \frac{0.72147}{\sin 2 I} = F(J_1)$	$t+h-s+p+90^\circ$	$-v$
M_2	$(\frac{1}{2} - \frac{1}{2} e^2) \cos^4 \frac{1}{2} I$.45426	1.00000	1.71277	$F(M_2) = \frac{0.91538}{\cos^4 \frac{1}{2} I}$	$2t+2h-2s$	$2\xi-2v$
M_3	$\frac{1}{16} e' \cos^4 \frac{1}{2} I$, (and shallow water)	.00599	.01319	.02259	$F(M_3) = \frac{0.87579}{\cos^6 \frac{1}{2} I} = F^{\frac{3}{2}}(M_2)$	$3t+3h-3s+180^\circ$	$3\xi-3v$
M_4	(Shallow water)				$F(M_4) = \frac{0.83792}{\cos^8 \frac{1}{2} I} = F^2(M_2)$	$4t+4h-4s$	$4\xi-4v$
M_5	(Shallow water)				$F(M_5) = \frac{0.76701}{\cos^{12} \frac{1}{2} I} = F^3(M_2)$	$6t+6h-6s$	$6\xi-6v$
M_6	(Shallow water)				$F(M_6) = \frac{0.70210}{\cos^{16} \frac{1}{2} I} = F^4(M_2)$	$8t+8h-8s$	$8\xi-8v$
N_2	$\frac{1}{2} e \cos^4 \frac{1}{2} I$.08796	.19363	.33165	$F(N_2) = \frac{0.91538}{\cos^4 \frac{1}{2} I} = F(M_2)$	$2t+2h-3s+p$	$2\xi-2v$
$2N$	$\frac{1}{2} e^2 \cos^4 \frac{1}{2} I$.01173	.02582	.04423	$F(2N) = \frac{0.91538}{\cos^4 \frac{1}{2} I} = F(M_2)$	$2t+2h-4s+2p$	$2\xi-2v$
O_1	$(\frac{1}{2} - \frac{1}{2} e^2) \sin I \cos^2 \frac{1}{2} I$.18856	.41509	.71096	$F(O_1) = \frac{0.38005}{\sin I \cos^2 \frac{1}{2} I}$	$t+h-2s-90^\circ$	$2\xi-v$
OO	$(\frac{1}{2} - \frac{1}{2} e^2) \sin I \sin^2 \frac{1}{2} I$.00812	.01788	.03062	$F(OO) = \frac{0.01638}{\sin I \sin^2 \frac{1}{2} I}$	$t+h+2s+90^\circ$	$-2\xi-v$
P_1	$(\frac{1}{2} - \frac{1}{2} e_1^2) \frac{T_1}{T^2} \sin \omega \cos^2 \frac{1}{2} \omega$.08775	.19317	.33086	$F(P_1) = \text{unity}$	$t-h-90^\circ$	Zero
Q_1	$\frac{1}{2} e \sin I \cos^2 \frac{1}{2} I$.03651	.08037	.13766	$F(Q_1) = \frac{0.38005}{\sin I \cos^2 \frac{1}{2} I} = F(O_1)$	$t+h-3s+p-90^\circ$	$2\xi-v$
R_2	$\frac{1}{2} e_1 \frac{T_1}{T^2} \cos^4 \frac{1}{2} \omega$.00178	.00392	.00671	$F(R_2) = \text{unity}$	$2t+h-h_1$	Zero
S_1	(Chiefly meteorological)				$F(S_1) = \text{unity}$	$t+180^\circ$	Zero
S_2	$(\frac{1}{2} - \frac{1}{2} e_1^2) \frac{T_1}{T^2} \cos^4 \frac{1}{2} \omega$.21137	.46531	.79696	$F(S_2) = \text{unity}$	$2t$	Zero
S_3	(Chiefly shallow water)				$F(S_3) = \text{unity}$	$3t+180^\circ$	Zero
S_4	(Shallow water), $(S_2/M_2)^2 M_4$				$F(S_4) = \text{unity}$	$4t$	Zero
T_2	$\frac{1}{2} e_1 \frac{T_1}{T^2} \cos^4 \frac{1}{2} \omega$.01243	.02736	.04687	$F(T_2) = \text{unity}$	$2t-h+p_1$	Zero
λ_2	$\frac{1}{16} m e \cos^4 \frac{1}{2} I$.00176 .00330 .00736 .01094 .01234 .01706	.00387 .00726 .01620 .02408 .02717 .03756	.00664 .01244 .02775 .04125 .04653 .06432	$F(\lambda_2) = \frac{0.91538}{\cos^4 \frac{1}{2} I} = F(M_2)$ $F(\mu_2) = \frac{0.91538}{\cos^4 \frac{1}{2} I} = F(M_2)$ $F(\nu_2) = \frac{0.91538}{\cos^4 \frac{1}{2} I} = F(M_2)$	$2t-s+p+180^\circ$ $2t+4h-4s$ $2t+4h-3s-p$	$2\xi-2v$ $2\xi-2v$ $2\xi-2v$

* The lower of these two figures gives the value when the coefficients in the evection and variation have their full values as derived from Lunar Theory.

† The coefficients of L_2 and M_1 are approximately expressed by the given formulæ; the true mean values are .01278 and .01531, respectively.
‡ The first of these two numbers is the mean value of the coefficients of the tide $\gamma-\sigma-\omega$; the second applies to the tide M_{11} , compounded from $\gamma-\sigma-\omega$ and $\gamma-c+\omega$.

TABLE 1.—The principal harmonic components,

Sym- bol.	Name of component, etc.	Speed per mean solar hour.		Speed ratios.				Synod c period.* 15/(m-c)
		Formula.	c	$\frac{c}{s_1}$	$\frac{s_1}{c}$	$\frac{c}{m_2}$	$\frac{c}{k_1}$	
MK	Compound tides	$3\gamma-2\sigma$	44'0251728	2'93501	0'34071	1'51894	2'92700	d.
2 MK		$3\gamma-4\sigma$	42'9271398	2'86181	0'34943	1'48106	2'85400	
MN		$4\gamma-5\sigma+\omega$	57'4238338	3'82826	0'26122	1'98122	3'81780	
MS†		$4\gamma-2\sigma-2\eta$	58'9841042	3'93227	0'25431	2'03505	3'92154	
2 MS		$2\gamma-4\sigma+2\eta$	27'9682084	1'86455	0'53632	0'96495	1'85946	
2 SM		$2\gamma+2\sigma-4\eta$	31'0158958	2'06773	0'48362	1'07010	2'06208	
Mf	Lunar fortnightly	2σ	1'0980330	0'07320	13'66079	0'03788	0'07300	
MSf†	Luni-solar synodic fortnightly	$2(\sigma-\eta)$	1'0158958	0'06773	14'76589	0'03505	0'06754	
Mm	Lunar monthly	$\sigma-\omega$	0'5443747	0'03629	27'55455	0'01878	0'03619	
Sa	Solar annual	η	0'0410686	0'00274	365'24219	0'00142	0'00273	
Ssa	Solar semiannual	2η	0'0821372	0'00548	182'62109	0'00283	0'00546	
γ	Right ascension of local meridian	γ	15'04106864	1'00274	0'99727	0'51894	1'00000	
η	Mean longitude of sun	η	0'04106864	0'00274	365'24219	0'00142	0'00273	
σ	Mean longitude of moon	σ	0'54901653	0'03660	27'32158	0'01894	0'03650	
ω	Mean longitude of lunar perigee	ω	0'00464183	0'00031	3231'48	0'00016	0'00031	
ϕ_1	Mean longitude of solar perigee		0'00000196	0'00000	0'00000	0'00000	
t	Time in hours after midnight, or do. multiplied by 15	$\gamma-\eta$	15'00000000	1'00000	1'00000	0'51753	0'99727	

c = speed of any component.

c₁ = speed of any diurnal component.k₁ = speed of the diurnal component K₁.m₁ = speed of the diurnal component M₁.m₂ = speed of the semidiurnal component M₂ = 2 m₁.s₁ = speed of the diurnal component S₁ = 15° per hour.* The diurnal components are synodic with M₁, and the semidiurnal components with M₂.† Suggested by Helmholtz's theory of compound sounds. B. A. A. S. Report 1869, p. 504. MS has been called the Helmholtz luni-solar tide. B. A. A. S. Report 1870, p. 149. Ferrel designated it by (MS)₄.

with their speeds, coefficients, etc.—Continued.

Sym- bol.	Coefficients.		Coeff. ratios.		Factors for reduction.	Equilibrium arguments.	
	Formula.	Mean value.	$\frac{C}{M_2}$	$\frac{C}{K_1}$	F	l'	λ
MK	(Shallow water)				$F(MK) = F(M_2) \times F(K_1)$	$3l+3h-2s+90^\circ$	$2\xi-2\nu-\nu'$
$2MK$	(Shallow water)				$F(2MK) = F(M_2) \times F(K_1)$	$3l+3h-4s-90^\circ$	$4\xi-4\nu+\nu'$
MN	(Shallow water)				$F(MN) = \frac{0.83792}{\cos^2 \frac{1}{2} l} = F(M_2)$	$4l+4h-5s+\phi$	$4\xi-4\nu$
MS	(Shallow water), $2(S_2/M_2) M_2$				$F(MS) = \frac{0.91538}{\cos^2 \frac{1}{2} l} = F(M_2)$	$4l+2h-2s$	$2\xi-2\nu$
$2MS$	(Shallow water)				$F(2MS) = \frac{0.83792}{\cos^2 \frac{1}{2} l} = F(M_2)$	$2l+4h-4s$	$4\xi-4\nu$
$2SM$	(Shallow water)				$F(2SM) = \frac{0.91538}{\cos^2 \frac{1}{2} l} = F(M_2)$	$2l-2h+2s$	$-2\xi+2\nu$
Mf	$(\frac{1}{2}-\frac{1}{2}e^2) \sin^2 l$	0.07827	0.17230	0.29511	$F(Mf) = \frac{0.15779}{\sin^2 l}$	$2s$	-2ξ
MSf	$m^2(1-\frac{1}{2}\sin^2 l)$, (and shallow water)	$\begin{cases} 0.00422^* \\ 0.00621 \end{cases}$	$\begin{cases} 0.00929 \\ 0.01367 \end{cases}$	$\begin{cases} 0.01591 \\ 0.02341 \end{cases}$	$F(MSf) = \frac{0.91538}{\cos^2 \frac{1}{2} l} = F(M_2)$	$-2h+2s$	$-2\xi+2\nu$
Mm	$e(1-\frac{1}{2}\sin^2 l)$	0.04136	0.09105	0.15595	$F(Mm) = \frac{0.75316}{1-1.5\sin^2 l}$	$s-\phi$	Zero
Sa	(Chiefly meteorological)				$F(Sa) = \text{unity}$	h	Zero
Ssa	$(\frac{1}{2}-\frac{1}{2}e_1^2) \frac{r_1}{r} \sin^2 \omega$	0.03643	0.08090	0.13736	$F(Ssa) = \text{unity}$	$2h$	Zero

C = Coefficient or theoretical amplitude of any component.
 M_2 = Coefficient or theoretical amplitude of the component M_2 .
 K_1 = Coefficient or theoretical amplitude of the component K_1 .

$$m = \frac{\text{mean motion of sun}}{\text{mean motion of moon}} = \frac{0.07480}{13.369}.$$

R is such that $\tan R = \frac{\sin 2P}{\frac{1}{2} \cot^2 \frac{1}{2} l - \cos 2P}$; see Table 8.

$$R' = \left(\frac{1}{1-12 \tan^2 \frac{1}{2} l \cos 2P} \right)^{\frac{1}{2}}; \text{ see Table 11.}$$

ν' is such that $\tan \nu' = \frac{\sin \nu \sin 2l}{0.334811 + \cos \nu \sin 2l}$; see Table 7.

e = eccentricity of moon's orbit = 0.0549.
 e_1 = eccentricity of earth's orbit = 0.0168.
 ω = obliquity of the ecliptic = $23^\circ 27' 3''$.

$$\frac{r_1}{r} = \frac{\text{mass of sun}}{\text{mass of moon}} \times \left(\frac{\text{mean dist. of moon}}{\text{mean dist. of sun}} \right)^3 = 0.46035 = \frac{1}{2.17220}.$$

Q is such that $\tan Q = \frac{1}{2} \tan P$; see Table 9.

$$Q' = \left(\frac{1}{2.5 + 1.5 \cos 2P} \right)^{\frac{1}{2}}; \text{ see Table 12.}$$

ν'' is such that $\tan 2\nu'' = \frac{\sin 2\nu \sin^2 l}{0.072634 + \cos 2\nu \sin^2 l}$; see Table 7.

l = inclination of lunar orbit to the plane of the earth's equator; it varies between $\omega-5^\circ 8' 8''$ and $\omega+5^\circ 8' 8''$, i. e. from $18^\circ 18' 5''$ to $28^\circ 36' 1''$. See Tables 6 and 7.

P = mean longitude of lunar perigee measured from the intersection of moon's orbit with the plane of the earth's equator. See Table 6.

d = earth's radius divided by moon's mean distance, i. e. the lunar parallax expressed in radians = $0.01659 = \frac{1}{60.27}$.

ξ = longitude in moon's orbit of the intersection of the lunar orbit with the plane of the earth's equator. See Table 7.

ν = right ascension of the intersection of the lunar orbit with the plane of the earth's equator. See Table 7.

The semidiurnals have a general coefficient $\cos^2 \lambda$; the diurnals, $\sin^2 \lambda$; those of long period, $\frac{1}{2}-\frac{1}{2}\sin^2 \lambda$; and $M_2 \cos^2 \lambda$. λ denotes the latitude.

*The lower of these two figures gives the value when the coefficients in the evection and variation have their full values as derived from Lunar Theory.

S. Ex. 8, pt. 2—13

TABLE 2.—Dependence of component speeds upon certain astronomical quantities (epoch 1900).

Solar day	$= 24 \cdot 000 \ 0000 = \frac{2 \times 360}{s_2}$	$s_2 = \frac{2 \times 360}{\text{solar day}} = 30 \cdot 000 \ 0000$
Lunar day	$= 24 \cdot 841 \ 2024 = \frac{2 \times 360}{m_2}$	$m_2 = \frac{2 \times 360}{\text{lunar day}} = 28 \cdot 984 \ 1042$
Tropical day	$= 23 \cdot 934 \ 4696 = \frac{2 \times 360}{k_2}$	$k_2 = \frac{2 \times 360}{\text{tropical day}} = 30 \cdot 082 \ 1373$

If c denote the speed of any given component C , then a component whose speed is

$$c \pm \frac{360}{\Pi}$$

has with C a synodic period Π hours in length. If C be the principal one of the two components, then the effect of the other is to introduce an inequality into C of a period equal to Π . The known motions of the moon and sun suggest what tidal inequalities may exist corresponding to periodic irregularities in the motions of these bodies, or in the motions of the elements of their orbits. The following tabulation shows that it is often necessary to use more than one component to account for what may be regarded as an irregularity in the principal component:

	Π		Speed.		
	Name.	Value (in hrs, days).	Formula.		Symbol.
			$c + \frac{360}{\Pi}$	$c - \frac{360}{\Pi}$	
$c = m_2 = 28 \cdot 984 \ 1042$	Half synodic month	$\frac{1}{2} (708 \cdot 734115) \text{ h.}$	$30 \cdot 0000000$	0	s_2
		$\frac{1}{2} (29 \cdot 5305881) \text{ d.}$			μ_2
	Half tropical month	$\frac{1}{2} (655 \cdot 717960) \text{ h.}$	$30 \cdot 0821373$	$27 \cdot 9682084$	k_2
		$\frac{1}{2} (27 \cdot 3215817) \text{ d.}$			
	Anomalistic month	$661 \cdot 309206 \text{ h.}$	$29 \cdot 5284789$		l_2
		$27 \cdot 5545503 \text{ d.}$		$28 \cdot 4397295$	n_2
$c = s_2 = 30$	Half anomalistic month	$\frac{1}{2} (661 \cdot 309206) \text{ h.}$			
		$\frac{1}{2} (27 \cdot 5545503) \text{ d.}$		$27 \cdot 8953549$	$2n$
	Evectional period	$763 \cdot 486534 \text{ h.}$	$29 \cdot 4556253$		λ_2
		$31 \cdot 8119389 \text{ d.}$		$28 \cdot 5125831$	v_2
	Half tropical year	$\frac{1}{2} (8765 \cdot 812722) \text{ h.}$	$30 \cdot 0821373$		k_1
		$\frac{1}{2} (365 \cdot 2421968) \text{ d.}$			
$c = k_1 = 15 \cdot 0410686$	Tropical year *	$8765 \cdot 812722 \text{ h.}$	$30 \cdot 0410686$		r_2
		$365 \cdot 2421968 \text{ d.}$		$29 \cdot 9589314$	t_2
	Half tropical month	$\frac{1}{2} (655 \cdot 717960) \text{ h.}$	$16 \cdot 1391017$		oo
		$\frac{1}{2} (27 \cdot 3215817) \text{ d.}$		$13 \cdot 9430356$	o_1
	Anomalistic month	$661 \cdot 309206 \text{ h.}$	$15 \cdot 5854433$		j_1
		$27 \cdot 5545503 \text{ d.}$			
$c = k_1 = 15 \cdot 0410686$	Half tropical year	$\frac{1}{2} (8765 \cdot 812722) \text{ h.}$		$14 \cdot 9589314$	p_1
		$\frac{1}{2} (365 \cdot 2421968) \text{ d.}$			
$c = 0$	Anomalistic month	$661 \cdot 309206 \text{ h.}$			
		$27 \cdot 5545503 \text{ d.}$		$13 \cdot 3986609$	q_1
$c = l_2$ $c = n_2$	Half evectional period in moon's parallax	$\frac{1}{2} (9882 \cdot 831893) \text{ h.}$		$29 \cdot 4556253$	λ_2
		$\frac{1}{2} (411 \cdot 7846622) \text{ d.}$	$28 \cdot 5125831$		v_2

* The tropical year is taken for convenience instead of the anomalistic year ($= 8766 \cdot 2295 \text{ h. or } 365 \cdot 25956 \text{ d.}$).

This table is based in part upon values given in Harkness' paper on the solar parallax, App. III, Washington Observations for 1885.

TABLE 3.—*Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them.*

Component.	1850	1851	1852	1853	1854	1855	1856	1857	1858	1859
J_1	29°66	116°00	204°03	307°53	38°07	129°39	221°21	327°32	59°40	151°25
K_1	3°47	1°64	0°92	2°20	3°31	5°03	7°13	10°45	12°78	14°92
K_2	187°75	183°76	181°83	183°99	186°04	189°52	193°95	200°87	205°84	210°36
L_2	155°70	350°62	161°48	330°94	177°72	26°05	194°57	353°33	204°19	54°37
$[L_2]$	149°54	338°59	167°84	346°01	175°74	5°69	195°78	14°65	204°84	34°96
M_1	6°83	273°54	165°21	50°56	346°93	274°81	170°80	55°36	347°23	281°70
$[M_1]$	330°16	239°05	149°63	49°56	322°66	236°54	150°91	53°45	328°08	242°49
M_2	299°79	40°11	140°64	217°03	318°04	59°26	160°63	237°71	339°18	80°58
M_3	269°69	240°17	210°96	145°54	117°05	88°89	60°95	356°57	328°78	300°86
M_4	239°58	80°22	281°28	74°06	276°07	118°52	321°26	115°43	318°37	161°15
M_5	179°38	120°34	61°93	291°08	234°11	177°77	121°89	353°14	297°55	241°73
M_6	119°17	160°45	202°57	148°11	192°14	237°03	282°52	230°86	270°74	322°30
N_2	270°04	281°64	293°45	268°05	280°33	292°83	305°48	280°78	293°52	306°19
$2 N$	240°29	163°17	86°25	319°06	242°63	166°40	90°33	323°84	247°86	171°81
O_1	299°88	42°59	143°81	218°48	317°67	56°29	154°57	227°33	325°45	63°71
OO	239°95	132°46	29°85	318°61	223°07	129°63	37°54	333°44	242°03	150°04
P_1	349°70	349°94	350°18	349°43	349°67	349°91	350°15	349°40	349°64	349°88
Q_1	270°13	284°11	296°62	269°50	279°96	289°87	299°42	270°39	279°79	289°32
R_2	359°93	359°67	359°42	0°15	359°89	359°63	359°38	0°11	359°85	359°60
$S_{1, 3}$	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00
$S_{2, 4, 6}$	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00
T_2	0°07	0°33	0°58	359°85	0°11	0°37	0°62	359°89	0°15	0°40
λ_2	148°79	59°63	330°69	228°92	140°45	52°19	324°09	223°02	135°01	46°93
μ_2	241°04	82°12	283°40	76°16	277°92	119°89	322°02	115°47	317°70	159°84
ν_2	270°79	200°59	130°60	25°14	315°62	246°32	177°17	72°41	3°36	294°22
MK	303°26	41°75	141°56	219°23	321°35	64°29	167°77	248°16	351°96	95°49
$2 MK$	236°12	78°59	280°37	71°86	272°76	113°49	314°13	104°98	305°59	146°23
MN	209°83	321°75	74°09	125°07	238°37	352°09	106°11	158°49	272°71	26°77
MS	299°79	40°11	140°64	217°03	318°04	59°26	160°63	237°71	339°18	80°58
$2 MS$	239°58	80°22	281°28	74°06	276°07	118°52	321°26	115°43	318°37	161°15
$2 SM$	60°21	319°89	219°36	142°97	41°96	300°74	199°37	122°29	20°82	279°42
Mf	240°03	134°94	33°02	320°06	222°70	126°67	31°48	323°06	228°29	133°17
MSf	60°21	319°89	219°36	142°97	41°96	300°74	199°37	122°29	20°82	279°42
Mm	29°75	118°47	207°20	308°98	37°71	126°43	215°15	316°94	45°66	134°38
Sa	280°30	280°06	279°82	280°57	280°33	280°09	279°85	280°60	280°36	280°12
Ssa	200°60	200°12	199°64	201°13	200°66	200°18	199°70	201°20	200°72	200°24
Elements.	Values at Greenwich, midnight beginning each year.									
h	280°30	280°06	279°82	280°57	280°33	280°09	279°85	280°60	280°36	280°12
s	129°67	259°06	28°44	171°00	300°39	69°77	199°16	341°72	111°10	240°49
p	99°92	140°58	181°25	222°02	262°68	303°35	344°01	24°78	65°44	106°11
	Values at the middle of each year, or for July 2 at Greenwich mean noon for common years, and at preceding midnight for leap years.									
p_1	280°37	280°39	280°40	280°42	280°44	280°46	280°47	280°49	280°51	280°52
P	110°60	149°33	189°70	231°38	273°98	317°24	0°98	44°92	88°82	132°53
N	136°54	117°21	97°85	78°50	59°17	39°84	20°49	1°13	341°81	322°48
I	20°02	21°57	23°28	24°97	26°44	27°59	28°33	28°60	28°39	27°71
Q	126°93	163°48	184°89	212°04	277°91	335°19	0°49	26°51	87°63	151°41
R	353°84	347°96	6°36	15°08	358°02	339°64	1°21	21°32	0°66	345°41
ξ	9°65	11°59	11°93	10°97	9°04	6°44	3°42	0°19	356°96	353°91
ν	10°39	12°54	12°99	12°02	9°96	7°13	3°79	0°21	356°62	353°25
ν'	6°83	8°42	8°90	8°37	7°02	5°06	2°72	0°15	357°58	355°20
$2 \nu''$	12°85	16°36	17°81	17°14	14°62	10°67	5°75	0°32	354°88	349°88

In making analyses or predictions it is not desirable to modify the values of $V_0 + u$ as here given either on account of the longitude of the station or the longitude of the time meridian. Such alteration should be made once for all in the epoch (C_0) of the particular component (C) as will adapt it to the tabular $V_0 + u$.

TABLE 3.—*Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.*

Component.	1860	1861	1862	1863	1864	1865	1866	1867	1868	1869
J_1	242° 65	347° 35	76° 95	165° 17	251° 71	350° 55	74° 00	157° 03	240° 76	340° 06
K_1	16° 70	18° 88	19° 28	18° 69	16° 99	15° 22	11° 71	7° 97	4° 62	3° 11
K_2	213° 98	218° 19	218° 62	216° 95	213° 19	209° 68	203° 16	196° 37	190° 05	186° 95
L_2	229° 00	26° 04	228° 97	72° 20	259° 39	62° 53	249° 41	86° 02	283° 89	100° 40
$[L_2]$	224° 92	43° 36	232° 88	62° 16	251° 23	68° 81	257° 60	86° 37	275° 18	92° 79
M_1	178° 86	61° 04	340° 98	280° 05	179° 71	56° 79	311° 75	235° 51	162° 59	47° 19
$[M_1]$	156° 44	57° 57	329° 72	240° 49	149° 59	44° 86	310° 86	216° 44	122° 73	18° 46
M_2	181° 82	258° 47	359° 27	99° 83	200° 17	275° 96	16° 03	116° 07	216° 16	291° 99
M_3	272° 72	207° 70	178° 90	149° 74	120° 25	53° 94	24° 05	354° 11	324° 25	257° 99
M_4	3° 63	156° 94	358° 53	199° 65	40° 33	191° 92	32° 06	232° 15	72° 33	223° 99
M_5	185° 45	55° 40	357° 80	299° 48	240° 50	107° 88	48° 10	348° 22	288° 49	155° 98
M_6	7° 26	313° 87	357° 06	39° 30	80° 66	23° 84	64° 13	104° 30	144° 66	87° 98
N_2	318° 71	293° 58	305° 65	317° 49	329° 11	303° 11	314° 46	325° 78	337° 15	311° 19
2 N	95° 61	328° 68	252° 04	175° 15	98° 05	330° 27	252° 89	175° 49	98° 14	330° 39
O_1	162° 27	236° 01	335° 93	77° 00	179° 52	258° 25	3° 60	109° 34	214° 42	292° 74
OO	56° 81	348° 91	250° 74	148° 66	41° 77	317° 16	201° 26	84° 13	329° 07	245° 78
P_1	350° 12	349° 37	349° 61	349° 85	350° 09	349° 34	349° 58	349° 82	350° 06	349° 31
Q_1	299° 17	271° 12	282° 32	294° 66	308° 46	285° 41	302° 03	319° 05	335° 40	311° 93
R_2	359° 34	0° 07	359° 82	359° 56	359° 30	0° 03	359° 78	359° 52	359° 27	359° 99
$S_{1, 2}$	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00
$S_{3, 4, 5}$	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00
T_2	0° 66	359° 93	0° 18	0° 44	0° 70	359° 97	0° 22	0° 48	0° 73	0° 01
λ_2	318° 69	217° 19	128° 51	39° 59	310° 46	208° 10	118° 69	29° 26	299° 87	197° 55
μ_2	1° 83	154° 86	356° 41	197° 72	38° 81	190° 98	31° 80	232° 60	73° 44	225° 64
ν_2	224° 94	119° 75	50° 02	340° 06	269° 87	163° 82	93° 37	22° 89	312° 45	206° 44
MK	198° 51	277° 35	18° 54	118° 51	217° 15	291° 18	27° 74	124° 04	220° 78	295° 10
2 MK	346° 93	138° 06	339° 25	180° 96	23° 34	176° 70	20° 36	224° 18	67° 71	220° 88
MN	140° 53	192° 04	304° 92	57° 31	169° 27	219° 07	330° 49	81° 86	193° 31	243° 19
MS	181° 82	258° 47	359° 27	99° 83	200° 17	275° 96	16° 03	116° 07	216° 16	291° 99
2 MS	3° 63	156° 94	358° 53	199° 65	40° 33	191° 92	32° 06	232° 15	72° 33	223° 99
2 SM	178° 18	101° 53	0° 73	260° 17	159° 83	84° 04	343° 97	243° 93	143° 84	68° 01
Mf	37° 27	326° 45	227° 40	125° 83	21° 13	299° 45	188° 83	77° 40	327° 33	246° 52
MSf	178° 18	101° 53	0° 73	260° 17	159° 83	84° 04	343° 97	243° 93	143° 84	68° 01
Mm	223° 11	324° 00	53° 61	142° 34	231° 06	332° 85	61° 57	150° 29	239° 01	340° 80
Sa	279° 88	280° 63	280° 39	280° 15	279° 91	280° 66	280° 42	280° 18	279° 94	280° 69
Ssa	199° 76	201° 26	200° 78	200° 30	199° 83	201° 32	200° 84	200° 37	199° 89	201° 38
Elements.	Values at Greenwich, midnight beginning each year.									
h	279° 88	280° 63	280° 39	280° 15	279° 91	280° 66	280° 42	280° 18	279° 94	280° 69
s	9° 87	152° 43	281° 82	51° 21	180° 59	323° 15	92° 54	221° 92	351° 31	133° 87
p	146° 77	187° 54	228° 21	268° 87	309° 53	350° 31	30° 97	71° 63	112° 29	153° 07
	Values at the middle of each year, or for July 2, at Greenwich mean noon for common years and at preceding midnight for leap years.									
p_1	280° 54	280° 56	280° 57	280° 59	280° 61	280° 63	280° 64	280° 66	280° 68	280° 69
P	175° 91	218° 66	260° 42	300° 91	339° 89	17° 21	53° 18	88° 74	125° 04	162° 79
N	303° 12	283° 77	264° 44	245° 11	225° 76	206° 40	187° 07	167° 75	148° 39	129° 03
I	26° 60	25° 15	23° 49	21° 77	20° 19	18° 98	18° 36	18° 46	19° 25	20° 58
Q	177° 95	201° 80	251° 35	320° 13	349° 62	8° 81	33° 73	87° 48	144° 51	171° 19
R	355° 92	17° 32	3° 91	349° 96	351° 83	6° 27	8° 19	0° 34	351° 29	352° 39
ξ	351° 24	349° 21	348° 12	348° 29	350° 03	353° 43	358° 12	3° 22	7° 64	10° 61
ν	350° 34	348° 17	347° 06	347° 32	349° 27	352° 95	357° 99	3° 45	8° 20	11° 43
ν'	353° 19	351° 75	351° 11	351° 46	352° 92	355° 44	358° 71	2° 21	5° 33	7° 58
2 ν''	345° 78	343° 07	342° 16	343° 35	346° 64	351° 64	357° 68	4° 10	9° 84	14° 43

From §62 we have for the modified epoch (ζ^0)

$$\zeta^0 = C^0 + 15 p L - c S$$

where p is to be put equal 1, 2, . . . according as C is diurnal, semidiurnal, e.c.: c is the hourly speed of C ; L , S denote the west longitude in hours of the station and of the time meridian used. The values of L and S should always accompany the work of analysis or prediction, thus enabling one to pass from ζ^0 to C^0 or C^0 to ζ^0 as the case may be.

TABLE 3.—*Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.*

Component.	1870	1871	1872	1873	1874	1875	1876	1877	1878	1879
J_1	67°07	155°69	245°61	350°54	82°08	174°02	266°11	12°19	103°89	195°03
K_1	1°71	1°41	2°04	4°40	6°29	8°51	10°86	14°12	16°15	17°73
K_2	183°73	182°64	183°58	188°21	192°12	196°81	201°81	208°64	212°86	216°00
L_2	270°95	100°60	307°85	139°35	296°92	121°83	334°74	170°64	331°40	151°77
$[L_2]$	281°92	111°26	300°85	119°34	309°36	139°50	329°70	148°55	338°61	168°50
M_1	297°71	201°00	145°32	47°99	301°76	202°48	145°46	55°40	309°99	207°85
$[M_1]$	288°03	199°20	111°67	13°03	287°13	201°62	116°27	18°77	293°03	206°72
M_2	32°39	133°02	233°88	310°59	51°88	153°30	254°77	331°84	73°18	174°35
M_3	228°59	199°52	170°81	105°88	77°82	49°95	22°16	317°76	289°77	261°52
M_4	64°79	266°03	107°75	261°18	103°76	306°60	149°55	303°68	146°36	348°70
M_6	97°18	39°05	341°63	211°76	155°63	99°90	44°32	275°53	219°55	163°04
M_8	129°58	172°06	215°50	162°35	207°51	253°20	299°10	247°37	292°73	337°39
N_2	322°87	334°77	346°91	321°83	334°40	347°10	359°85	335°13	347°75	0°19
$2 N$	253°35	176°52	99°94	333°08	256°92	180°90	104°93	338°42	262°32	186°04
O_1	34°84	135°57	235°24	308°80	47°27	145°48	243°58	316°37	54°72	153°47
OO	140°26	39°33	302°04	234°76	141°93	50°16	318°80	254°59	162°20	68°28
P_1	349°55	349°79	350°03	349°28	349°52	349°75	349°99	349°25	349°49	349°73
Q_1	325°32	337°32	348°27	320°05	329°80	339°28	348°66	319°65	329°29	339°32
R_2	359°74	359°49	359°23	359°96	359°70	359°45	359°19	359°92	359°67	359°41
$S_{1, 3}$	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00
$S_{2, 4, 6}$	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00
T_2	0°26	0°51	0°77	0°04	0°30	0°55	0°81	0°08	0°33	0°59
λ_2	108°47	19°62	291°00	189°56	101°37	13°32	285°32	184°23	96°09	7°78
μ_2	66°79	268°17	109°78	262°86	104°91	307°08	149°31	302°75	144°84	346°76
v_2	136°32	66°42	356°75	251°62	182°39	113°28	44°23	299°46	230°27	160°91
MK	34°11	134°42	235°92	314°99	58°17	161°81	265°63	345°97	89°33	192°07
$2 MK$	63°07	264°62	105°71	256°78	97°46	298°09	138°69	289°56	130°21	330°97
MN	355°27	107°79	220°78	272°42	26°28	140°40	254°63	306°97	60°93	174°54
MS	32°39	133°02	233°88	310°59	51°88	153°30	254°77	331°84	73°18	174°35
$2 MS$	64°79	266°03	107°75	261°18	103°76	306°60	149°55	303°68	146°36	348°70
$2 SM$	327°61	226°98	126°12	49°41	308°12	206°70	105°23	28°16	286°82	185°65
Mf	142°71	41°88	303°40	232°98	137°33	42°34	307°61	239°11	143°74	47°41
MSf	327°61	226°98	126°12	49°41	308°12	206°70	105°23	28°16	286°82	185°65
Mm	69°52	158°25	246°97	348°75	77°48	166°20	254°92	356°71	85°43	174°15
Sa	280°45	280°21	279°97	280°72	280°48	280°25	280°01	280°75	280°51	280°27
Ssa	200°90	200°43	199°95	201°44	200°97	200°49	200°01	201°50	201°03	200°55
Elements.	Values at Greenwich, midnight beginning each year.									
k	280°45	280°21	279°97	280°72	280°48	280°25	280°01	280°75	280°51	280°27
s	263°25	32°64	162°02	304°58	73°97	203°35	332°74	115°30	244°68	14°07
p	193°73	234°39	275°05	315°83	356°49	37°15	77°81	118°59	159°25	199°91
	Values at the middle of each year, or for July 2, at Greenwich mean noon for common years and at preceding midnight for leap years.									
p_1	280°71	280°73	280°75	280°76	280°78	280°80	280°81	280°83	280°85	280°87
P	202°16	243°03	285°12	328°07	11°52	55°30	99°27	143°18	186°77	229°88
N	109°71	90°38	71°03	51°67	32°34	13°01	353°66	334°30	314°97	295°65
I	22°22	23°95	25°57	26°93	27°93	28°49	28°57	28°18	27°32	26°07
Q	191°51	224°49	298°38	342°69	5°82	35°83	108°07	159°48	183°40	210°68
R	10°97	10°66	353°00	339°99	12°44	17°67	354°95	337°91	7°22	16°73
ξ	11°90	11°70	10°32	8°09	5°30	2°18	358°93	355°74	352°81	350°37
v	12°90	12°77	11°33	8°94	5°88	2°43	358°81	355°27	352°05	349°40
v'	8°74	8°81	7°93	6°32	4°19	1°74	359°15	356°63	354°37	352°55
$2 v''$	17°18	17°79	16°36	13°23	8°85	3°68	358°20	352°87	348°16	344°55

If the first day of the series to be analyzed be other than January 1, the change in the $V_0 + u$ as here tabulated is obtained from Tables 4, 6, 7, 8, and 9. In the case of prediction, only Table 4 is required, because u may be regarded as constant for any fraction of a year.

TABLE 3.—Equilibrium arguments ($\Gamma_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.

Component.	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889
J_1	285°32	28°47	116°06	201°92	286°12	23°31	106°49	190°75	276°68	18°39
K_1	18°64	19°66	18°65	16°53	13°43	10°74	7°07	4°01	1°94	1°90
K_2	217°59	219°20	216°70	212°21	206°24	201°48	194°79	188°87	184°45	184°02
L_2	359°19	189°85	8°80	184°51	17°70	204°53	38°89	219°81	35°38	219°58
$[L_2]$	358°17	176°28	5°47	194°47	23°31	200°77	29°54	218°40	47°40	225°28
M_1	142°24	58°16	312°82	203°44	108°15	31°14	301°37	192°56	83°72	344°54
$[M_1]$	119°56	19°14	289°29	197°71	104°45	358°07	263°80	170°62	79°11	337°25
M_2	275°29	351°62	92°08	192°36	292°48	8°15	108°20	208°33	308°61	24°71
M_3	232°94	167°42	138°13	108°54	78°72	12°22	342°30	312°50	282°91	217°06
M_4	190°58	343°23	184°17	24°72	224°96	16°30	216°40	5°67	257°22	49°42
M_6	105°88	334°85	276°25	217°07	157°44	24°44	324°61	265°00	205°83	72°12
M_8	21°17	326°46	8°34	49°43	89°92	32°59	72°81	113°34	154°44	98°83
N_2	12°41	346°95	358°70	10°25	21°65	355°53	6°86	18°27	29°82	4°13
$2N$	109°54	342°29	265°31	188°14	110°82	342°91	265°52	188°21	111°04	343°56
O_7	252°85	327°81	69°41	172°54	277°19	357°46	103°06	207°65	310°72	26°89
OO	332°03	259°80	155°95	47°08	293°37	203°93	87°23	333°72	225°07	148°75
P_1	349°96	349°22	349°46	349°69	349°93	349°19	349°43	349°66	349°90	349°16
Q_1	349°97	323°15	336°02	350°43	6°36	344°84	1°72	17°59	31°93	6°31
R_2	359°15	359°88	359°63	359°37	359°11	359°84	359°59	359°33	359°08	359°81
$S_{1, 3}$	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00
$S_{2, 4, 6}$	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00
T_2	0°85	0°12	0°37	0°63	0°89	0°16	0°41	0°67	0°92	0°19
λ_2	279°25	177°42	88°41	359°21	269°85	167°37	77°94	348°60	259°40	157°34
μ_2	188°46	341°15	182°37	23°40	224°27	16°31	217°12	58°00	259°03	51°50
v_2	91°33	345°82	275°76	205°51	135°11	28°93	318°46	248°07	177°82	72°08
MK	293°93	11°28	110°73	208°89	305°91	18°88	115°28	212°34	310°55	26°67
$2MK$	171°95	323°57	165°52	8°18	211°53	5°56	209°33	52°66	255°28	47°46
MN	287°71	338°57	90°78	202°61	314°13	3°68	115°06	226°60	338°43	28°84
MS	275°29	351°62	92°08	192°36	292°48	8°15	108°20	208°33	308°61	24°71
$2MS$	190°58	343°23	184°17	24°72	224°96	16°30	216°40	56°67	257°22	49°42
$2SM$	84°71	8°38	267°92	167°64	67°52	351°85	251°80	151°67	51°39	335°29
Mf	309°59	236°00	133°27	27°27	278°09	193°23	82°09	333°04	227°17	150°93
MSf	84°71	8°38	267°92	167°64	67°52	351°85	251°80	151°67	51°39	335°29
Mm	262°88	4°66	93°39	182°11	270°83	12°62	101°34	190°06	278°79	20°57
Sa	280°04	280°78	280°54	280°31	280°07	280°81	280°57	280°34	280°10	280°84
Ssa	200°07	201°57	201°09	200°61	200°13	201°63	201°15	200°67	200°20	201°69
Elements.	Values at Greenwich, midnight beginning each year.									
h	280°04	280°78	280°54	280°31	280°07	280°81	280°57	280°34	280°10	280°84
s	143°45	286°01	55°40	184°79	314°17	96°73	226°12	355°50	124°89	267°45
p	240°58	281°35	322°01	2°68	43°34	84°11	124°77	165°44	206°10	246°87
	Values at the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years.									
p	280°88	280°90	280°92	280°93	280°95	280°97	280°99	281°00	281°02	281°04
P	272°30	313°67	353°58	31°86	68°60	104°33	140°03	176°79	215°19	255°22
N	276°29	256°94	237°61	218°28	198°93	179°57	160°24	140°91	121°56	102°21
I	24°53	22°82	21°12	19°66	18°66	18°31	18°69	19°71	21°19	22°89
Q	274°60	332°35	356°78	17°26	51°91	117°06	157°27	178°39	199°42	242°19
R	358°98	346°43	356°67	9°96	5°61	356°24	350°65	358°58	12°01	5°71
ξ	348°66	348°02	348°77	351°15	355°13	0°11	5°07	8°98	11°30	11°98
v	347°60	347°87	350°49	350°49	354°78	0°12	5°43	9°65	12°21	13°03
v'	351°40	351°12	351°89	353°77	356°64	0°08	3°50	6°33	8°16	8°89
$2v''$	342°48	342°37	344°39	348°40	353°90	0°14	6°36	11°80	15°74	17°67

TABLE 3.—*Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.*

Component.	1890	1891	1892	1893	1894	1895	1896	1897	1898	1899
J_1	107°55	197°88	289°05	34°83	126°86	218°95	310°88	56°45	147°29	237°16
K_1	2°04	2°98	4°58	7°61	9°90	12°24	14°45	17°31	18°65	19°25
K_2	183°75	185°40	188°60	194°35	199°71	204°70	209°37	215°21	217°80	218°67
L_2	65°75	260°29	55°86	240°32	93°98	295°30	86°65	268°40	118°43	317°10
$[L_2]$	54°72	244°40	74°29	253°04	83°22	273°42	103°56	282°25	112°05	301°63
M_1	287°73	192°32	85°29	340°37	288°59	199°79	93°36	344°85	289°57	205°07
$[M_1]$	248°96	161°84	75°57	337°77	252°35	167°00	81°48	343°48	256°88	169°30
M_2	125°42	226°38	327°55	44°52	145°97	247°45	348°86	65°76	166°84	267°70
M_3	188°14	159°57	131°33	66°77	38°95	11°17	343°29	278°64	250°26	221°55
M_4	250°85	92°76	295°10	89°03	291°94	134°89	337°72	131°52	333°68	175°40
M_6	16°27	319°14	262°66	133°55	77°91	22°34	326°58	197°29	140°53	83°09
M_8	141°70	185°52	230°21	178°06	223°88	269°78	315°44	263°05	307°37	350°79
N_2	16°13	28°36	40°81	15°99	28°72	41°47	54°16	29°28	41°64	53°77
$2N$	266°83	190°34	114°07	347°46	271°47	195°50	119°47	352°80	276°43	199°84
O_1	127°17	226°52	325°24	38°21	136°37	234°47	332°68	45°80	144°76	244°47
OO	49°35	313°21	219°38	154°39	62°85	331°47	239°68	174°14	79°41	341°99
P_1	349°39	349°63	349°87	349°13	349°36	349°60	349°84	349°09	349°33	349°57
Q_1	17°87	28°50	38°50	9°69	19°11	28°50	37°98	9°31	19°55	30°54
R_2	359°55	359°30	359°04	359°77	359°51	359°26	359°00	359°73	359°48	359°22
$S_{1, 3}$	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00
$S_{2, 4, 6}$	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00
T_2	0°45	0°70	0°96	0°23	0°49	0°74	1°00	0°27	0°52	0°78
λ_2	68°58	340°06	251°76	150°56	62°54	334°54	246°48	145°23	56°83	328°21
μ_2	252°97	94°68	296°60	89°94	292°15	134°38	336°54	129°82	331°65	173°26
v_2	2°27	292°70	223°35	118°47	49°40	340°35	271°24	166°30	96°85	27°19
MK	127°46	229°36	332°13	52°13	155°87	259°68	3°31	83°07	185°50	286°95
$2MK$	248°81	89°78	290°53	81°42	282°04	122°65	323°27	114°21	315°03	156°14
MN	141°55	254°74	8°36	60°50	174°69	288°92	43°02	95°04	208°48	321°47
MS	125°42	226°38	327°55	44°52	145°97	247°45	348°86	65°76	166°84	267°70
$2MS$	250°85	92°76	295°10	89°03	291°94	134°89	337°72	131°52	333°68	175°40
$2SM$	234°58	133°62	32°45	315°48	214°03	112°55	11°14	294°24	193°16	92°30
Mf	51°09	313°34	217°07	148°09	53°24	318°50	223°50	154°17	57°33	318°76
MSf	234°58	133°62	32°45	315°48	214°03	112°55	11°14	294°24	193°16	92°30
Mm	109°30	198°02	286°74	28°53	117°25	205°97	294°70	36°48	125°21	213°93
Sa	280°61	280°37	280°13	280°87	280°64	280°40	280°16	280°91	280°67	280°43
Ssa	201°21	200°74	200°26	201°75	201°27	200°80	200°32	201°82	201°33	200°86
Elements.	Values at Greenwich, midnight beginning each year.									
A	280°61	280°37	280°13	280°87	280°64	280°40	280°16	280°91	280°67	280°43
s	36°83	166°22	295°60	78°16	207°55	336°93	106°32	248°88	18°26	147°65
ϕ	287°54	328°20	8°86	49°63	90°30	130°96	171°62	212°40	253°06	293°72
	Values at the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years.									
p	281°05	281°07	281°09	281°11	281°12	281°14	281°16	281°17	281°19	281°21
P	296°58	338°99	22°18	65°85	109°70	153°61	197°44	240°93	283°79	325°79
N	82°88	63°55	44°19	24°84	5°51	346°18	326°83	307°47	288°15	268°82
I	24°60	26°13	27°37	28°20	28°58	28°48	27°90	26°88	25°50	23°88
Q	315°02	349°13	11°52	48°11	125°61	166°07	188°93	221°97	296°15	341°22
R	348°97	344°10	18°44	12°73	349°24	338°12	16°90	13°84	353°62	344°53
ξ	11°29	9°55	7°07	4°12	0°93	357°68	354°57	351°79	349°60	348°27
v	12°35	10°51	7°82	4°57	1°03	357°43	353°98	350°94	348°58	347°20
v'	8°57	7°39	5°55	3°26	0°74	358°16	355°71	353°59	352°01	351°18
$2v''$	17°47	15°33	11°66	6°91	1°57	356°10	350°95	346°60	343°54	342°19

TABLE 3.—Equilibrium arguments ($T_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.

Component.	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909
J_1	325°72	52°66	137°84	221°53	304°55	42°08	126°95	213°56	301°84	45°54
K_1	18°90	17°46	14°93	11°56	7°81	5°30	2°59	0°93	0°39	1°81
K_2	217°49	214°18	209°05	202°71	195°90	191°37	185°96	182°27	180°69	183°17
L_2	127°09	309°02	147°11	345°00	174°12	339°64	162°47	1°20	204°78	17°54
$[L_2]$	130°96	320°07	149°00	337°81	166°58	344°05	172°95	2°03	191°32	9°53
M_1	97°46	351°64	271°67	203°30	101°59	338°11	232°12	158°08	90°59	336°18
$[M_1]$	80°41	349°91	257°65	163°90	69°47	323°42	230°85	140°02	50°85	310°97
M_2	8°31	108°70	208°90	309°00	49°04	124°72	224°90	325°26	65°83	142°25
M_3	192°46	163°04	133°35	103°49	73°55	7°09	337°36	307°89	278°74	213°37
M_4	16°62	217°39	57°80	257°99	98°07	249°45	89°81	290°52	131°65	284°50
M_5	24°92	326°09	266°71	206°99	147°11	14°17	314°71	255°77	197°48	66°74
M_6	33°23	74°78	115°61	155°98	196°14	138°90	179°62	221°03	263°30	208°99
N_2	65°66	77°32	88°81	100°18	111°50	85°40	96°85	108°49	120°33	94°97
$2 N$	123°01	45°95	328°71	251°36	173°96	46°07	328°81	251°72	174°84	47°69
Q^1	345°24	87°41	191°16	296°28	42°03	121°97	226°01	328°47	69°48	143°99
QO	240°89	135°15	24°33	269°15	151°99	63°53	311°79	205°11	103°20	32°52
P_1	349°81	350°05	350°29	350°53	350°76	350°02	350°26	350°49	350°73	349°99
Q_1	42°59	56°04	71°06	87°46	104°49	82°65	97°96	111°70	123°99	96°71
R_2	358°97	358°71	358°45	358°20	357°94	358°67	358°41	358°16	357°90	358°63
$S_{1, 2}$	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00
$S_{2, 4, 6}$	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00
T_2	1°03	1°29	1°55	1°80	2°06	1°33	1°59	1°84	2°10	1°37
λ_2	239°35	150°26	60°99	331°61	242°17	139°70	50°41	321°9	232°38	130°64
μ_2	14°62	215°76	56°72	257°57	98°36	250°42	91°35	292°46	133°78	286°57
v_2	317°27	247°13	176°81	106°39	35°90	289°75	219°40	149°23	79°27	333°85
MK	27°21	126°16	223°83	320°55	56°84	130°02	227°49	326°19	66°21	144°06
$2 MK$	357°71	199°93	42°87	246°44	90°26	244°15	87°22	289°58	131°27	282°69
MN	73°97	186°02	297°71	49°17	160°53	210°12	321°76	73°75	186°16	237°21
MS	8°31	108°70	208°90	309°00	49°04	124°72	224°90	325°26	65°83	142°25
$2 MS$	16°62	217°39	57°80	257°99	98°07	249°45	89°81	290°52	131°65	284°50
$2 SM$	351°69	251°30	151°10	51°00	310°96	235°28	135°10	34°74	294°17	217°75
Mf	217°82	113°87	6°59	256°44	144°98	60°78	312°89	208°32	106°86	34°26
MSf	351°69	251°30	151°10	51°00	310°96	235°28	135°10	34°74	294°17	217°75
Mm	302°65	31°37	120°09	208°82	297°54	39°33	128°05	216°77	305°49	47°28
Sa	280°19	279°95	279°71	279°47	279°24	279°98	279°74	279°51	279°27	280°01
Ssa	200°38	199°90	199°42	198°95	198°47	199°96	199°49	199°01	198°53	200°03
Elements.	Values at Greenwich, midnight beginning each year.									
λ	280°19	279°95	279°71	279°47	279°24	279°98	279°74	279°51	279°27	280°01
s	277°03	46°42	175°80	305°19	74°57	217°13	346°52	115°90	245°29	27°85
p	334°38	15°05	55°71	96°37	137°03	177°81	218°47	259°13	299°80	340°57
Values at the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years.										
p_1	281°23	281°24	281°26	281°28	281°29	281°31	281°33	281°35	281°36	281°38
P	6°59	45°89	83°53	119°73	155°34	191°39	228°73	267°72	308°32	350°18
N	249°49	230°16	210°83	191°51	172°15	152°80	133°47	114°14	94°79	75°43
I	22°15	20°52	19°21	18°44	18°37	19°01	20°24	21°83	23°56	25°22
Q	3°31	27°29	77°22	138°80	167°07	185°75	209°67	265°45	327°68	355°06
R	3°87	11°05	1°89	352°82	352°45	4°41	10°48	0°83	346°54	351°99
ξ	348°12	349°48	352°51	356°97	2°08	6°75	10°07	11°74	11°86	10°72
v	347°13	348°67	351°97	356°76	2°23	7°23	10°85	12°72	12°92	11°76
v'	351°29	352°49	354°78	357°92	1°43	4°68	7°16	8°57	8°88	8°21
$2 v''$	342°89	345°72	350°37	356°24	2°57	8°59	13°53	16°73	17°84	16°85

TABLE 3.—Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.

Component.	1910	1911	1912	1913	1914	1915	1916	1917	1918	1919
I_1	136°22	227°63	319°51	65°64	157°69	249°49	340°79	85°35	174°75	262°72
K_1	3°03	4°82	6°99	10°30	12°61	14°72	16°41	18°49	18°74	17°98
K_2	185°47	189°13	193°69	200°64	205°56	209°97	213°40	217°36	217°46	215°46
L_2	179°69	22°00	235°00	55°42	206°55	48°99	260°18	81°51	245°33	75°84
$[L_2]$	199°29	29°27	219°39	38°26	228°44	58°54	248°48	66°88	256°36	85°61
M_1	229°57	146°18	92°11	343°35	237°05	149°20	96°33	350°12	242°01	142°58
$[M_1]$	224°21	138°19	52°61	315°17	229°78	144°14	57°99	318°97	230°93	141°45
M_2	243°29	344°54	85°94	163°03	264°49	5°86	107°07	183°69	284°45	24°97
M_3	184°94	156°81	128°91	64°54	36°73	8°79	340°61	275°53	246°67	217°46
M_4	126°58	329°08	171°88	326°06	168°97	11°72	214°15	7°38	208°89	49°94
M_5	9°88	313°63	257°82	129°08	73°46	17°59	321°22	191°07	133°34	74°92
M_6	253°17	298°17	343°76	292°11	337°94	23°45	68°30	14°76	57°78	99°89
N_2	107°29	119°82	132°49	107°79	120°53	133°18	145°67	120°50	132°53	144°34
$2N$	331°29	255°09	179°04	52°56	336°57	260°50	184°27	57°31	340°62	263°71
O_1	243°07	341°63	79°88	152°62	250°75	349°05	87°69	161°53	261°61	2°89
OO	297°36	204°19	112°24	48°19	316°73	224°58	131°09	62°79	324°07	221°28
P_1	350°23	350°47	350°70	349°96	350°20	350°43	350°67	349°93	350°17	350°40
Q_1	107°07	116°91	126°43	97°39	106°79	116°37	126°28	98°34	109°70	122°26
R_2	358°38	358°12	357°87	358°60	358°34	358°08	357°83	358°56	358°30	358°05
$S_1, 3$	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00
$S_2, 4, 6$	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00
T_2	1°62	1°88	2°13	1°40	1°66	1°92	2°17	1°44	1°70	1°95
λ_2	42°21	313°99	225°91	124°84	36°83	308°72	220°46	118°92	30°20	301°25
μ_2	128°37	330°37	172°52	325°98	168°19	10°32	212°29	5°27	206°78	48°06
v_2	264°37	195°10	125°97	21°22	312°15	243°00	173°69	68°46	358°69	288°69
MK	246°32	349°37	92°93	173°33	277°10	20°58	123°49	202°17	303°18	42°95
$2MK$	123°55	324°26	164°89	315°75	156°36	357°01	197°73	348°89	190°15	31°97
MN	350°58	104°36	218°43	270°82	25°01	139°05	252°75	304°19	56°98	169°31
MS	243°29	344°54	85°94	160°03	264°49	5°86	107°07	183°69	284°45	24°97
$2MS$	126°58	329°08	171°88	326°06	168°97	11°72	214°15	7°38	208°89	49°94
$2SM$	116°71	15°46	274°06	196°97	95°51	354°14	352°93	176°31	75°55	335°03
Mf	297°14	201°28	106°18	37°79	302°99	207°77	111°70	40°63	301°23	199°19
MSf	116°71	15°46	274°06	196°97	95°51	354°14	252°93	176°31	75°55	335°03
Mm	136°00	224°73	313°45	55°23	143°96	232°68	321°40	63°19	151°91	240°63
Sa	279°77	279°53	279°30	280°04	279°80	279°57	279°33	280°07	279°83	279°60
Ssa	199°55	199°07	198°59	200°09	199°61	199°13	198°66	200°15	199°67	199°19
Elements.	Values at Greenwich, midnight beginning each year.									
h	279°77	279°53	279°30	280°04	279°80	279°57	279°33	280°07	279°83	279°60
s	157°24	286°62	56°01	198°57	327°95	97°34	226°72	9°28	138°67	268°05
θ	21°23	61°89	102°56	143°33	183°99	224°66	265°32	306°09	346°76	27°42
Values at the middle of each year, or for July 2, at Greenwich mean noon for common years and at preceding midnight for leap years.										
p_1	281°40	281°41	281°43	281°45	281°47	281°48	281°50	281°52	281°53	281°55
P	72°90	76°25	120°03	163°99	207°87	251°54	294°84	337°46	19°03	59°29
N	56°10	36°77	17°42	358°06	338°74	319°41	300°05	280°70	261°37	242°04
I	26°65	27°74	28°40	28°60	28°31	27°55	26°39	24°90	23°21	21°50
Q	17°93	63°91	139°14	171°83	194°81	236°27	312°79	348°28	9°79	40°09
R	19°60	7°27	344°39	342°84	21°89	9°55	348°29	345°37	11°03	9°76
E	8.66	5.98	2.91	359°67	356°46	353°45	350°87	348°97	348°05	348°45
v	9°56	6°63	3°24	359°64	356°07	352°75	349°94	347°91	347°00	347°51
v'	6°75	4°71	2°31	359°74	357°19	354°85	352°91	351°59	351°10	351°62
$2v''$	14°08	9°94	4°90	359°45	354°05	349°16	345°25	342°79	342°21	343°74

TABLE 3.—*Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.*

Component.	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929
I_1	348° 98	87° 57	170° 87	253° 94	337° 87	77° 45	164° 74	253° 58	343° 67	88° 73
K_1	16° 11	14° 19	10° 60	6° 88	3° 63	2° 29	1° 08	0° 93	1° 70	4° 15
K_2	211° 39	207° 66	201° 05	194° 28	188° 11	185° 27	182° 37	181° 62	182° 87	187° 73
L_2	277° 13	101° 57	284° 24	101° 71	291° 23	119° 89	319° 09	133° 44	306° 72	141° 37
$[L_2]$	274° 64	92° 19	280° 98	109° 75	298° 58	116° 22	305° 37	134° 75	324° 38	142° 91
M_1	78° 36	344° 03	235° 99	124° 59	25° 11	309° 86	226° 49	119° 97	15° 31	292° 00
$[M_1]$	50° 27	305° 28	211° 14	116° 77	23° 25	279° 26	189° 10	100° 50	13° 15	274° 63
M_2	125° 28	201° 05	301° 11	41° 16	141° 27	217° 12	317° 55	58° 21	159° 11	235° 86
M_3	187° 93	121° 58	91° 67	61° 74	31° 90	325° 68	296° 33	267° 32	238° 67	173° 79
M_4	250° 57	42° 10	242° 22	82° 32	282° 53	74° 24	275° 10	116° 42	318° 23	111° 72
M_5	15° 85	243° 16	183° 33	123° 48	63° 80	291° 37	232° 65	174° 64	117° 34	347° 57
M_6	141° 14	84° 21	124° 44	164° 64	205° 06	148° 49	190° 20	232° 85	276° 46	223° 43
N_2	155° 93	129° 91	141° 24	152° 57	163° 96	138° 02	149° 73	161° 67	173° 85	148° 81
$2 N$	186° 57	58° 77	341° 38	263° 99	186° 65	58° 93	341° 91	265° 13	188° 59	61° 75
O_1	105° 66	184° 63	290° 10	35° 81	140° 70	218° 77	320° 63	61° 17	160° 71	234° 18
OO	113° 58	28° 22	271° 91	154° 90	40° 43	317° 94	213° 20	112° 91	16° 11	309° 18
P_1	350° 64	349° 89	350° 13	350° 37	350° 61	349° 86	350° 10	350° 34	350° 58	349° 83
Q_1	136° 31	113° 49	130° 24	147° 22	163° 39	139° 67	152° 81	164° 63	175° 44	147° 13
R_2	357° 79	358° 52	358° 26	358° 01	357° 75	358° 48	358° 23	357° 97	357° 72	358° 45
$S_1, 3$	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00
$S_2, 4, 6$	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00
T_2	2° 21	1° 48	1° 74	1° 99	2° 25	1° 52	1° 77	2° 03	2° 28	1° 55
λ_2	212° 08	109° 70	20° 28	290° 85	201° 48	99° 18	10° 14	281° 32	192° 75	91° 33
μ_2	249° 13	41° 26	242° 07	82° 88	283° 74	65° 96	277° 14	118° 56	320° 22	113° 33
ν_2	218° 48	112° 41	41° 94	331° 47	261° 05	155° 06	84° 96	15° 10	305° 48	200° 38
MK	141° 39	215° 24	311° 71	48° 04	144° 90	219° 42	318° 63	59° 15	160° 81	240° 01
$2 MK$	234° 46	27° 91	231° 62	75° 44	278° 90	71° 95	274° 02	115° 49	316° 53	107° 56
MN	281° 21	330° 96	82° 35	193° 73	305° 22	355° 15	107° 28	219° 88	332° 96	24° 66
MS	125° 28	201° 05	301° 11	41° 16	141° 27	217° 12	317° 55	58° 21	159° 11	235° 86
$2 MS$	250° 57	42° 10	242° 22	82° 32	282° 53	74° 24	275° 10	116° 42	318° 23	111° 72
$2 SM$	234° 72	158° 95	58° 89	318° 84	218° 73	142° 88	42° 45	301° 70	200° 89	124° 14
Mf	93° 96	11° 80	260° 90	149° 54	39° 86	319° 59	216° 28	115° 87	17° 70	307° 50
MSf	234° 72	158° 95	58° 89	318° 84	218° 73	142° 88	42° 45	301° 79	200° 89	124° 14
Mm	329° 36	71° 14	159° 87	248° 59	337° 31	79° 10	167° 82	256° 54	345° 26	87° 05
Sa	279° 36	280° 11	279° 87	279° 63	279° 39	280° 14	279° 90	279° 66	279° 42	280° 17
Ssa	198° 72	200° 21	199° 73	199° 26	198° 78	200° 27	199° 79	199° 32	198° 84	200° 33
Elements.	Values at Greenwich, midnight beginning each year.									
h	279° 36	280° 11	279° 87	279° 63	279° 39	280° 14	279° 90	279° 66	279° 42	280° 17
s	37° 44	180° 00	309° 38	78° 77	208° 15	350° 71	120° 10	249° 48	18° 87	161° 43
p	68° 08	108° 86	149° 52	190° 18	230° 84	271° 62	312° 28	352° 94	33° 61	74° 38
	Values at the middle of each year, or for July 2, at Greenwich, mean noon for common years and at preceding midnight for leap years.									
p_1	281° 57	281° 59	281° 60	281° 62	281° 64	281° 65	281° 67	281° 69	281° 70	281° 72
P	98° 01	135° 09	170° 92	206° 52	243° 01	281° 03	320° 65	1° 73	43° 97	87° 03
Λ	222° 69	203° 33	184° 00	164° 68	145° 32	125° 97	106° 64	87° 31	67° 96	48° 60
I	19° 96	18° 83	18° 32	18° 54	19° 43	20° 83	22° 50	24° 22	25° 81	27° 12
Q	105° 72	153° 50	175° 43	194° 01	224° 47	291° 29	337° 71	0° 87	25° 75	84° 07
R	357° 51	350° 62	356° 73	8° 04	7° 35	356° 33	346° 28	1° 32	17° 66	1° 54
ξ	350° 46	354° 10	358° 93	4° 00	8° 22	10° 92	11° 96	11° 55	10° 02	7° 68
ν	349° 74	353° 68	358° 86	4° 28	8° 83	11° 78	12° 98	12° 62	11° 01	8° 49
ν'	353° 25	355° 92	359° 27	2° 75	5° 75	7° 84	8° 82	8° 73	7° 72	6° 01
$2 \nu''$	347° 33	352° 55	358° 68	4° 98	10° 67	15° 01	17° 42	17° 69	15° 97	12° 61

TABLE 3. — *Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.*

Component.	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939
J_1	180° 35	272° 33	4° 43	110° 47	202° 10	293° 12	23° 24	126° 18	213° 51	299° 09
K_1	6° 11	8° 36	10° 71	13° 95	15° 92	17° 40	18° 19	19° 06	17° 87	15° 58
K_2	191° 79	196° 56	201° 57	208° 33	212° 41	215° 33	216° 64	217° 91	215° 07	210° 31
L_2	352° 21	170° 08	330° 77	167° 92	18° 80	201° 72	5° 95	193° 11	35° 89	227° 40
$[L_2]$	332° 94	163° 10	353° 30	172° 15	2° 18	192° 04	21° 67	199° 74	28° 90	217° 87
M_1	228° 28	126° 48	21° 61	293° 28	234° 44	134° 12	26° 99	285° 68	227° 36	132° 55
$[M_1]$	188° 80	103° 33	17° 99	280° 46	194° 64	108° 22	20° 90	280° 26	190° 14	98° 28
M_2	337° 17	78° 60	180° 08	257° 14	358° 45	99° 59	200° 50	276° 78	17° 21	117° 46
M_3	145° 75	117° 91	90° 13	25° 71	357° 68	329° 38	300° 74	235° 17	205° 82	176° 19
M_4	314° 34	157° 21	0° 17	154° 28	356° 90	199° 18	40° 99	193° 56	34° 43	234° 92
M_6	291° 51	235° 81	180° 25	51° 42	355° 36	298° 76	241° 49	110° 34	51° 64	352° 39
M	268° 68	314° 42	0° 34	308° 56	353° 81	38° 35	81° 98	27° 12	68° 86	109° 85
N_2	161° 40	174° 11	186° 87	162° 13	174° 72	187° 14	199° 32	173° 82	185° 53	197° 06
$2 N$	345° 62	269° 61	193° 65	67° 13	351° 00	274° 69	108° 15	70° 86	353° 85	276° 65
O_1	332° 59	70° 77	168° 87	241° 68	340° 08	78° 92	178° 43	253° 56	355° 39	98° 78
OO	216° 56	124° 89	33° 55	329° 25	236° 66	142° 43	45° 71	332° 87	228° 26	118° 58
P_1	350° 07	350° 31	350° 55	349° 80	350° 04	350° 28	350° 52	349° 77	350° 01	350° 25
Q_1	156° 82	166° 28	175° 65	146° 67	156° 35	166° 47	177° 25	150° 60	163° 71	178° 38
R_2	358° 19	357° 93	357° 68	358° 41	358° 15	357° 90	357° 64	358° 37	358° 11	357° 87
S_{11}	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00	180° 00
$S_{21}, 41, 61$	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00	0° 00
T_2	1° 81	2° 07	2° 32	1° 59	1° 85	2° 10	2° 36	1° 63	1° 89	2° 14
λ_2	3° 17	275° 13	187° 13	86° 03	357° 87	269° 53	180° 96	79° 09	350° 05	260° 82
μ_2	315° 39	157° 58	359° 82	153° 24	355° 30	197° 20	38° 86	191° 51	32° 70	233° 70
v_2	131° 17	62° 08	353° 03	248° 25	179° 03	109° 65	40° 03	294° 47	224° 38	154° 10
MK	343° 28	86° 96	190° 79	271° 09	14° 37	116° 99	218° 69	295° 84	35° 09	133° 04
$2 MK$	308° 23	148° 85	349° 46	140° 33	340° 99	181° 77	22° 80	174° 50	16° 56	219° 35
MN	138° 57	252° 71	6° 95	59° 27	173° 18	286° 73	39° 82	90° 60	202° 75	314° 52
MS	337° 17	78° 60	180° 08	257° 14	358° 45	99° 59	200° 50	276° 78	17° 21	117° 46
$2 MS$	314° 34	157° 21	0° 17	154° 28	356° 90	199° 18	40° 99	193° 56	34° 43	234° 92
$2 SM$	22° 83	281° 40	179° 92	102° 86	1° 55	260° 41	159° 50	83° 22	342° 79	242° 54
Mf	211° 08	117° 06	22° 34	313° 78	218° 29	121° 76	23° 64	309° 66	206° 43	99° 90
MSf	22° 83	281° 40	179° 92	102° 86	1° 55	260° 41	159° 50	83° 22	342° 79	242° 54
Mm	175° 77	264° 49	353° 22	95° 01	183° 73	272° 45	1° 17	102° 96	191° 68	280° 40
Sa	279° 93	279° 69	279° 45	280° 20	279° 96	279° 72	279° 48	280° 23	279° 99	279° 75
Ssa	199° 86	199° 38	199° 90	200° 40	199° 92	199° 44	198° 96	200° 46	199° 98	199° 50
Elements.	Values at Greenwich, midnight beginning each year.									
h	279° 93	279° 69	279° 45	280° 20	279° 96	279° 72	279° 48	280° 23	279° 99	279° 75
s	290° 82	60° 20	189° 59	332° 15	101° 53	230° 92	0° 30	142° 86	272° 25	41° 63
p	115° 04	155° 70	196° 37	237° 14	277° 80	318° 47	359° 13	39° 90	80° 57	121° 23
Values at the middle of each year, or for July 2, at Greenwich mean noon for common years and at preceding midnight for leap years.										
p_1	281° 74	281° 76	281° 77	281° 79	281° 81	281° 82	281° 84	281° 86	281° 88	281° 89
P	130° 55	174° 37	218° 33	262° 22	305° 75	348° 76	31° 03	72° 20	111° 87	149° 88
N	29° 27	9° 94	350° 59	331° 23	311° 91	292° 58	273° 22	253° 87	234° 54	215° 21
I	28° 05	28° 54	28° 54	28° 07	27° 15	25° 85	24° 26	22° 54	20° 87	19° 47
Q	149° 70	177° 18	201° 57	254° 71	325° 22	354° 33	16° 75	57° 29	128° 75	163° 82
R	340° 73	353° 02	22° 54	4° 23	343° 38	350° 32	15° 72	6° 63	353° 01	350° 47
ξ	4° 83	1° 67	358° 42	355° 25	352° 39	350° 04	348° 48	348° 04	349° 03	351° 68
v	5° 35	1° 86	358° 24	354° 74	351° 59	349° 05	347° 41	347° 01	348° 17	351° 07
v''	3° 81	1° 33	358° 74	356° 25	354° 04	352° 32	351° 29	351° 17	352° 12	354° 17
$2 v''$	8° 07	2° 82	357° 33	352° 06	347° 50	344° 11	342° 33	342° 55	344° 90	349° 19

The values of the mean longitudes were obtained from formulæ given by Darwin in B. A. A. S. Report, 1883, p. 87:

$$\begin{aligned}
 s &= 150^\circ 04' 19'' + [13 \times 360^\circ + 132^\circ 67' 90''] T + 13^\circ 17' 64'' D + 0^\circ 54' 90'' 165 H, \\
 p &= 240^\circ 63' 22'' + 40^\circ 69' 03'' T + 0^\circ 11' 14'' D + 0^\circ 00' 46'' 18 H, \\
 h &= 280^\circ 52' 87'' + 360^\circ 00' 76'' T + 0^\circ 98' 56'' D + 0^\circ 04' 10'' 86 H, \\
 p_1 &= 280^\circ 87' 48'' + 0^\circ 01' 17'' T + 0^\circ 00' 00'' 47 D, \\
 N &= 285^\circ 95' 69'' - 19^\circ 34' 14'' T - 0^\circ 05' 29'' 10 D,
 \end{aligned}$$

TABLE 3.—Equilibrium arguments ($V_0 + u$) at the midnight preceding January 1 of each year, from 1850 to 1950, for the meridian of Greenwich, together with the elements used in computing them—Continued.

Component.	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949	1950
J_1	23°06	120°18	203°46	287°96	14°18	116°15	205°52	296°00	27°28	133°12	225°17
K_1	12°35	9°62	6°02	3°09	1°19	1°39	1°62	2°68	4°37	7°45	9°75
K_2	204°17	199°37	192°74	187°02	182°90	182°80	182°85	184°79	188°19	194°55	199°47
L_2	44°29	215°18	49°56	248°99	82°00	240°19	64°35	271°82	118°73	270°72	86°64
$[L_2]$	46°69	224°15	52°93	241°80	70°83	248°75	78°23	267°94	97°87	276°64	106°82
M_1	21°99	260°90	173°39	109°54	12°89	251°95	151°20	91°30	14°50	257°36	155°34
$[M_1]$	4°80	258°35	164°19	71°25	340°01	238°41	150°34	63°37	337°20	239°47	154°08
M_2	217°57	293°23	33°29	133°44	233°75	309°88	50°63	151°63	252°83	329°82	71°28
M_3	146°35	79°85	49°93	20°16	350°62	284°82	255°95	227°44	199°25	134°73	106°92
M_4	75°14	226°46	66°58	266°88	107°50	259°76	101°27	303°25	145°67	299°64	142°56
M_5	292°70	159°70	99°87	40°33	341°24	209°65	151°90	94°88	38°50	269°46	213°83
M_6	150°27	92°93	133°16	173°77	214°99	159°53	202°54	246°50	291°34	239°28	285°11
N_2	208°44	182°32	193°65	205°08	216°67	191°01	203°04	215°31	227°80	203°00	215°73
$2N$	199°31	71°40	354°02	276°73	199°59	72°14	355°45	279°00	202°76	76°17	0°19
O_1	203°64	283°97	29°47	133°84	236°65	312°60	52°71	151°94	250°59	323°52	61°65
OO	4°22	274°55	158°19	45°38	297°54	221°96	123°15	27°42	293°89	229°07	137°59
P_1	350°49	349°74	349°98	350°22	350°46	349°71	349°95	350°19	350°43	349°68	349°92
Q_1	194°51	173°06	189°83	205°48	219°57	193°73	205°12	215°63	225°56	196°70	206°11
R_2	357°60	358°33	358°08	357°82	357°57	358°29	358°04	357°78	357°53	358°26	358°00
$S_{1,2}$	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00	180°00
$S_{3,4,6}$	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00	0°00
T_2	2°40	1°67	1°92	2°18	2°43	1°71	1°96	2°22	2°47	1°74	2°00
λ_2	171°45	68°96	339°54	250°22	161°05	59°62	330°30	241°82	153°55	52°38	324°36
μ_2	74°56	226°59	67°40	268°31	109°37	261°87	103°38	305°12	147°08	300°44	142°65
ν_2	83°69	337°51	267°04	196°67	126°45	20°74	310°97	241°43	172°12	67°26	358°19
MK	229°92	302°85	39°31	136°53	234°94	311°27	52°25	154°31	257°20	337°27	81°03
$2MK$	62°78	216°84	60°56	263°79	106°30	258°38	99°65	300°57	141°30	292°19	132°80
MN	66°01	115°55	226°94	338°53	90°41	140°89	253°68	6°94	120°63	272°82	287°01
MS	217°57	293°23	33°29	133°44	233°75	309°88	50°63	151°63	252°83	329°82	71°28
$2MS$	75°14	226°46	66°58	266°88	107°50	259°76	101°27	303°25	145°67	299°64	142°56
$2SM$	142°43	66°77	326°71	226°56	126°25	50°12	309°37	208°37	107°17	30°18	288°72
Mf	350°29	265°29	154°36	45°77	300°44	224°68	125°22	27°74	291°65	222°77	127°97
MSf	142°43	66°77	326°71	226°56	126°25	50°12	309°37	208°37	107°17	30°18	288°72
Mm	9°13	110°91	199°64	288°36	17°08	118°87	207°59	296°31	25°04	126°82	215°55
Sa	279°51	280°26	280°02	279°78	279°54	280°29	280°05	279°81	279°57	280°32	280°08
Ssa	199°02	200°52	200°04	199°56	199°09	200°58	200°10	199°62	199°15	200°64	200°16
Elements.	Values at Greenwich, midnight beginning each year.										
h	279°51	280°26	280°02	279°78	279°54	280°29	280°05	279°81	279°57	280°32	280°08
s	171°02	313°58	82°96	212°35	341°73	124°29	253°68	23°06	152°45	295°01	64°40
p	161°89	202°66	243°33	283°99	324°65	5°43	46°09	86°75	127°41	168°19	208°85
	Values at the middle of each year, or for July 2, at Greenwich mean noon for common years and at preceding midnight for leap years.										
p_1	281°91	281°93	281°94	281°96	281°98	281°99	282°01	282°03	282°05	282°06	282°08
P	186°41	222°06	257°87	294°86	333°53	13°80	55°35	97°89	141°17	184°89	228°77
N	195°86	176°50	157°17	137°85	118°49	99°14	79°81	60°48	41°13	21°77	2°44
I	18°55	18°32	18°81	19°92	21°45	23°17	24°86	26°35	27°53	28°29	28°60
Q	183°21	204°29	246°75	312°82	346°02	7°01	35°89	105°49	158°08	182°45	209°71
R	2°41	8°96	3°36	352°81	348°82	8°56	13°88	356°12	339°14	5°92	20°18
ξ	355°87	0°93	5°78	9°46	11°51	11°95	11°07	9°19	6°63	3°63	0°41
ν	355°58	1°00	6°19	10°17	12°45	13°01	12°12	10°13	7°33	4°03	0°46
ν'	357°16	0°64	4°00	6°69	8°35	8°90	8°43	7°13	5°21	2°87	0°33
$2\nu''$	354°85	1°15	7°30	12°54	16°19	17°78	17°25	14°84	10°96	6°09	0°70

where T is the number of Julian years of 365 $\frac{1}{4}$ mean solar days; D , the number of mean solar days; H , the number of mean solar hours after Greenwich mean noon, January 1, 1880. On account of the slowness of the secular changes in the coefficients of T , D , or H , the epoch of this table may be regarded as 1900. See Hansen's *Tables de la Lune*, p. 15, from which these formulæ may be obtained by putting $t = 80$. Newcomb's corrections (*Washington Observations*, Vol. 22 (1875), App. II, pp. 268, 274) are not of sufficient magnitude to affect the values in Table 3.

TABLE 4.—For adapting the uniformly varying portion (V_0) of the equilibrium arguments of Table 3 to Greenwich midnight, beginning any day throughout the year.

Months.	K_1	K_2	L_2	M_2	M_3	M_4	N_4	N_5	N_2	O_1	P_1	Q_1
Jan. 1	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°	0°
Feb. 1	30°56	61°11	9°19	324°17	306°26	288°35	252°52	216°69	279°16	293°62	329°44	248°60
Mar. 1	Com. yr. 58°15	116°31	52°33	1°49	2°24	2°98	4°48	5°97	310°66	303°34	301°85	252°50
	Leap yr. 59°14	118°28	41°01	337°11	325°66	314°22	291°33	268°44	273°21	277°97	300°86	214°07
Apr. 1	Com. yr. 88°71	177°42	61°54	325°66	308°50	291°33	257°00	222°66	229°82	236°96	271°29	141°11
	Leap yr. 89°69	179°39	50°20	301°28	271°93	242°57	183°85	125°13	192°37	211°59	270°31	102°68
May 1	Com. yr. 118°28	236°56	82°02	314°22	291°33	268°44	222°66	176°88	186°42	195°94	241°72	68°14
	Leap yr. 119°26	238°53	70°70	289°84	254°76	219°68	149°52	79°36	148°98	170°58	240°74	29°71
June 1	Com. yr. 148°83	297°66	91°21	278°39	237°59	196°79	115°18	33°58	105°58	129°56	211°17	316°75
	Leap yr. 149°82	299°64	79°89	254°01	201°02	148°02	42°04	296°05	68°13	104°19	210°18	278°32
July 1	Com. yr. 178°40	356°80	111°71	266°95	220°42	173°90	80°85	347°80	62°18	88°55	181°60	243°78
	Leap yr. 179°39	358°78	100°40	242°57	183°85	125°14	7°70	250°27	24°74	63°18	180°61	205°15
Aug. 1	Com. yr. 208°96	57°91	120°90	231°12	166°68	102°24	333°37	204°49	341°34	22°16	151°04	132°39
	Leap yr. 209°94	59°89	109°58	206°74	130°11	53°48	260°22	106°96	303°90	356°80	150°06	93°96
Sept. 1	Com. yr. 239°51	119°02	130°09	195°30	112°94	30°59	225°89	61°18	260°50	315°78	120°49	20°99
	Leap yr. 240°50	121°00	118°77	170°92	76°37	341°83	152°74	323°66	223°06	290°42	119°50	342°56
Oct. 1	Com. yr. 269°08	178°16	150°59	183°85	95°78	7°70	191°55	15°40	217°11	274°77	90°92	308°03
	Leap yr. 270°07	180°13	139°28	159°47	59°20	318°94	118°41	277°88	179°66	249°40	89°93	269°59
Nov. 1	Com. yr. 299°64	239°27	159°78	148°02	42°04	296°05	84°07	232°10	136°27	208°39	60°36	196°63
	Leap yr. 300°62	241°24	148°47	123°64	5°46	247°29	10°93	134°57	98°82	183°02	59°38	158°20
Dec. 1	Com. yr. 329°21	298°41	180°29	136°58	24°87	273°16	49°74	186°32	92°87	167°37	30°79	123°67
	Leap yr. 330°19	300°38	168°97	112°20	348°30	224°40	336°59	88°79	55°43	142°01	29°81	85°23
Dec. 32	Com. yr. 359°76	359°52	189°48	100°75	331°13	201°51	302°26	43°01	12°03	100°99	0°24	12°27
	Leap yr. 0°75	1°49	178°16	76°37	294°56	152°74	229°11	305°49	334°58	75°62	359°25	333°84
Day of month												
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0°99	1°97	348°68	335°62	323°43	311°24	286°86	262°47	322°55	334°63	— 0°99	321°57
3	1°97	3°94	337°37	311°24	286°86	262°47	213°71	164°95	285°11	309°27	— 1°97	283°14
4	2°96	5°91	326°05	286°86	250°28	213°71	140°57	67°42	247°66	283°90	— 2°96	244°70
5	3°94	7°88	314°73	262°47	213°71	164°95	67°42	329°90	210°21	258°53	— 3°94	206°27
6	4°93	9°86	303°42	238°09	177°14	116°18	354°28	232°37	172°77	233°16	— 4°93	167°84
7	5°91	11°83	292°10	213°71	140°57	67°42	281°13	134°84	135°32	207°80	— 5°91	129°41
8	6°90	13°80	280°78	189°33	103°99	18°66	207°99	37°32	97°88	182°43	— 6°90	90°98
9	7°88	15°77	269°47	164°95	67°42	329°90	134°84	299°79	60°43	157°06	— 7°88	52°54
10	8°87	17°74	258°15	140°57	30°85	281°13	61°70	202°27	22°98	131°70	— 8°87	14°11
11	9°86	19°71	246°84	116°18	354°28	232°37	348°56	104°74	345°54	106°33	— 9°86	335°68
12	10°84	21°68	235°52	91°80	317°70	183°61	275°41	7°21	308°09	80°96	— 10°84	297°25
13	11°83	23°66	224°20	67°42	281°13	134°84	202°27	269°60	270°64	55°59	— 11°83	258°81
14	12°81	25°63	212°88	43°04	244°56	86°08	129°12	172°16	233°20	30°23	— 12°81	220°38
15	13°80	27°60	201°57	18°66	207°99	37°32	55°98	74°64	195°75	4°86	— 13°80	181°95
16	14°78	29°57	190°25	354°28	171°42	348°56	342°83	337°11	158°30	339°49	— 14°78	143°52
17	15°77	31°54	178°94	329°90	134°84	299°79	269°69	239°58	120°86	314°13	— 15°77	105°09
18	16°76	33°51	167°62	305°52	98°27	251°03	196°54	142°06	83°41	288°76	— 16°76	66°65
19	17°74	35°48	156°30	281°13	61°70	202°27	123°40	44°53	45°96	263°39	— 17°74	28°22
20	18°73	37°46	144°99	256°75	25°13	153°50	50°26	307°01	8°52	238°02	— 18°73	349°79
21	19°71	39°43	133°67	232°37	348°56	104°74	337°11	209°48	331°07	212°66	— 19°71	311°36
22	20°70	41°40	122°35	207°99	311°98	55°98	263°97	111°95	293°62	187°29	— 20°70	272°92
23	21°68	43°37	111°04	183°61	275°41	7°21	190°82	14°43	256°18	161°92	— 21°68	234°49
24	22°67	45°34	99°72	159°23	238°84	318°45	117°68	276°90	218°73	136°56	— 22°67	196°06
25	23°66	47°31	88°40	134°84	202°27	269°69	44°53	179°38	181°28	111°19	— 23°66	157°63
26	24°64	49°28	77°09	110°46	165°69	220°92	331°39	81°85	143°84	85°82	— 24°64	119°20
27	25°63	51°25	65°77	86°08	129°12	172°16	258°24	344°32	106°39	60°45	— 25°63	80°76
28	26°61	53°22	54°45	61°70	92°55	123°40	185°10	246°80	68°94	35°09	— 26°61	42°33
29	27°60	55°20	43°14	37°32	55°98	74°64	111°95	149°27	31°50	9°72	— 27°60	3°90
30	28°58	57°17	31°82	12°94	25°87	19°40	38°81	51°75	354°05	344°35	— 28°58	325°47
31	29°57	59°14	20°50	348°56	342°83	337°11	325°66	314°22	316°60	318°99	— 29°57	287°04

The upper line for each month is for common years, and the lower line for leap years. For longitude corrections see Table 5, and for the portion (μ) of the equilibrium arguments of Table 3, which depend upon the longitude of the moon's node, see Tables 6 and 7. The changes for other components may be found from those above, as follows:

$$J_1 = L_2 - O_1 \quad [L_2] = L_2 \quad M_1 = A - s \quad [M_1] = K_1 + \lambda_2 \quad 2N = N_2 + \lambda_2 \quad OO = K_2 - O_1 \quad R_2 = K_1 \quad S_{1,2} = 0 \\ S_{2,4,6} = 0 \quad T_2 = P_1 \quad MN = M_1 + N_2 \quad MS = M_2 \quad 2MS = M_1 \quad 2SM = MSf \quad Sa = K_1 \quad S_{2a} = K_2$$

TABLE 4.—For adapting the uniformly varying portion (V_0) of the equilibrium arguments of Table 3 to Greenwich midnight, beginning any day throughout the year—Continued.

Months.	λ_2	μ_2	ν_2	M K	\pm M K	M S f	M f	M m	A	s	f	N^*
Jan. 1	0	0	0	0	0	0	0	0	0	0	0	{ C.+9°66 L.+9°69
Feb. 1	314°98	288°35	333°36	354°73	257°79	35°83	96°94	45°02	30°56	48°47	3°45	+8°02 +8°05
Mar. 1	{ Com. yr. 309°16 Leap yr. 296°10	{ 2°98 314°22	{ 53°82 18°12	{ 59°64 36°25	{ 304°83 255°08	{ 358°51 22°89	{ 114°82 141°17	{ 50°84 63°90	{ 58°15 59°14	{ 57°41 70°58	{ 6°57 6°68	+6°54 +6°51
Apr. 1	{ Com. yr. 264°15 Leap yr. 251°09	{ 291°33 242°57	{ 27°18 351°48	{ 54°37 30°98	{ 202°62 152°87	{ 34°34 58°72	{ 211°75 238°10	{ 95°85 108°91	{ 88°71 89°69	{ 105°88 119°05	{ 10°03 10°14	+4°90 +4°87
May 1	{ Com. yr. 232°20 Leap yr. 219°14	{ 268°44 219°68	{ 36°24 0°54	{ 72°50 49°10	{ 150°16 100°41	{ 45°78 70°16	{ 282°34 308°69	{ 127°80 140°86	{ 118°28 119°26	{ 141°17 154°34	{ 13°37 13°48	+3°31 +3°28
June 1	{ Com. yr. 187°19 Leap yr. 174°12	{ 196°79 148°02	{ 9°60 333°90	{ 67°23 45°83	{ 47°96 358°21	{ 81°61 105°99	{ 19°27 45°62	{ 172°81 185°88	{ 148°83 149°82	{ 189°64 202°81	{ 16°82 16°93	+1°67 +1°64
July 1	{ Com. yr. 155°24 Leap yr. 142°17	{ 173°90 125°14	{ 18°66 342°96	{ 85°35 61°96	{ 355°50 305°75	{ 93°05 117°43	{ 89°86 116°21	{ 204°76 217°83	{ 178°40 179°39	{ 224°93 238°10	{ 20°16 20°28	+0°08 +0°05
Aug. 1	{ Com. yr. 110°22 Leap yr. 97°16	{ 102°24 53°48	{ 352°02 316°32	{ 80°08 56°68	{ 253°29 203°54	{ 128°88 153°26	{ 186°79 213°14	{ 249°78 262°84	{ 208°96 209°94	{ 273°40 286°57	{ 23°62 23°73	-1°56 -1°59
Sept. 1	{ Com. yr. 65°21 Leap yr. 52°14	{ 30°59 341°83	{ 325°38 289°69	{ 74°81 51°41	{ 151°08 101°33	{ 164°70 189°08	{ 283°73 310°08	{ 294°79 307°86	{ 239°51 240°50	{ 321°86 335°04	{ 27°07 27°18	-3°20 -3°23
Oct. 1	{ Com. yr. 33°26 Leap yr. 20°19	{ 7°70 318°94	{ 334°44 298°75	{ 92°93 69°54	{ 98°62 48°88	{ 176°15 200°53	{ 354°31 20°66	{ 326°74 339°81	{ 269°08 270°07	{ 357°16 10°33	{ 30°41 30°52	-4°79 -4°82
Nov. 1	{ Com. yr. 348°24 Leap yr. 335°18	{ 296°05 247°29	{ 307°81 272°11	{ 87°66 64°26	{ 356°41 306°66	{ 211°98 236°36	{ 91°25 117°60	{ 11°76 24°82	{ 299°64 300°62	{ 45°62 58°80	{ 33°87 33°98	-6°43 -6°46
Dec. 1	{ Com. yr. 316°29 Leap yr. 303°23	{ 273°16 224°40	{ 316°87 281°17	{ 105°79 82°39	{ 303°95 254°20	{ 223°42 247°80	{ 161°83 188°19	{ 43°71 56°77	{ 329°21 330°19	{ 80°92 94°09	{ 37°21 37°32	-8°02 -8°05
Dec. 32	{ Com. yr. 271°28 Leap yr. 258°21	{ 201°51 152°74	{ 290°23 254°53	{ 100°51 77°12	{ 201°74 152°00	{ 259°25 283°63	{ 258°77 285°12	{ 88°72 101°79	{ 359°76 0°75	{ 129°38 142°56	{ 40°66 40°77	-9°66 -9°69
Day of month.	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0
2	346°93	311°24	324°30	336°60	310°25	24°38	26°35	13°07	0°99	13°18	0°11	-0°05
3	333°87	262°47	288°60	313°21	260°50	48°76	52°71	26°13	1°97	26°35	0°22	-0°11
4	320°80	213°71	252°91	289°81	210°75	73°14	79°06	39°20	2°96	39°53	0°33	-0°16
5	307°74	164°95	217°21	266°42	161°00	97°53	105°41	52°26	3°94	52°71	0°45	-0°21
6	294°68	116°18	181°51	243°02	111°26	121°91	131°76	65°32	4°93	65°88	0°56	-0°26
7	281°61	67°42	145°81	219°62	61°51	146°29	158°12	78°39	5°91	79°06	0°67	-0°32
8	268°54	18°66	110°11	196°23	11°76	170°67	184°47	91°46	6°90	92°23	0°78	-0°37
9	255°48	329°90	74°42	172°83	322°01	195°05	210°82	104°52	7°88	105°41	0°89	-0°42
10	242°42	281°13	38°72	149°44	272°26	219°43	237°18	117°58	8°87	118°59	1°00	-0°48
11	229°35	232°37	3°02	126°04	222°51	243°82	263°53	130°65	9°86	131°76	1°11	-0°53
12	216°28	183°61	327°32	102°65	172°76	268°20	289°88	143°72	10°84	144°94	1°23	-0°58
13	203°22	134°84	291°62	79°25	123°02	292°58	316°23	156°78	11°83	158°12	1°34	-0°64
14	190°16	86°08	255°93	55°85	73°27	316°96	342°59	169°84	12°81	171°29	1°45	-0°69
15	177°09	37°32	220°23	32°46	23°52	341°34	8°94	182°91	13°80	184°47	1°56	-0°74
16	164°02	348°56	184°53	9°06	333°77	5°72	35°29	195°98	14°78	197°65	1°67	-0°79
17	150°96	299°79	148°83	345°67	284°02	30°10	61°64	209°04	15°77	210°82	1°78	-0°85
18	137°90	251°03	113°13	322°27	234°27	54°48	88°00	222°10	16°76	224°00	1°89	-0°90
19	124°83	202°27	77°44	298°88	184°52	78°87	114°35	235°17	17°74	237°18	2°01	-0°95
20	111°76	153°50	41°74	275°48	134°78	103°25	140°70	248°24	18°73	250°35	2°12	-1°01
21	98°70	104°74	6°04	252°08	85°03	127°63	167°06	261°30	19°71	263°53	2°23	-1°06
22	85°64	55°98	330°34	228°69	35°28	152°01	193°41	274°36	20°70	276°70	2°34	-1°11
23	72°57	7°21	291°64	205°29	345°53	176°39	219°76	287°43	21°68	289°88	2°45	-1°16
24	59°50	318°45	258°95	181°90	295°78	200°77	246°11	300°50	22°67	303°06	2°56	-1°22
25	46°44	269°69	223°25	158°50	246°03	225°16	272°47	313°56	23°66	316°23	2°67	-1°27
26	33°38	220°92	187°55	135°10	196°28	249°54	298°82	326°62	24°64	329°41	2°79	-1°32
27	20°31	172°16	151°85	111°71	146°54	273°92	325°17	339°69	25°63	342°59	2°90	-1°38
28	7°24	123°40	116°15	88°31	96°79	298°30	351°52	352°76	26°61	355°76	3°01	-1°43
29	354°18	74°64	80°46	64°92	47°04	322°68	17°88	5°82	27°60	8°94	3°12	-1°48
30	341°12	25°87	44°76	41°52	357°29	347°06	44°23	18°88	28°58	22°12	3°23	-1°54
31	328°05	337°11	9°06	18°12	307°54	11°44	70°58	31°95	29°57	35°29	3°34	-1°59

* This column gives the longitude of the moon's ascending node for any day when applied to the N of Table 3.

The change in the dial reading of any component (except M_2 and S_a) on the Ferrel machine = \pm (tabular value for component - tabular value for M_2), the upper sign being used when the speed of the component is greater than that of M_2 . The changes for M_2 and S_a are given directly by this table.

TABLE 5.—For adapting the uniformly varying portion (V_0) of the equilibrium arguments of Table 3 to local midnight for any degree of west longitude.

West longitude.	K_1	K_2	L_2	M_2	M_3	M_4	M_5	M_6	N_2	O_1	P_1	Q_1
$\frac{1}{4}$	0°00	0°00	— 0°01	— 0°02	— 0°03	— 0°03	— 0°05	— 0°07	— 0°03	— 0°02	0°00	— 0°03
$\frac{1}{2}$	0°00	0°00	— 0°02	— 0°03	— 0°05	— 0°07	— 0°10	— 0°14	— 0°05	— 0°04	0°00	— 0°05
$\frac{3}{4}$	0°00	0°00	— 0°02	— 0°05	— 0°08	— 0°10	— 0°15	— 0°20	— 0°08	— 0°05	0°00	— 0°08
1	0°00	0°01	— 0°03	— 0°07	— 0°10	— 0°14	— 0°21	— 0°27	— 0°10	— 0°07	0°00	— 0°11
2	0°01	0°01	— 0°06	— 0°14	— 0°20	— 0°27	— 0°41	— 0°54	— 0°21	— 0°14	— 0°01	— 0°21
3	0°01	0°02	— 0°09	— 0°20	— 0°30	— 0°41	— 0°61	— 0°81	— 0°31	— 0°21	— 0°01	— 0°32
4	0°01	0°02	— 0°13	— 0°27	— 0°41	— 0°54	— 0°81	— 1°08	— 0°42	— 0°28	— 0°01	— 0°43
5	0°01	0°03	— 0°16	— 0°34	— 0°51	— 0°68	— 1°02	— 1°35	— 0°52	— 0°35	— 0°01	— 0°53
6	0°02	0°03	— 0°19	— 0°41	— 0°61	— 0°81	— 1°22	— 1°63	— 0°62	— 0°42	— 0°02	— 0°64
7	0°02	0°04	— 0°22	— 0°47	— 0°71	— 0°95	— 1°42	— 1°90	— 0°73	— 0°49	— 0°02	— 0°75
8	0°02	0°04	— 0°25	— 0°54	— 0°81	— 1°08	— 1°63	— 2°17	— 0°83	— 0°56	— 0°02	— 0°85
9	0°02	0°05	— 0°28	— 0°61	— 0°91	— 1°22	— 1°83	— 2°44	— 0°94	— 0°63	— 0°02	— 0°96
10	0°03	0°05	— 0°31	— 0°68	— 1°02	— 1°35	— 2°03	— 2°71	— 1°04	— 0°70	— 0°03	— 1°07
20	0°05	0°11	— 0°63	— 1°35	— 2°03	— 2°71	— 4°06	— 5°42	— 2°08	— 1°41	— 0°05	— 2°14
30	0°08	0°16	— 0°94	— 2°03	— 3°05	— 4°06	— 6°10	— 8°13	— 3°12	— 2°11	— 0°08	— 3°20
40	0°11	0°22	— 1°26	— 2°71	— 4°06	— 5°42	— 8°13	— 10°84	— 4°16	— 2°82	— 0°11	— 4°27
50	0°14	0°27	— 1°57	— 3°39	— 5°08	— 6°77	— 10°16	— 13°55	— 5°20	— 3°52	— 0°14	— 5°34
60	0°16	0°33	— 1°89	— 4°06	— 6°10	— 8°13	— 12°19	— 16°25	— 6°24	— 4°23	— 0°16	— 6°41
70	0°19	0°38	— 2°20	— 4°74	— 7°11	— 9°48	— 14°22	— 18°96	— 7°28	— 4°93	— 0°19	— 7°47
80	0°22	0°44	— 2°51	— 5°42	— 8°13	— 10°84	— 16°25	— 21°67	— 8°32	— 5°64	— 0°22	— 8°54
90	0°25	0°49	— 2°83	— 6°10	— 9°14	— 12°19	— 18°29	— 24°38	— 9°36	— 6°34	— 0°25	— 9°61
100	0°27	0°55	— 3°14	— 6°77	— 10°16	— 13°55	— 20°32	— 27°09	— 10°40	— 7°05	— 0°27	— 10°68
110	0°30	0°60	— 3°46	— 7°45	— 11°17	— 14°90	— 22°35	— 29°80	— 11°44	— 7°75	— 0°30	— 11°74
120	0°33	0°66	— 3°77	— 8°13	— 12°19	— 16°25	— 24°38	— 32°51	— 12°48	— 8°46	— 0°33	— 12°81
130	0°36	0°71	— 4°09	— 8°80	— 13°21	— 17°61	— 26°41	— 35°22	— 13°52	— 9°16	— 0°36	— 13°88
140	0°38	0°77	— 4°40	— 9°48	— 14°22	— 18°96	— 28°45	— 37°93	— 14°56	— 9°87	— 0°38	— 14°95
150	0°41	0°82	— 4°72	— 10°16	— 15°24	— 20°32	— 30°48	— 40°64	— 15°60	— 10°57	— 0°41	— 16°01
160	0°44	0°88	— 5°03	— 10°84	— 16°25	— 21°67	— 32°51	— 43°34	— 16°64	— 11°27	— 0°44	— 17°08
170	0°47	0°93	— 5°34	— 11°51	— 17°27	— 23°03	— 34°54	— 46°05	— 17°58	— 11°98	— 0°47	— 18°15
180	0°49	0°99	— 5°66	— 12°19	— 18°29	— 24°38	— 36°57	— 48°76	— 18°72	— 12°68	— 0°49	— 19°22
190	0°52	1°04	— 5°97	— 12°87	— 19°30	— 25°74	— 38°60	— 51°47	— 19°76	— 13°39	— 0°52	— 20°28
200	0°55	1°10	— 6°29	— 13°55	— 20°32	— 27°09	— 40°64	— 54°18	— 20°80	— 14°09	— 0°55	— 21°35
210	0°57	1°15	— 6°60	— 14°22	— 21°33	— 28°45	— 42°67	— 56°89	— 21°84	— 14°80	— 0°57	— 22°42
220	0°60	1°20	— 6°92	— 14°90	— 22°35	— 29°80	— 44°70	— 59°60	— 22°88	— 15°50	— 0°60	— 23°49
230	0°63	1°26	— 7°23	— 15°58	— 23°37	— 31°15	— 46°73	— 62°31	— 23°92	— 16°21	— 0°63	— 24°55
240	0°66	1°31	— 7°54	— 16°25	— 24°38	— 32°51	— 48°76	— 65°02	— 24°96	— 16°91	— 0°66	— 25°62
250	0°68	1°37	— 7°86	— 16°93	— 25°40	— 33°86	— 50°79	— 67°73	— 26°00	— 17°62	— 0°68	— 26°69
260	0°71	1°42	— 8°17	— 17°61	— 26°41	— 35°22	— 52°83	— 70°44	— 27°04	— 18°32	— 0°71	— 27°76
270	0°74	1°48	— 8°49	— 18°29	— 27°43	— 36°57	— 54°86	— 73°14	— 28°08	— 19°03	— 0°74	— 28°82
280	0°77	1°53	— 8°80	— 18°96	— 28°45	— 37°93	— 56°89	— 75°85	— 29°13	— 19°73	— 0°77	— 29°89
290	0°79	1°59	— 9°12	— 19°64	— 29°46	— 39°28	— 58°92	— 78°56	— 30°17	— 20°43	— 0°79	— 30°96
300	0°82	1°64	— 9°43	— 20°32	— 30°48	— 40°64	— 60°95	— 81°27	— 31°21	— 21°14	— 0°82	— 32°03
310	0°85	1°70	— 9°74	— 21°00	— 31°49	— 41°99	— 62°99	— 83°98	— 32°25	— 21°84	— 0°85	— 33°09
320	0°88	1°75	— 10°06	— 21°67	— 32°51	— 43°34	— 65°02	— 86°69	— 33°29	— 22°55	— 0°88	— 34°16
330	0°90	1°81	— 10°37	— 22°35	— 33°52	— 44°70	— 67°05	— 89°40	— 34°33	— 23°25	— 0°90	— 35°23
340	0°93	1°86	— 10°69	— 23°03	— 34°54	— 46°05	— 69°08	— 92°11	— 35°37	— 23°96	— 0°93	— 36°30
350	0°96	1°92	— 11°00	— 23°70	— 35°56	— 47°41	— 71°11	— 94°82	— 36°41	— 24°66	— 0°96	— 37°36
360	0°99	1°97	— 11°32	— 24°38	— 36°57	— 48°76	— 73°14	— 97°53	— 37°45	— 25°37	— 0°99	— 38°43

The changes for other components may be found from those above as follows:

$$\begin{array}{l}
 J_1 = L_2 - O_1 \quad [L_2] = L_2 \quad M_1 = h - s \quad [M_1] = K_1 + \lambda_2 \quad 2N = N_2 + \lambda_2 \quad OO = K_2 - O_1 \quad R_2 = K_1 \quad S_1, s = 0 \\
 S_2, 4, 6 = 0 \quad T_2 = P_1 \quad MN = M_2 + N_2 \quad MS = M_3 \quad 2MS = M_4 \quad 2SM = MSf \quad Sa = K_1 \quad Ssa = K_2
 \end{array}$$

TABLE 5.—For adapting the uniformly varying portion (V_0) of the equilibrium arguments of Table 3 to local midnight for any degree of west longitude—Continued.

West longitude.	λ_1	μ_1	ν_1	MK	${}_2$ MK	MSf	Mf	Mm	k	s	p	N
$\frac{1}{4}$	— 0°01	— 0°03	— 0°02	— 0°02	— 0°03	0°02	0°02	0°01	0°00	0°01	0°00	0°00
$\frac{1}{2}$	— 0°02	— 0°07	— 0°05	— 0°03	— 0°07	0°03	0°04	0°02	0°00	0°02	0°00	0°00
$\frac{3}{4}$	— 0°03	— 0°10	— 0°07	— 0°05	— 0°10	0°05	0°05	0°03	0°00	0°03	0°00	0°00
1	— 0°04	— 0°14	— 0°10	— 0°06	— 0°14	0°07	0°07	0°04	0°00	0°04	0°00	0°00
2	— 0°07	— 0°27	— 0°20	— 0°13	— 0°28	0°14	0°15	0°07	0°01	0°07	0°00	0°00
3	— 0°11	— 0°41	— 0°30	— 0°19	— 0°41	0°20	0°22	0°11	0°01	0°11	0°00	0°00
4	— 0°15	— 0°54	— 0°40	— 0°26	— 0°55	0°27	0°29	0°15	0°01	0°15	0°00	0°00
5	— 0°18	— 0°68	— 0°50	— 0°32	— 0°69	0°34	0°37	0°18	0°01	0°18	0°00	0°00
6	— 0°22	— 0°81	— 0°59	— 0°39	— 0°83	0°41	0°44	0°22	0°02	0°22	0°00	0°00
7	— 0°25	— 0°95	— 0°69	— 0°45	— 0°97	0°47	0°51	0°25	0°02	0°26	0°00	0°00
8	— 0°29	— 1°08	— 0°79	— 0°52	— 1°11	0°54	0°59	0°29	0°02	0°29	0°00	0°00
9	— 0°33	— 1°22	— 0°89	— 0°58	— 1°24	0°61	0°66	0°33	0°02	0°33	0°00	0°00
10	— 0°36	— 1°35	— 0°99	— 0°65	— 1°38	0°68	0°73	0°36	0°03	0°37	0°00	0°00
20	— 0°73	— 2°71	— 1°98	— 1°30	— 2°76	1°35	1°46	0°73	0°05	0°73	0°01	0°00
30	— 1°09	— 4°06	— 2°97	— 1°95	— 4°15	2°03	2°20	1°09	0°08	1°10	0°01	0°00
40	— 1°45	— 5°42	— 3°97	— 2°60	— 5°53	2°71	2°93	1°45	0°11	1°46	0°01	— 0°01
50	— 1°81	— 6°77	— 4°96	— 3°25	— 6°91	3°39	3°66	1°81	0°14	1°83	0°02	— 0°01
60	— 2°18	— 8°13	— 5°95	— 3°90	— 8°29	4°06	4°39	2°18	0°16	2°20	0°02	— 0°01
70	— 2°54	— 9°48	— 6°94	— 4°55	— 9°67	4°74	5°12	2°54	0°19	2°56	0°02	— 0°01
80	— 2°90	— 10°84	— 7°93	— 5°20	— 11°06	5°42	5°86	2°90	0°22	2°93	0°02	— 0°01
90	— 3°27	— 12°19	— 8°92	— 5°85	— 12°44	6°10	6°59	3°27	0°25	3°29	0°03	— 0°01
100	— 3°63	— 13°55	— 9°92	— 6°50	— 13°82	6°77	7°32	3°63	0°27	3°66	0°03	— 0°01
110	— 3°99	— 14°90	— 10°91	— 7°15	— 15°20	7°45	8°05	3°99	0°30	4°03	0°03	— 0°02
120	— 4°36	— 16°25	— 11°90	— 7°80	— 16°58	8°13	8°78	4°36	0°33	4°39	0°04	— 0°02
130	— 4°72	— 17°61	— 12°89	— 8°45	— 17°96	8°80	9°52	4°72	0°36	4°76	0°04	— 0°02
140	— 5°08	— 18°96	— 13°88	— 9°10	— 19°35	9°48	10°25	5°08	0°38	5°12	0°04	— 0°02
150	— 5°44	— 20°32	— 14°87	— 9°75	— 20°73	10°16	10°98	5°44	0°41	5°49	0°05	— 0°02
160	— 5°81	— 21°67	— 15°87	— 10°40	— 22°11	10°84	11°71	5°81	0°44	5°86	0°05	— 0°02
170	— 6°17	— 23°03	— 16°86	— 11°05	— 23°49	11°51	12°44	6°17	0°47	6°22	0°05	— 0°03
180	— 6°53	— 24°38	— 17°85	— 11°70	— 24°87	12°19	13°18	6°53	0°49	6°59	0°06	— 0°03
190	— 6°90	— 25°74	— 18°84	— 12°35	— 26°26	12°87	13°91	6°90	0°52	6°95	0°06	— 0°03
200	— 7°26	— 27°09	— 19°83	— 13°00	— 27°64	13°55	14°64	7°26	0°55	7°32	0°06	— 0°03
210	— 7°62	— 28°45	— 20°82	— 13°65	— 29°02	14°22	15°37	7°62	0°57	7°69	0°06	— 0°03
220	— 7°98	— 29°80	— 21°82	— 14°30	— 30°40	14°99	16°10	7°98	0°60	8°05	0°07	— 0°03
230	— 8°35	— 31°15	— 22°81	— 14°95	— 31°78	15°58	16°84	8°35	0°63	8°42	0°07	— 0°03
240	— 8°71	— 32°51	— 23°80	— 15°60	— 33°17	16°25	17°57	8°71	0°66	8°78	0°07	— 0°04
250	— 9°07	— 33°86	— 24°79	— 16°25	— 34°55	16°93	18°30	9°07	0°68	9°15	0°08	— 0°04
260	— 9°44	— 35°22	— 25°78	— 16°90	— 35°93	17°61	19°03	9°44	0°71	9°52	0°08	— 0°04
270	— 9°80	— 36°57	— 26°77	— 17°55	— 37°31	18°29	19°76	9°80	0°74	9°88	0°08	— 0°04
280	— 10°16	— 37°93	— 27°77	— 18°20	— 38°69	18°96	20°50	10°16	0°77	10°25	0°09	— 0°04
290	— 10°52	— 39°28	— 28°76	— 18°85	— 40°08	19°64	21°23	10°52	0°79	10°61	0°09	— 0°04
300	— 10°89	— 40°64	— 29°75	— 19°50	— 41°46	20°32	21°96	10°89	0°82	10°98	0°09	— 0°04
310	— 11°25	— 41°99	— 30°74	— 20°15	— 42°84	21°00	22°69	11°25	0°85	11°35	0°10	— 0°05
320	— 11°61	— 43°34	— 31°73	— 20°80	— 44°22	21°67	23°42	11°61	0°88	11°71	0°10	— 0°05
330	— 11°98	— 44°70	— 32°72	— 21°45	— 45°60	22°35	24°16	11°98	0°90	12°08	0°10	— 0°05
340	— 12°34	— 46°05	— 33°71	— 22°10	— 46°98	23°03	24°89	12°34	0°93	12°44	0°11	— 0°05
350	— 12°70	— 47°41	— 34°71	— 22°75	— 48°37	23°70	25°62	12°70	0°96	12°81	0°11	— 0°05
360	— 13°07	— 48°76	— 35°70	— 23°40	— 49°75	24°38	26°35	13°07	0°99	13°18	0°11	— 0°05

TABLE 6.—Values of N , I , and P for Greenwich midnight, beginning each month, from 1850 to 1949.

Month.	N	I	P	N	I	P	N	I	P	N	I	P
	1850			1855			1860			1865		
Jan. 1	146°201	19°379	91°861	49°506	27°066	295°540	312°812	27°201	154°257	216°065	19°518	358°775
Feb. 1	144°559	19°479	95°015	47°865	27°162	299°217	311°171	27°105	157°936	214°424	19°416	1°933
Mar. 1	143°076	19°572	97°875	46°382	27°247	302°542	309°635	27°014	161°374	212°941	19°329	4°777
Apr. 1	141°435	19°679	101°049	44°741	27°339	306°226	307°994	26°913	165°045	211°300	19°234	7°913
May 1	139°846	19°785	104°135	43°152	27°424	309°794	306°405	26°813	168°594	209°711	19°146	10°938
June 1	138°205	19°899	107°335	41°510	27°509	313°485	304°763	26°707	172°258	208°069	19°060	14°054
July 1	136°616	20°011	110°442	39°922	27°589	317°059	303°175	26°603	175°797	206°481	18°980	17°061
Aug. 1	134°974	20°130	113°664	38°280	27°660	320°756	301°533	26°491	179°452	204°839	18°902	20°157
Sept. 1	133°333	20°253	116°900	36°639	27°746	324°456	299°892	26°378	183°102	203°198	18°828	23°246
Oct. 1	131°744	20°373	120°041	35°050	27°817	328°038	298°303	26°266	186°629	201°609	18°761	26°223
Nov. 1	130°103	20°500	123°301	33°408	27°888	331°744	296°661	26°149	190°269	199°967	18°666	29°294
Dec. 1	128°514	20°625	126°468	31°820	27°952	335°331	295°073	26°032	193°787	198°379	18°638	32°258
	1851			1856			1861			1866		
Jan. 1	126°872	20°755	129°753	30°178	28°017	339°040	293°431	25°911	197°415	196°737	18°583	35°312
Feb. 1	125°231	20°889	133°047	28°537	28°078	342°752	291°790	25°787	201°038	195°096	18°532	38°360
Mar. 1	123°748	21°011	136°034	27°001	28°132	346°226	290°307	25°673	204°309	193°613	18°490	41°108
Apr. 1	122°107	21°148	139°353	25°360	28°187	349°941	288°666	25°545	207°919	191°972	18°449	44°144
May 1	120°518	21°281	142°575	23°771	28°237	353°539	287°077	25°419	211°410	190°383	18°415	47°078
June 1	118°876	21°421	145°918	22°129	28°285	357°258	285°435	25°288	215°011	188°741	18°383	50°106
July 1	117°288	21°558	149°164	20°541	28°329	0°858	283°847	25°159	218°489	187°153	18°359	53°031
Aug. 1	115°646	21°700	152°528	18°899	28°370	4°580	282°205	25°025	222°076	185°511	18°338	56°051
Sept. 1	114°005	21°845	155°905	17°258	28°408	8°303	280°564	24°888	225°656	183°870	18°323	59°070
Oct. 1	112°416	21°984	159°183	15°669	28°443	11°909	278°975	24°755	229°115	182°281	18°314	61°089
Nov. 1	110°774	22°129	162°582	14°027	28°474	15°633	277°333	24°617	232°682	180°639	18°309	65°004
Dec. 1	109°186	22°270	165°882	12°439	28°501	19°239	275°745	24°481	236°126	179°051	18°309	67°923
	1852			1857			1862			1867		
Jan. 1	107°544	22°416	169°301	10°797	28°526	22°968	274°103	24°340	239°678	177°409	18°315	70°939
Feb. 1	105°903	22°563	172°732	9°156	28°547	26°695	272°462	24°197	243°221	175°768	18°326	74°955
Mar. 1	104°367	22°701	175°951	7°673	28°563	30°064	270°979	24°068	246°415	174°285	18°341	76°682
Apr. 1	102°726	22°848	179°401	6°032	28°578	33°793	269°338	23°924	249°943	172°644	18°362	79°676
May 1	101°137	22°991	182°750	4°443	28°589	37°402	267°749	23°783	253°349	171°054	18°387	82°628
June 1	99°495	23°137	186°221	2°801	28°597	41°132	266°107	23°638	256°860	169°413	18°419	85°657
July 1	97°907	23°279	189°588	1°213	28°601	44°741	264°519	23°497	260°249	167°825	18°454	88°591
Aug. 1	96°265	23°426	193°076	359°571	28°602	48°471	262°877	23°350	263°742	166°183	18°496	91°628
Sept. 1	94°624	23°573	196°574	357°930	28°599	52°202	261°236	23°203	267°226	164°542	18°542	94°671
Oct. 1	93°035	23°714	199°968	356°341	28°593	55°842	259°647	23°060	270°589	162°953	18°592	97°623
Nov. 1	91°393	23°859	203°484	354°699	28°583	59°542	258°005	22°914	274°054	161°311	18°649	100°679
Dec. 1	89°805	23°999	206°895	353°111	28°571	63°150	256°417	22°771	277°398	159°723	18°707	103°644
	1853			1858			1863			1868		
Jan. 1	88°163	24°143	210°426	351°460	28°555	66°879	254°774	22°624	280°842	158°080	18°774	106°715
Feb. 1	86°522	24°285	213°966	349°828	28°535	70°607	253°133	22°477	284°277	156°439	18°844	109°795
Mar. 1	85°039	24°414	217°172	348°345	28°514	73°975	251°650	22°345	287°371	154°903	18°914	112°686
Apr. 1	83°397	24°555	220°727	346°704	28°487	77°701	250°009	22°199	290°785	153°262	18°992	115°783
May 1	81°809	24°689	224°174	345°115	28°458	81°307	248°420	22°058	294°079	151°673	19°073	118°791
June 1	80°167	24°827	227°746	343°473	28°424	85°032	246°778	21°913	297°473	150°031	19°160	121°908
July 1	78°578	24°960	231°207	341°885	28°389	88°636	245°190	21°774	300°746	148°443	19°248	124°935
Aug. 1	76°937	25°095	234°790	340°243	28°349	92°358	243°548	21°631	304°117	146°801	19°343	128°074
Sept. 1	75°295	25°229	238°382	338°602	28°306	96°079	241°907	21°489	307°476	145°160	19°442	131°224
Oct. 1	73°707	25°357	241°863	337°013	28°260	99°679	240°318	21°353	310°716	143°571	19°541	134°283
Nov. 1	72°065	25°488	245°467	335°371	28°211	103°398	238°676	21°214	314°052	141°929	19°647	137°456
Dec. 1	70°476	25°612	248°960	333°783	28°159	106°994	237°088	21°081	317°270	140°341	19°752	140°538
	1854			1859			1864			1869		
Jan. 1	68°834	25°746	252°587	332°140	28°102	110°709	235°446	20°945	320°582	138°699	19°864	143°733
Feb. 1	67°193	25°864	256°198	330°499	28°043	114°421	233°805	20°810	323°883	137°058	19°980	146°940
Mar. 1	65°710	25°974	259°475	329°016	27°986	117°773	232°269	20°687	326°960	135°575	20°087	149°848
Apr. 1	64°069	26°096	263°107	327°375	27°920	121°481	230°628	20°556	330°237	133°934	20°208	153°078
May 1	62°480	26°210	266°626	325°786	27°853	125°067	229°039	20°433	333°397	132°345	20°328	156°216
June 1	60°838	26°328	270°281	324°144	27°781	128°771	227°397	20°307	336°650	130°703	20°453	159°472
July 1	59°250	26°438	273°799	322°556	27°709	132°353	225°809	20°189	339°786	129°115	20°577	162°633
Aug. 1	57°608	26°550	277°451	320°914	27°629	136°050	224°167	20°068	343°015	127°473	20°707	165°911
Sept. 1	55°967	26°659	281°107	319°273	27°549	139°746	222°526	19°951	346°232	125°832	20°840	169°205
Oct. 1	54°378	26°763	284°651	317°684	27°468	143°319	220°937	19°840	349°332	124°243	20°970	172°399
Nov. 1	52°736	26°867	288°316	316°042	27°380	147°008	219°295	19°728	352°526	122°601	21°106	175°715
Dec. 1	51°148	26°966	291°867	314°454	27°293	150°575	217°707	19°623	355°605	121°013	21°240	178°934

N = the mean longitude of the moon's ascending node. I = the inclination of the lunar orbit to the plane of the earth's equator. P = the mean longitude of the lunar perigee measured from the intersection of moon's orbit with the plane of the earth's equator.

TABLE 6.—Values of *N*, *I*, and *P* for Greenwich midnight, beginning each month, from 1850 to 1949—Continued.

Month.	<i>N</i>	<i>I</i>	<i>P</i>	<i>N</i>	<i>I</i>	<i>P</i>	<i>N</i>	<i>I</i>	<i>P</i>	<i>N</i>	<i>I</i>	<i>P</i>
	1870			1875			1880			1885		
Jan. 1	119°371	21°379	182°274	22°677	28°270	33°381	285°983	25°332	251°173	189°236	18°393	86°557
Feb. 1	117°730	21°520	185°623	21°036	28°316	37°100	284°342	25°200	254°770	187°595	18°366	89°582
Mar. 1	116°247	21°649	188°660	19°553	28°355	40°462	282°806	25°074	258°128	186°112	18°345	92°312
Apr. 1	114°606	21°793	192°032	17°912	28°394	44°184	281°165	24°939	261°710	184°471	18°327	95°329
May 1	113°017	21°930	195°306	16°323	28°429	47°788	279°576	24°806	265°171	182°882	18°317	98°249
June 1	111°375	22°076	198°703	14°681	28°462	51°514	277°934	24°667	268°741	181°240	18°309	101°265
July 1	109°787	22°217	201°997	13°093	28°490	55°120	276°346	24°532	272°188	179°652	18°309	104°183
Aug. 1	108°145	22°363	205°414	11°451	28°517	58°846	274°704	24°392	275°742	178°010	18°312	107°198
Sept. 1	106°504	22°510	208°839	9°810	28°539	62°574	273°063	24°249	279°289	176°369	18°321	110°214
Oct. 1	104°915	22°652	212°166	8°221	28°558	66°183	271°474	24°111	282°711	174°780	18°335	113°135
Nov. 1	103°273	22°798	215°613	6°579	28°573	69°913	269°832	23°967	286°244	173°138	18°355	116°155
Dec. 1	101°685	22°941	218°958	4°991	28°585	73°521	268°244	23°828	289°652	171°550	18°379	119°080
	1871			1876			1881			1886		
Jan. 1	100°043	23°088	222°425	3°349	28°595	77°249	266°602	23°682	293°166	169°908	18°409	122°107
Feb. 1	98°402	23°236	225°901	1°708	28°600	80°981	264°961	23°536	296°671	168°267	18°444	125°138
Mar. 1	96°919	23°368	229°050	0°172	28°602	84°471	263°478	23°403	299°829	166°784	18°480	128°881
Apr. 1	95°278	23°515	232°543	358°531	28°600	88°203	261°837	23°257	303°316	165°143	18°525	130°920
May 1	93°689	23°656	235°934	356°942	28°596	91°811	260°248	23°114	306°682	163°554	18°574	133°867
June 1	92°047	23°802	239°446	355°300	28°587	95°541	258°606	22°968	310°151	161°912	18°627	136°923
July 1	90°459	23°942	242°853	353°712	28°576	99°149	257°018	22°823	313°498	160°324	18°685	139°886
Aug. 1	88°817	24°086	246°382	352°070	28°561	102°878	255°376	22°678	316°946	158°682	18°749	142°954
Sept. 1	87°176	24°229	249°920	350°429	28°542	106°608	253°735	22°531	320°386	157°041	18°817	146°031
Oct. 1	85°587	24°367	253°350	348°840	28°521	110°215	252°146	22°389	323°704	155°452	18°888	149°017
Nov. 1	83°945	24°508	256°904	347°198	28°495	113°943	250°504	22°242	327°123	153°810	18°965	152°113
Dec. 1	82°357	24°643	260°349	345°610	28°467	117°549	248°916	22°102	330°419	152°222	19°044	155°116
	1872			1877			1882			1887		
Jan. 1	80°715	24°782	263°916	343°967	28°435	121°273	247°273	21°956	333°815	150°579	19°131	158°230
Feb. 1	79°074	24°919	267°492	342°326	28°399	124°997	245°632	21°813	337°200	148°938	19°220	161°355
Mar. 1	77°538	25°046	271°843	340°843	28°364	128°361	244°149	21°683	340°249	147°455	19°305	164°187
Apr. 1	75°897	25°180	274°431	339°202	28°322	132°082	242°508	21°541	343°612	145°814	19°463	167°332
May 1	74°308	25°309	277°910	337°613	28°278	135°682	240°919	21°404	346°856	144°225	19°500	170°387
June 1	72°666	25°440	281°512	335°971	28°229	139°401	239°277	21°264	350°197	142°583	19°604	173°554
July 1	71°078	25°565	285°003	334°383	28°179	142°998	237°689	21°131	353°418	140°995	19°708	176°631
Aug. 1	69°436	25°693	288°615	332°741	28°123	146°708	236°047	20°994	356°735	139°353	19°819	179°822
Sept. 1	67°795	25°819	292°236	331°100	28°065	150°426	234°406	20°859	0°040	137°712	19°934	183°025
Oct. 1	66°206	25°938	295°744	329°511	28°005	154°017	232°817	20°731	3°227	136°123	20°047	186°136
Nov. 1	64°564	26°059	299°375	327°869	27°940	157°727	231°175	20°600	6°509	134°481	20°167	189°363
Dec. 1	62°976	26°175	302°893	326°281	27°874	161°314	229°587	20°475	9°672	132°893	20°307	192°461
	1873			1878			1883			1888		
Jan. 1	61°333	26°292	305°534	324°639	27°803	165°019	227°945	20°349	12°930	131°251	20°411	195°747
Feb. 1	59°692	26°408	310°180	322°998	27°729	168°720	226°304	20°226	16°174	129°610	20°538	199°021
Mar. 1	58°209	26°509	313°478	321°515	27°659	172°062	224°821	20°116	19°095	128°074	20°660	202°075
Apr. 1	56°568	26°620	317°133	319°874	27°579	175°757	223°180	19°997	22°316	126°433	20°791	205°361
May 1	54°979	26°724	321°674	318°285	27°501	179°331	221°591	19°884	25°422	124°844	20°921	208°553
June 1	53°337	26°829	324°338	316°643	27°413	183°020	219°949	19°772	28°620	123°202	21°056	211°864
July 1	51°749	26°929	327°889	315°055	27°327	186°590	218°361	19°666	31°704	121°614	21°189	215°081
Aug. 1	50°107	27°030	331°560	313°413	27°235	190°273	216°719	19°559	34°878	119°972	21°328	218°413
Sept. 1	48°466	27°127	335°236	311°772	27°141	193°954	215°078	19°457	38°041	118°331	21°468	221°760
Oct. 1	46°877	27°219	338°796	310°183	27°047	197°511	213°489	19°360	41°091	116°742	21°606	225°009
Nov. 1	45°235	27°311	342°480	308°541	26°947	201°185	211°847	19°264	44°231	115°100	21°748	228°378
Dec. 1	43°647	27°397	346°047	306°953	26°848	204°735	210°259	19°176	47°261	113°512	21°887	231°649
	1874			1879			1884			1889		
Jan. 1	42°005	27°484	349°736	305°311	26°743	208°399	208°617	19°088	50°379	111°870	22°032	235°040
Feb. 1	40°364	27°567	353°429	303°670	26°635	212°059	206°976	19°004	53°489	110°229	22°177	238°442
Mar. 1	38°881	27°642	356°768	302°187	26°536	215°361	205°440	18°930	56°390	108°746	22°309	241°524
Apr. 1	37°240	27°718	0°465	300°546	26°424	219°012	203°799	18°854	59°480	107°105	22°455	244°947
May 1	35°651	27°790	4°047	298°957	26°313	222°542	202°210	18°786	62°462	105°516	22°598	248°269
June 1	34°009	27°862	7°752	297°313	26°196	226°184	200°568	18°719	65°535	103°874	22°744	251°713
July 1	32°421	27°928	11°338	295°727	26°081	229°703	198°980	18°659	68°501	102°286	22°887	255°055
Aug. 1	30°779	27°994	15°046	294°085	25°959	233°334	197°338	18°602	71°558	100°644	23°034	258°518
Sept. 1	29°138	28°056	18°757	292°444	25°837	236°961	195°697	18°548	74°610	99°003	23°182	261°991
Oct. 1	27°549	28°113	22°349	290°855	25°715	240°464	194°108	18°504	77°555	97°414	23°324	265°361
Nov. 1	25°907	28°169	26°066	289°213	25°588	244°080	192°474	18°461	80°594	95°772	23°470	268°853
Dec. 1	24°319	28°220	29°663	287°625	25°463	247°571	190°878	18°425	83°530	94°184	23°612	272°241

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TABLE 6.—*Values of N, I, and P for Greenwich midnight, beginning each month, from 1850 to 1949—Continued.*

Month.	N	I	P	N	I	P	N	I	P	N	I	P
	1890			1895			1900			1905		
Jan. I	92°542	23°757	275°751	355°848	28°590	131°659	259°154	23°017	346°357	162°460	18°608	173°267
Feb. I	90°901	23°903	279°267	354°207	28°580	135°388	257°513	22°869	349°819	160°819	18°666	176°328
Mar. I	89°418	24°033	282°454	352°724	28°568	138°754	256°030	22°737	352°938	159°336	18°723	179°055
Apr. I	87°777	24°176	285°988	351°083	28°550	142°486	254°389	22°589	356°380	157°695	18°790	182°168
May I	86°188	24°314	289°415	349°494	28°530	146°104	252°800	22°447	359°702	156°106	18°858	185°151
June I	84°546	24°456	292°966	347°852	28°506	149°822	251°158	22°301	3°124	154°464	18°935	188°242
July I	82°958	24°593	296°409	346°264	28°479	153°428	249°570	22°159	6°426	152°876	19°011	191°243
Aug. I	81°316	24°730	299°974	344°622	28°448	157°153	247°928	22°015	9°826	151°234	19°096	194°353
Sept. I	79°675	24°868	303°547	342°981	28°414	160°878	246°287	21°869	13°217	149°593	19°184	197°474
Oct. I	78°086	25°001	307°010	341°392	28°377	164°482	244°698	21°730	16°487	148°004	19°273	200°504
Nov. I	76°444	25°136	310°597	339°750	28°337	168°204	243°056	21°588	19°854	146°362	19°369	203°645
Dec. I	74°856	25°264	313°074	338°162	28°294	171°805	241°468	21°451	23°102	144°774	19°466	206°696
	1891			1896			1901			1906		
Jan. I	73°214	25°396	317°673	336°519	28°246	175°523	239°825	21°311	26°447	143°131	19°568	209°861
Feb. I	71°573	25°526	321°278	334°878	28°195	179°240	238°184	21°172	29°779	141°490	19°676	213°035
Mar. I	70°090	25°642	324°540	333°342	28°144	182°717	236°701	21°048	32°780	140°007	19°775	215°915
Apr. I	68°449	25°769	328°157	331°701	28°086	186°430	235°060	20°913	36°089	138°366	19°887	219°113
May I	66°860	25°889	331°663	330°112	28°028	190°023	233°471	20°784	39°280	136°777	20°000	222°205
June I	65°218	26°011	335°291	328°470	28°963	193°733	231°829	20°652	42°567	135°135	20°119	225°441
July I	63°630	26°128	338°808	326°882	27°899	197°321	230°241	20°527	45°735	133°547	20°237	228°570
Aug. I	61°988	26°246	342°446	325°240	27°830	201°026	228°599	20°400	49°007	131°905	20°360	231°815
Sept. I	60°347	26°362	346°091	323°599	27°757	204°728	226°958	20°275	52°247	130°264	20°497	235°075
Oct. I	58°758	26°471	349°622	322°010	27°682	208°309	225°369	20°156	55°380	128°675	20°612	238°239
Nov. I	57°116	26°583	353°276	320°368	27°603	212°007	223°727	20°036	58°606	127°033	20°743	241°521
Dec. I	55°528	26°688	356°815	318°780	27°524	215°582	222°139	19°923	61°716	125°445	20°872	244°709
	1892			1897			1902			1907		
Jan. I	53°885	26°794	0°478	317°138	27°439	219°273	220°497	19°809	64°918	123°803	21°007	248°015
Feb. I	52°244	26°898	4°145	315°497	27°351	222°960	218°856	19°698	68°107	122°162	21°143	251°333
Mar. I	50°708	26°993	7°579	314°014	27°269	226°290	217°373	19°602	70°979	120°679	21°268	254°341
Apr. I	49°067	27°091	11°253	312°373	27°176	229°970	215°732	19°497	74°146	119°038	21°408	257°682
May I	47°478	27°185	14°812	310°784	27°081	233°529	214°143	19°400	77°200	117°449	21°544	260°926
June I	45°836	27°277	18°494	309°142	26°984	237°204	212°501	19°302	80°345	115°807	21°687	264°290
July I	44°248	27°365	22°061	307°554	26°885	240°756	210°913	19°212	83°379	114°219	21°826	267°557
Aug. I	42°606	27°453	25°748	305°912	26°782	244°421	209°271	19°123	86°502	112°577	21°969	270°942
Sept. I	40°965	27°537	29°440	304°271	26°675	248°084	207°630	19°037	89°616	110°936	22°115	274°340
Oct. I	39°376	27°616	33°016	302°682	26°569	251°623	206°041	18°958	92°619	109°347	22°256	277°638
Nov. I	37°734	27°695	36°714	301°040	26°455	255°276	204°399	18°881	95°714	107°705	22°402	281°058
Dec. I	36°146	27°768	40°295	299°452	26°348	258°807	202°811	18°811	98°699	106°117	22°544	284°376
	1893			1898			1903			1908		
Jan. I	34°504	27°841	43°998	297°810	26°231	262°450	201°169	18°743	101°774	104°475	22°691	287°816
Feb. I	32°863	27°909	47°703	296°169	26°113	266°087	199°528	18°679	104°842	102°834	22°838	291°266
Mar. I	31°380	27°970	51°054	294°686	25°004	269°370	198°045	18°626	107°607	101°298	22°976	294°503
Apr. I	29°739	28°034	54°763	293°045	25°881	272°997	196°404	18°572	110°660	99°657	23°123	297°971
May I	28°150	28°093	58°355	291°456	25°762	276°503	194°815	18°524	113°609	98°068	23°265	301°338
June I	26°508	28°149	62°070	289°814	25°634	280°120	193°173	18°479	116°650	96°426	23°413	304°826
July I	24°920	28°202	65°666	288°226	25°511	283°614	191°585	18°441	119°587	94°838	23°554	308°210
Aug. I	23°278	28°252	69°383	286°584	25°381	287°218	189°943	18°406	122°617	93°196	23°700	311°715
Sept. I	21°637	28°299	73°103	284°943	25°238	290°818	188°302	18°377	125°644	91°555	23°845	315°231
Oct. I	20°048	28°342	76°703	283°354	25°119	294°294	186°713	18°353	128°569	89°966	23°985	318°640
Nov. I	18°406	28°382	80°426	281°712	24°984	297°879	185°071	18°334	131°589	88°324	24°128	322°172
Dec. I	16°818	28°418	84°030	280°124	24°851	301°342	183°483	18°320	134°509	86°736	24°267	325°597
	1894			1899			1904			1909		
Jan. I	15°176	28°452	87°754	278°482	24°713	304°915	181°841	18°312	137°525	85°094	24°409	329°145
Feb. I	13°535	28°482	91°479	276°841	24°574	308°478	180°200	18°308	140°540	83°453	24°549	332°700
Mar. I	12°052	28°507	94°846	275°358	24°448	311°692	178°664	18°310	143°361	81°970	24°676	335°918
Apr. I	10°411	28°531	98°573	273°717	24°306	315°241	177°023	18°317	146°376	80°329	24°814	339°488
May I	8°822	28°551	102°181	272°128	24°168	318°668	175°434	18°329	149°296	78°740	24°946	342°949
June I	7°180	28°568	105°915	270°486	24°025	322°203	173°792	18°346	152°315	77°098	25°082	346°532
July I	5°592	28°582	109°520	268°898	23°885	325°614	172°204	18°369	155°239	75°510	25°212	350°007
Aug. I	3°950	28°591	113°248	267°256	23°740	329°131	170°562	18°397	158°264	73°868	25°345	353°603
Sept. I	2°309	28°598	116°979	265°615	23°594	332°639	168°921	18°430	161°296	72°227	25°475	357°206
Oct. I	0°720	28°602	120°589	264°026	23°452	336°026	167°332	18°466	164°229	70°638	25°599	0°699
Nov. I	359°078	28°601	124°320	262°384	23°306	339°517	165°690	18°510	167°269	68°996	25°726	4°314
Dec. I	357°490	28°598	127°929	260°796	23°164	342°886	164°102	18°556	170°215	67°408	25°848	7°819

TABLE 6.—Values of *N*, *I*, and *P* for Greenwich midnight, beginning each month, from 1850 to 1949—Continued.

Month.	<i>N</i>	<i>I</i>	<i>P</i>	<i>N</i>	<i>I</i>	<i>P</i>	<i>N</i>	<i>I</i>	<i>P</i>	<i>N</i>	<i>I</i>	<i>P</i>
	1910			1915			1920			1925		
Jan. 1	65°765	25°970	11°446	329°071	27°988	229°741	232°377	20°695	78°837	135°630	20°083	261°833
Feb. 1	64°124	20°092	15°077	327°430	27°922	233°449	230°736	20°565	82°115	133°989	20°204	265°064
Mar. 1	62°641	26°199	18°363	325°947	27°860	236°797	229°200	20°446	85°171	132°506	20°315	267°992
Apr. 1	61°000	26°316	22°004	324°306	27°788	240°500	227°559	20°318	88°425	130°865	20°441	271°245
May 1	59°411	26°427	25°534	322°717	27°716	244°082	225°970	20°201	91°562	129°276	20°564	274°406
June 1	57°769	26°539	29°186	321°075	27°638	247°781	224°328	20°080	94°793	127°634	20°695	277°684
July 1	56°181	26°645	32°723	319°487	27°559	251°357	222°740	19°966	97°907	126°046	20°822	280°868
Aug. 1	54°539	26°753	36°384	317°845	27°476	255°049	221°098	19°851	101°112	124°404	20°957	284°169
Sept. 1	52°898	26°865	40°049	316°204	27°389	258°739	219°457	19°739	104°307	122°763	21°093	287°483
Oct. 1	51°309	26°956	43°600	314°615	27°302	262°307	217°868	19°634	107°387	121°174	21°226	290°701
Nov. 1	49°667	27°056	47°273	312°973	27°210	265°990	216°226	19°528	110°558	119°532	21°365	294°039
Dec. 1	48°079	27°149	50°831	311°385	27°118	269°550	214°638	19°430	113°616	117°944	21°502	297°280
	1911			1916			1921			1926		
Jan. 1	46°437	27°244	54°512	309°743	27°021	273°224	212°996	19°332	116°765	116°302	21°644	300°641
Feb. 1	44°796	27°335	58°195	308°102	26°919	276°897	211°355	19°237	119°902	114°661	21°787	304°012
Mar. 1	43°313	27°415	61°526	306°566	26°823	280°328	209°872	19°155	122°727	113°178	21°916	307°068
Apr. 1	41°672	27°501	65°216	304°925	26°718	283°991	208°231	19°068	125°843	111°537	22°062	310°461
May 1	40°083	27°581	68°790	303°336	26°613	287°532	206°642	18°988	128°850	109°948	22°203	313°756
June 1	38°441	27°661	72°487	301°694	26°502	291°187	205°000	18°909	131°948	108°306	22°348	317°171
July 1	36°853	27°736	76°066	300°106	26°393	294°719	203°412	18°837	134°937	106°718	22°490	320°486
Aug. 1	35°211	27°810	79°768	298°464	26°277	298°365	201°770	18°767	138°016	105°076	22°637	323°922
Sept. 1	33°570	27°881	83°473	296°823	26°160	302°005	200°129	18°702	141°087	103°435	22°784	327°368
Oct. 1	31°981	27°946	87°060	295°234	26°044	305°523	198°540	18°643	144°052	101°846	22°927	330°712
Nov. 1	30°339	28°011	90°770	293°592	25°923	309°153	196°898	18°588	147°107	100°204	23°074	334°179
Dec. 1	28°751	28°070	94°362	292°004	25°803	312°661	195°310	18°538	150°058	98°616	23°217	337°541
	1912			1917			1922			1927		
Jan. 1	27°109	28°128	98°074	290°362	25°677	316°280	193°668	18°492	153°099	96°974	23°363	341°028
Feb. 1	25°468	28°184	101°790	288°721	25°549	319°892	192°027	18°451	156°136	95°333	23°510	344°521
Mar. 1	23°932	28°232	105°268	287°238	25°432	323°151	190°544	18°418	158°875	93°850	23°641	347°686
Apr. 1	22°291	28°281	108°986	285°597	25°301	326°751	188°903	18°386	161°903	92°209	23°787	351°166
May 1	20°702	28°325	112°586	284°008	25°173	330°230	187°314	18°361	164°829	90°620	23°927	354°602
June 1	19°060	28°367	116°308	282°366	25°038	333°819	185°672	18°340	167°850	88°978	24°072	358°131
July 1	17°472	28°404	119°911	280°778	24°906	337°284	184°084	18°325	170°771	87°390	24°210	1°553
Aug. 1	15°830	28°439	123°635	279°136	24°769	340°858	182°442	18°315	173°787	85°748	24°353	5°096
Sept. 1	14°189	28°471	127°361	277°495	24°631	344°426	180°801	18°309	176°803	84°107	24°494	8°649
Oct. 1	12°600	28°498	130°966	275°906	24°495	347°871	179°212	18°309	179°721	82°518	24°629	12°094
Nov. 1	10°958	28°524	134°695	274°264	24°354	351°424	177°570	18°315	182°737	80°876	24°768	15°662
Dec. 1	9°370	28°545	138°302	272°676	24°215	354°853	175°982	18°324	185°655	79°288	24°902	19°121
	1913			1918			1923			1928		
Jan. 1	7°728	28°563	142°032	271°034	24°073	358°389	174°340	18°340	188°674	77°646	25°037	22°701
Feb. 1	6°087	28°578	145°760	269°393	23°929	1°917	172°699	18°361	190°695	76°005	25°172	26°290
Mar. 1	4°604	28°588	149°130	267°910	23°798	5°097	171°216	18°385	194°426	74°469	25°296	29°653
Apr. 1	2°963	28°596	152°858	266°269	23°652	8°608	169°575	18°415	197°453	72°828	25°427	33°253
May 1	1°374	28°600	156°468	264°680	23°510	11°999	167°986	18°450	200°387	71°239	25°552	36°744
June 1	359°732	28°602	160°199	263°038	23°364	15°493	166°344	18°491	203°425	69°597	25°680	40°356
July 1	358°144	28°599	163°809	261°450	23°223	18°866	164°756	18°534	206°368	68°009	25°802	43°859
Aug. 1	356°502	28°594	167°539	259°808	23°075	22°341	163°114	18°587	209°417	66°367	25°926	47°483
Sept. 1	354°861	28°584	171°268	258°167	22°929	25°807	161°473	18°643	212°473	64°726	26°047	51°113
Oct. 1	353°272	28°572	174°877	256°578	22°785	29°152	159°884	18°701	215°437	63°137	26°163	54°631
Nov. 1	351°630	28°557	178°607	254°936	22°638	32°599	158°242	18°772	218°508	61°495	26°280	58°272
Dec. 1	350°042	28°537	182°215	253°348	22°496	35°924	156°654	18°834	221°488	59°907	26°393	61°800
	1914			1919			1924			1929		
Jan. 1	348°400	28°514	185°944	251°706	22°350	39°350	155°011	18°908	224°575	58°264	26°505	65°450
Feb. 1	346°759	28°488	189°670	250°065	22°204	42°765	153°370	18°987	227°673	56°623	26°616	69°104
Mar. 1	345°276	28°461	193°036	248°582	22°072	45°841	151°834	19°065	230°580	55°140	26°714	72°409
Apr. 1	343°635	28°428	196°761	246°941	21°927	49°235	150°193	19°152	233°696	53°499	26°819	76°072
May 1	342°046	28°393	200°364	245°352	21°788	52°508	148°604	19°239	236°714	51°910	26°919	79°622
June 1	340°404	28°354	204°088	243°710	21°645	55°881	146°962	19°334	239°860	50°268	27°020	83°294
July 1	338°816	28°312	207°689	242°122	21°507	59°133	145°374	19°429	242°907	48°680	27°114	86°851
Aug. 1	337°174	28°265	211°408	240°480	21°366	62°482	143°732	19°531	246°067	47°038	27°210	90°529
Sept. 1	335°533	28°216	215°127	238°839	21°228	65°820	142°091	19°636	249°238	45°397	27°302	94°212
Oct. 1	333°944	28°164	218°723	237°250	21°094	69°038	140°502	19°742	252°318	43°808	27°388	97°779
Nov. 1	332°302	28°108	222°438	235°608	20°958	72°353	138°860	19°854	255°513	42°166	27°475	101°469
Dec. 1	330°714	28°051	226°031	234°020	20°828	75°548	137°272	19°905	258°616	40°578	27°556	105°042

TABLE 6.—*Values of N, I, and P for Greenwich midnight, beginning each month, from 1850 to 1949—Continued.*

Month.	N	I	P	N	I	P	N	I	P	N	I	P
1930												
Jan. 1	38° 936	27° 637	108° 738	302° 242	26° 539	327° 333	205° 548	18° 935	168° 280	108° 801	22° 305	353° 506
Feb. 1	37° 295	27° 715	112° 436	300° 601	26° 428	330° 985	203° 907	18° 859	171° 372	107° 160	22° 451	356° 928
Mar. 1	35° 812	27° 783	115° 780	299° 118	26° 324	334° 280	202° 371	18° 793	174° 256	105° 677	22° 583	0° 029
Apr. 1	34° 171	27° 855	119° 483	297° 477	26° 207	337° 922	200° 730	18° 725	177° 329	104° 036	22° 731	3° 470
May 1	32° 582	27° 922	123° 069	295° 888	26° 093	341° 441	199° 141	18° 665	180° 296	102° 447	22° 873	6° 811
June 1	30° 940	28° 003	126° 778	294° 246	25° 971	345° 074	197° 499	18° 607	183° 355	100° 805	23° 020	10° 275
July 1	29° 352	28° 048	130° 368	292° 658	25° 853	348° 584	195° 911	18° 556	186° 308	99° 217	23° 162	13° 634
Aug. 1	27° 710	28° 107	134° 081	291° 016	25° 727	352° 204	194° 269	18° 509	189° 352	97° 575	23° 309	17° 115
Sept. 1	26° 069	28° 164	137° 796	289° 375	25° 600	355° 820	192° 628	18° 465	192° 392	95° 934	23° 456	20° 606
Oct. 1	24° 480	28° 216	141° 392	287° 786	25° 476	359° 313	191° 039	18° 429	195° 327	94° 345	23° 598	23° 993
Nov. 1	22° 838	28° 265	145° 111	286° 144	25° 346	2° 916	189° 397	18° 396	198° 356	92° 703	23° 743	27° 502
Dec. 1	21° 250	28° 310	148° 710	284° 556	25° 217	6° 396	187° 809	18° 369	201° 284	91° 115	23° 884	30° 905
1931												
1936												
Jan. 1	19° 608	28° 353	152° 432	282° 914	25° 083	9° 987	186° 167	18° 346	204° 306	89° 473	24° 028	34° 431
Feb. 1	17° 967	28° 393	156° 154	281° 273	24° 947	13° 570	184° 526	18° 328	207° 324	87° 832	24° 171	37° 965
Mar. 1	16° 484	28° 425	159° 519	279° 737	24° 819	16° 916	183° 043	18° 317	210° 050	86° 349	24° 300	41° 164
Apr. 1	14° 843	28° 459	163° 243	278° 096	24° 681	20° 486	181° 402	18° 311	213° 066	84° 708	24° 442	44° 714
May 1	13° 254	28° 487	166° 849	276° 507	24° 546	23° 934	179° 813	18° 308	215° 983	83° 119	24° 578	48° 155
June 1	11° 612	28° 514	170° 576	274° 865	24° 405	27° 489	178° 171	18° 312	218° 998	81° 477	24° 717	51° 721
July 1	10° 024	28° 537	174° 184	273° 277	24° 268	30° 922	176° 583	18° 320	221° 917	79° 889	24° 850	55° 177
Aug. 1	8° 382	28° 556	177° 912	271° 635	24° 125	34° 462	174° 941	18° 333	224° 934	78° 247	24° 987	58° 755
Sept. 1	6° 741	28° 572	181° 642	269° 994	23° 981	37° 992	173° 300	18° 353	227° 955	76° 606	25° 122	62° 340
Oct. 1	5° 152	28° 584	185° 250	268° 405	23° 842	41° 402	171° 711	18° 377	230° 879	75° 017	25° 252	65° 817
Nov. 1	3° 510	28° 594	188° 980	266° 763	23° 696	44° 917	170° 069	18° 406	233° 906	73° 375	25° 384	68° 416
Dec. 1	1° 922	28° 599	192° 674	265° 175	23° 555	48° 310	168° 481	18° 440	236° 839	71° 787	25° 511	72° 904
1932												
1937												
Jan. 1	0° 280	28° 602	196° 319	263° 533	23° 409	51° 807	166° 839	18° 479	239° 873	70° 145	25° 638	76° 515
Feb. 1	358° 639	28° 600	200° 050	261° 892	23° 261	55° 294	165° 198	18° 523	242° 914	68° 504	25° 765	80° 132
Mar. 1	357° 103	28° 596	203° 539	260° 409	23° 130	58° 437	163° 715	18° 568	245° 666	67° 021	25° 876	83° 404
Apr. 1	355° 462	28° 589	207° 268	258° 768	22° 983	61° 906	162° 074	18° 621	248° 719	65° 380	25° 999	87° 031
May 1	353° 873	28° 577	210° 877	257° 179	22° 839	65° 254	160° 485	18° 679	251° 680	63° 791	26° 117	90° 547
June 1	352° 231	28° 562	214° 607	255° 537	22° 692	68° 704	158° 843	18° 742	254° 749	62° 149	26° 234	94° 186
July 1	350° 643	28° 545	218° 216	253° 949	22° 550	72° 033	157° 255	18° 808	257° 725	60° 561	26° 347	97° 712
Aug. 1	349° 001	28° 523	221° 943	252° 307	22° 403	75° 463	155° 613	18° 881	260° 809	58° 919	26° 461	101° 360
Sept. 1	347° 360	28° 498	225° 671	250° 666	22° 257	78° 882	153° 972	18° 958	263° 904	57° 278	26° 570	105° 013
Oct. 1	345° 771	28° 470	229° 277	249° 077	22° 116	82° 180	152° 353	19° 036	266° 907	55° 689	26° 678	108° 552
Nov. 1	344° 129	28° 439	233° 003	247° 435	21° 970	85° 579	150° 741	19° 122	270° 021	54° 047	26° 784	112° 215
Dec. 1	342° 541	28° 404	236° 607	245° 847	21° 832	88° 855	149° 153	19° 208	273° 043	52° 459	26° 885	115° 762
1933												
1938												
Jan. 1	340° 899	28° 366	240° 330	244° 205	21° 688	92° 231	147° 510	19° 302	276° 178	50° 816	26° 986	119° 433
Feb. 1	339° 258	28° 324	244° 052	242° 564	21° 546	95° 594	145° 869	19° 399	279° 322	49° 175	27° 085	123° 107
Mar. 1	337° 775	28° 283	247° 412	241° 081	21° 418	98° 623	144° 386	19° 490	282° 173	47° 639	27° 175	126° 548
Apr. 1	336° 134	28° 234	251° 131	239° 440	21° 278	101° 965	142° 745	19° 594	285° 339	45° 998	27° 268	130° 228
May 1	334° 545	28° 184	254° 728	237° 851	21° 144	105° 187	141° 156	19° 698	288° 415	44° 409	27° 356	133° 795
June 1	332° 903	28° 129	258° 444	236° 209	21° 008	108° 506	139° 514	19° 808	291° 605	42° 767	27° 444	137° 494
July 1	331° 315	28° 072	262° 037	234° 621	20° 877	111° 705	137° 926	19° 920	294° 703	41° 179	27° 526	141° 055
Aug. 1	329° 673	28° 012	265° 748	232° 979	20° 744	115° 000	136° 284	20° 036	297° 916	39° 537	27° 608	144° 750
Sept. 1	328° 032	27° 946	269° 458	231° 338	20° 613	118° 283	134° 643	20° 155	301° 141	37° 896	27° 687	148° 447
Oct. 1	326° 443	27° 881	273° 045	229° 749	20° 488	121° 447	133° 054	20° 274	304° 275	36° 307	27° 761	152° 028
Nov. 1	324° 801	27° 810	276° 750	228° 107	20° 361	124° 706	131° 412	20° 399	307° 524	34° 665	27° 834	155° 731
Dec. 1	323° 213	27° 739	280° 333	226° 519	20° 241	127° 847	129° 824	20° 522	310° 681	33° 077	27° 901	159° 316
1934												
1939												
Jan. 1	321° 570	27° 661	284° 032	224° 876	20° 120	131° 082	128° 182	20° 651	313° 946	31° 435	27° 968	163° 025
Feb. 1	319° 929	27° 581	287° 728	223° 235	20° 001	134° 303	126° 541	20° 783	317° 241	29° 794	28° 032	166° 737
Mar. 1	318° 446	27° 507	291° 065	221° 752	19° 896	137° 203	125° 005	20° 908	320° 325	28° 311	28° 086	170° 088
Apr. 1	316° 805	27° 421	294° 755	220° 111	19° 783	140° 402	123° 364	21° 043	323° 634	26° 670	28° 143	173° 801
May 1	315° 216	27° 336	298° 323	218° 522	19° 676	143° 487	121° 775	21° 176	326° 849	25° 081	28° 196	177° 397
June 1	313° 574	27° 244	302° 007	216° 880	19° 569	146° 662	120° 133	21° 315	330° 182	23° 439	28° 247	181° 115
July 1	311° 986	27° 153	305° 570	215° 292	19° 470	149° 725	118° 545	21° 450	333° 419	21° 851	28° 293	184° 714
Aug. 1	310° 344	27° 057	309° 246	213° 650	19° 370	152° 877	116° 903	21° 592	336° 775	20° 209	28° 338	188° 433
Sept. 1	308° 703	26° 957	312° 919	212° 009	19° 274	156° 019	115° 262	21° 734	340° 143	18° 568	28° 378	192° 157
Oct. 1	307° 114	26° 858	316° 470	210° 420	19° 185	159° 049	113° 673	21° 873	343° 413	16° 979	28° 414	195° 760
Nov. 1	305° 472	26° 753	320° 136	208° 778	19° 095	162° 179	112° 031	22° 018	346° 803	15° 337	28° 449	199° 486
Dec. 1	303° 884	26° 649	323° 678	207° 190	19° 015	165° 180	110° 443	22° 158	350° 094	13° 749	28° 479	203° 091
1940												
1941												
Jan. 1	19° 608	28° 353	152° 432	282° 914	25° 083	9° 987	186° 167	18° 346	204° 306	89° 473	24° 028	34° 431
Feb. 1	17° 967	28° 393	156° 154	281° 273	24° 947	13° 570	184° 526	18° 328	207° 324	87° 832	24° 171	37° 965
Mar. 1	16° 484	28° 425	159° 519	279° 737	24° 819	16° 916	183° 043	18° 317	210° 050	86° 349	24° 300	41° 164
Apr. 1	14° 843	28° 459	163° 243	278° 096	24° 681	20° 486	181° 402	18° 311	213° 066	84° 708	24° 442	44° 714
May 1	13° 254	28° 487	166° 849	276° 507	24° 546	23° 934	179° 813	18° 308	215° 983	83° 119	24° 578	48° 155
June 1	11° 612	28° 514	170° 576	274° 865	24° 405	27° 489	178° 171	18° 312	218° 998	81° 477	24° 717	51° 721
July 1	10° 024	28° 537	174° 184	273° 277	24° 268	30° 922	176° 583	18° 320	221° 917	79° 889	24° 850	55° 177
Aug. 1	8° 382	28° 556	177° 912	271° 635	24° 125	34° 462	174° 941	18° 333	224° 934	78° 247	24° 987	58° 755
Sept. 1	6° 741	28° 572	181° 642	269° 994	23° 981	37° 992	173° 300	18° 353	227° 955	76° 606	25° 122	62° 340
Oct. 1	5° 152	28° 584	185° 250	268° 405	23° 842	41° 402	171° 711	18° 377	230° 879	75° 017	25° 252	65° 817
Nov. 1	3° 510	28° 594	188° 980	26								

TABLE 7.—Values of I , ν , ξ , ν' , and $2\nu''$, corresponding to each half degree of N .

N	I	ν	ξ	ν'	$2\nu''$	N	N	I	ν	ξ	ν'	$2\nu''$	N
0°	28°602	0°000	0°000	0°000	0°000	360°	30°	28°024	5°478	4°939	3°903	8°249	330°
0°5	28°602	0°094	0°084	0°067	0°143	359°5	30°5	28°005	5°564	5°017	3°964	8°377	329°5
1°	28°601	0°188	0°169	0°134	0°285	359°0	31°	27°985	5°651	5°095	4°025	8°504	329°0
1°5	28°600	0°281	0°253	0°201	0°427	358°5	31°5	27°965	5°736	5°173	4°085	8°631	328°5
2°	28°599	0°375	0°337	0°268	0°569	358°0	32°	27°945	5°822	5°251	4°146	8°757	328°0
2°5	28°598	0°468	0°421	0°335	0°711	357°5	32°5	27°925	5°907	5°328	4°206	8°883	327°5
3°	28°596	0°562	0°506	0°402	0°853	357°0	33°	27°904	5°992	5°406	4°266	9°008	327°0
3°5	28°594	0°656	0°590	0°469	0°995	356°5	33°5	27°884	6°077	5°482	4°326	9°133	326°5
4°	28°591	0°749	0°674	0°536	1°137	356°0	34°	27°862	6°162	5°559	4°386	9°257	326°0
4°5	28°589	0°843	0°758	0°603	1°279	355°5	34°5	27°841	6°246	5°636	4°445	9°381	325°5
5°	28°585	0°936	0°842	0°670	1°421	355°0	35°	27°819	6°330	5°712	4°504	9°504	325°0
5°5	28°582	1°030	0°926	0°737	1°563	354°5	35°5	27°797	6°414	5°788	4°563	9°626	324°5
6°	28°578	1°123	1°010	0°804	1°705	354°0	36°	27°775	6°497	5°864	4°621	9°748	324°0
6°5	28°574	1°217	1°094	0°871	1°847	353°5	36°5	27°752	6°580	5°939	4°679	9°869	323°5
7°	28°570	1°310	1°178	0°938	1°989	353°0	37°	27°729	6°663	6°015	4°737	9°989	323°0
7°5	28°565	1°403	1°262	1°004	2°130	352°5	37°5	27°706	6°745	6°090	4°797	10°109	322°5
8°	28°560	1°496	1°346	1°070	2°271	352°0	38°	27°682	6°828	6°164	4°855	10°228	322°0
8°5	28°555	1°590	1°430	1°137	2°413	351°5	38°5	27°658	6°909	6°239	4°912	10°347	321°5
9°	28°549	1°683	1°514	1°203	2°554	351°0	39°	27°634	6°991	6°313	4°969	10°465	321°0
9°5	28°543	1°776	1°598	1°270	2°695	350°5	39°5	27°610	7°072	6°387	5°026	10°583	320°5
10°	28°537	1°869	1°681	1°337	2°836	350°0	40°	27°585	7°153	6°461	5°083	10°700	320°0
10°5	28°530	1°962	1°765	1°403	2°977	349°5	40°5	27°560	7°234	6°534	5°139	10°816	319°5
11°	28°523	2°054	1°848	1°469	3°117	349°0	41°	27°535	7°314	6°608	5°195	10°932	319°0
11°5	28°516	2°147	1°932	1°535	3°257	348°5	41°5	27°510	7°394	6°680	5°251	11°047	318°5
12°	28°508	2°240	2°015	1°601	3°397	348°0	42°	27°484	7°473	6°753	5°307	11°161	318°0
12°5	28°500	2°332	2°099	1°667	3°536	347°5	42°5	27°458	7°553	6°825	5°362	11°275	317°5
13°	28°492	2°424	2°182	1°733	3°676	347°0	43°	27°432	7°631	6°897	5°416	11°388	317°0
13°5	28°483	2°517	2°265	1°799	3°816	346°5	43°5	27°405	7°710	6°969	5°471	11°500	316°5
14°	28°475	2°609	2°348	1°864	3°955	346°0	44°	27°378	7°788	7°040	5°526	11°612	316°0
14°5	28°465	2°701	2°431	1°930	4°094	345°5	44°5	27°351	7°866	7°111	5°580	11°723	315°5
15°	28°456	2°793	2°514	1°996	4°233	345°0	45°	27°324	7°943	7°182	5°634	11°833	315°0
15°5	28°446	2°885	2°596	2°061	4°372	344°5	45°5	27°296	8°020	7°253	5°687	11°942	314°5
16°	28°436	2°977	2°679	2°127	4°510	344°0	46°	27°268	8°097	7°323	5°740	12°051	314°0
16°5	28°425	3°068	2°762	2°192	4°648	343°5	46°5	27°240	8°173	7°392	5°793	12°159	313°5
17°	28°414	3°160	2°844	2°257	4°786	343°0	47°	27°212	8°249	7°462	5°846	12°266	313°0
17°5	28°403	3°251	2°927	2°322	4°923	342°5	47°5	27°183	8°324	7°531	5°898	12°372	312°5
18°	28°392	3°342	3°009	2°387	5°060	342°0	48°	27°154	8°399	7°600	5°950	12°477	312°0
18°5	28°380	3°433	3°091	2°452	5°197	341°5	48°5	27°125	8°474	7°668	6°002	12°582	311°5
19°	28°368	3°524	3°173	2°517	5°334	341°0	49°	27°095	8°548	7°736	6°053	12°686	311°0
19°5	28°356	3°615	3°255	2°581	5°471	340°5	49°5	27°066	8°622	7°804	6°104	12°789	310°5
20°	28°343	3°705	3°337	2°646	5°607	340°0	50°	27°036	8°695	7°871	6°154	12°892	310°0
20°5	28°330	3°796	3°418	2°710	5°743	339°5	50°5	27°006	8°768	7°938	6°205	12°994	309°5
21°	28°317	3°886	3°500	2°774	5°878	339°0	51°	26°975	8°841	8°005	6°255	13°095	309°0
21°5	28°303	3°976	3°581	2°838	6°013	338°5	51°5	26°944	8°913	8°071	6°305	13°195	308°5
22°	28°289	4°066	3°662	2°902	6°148	338°0	52°	26°913	8°985	8°137	6°354	13°294	308°0
22°5	28°275	4°156	3°743	2°966	6°282	337°5	52°5	26°882	9°056	8°203	6°403	13°392	307°5
23°	28°260	4°245	3°824	3°030	6°416	337°0	53°	26°851	9°127	8°268	6°452	13°489	307°0
23°5	28°245	4°335	3°905	3°093	6°550	336°5	53°5	26°819	9°197	8°333	6°500	13°586	306°5
24°	28°230	4°424	3°985	3°156	6°683	336°0	54°	26°787	9°267	8°397	6°547	13°682	306°0
24°5	28°215	4°513	4°066	3°219	6°816	335°5	54°5	26°755	9°336	8°461	6°594	13°777	305°5
25°	28°199	4°602	4°146	3°282	6°948	335°0	55°	26°723	9°405	8°525	6°641	13°871	305°0
25°5	28°183	4°690	4°226	3°345	7°080	334°5	55°5	26°690	9°474	8°588	6°688	13°964	304°5
26°	28°166	4°779	4°306	3°408	7°212	334°0	56°	26°657	9°542	8°651	6°735	14°057	304°0
26°5	28°149	4°867	4°386	3°471	7°343	333°5	56°5	26°624	9°609	8°713	6°781	14°148	303°5
27°	28°132	4°955	4°466	3°534	7°474	333°0	57°	26°591	9°676	8°775	6°827	14°238	303°0
27°5	28°115	5°043	4°545	3°596	7°604	332°5	57°5	26°557	9°743	8°836	6°872	14°327	302°5
28°	28°097	5°130	4°624	3°658	7°734	332°0	58°	26°523	9°809	8°897	6°917	14°415	302°0
28°5	28°079	5°218	4°703	3°720	7°863	331°5	58°5	26°489	9°874	8°958	6°961	14°502	301°5
29°	28°061	5°305	4°782	3°781	7°992	331°0	59°	26°455	9°939	9°018	7°005	14°588	301°0
29°5	28°043	5°391	4°861	3°842	8°121	330°5	59°5	26°421	10°004	9°078	7°048	14°673	300°5
30°	28°024	5°478	4°939	3°903	8°249	330°0	60°	26°386	10°068	9°137	7°091	14°757	300°0

See Tables 1 and 5 for explanation of symbols. I is always positive; ν , ξ , ν' , and $2\nu''$ are positive when N is between 0° and 180° , and negative when N is between 180° and 360° . N is the longitude of the moon's ascending node.

$u(J_1) = -\nu$; $u(K_1) = -\nu'$; $u(K_2) = -2\nu''$; $u(L_2) = 2\xi - 2\nu - R$; see Table 8 for R ; $u(M_1) = \xi - \nu + Q$; see Table 9 for Q ; $u(M_2) = 2\xi - 2\nu$; $u(M_3) = 3\xi - 3\nu$; $u(M_4) = 4\xi - 4\nu$; $u(M_5) = 5\xi - 5\nu$; $u(M_6) = 6\xi - 6\nu$; $u(M_7) = 7\xi - 7\nu$; $u(N_2) = 2\xi - 2\nu = u(M_2)$; $u(2N) = 2\xi - 2\nu = u(M_2)$; $u(O_1) = 2\xi - \nu$; $u(OO) = -2\xi - \nu$; $u(P_1) = 0$; $u(Q_1) = 2\xi - \nu = u(O_1)$; $u(R_2) = 0$; $u(S_{1,2,3,4}) = 0$; $u(T_2) = 0$; $u(\mu_2) = u(\nu_2) = 2\xi - 2\nu = u(M_2)$; $u(MK) = 2\xi - 2\nu - \nu' = u(M_2) + u(K_1)$; $u(2MK) = 4\xi - 4\nu + \nu' = u(M_2) - u(K_1)$; $u(MN) = 4\xi - 4\nu = u(M_2)$; $u(MS) = 2\xi - 2\nu = u(M_2)$; $u(2MS) = 4\xi - 4\nu = u(M_2)$; $u(2SM) = -2\xi + 2\nu = -u(M_2)$; $u(Mf) = -2\xi$; $u(MSf) = -2\xi + 2\nu = -u(M_2)$; $u(Mm) = 0$; $u(Sa) = 0$; $u(SSa) = 0$.

TABLE 7.—Values of I , ν , ξ , ν' , and $2\nu''$, corresponding to each half degree of N —Continued.

N	I	ν	ξ	ν'	$2\nu''$	N	N	I	ν	ξ	ν'	$2\nu''$	N
60°0	26°386	10°068	9°137	7°091	14°757	300°0	90°0	23°982	12°751	11°681	8°797	17°783	270°0
60°5	26°351	10°131	9°196	7°134	14°841	299°5	90°5	23°938	12°772	11°704	8°808	17°795	269°5
61°0	26°316	10°194	9°255	7°177	14°924	299°0	91°0	23°894	12°793	11°725	8°819	17°805	269°0
61°5	26°280	10°256	9°313	7°219	15°006	298°5	91°5	23°850	12°814	11°745	8°829	17°814	268°5
62°0	26°245	10°318	9°370	7°261	15°087	298°0	92°0	23°806	12°833	11°765	8°839	17°822	268°0
62°5	26°209	10°379	9°427	7°302	15°167	297°5	92°5	23°761	12°851	11°784	8°848	17°829	267°5
63°0	26°173	10°440	9°484	7°343	15°246	297°0	93°0	23°717	12°869	11°802	8°856	17°834	267°0
63°5	26°137	10°500	9°539	7°383	15°324	296°5	93°5	23°673	12°886	11°819	8°863	17°837	266°5
64°0	26°101	10°560	9°595	7°423	15°401	296°0	94°0	23°628	12°901	11°835	8°870	17°839	266°0
64°5	26°064	10°619	9°650	7°462	15°477	295°5	94°5	23°584	12°916	11°851	8°876	17°840	265°5
65°0	26°027	10°677	9°705	7°501	15°551	295°0	95°0	23°539	12°930	11°866	8°882	17°840	265°0
65°5	25°990	10°735	9°759	7°539	15°624	294°5	95°5	23°495	12°943	11°880	8°887	17°838	264°5
66°0	25°953	10°793	9°812	7°577	15°696	294°0	96°0	23°450	12°955	11°893	8°891	17°835	264°0
66°5	25°916	10°849	9°865	7°614	15°767	293°5	96°5	23°406	12°966	11°905	8°895	17°830	263°5
67°0	25°878	10°906	9°918	7°651	15°837	293°0	97°0	23°361	12°976	11°916	8°898	17°824	263°0
67°5	25°841	10°961	9°970	7°688	15°906	292°5	97°5	23°316	12°985	11°927	8°900	17°816	262°5
68°0	25°803	11°016	10°021	7°724	15°974	292°0	98°0	23°271	12°994	11°936	8°902	17°807	262°0
68°5	25°765	11°070	10°072	7°760	16°042	291°5	98°5	23°227	13°001	11°945	8°903	17°796	261°5
69°0	25°726	11°124	10°123	7°795	16°109	291°0	99°0	23°182	13°007	11°953	8°903	17°784	261°0
69°5	25°688	11°177	10°173	7°830	16°174	290°5	99°5	23°137	13°013	11°960	8°902	17°770	260°5
70°0	25°649	11°230	10°222	7°864	16°238	290°0	100°0	23°092	13°017	11°966	8°901	17°755	260°0
70°5	25°610	11°282	10°271	7°898	16°300	289°5	100°5	23°047	13°021	11°971	8°899	17°739	259°5
71°0	25°571	11°333	10°319	7°932	16°361	289°0	101°0	23°003	13°023	11°975	8°896	17°721	259°0
71°5	25°532	11°383	10°366	7°965	16°421	288°5	101°5	22°958	13°024	11°979	8°892	17°702	258°5
72°0	25°493	11°433	10°414	7°997	16°480	288°0	102°0	22°913	13°025	11°981	8°888	17°681	258°0
72°5	25°453	11°482	10°460	8°029	16°538	287°5	102°5	22°868	13°024	11°983	8°883	17°659	257°5
73°0	25°413	11°531	10°506	8°061	16°594	287°0	103°0	22°823	13°023	11°983	8°878	17°636	257°0
73°5	25°374	11°579	10°551	8°092	16°649	286°5	103°5	22°778	13°020	11°983	8°872	17°611	256°5
74°0	25°334	11°626	10°596	8°122	16°703	286°0	104°0	22°734	13°017	11°982	8°865	17°584	256°0
74°5	25°293	11°673	10°640	8°152	16°756	285°5	104°5	22°689	13°012	11°979	8°858	17°556	255°5
75°0	25°253	11°719	10°684	8°181	16°808	285°0	105°0	22°644	13°006	11°976	8°850	17°526	255°0
75°5	25°213	11°764	10°726	8°209	16°859	284°5	105°5	22°599	13°000	11°972	8°841	17°495	254°5
76°0	25°172	11°809	10°769	8°237	16°909	284°0	106°0	22°554	12°992	11°967	8°831	17°463	254°0
76°5	25°131	11°852	10°811	8°264	16°958	283°5	106°5	22°510	12°983	11°961	8°821	17°430	253°5
77°0	25°090	11°895	10°852	8°291	17°005	283°0	107°0	22°465	12°974	11°954	8°810	17°395	253°0
77°5	25°049	11°938	10°892	8°318	17°051	282°5	107°5	22°420	12°963	11°946	8°799	17°358	252°5
78°0	25°008	11°980	10°932	8°344	17°096	282°0	108°0	22°376	12°951	11°937	8°787	17°320	252°0
78°5	24°966	12°021	10°971	8°370	17°140	281°5	108°5	22°331	12°938	11°927	8°774	17°281	251°5
79°0	24°925	12°061	11°009	8°395	17°183	281°0	109°0	22°287	12°924	11°916	8°760	17°240	251°0
79°5	24°883	12°100	11°047	8°420	17°225	280°5	109°5	22°242	12°909	11°904	8°745	17°197	250°5
80°0	24°841	12°139	11°084	8°444	17°265	280°0	110°0	22°198	12°892	11°891	8°729	17°153	250°0
80°5	24°800	12°177	11°121	8°467	17°303	279°5	110°5	22°153	12°875	11°877	8°713	17°107	249°5
81°0	24°757	12°214	11°157	8°490	17°340	279°0	111°0	22°109	12°857	11°862	8°696	17°060	249°0
81°5	24°715	12°251	11°192	8°512	17°376	278°5	111°5	22°065	12°837	11°846	8°678	17°011	248°5
82°0	24°673	12°287	11°227	8°533	17°410	278°0	112°0	22°021	12°817	11°829	8°659	16°961	248°0
82°5	24°631	12°322	11°261	8°554	17°443	277°5	112°5	21°976	12°795	11°811	8°639	16°910	247°5
83°0	24°588	12°356	11°294	8°574	17°475	277°0	113°0	21°932	12°772	11°792	8°619	16°858	247°0
83°5	24°545	12°389	11°326	8°594	17°505	276°5	113°5	21°888	12°748	11°772	8°599	16°805	246°5
84°0	24°503	12°422	11°358	8°613	17°534	276°0	114°0	21°845	12°723	11°750	8°578	16°750	246°0
84°5	24°460	12°454	11°389	8°631	17°562	275°5	114°5	21°801	12°697	11°728	8°556	16°693	245°5
85°0	24°417	12°485	11°419	8°649	17°589	275°0	115°0	21°757	12°670	11°705	8°533	16°635	245°0
85°5	24°374	12°515	11°449	8°666	17°614	274°5	115°5	21°713	12°642	11°681	8°509	16°576	244°5
86°0	24°331	12°545	11°478	8°683	17°638	274°0	116°0	21°670	12°612	11°655	8°484	16°515	244°0
86°5	24°287	12°573	11°506	8°700	17°661	273°5	116°5	21°627	12°581	11°629	8°458	16°453	243°5
87°0	24°244	12°601	11°533	8°716	17°683	273°0	117°0	21°583	12°550	11°601	8°432	16°389	243°0
87°5	24°200	12°628	11°560	8°731	17°703	272°5	117°5	21°540	12°517	11°572	8°405	16°323	242°5
88°0	24°157	12°654	11°586	8°745	17°722	272°0	118°0	21°497	12°483	11°543	8°378	16°256	242°0
88°5	24°113	12°680	11°611	8°759	17°739	271°5	118°5	21°454	12°447	11°512	8°350	16°188	241°5
89°0	24°070	12°704	11°635	8°772	17°755	271°0	119°0	21°411	12°411	11°480	8°321	16°118	241°0
89°5	24°026	12°728	11°659	8°785	17°770	270°5	119°5	21°368	12°373	11°447	8°291	16°047	240°5
90°0	23°982	12°751	11°681	8°797	17°783	270°0	120°0	21°326	12°335	11°413	8°260	15°975	240°0

TABLE 7.—Values of I , ν , ξ , ν' , and $2\nu''$, corresponding to each half degree of N —Continued.

N	I	ν	ξ	ν'	$2\nu''$	N	N	I	ν	ξ	ν'	$2\nu''$	N
°	°	°	°	°	°	°	°	°	°	°	°	°	°
120°0	21°326	12°335	11°413	8°260	15°975	240°0	150°0	19°162	7°854	7°324	5°096	9°392	210°0
120°5	21°283	12°295	11°378	8°229	15°902	239°5	150°5	19°135	7°745	7°223	5°024	9°251	209°5
121°0	21°241	12°254	11°342	8°197	15°827	239°0	151°0	19°108	7°635	7°120	4°951	9°109	209°0
121°5	21°199	12°211	11°304	8°165	15°751	238°5	151°5	19°082	7°523	7°017	4°878	8°966	208°5
122°0	21°157	12°168	11°266	8°132	15°673	238°0	152°0	19°056	7°411	6°913	4°805	8°823	208°0
122°5	21°115	12°123	11°226	8°098	15°594	237°5	152°5	19°030	7°298	6°809	4°730	8°679	207°5
123°0	21°073	12°078	11°186	8°063	15°513	237°0	153°0	19°005	7°184	6°703	4°655	8°534	207°0
123°5	21°032	12°031	11°144	8°028	15°431	236°5	153°5	18°981	7°069	6°596	4°579	8°389	206°5
124°0	20°990	11°983	11°101	7°992	15°347	236°0	154°0	18°956	6°953	6°488	4°502	8°243	206°0
124°5	20°949	11°933	11°057	7°955	15°262	235°5	154°5	18°933	6°836	6°380	4°424	8°096	205°5
125°0	20°908	11°883	11°012	7°917	15°176	235°0	155°0	18°909	6°718	6°270	4°346	7°948	205°0
125°5	20°867	11°831	10°965	7°878	15°089	234°5	155°5	18°886	6°599	6°160	4°268	7°800	204°5
126°0	20°826	11°778	10°918	7°839	15°001	234°0	156°0	18°863	6°480	6°049	4°189	7°651	204°0
126°5	20°786	11°724	10°869	7°799	14°911	233°5	156°5	18°841	6°359	5°937	4°110	7°502	203°5
127°0	20°746	11°669	10°820	7°758	14°820	233°0	157°0	18°819	6°238	5°824	4°030	7°352	203°0
127°5	20°705	11°612	10°769	7°716	14°728	232°5	157°5	18°798	6°116	5°710	3°950	7°201	202°5
128°0	20°666	11°555	10°717	7°674	14°635	232°0	158°0	18°777	5°993	5°596	3°869	7°050	202°0
128°5	20°626	11°496	10°664	7°631	14°540	231°5	158°5	18°756	5°869	5°480	3°787	6°898	201°5
129°0	20°586	11°436	10°610	7°587	14°444	231°0	159°0	18°736	5°744	5°364	3°705	6°745	201°0
129°5	20°547	11°374	10°554	7°542	14°347	230°5	159°5	18°716	5°618	5°247	3°623	6°592	200°5
130°0	20°508	11°312	10°498	7°496	14°248	230°0	160°0	18°697	5°492	5°130	3°541	6°438	200°0
130°5	20°469	11°248	10°440	7°449	14°148	229°5	160°5	18°678	5°365	5°011	3°458	6°284	199°5
131°0	20°430	11°184	10°382	7°401	14°048	229°0	161°0	18°660	5°237	4°892	3°375	6°130	199°0
131°5	20°392	11°118	10°322	7°353	13°946	228°5	161°5	18°642	5°109	4°773	3°291	5°975	198°5
132°0	20°353	11°050	10°261	7°304	13°842	228°0	162°0	18°624	4°980	4°652	3°207	5°819	198°0
132°5	20°315	10°982	10°199	7°255	13°737	227°5	162°5	18°607	4°850	4°531	3°122	5°663	197°5
133°0	20°278	10°912	10°135	7°205	13°631	227°0	163°0	18°591	4°719	4°409	3°037	5°506	197°0
133°5	20°240	10°841	10°071	7°154	13°524	226°5	163°5	18°575	4°588	4°287	2°952	5°349	196°5
134°0	20°203	10°769	10°005	7°102	13°416	226°0	164°0	18°559	4°456	4°164	2°866	5°192	196°0
134°5	20°166	10°696	9°939	7°050	13°306	225°5	164°5	18°544	4°323	4°040	2°780	5°034	195°5
135°0	20°129	10°622	9°871	6°998	13°195	225°0	165°0	18°529	4°190	3°916	2°694	4°875	195°0
135°5	20°092	10°546	9°802	6°945	13°083	224°5	165°5	18°515	4°056	3°791	2°607	4°716	194°5
136°0	20°056	10°469	9°732	6°891	12°970	224°0	166°0	18°501	3°922	3°665	2°520	4°557	194°0
136°5	20°020	10°391	9°660	6°837	12°856	223°5	166°5	18°487	3°787	3°539	2°433	4°397	193°5
137°0	19°984	10°312	9°588	6°782	12°741	223°0	167°0	18°475	3°651	3°413	2°345	4°238	193°0
137°5	19°949	10°232	9°515	6°727	12°624	222°5	167°5	18°462	3°515	3°286	2°257	4°078	192°5
138°0	19°913	10°150	9°440	6°671	12°506	222°0	168°0	18°450	3°379	3°158	2°169	3°918	192°0
138°5	19°878	10°068	9°364	6°614	12°387	221°5	168°5	18°439	3°242	3°030	2°081	3°757	191°5
139°0	19°844	9°984	9°287	6°556	12°268	221°0	169°0	18°428	3°104	2°902	1°992	3°596	191°0
139°5	19°809	9°899	9°209	6°498	12°148	220°5	169°5	18°417	2°966	2°773	1°903	3°435	190°5
140°0	19°775	9°813	9°130	6°439	12°027	220°0	170°0	18°407	2°828	2°644	1°814	3°273	190°0
140°5	19°742	9°725	9°050	6°379	11°904	219°5	170°5	18°398	2°689	2°514	1°725	3°110	189°5
141°0	19°708	9°637	8°969	6°318	11°780	219°0	171°0	18°388	2°550	2°384	1°635	2°948	189°0
141°5	19°675	9°547	8°887	6°256	11°655	218°5	171°5	18°380	2°410	2°254	1°546	2°786	188°5
142°0	19°642	9°457	8°803	6°193	11°529	218°0	172°0	18°372	2°270	2°123	1°456	2°623	188°0
142°5	19°610	9°365	8°719	6°129	11°402	217°5	172°5	18°364	2°130	1°992	1°365	2°460	187°5
143°0	19°577	9°272	8°633	6°064	11°274	217°0	173°0	18°357	1°989	1°860	1°275	2°297	187°0
143°5	19°545	9°177	8°546	5°999	11°145	216°5	173°5	18°350	1°848	1°729	1°185	2°133	186°5
144°0	19°514	9°082	8°458	5°933	11°016	216°0	174°0	18°344	1°707	1°597	1°094	1°970	186°0
144°5	19°483	8°986	8°370	5°866	10°886	215°5	174°5	18°338	1°566	1°464	1°003	1°806	185°5
145°0	19°452	8°888	8°280	5°798	10°755	215°0	175°0	18°333	1°424	1°332	0°912	1°642	185°0
145°5	19°421	8°790	8°189	5°730	10°623	214°5	175°5	18°328	1°282	1°199	0°821	1°478	184°5
146°0	19°391	8°690	8°097	5°661	10°490	214°0	176°0	18°324	1°140	1°067	0°730	1°314	184°0
146°5	19°361	8°589	8°004	5°592	10°356	213°5	176°5	18°320	0°998	0°934	0°639	1°150	183°5
147°0	19°332	8°487	7°910	5°522	10°221	213°0	177°0	18°317	0°856	0°801	0°548	0°986	183°0
147°5	19°302	8°384	7°814	5°452	10°084	212°5	177°5	18°315	0°713	0°667	0°457	0°822	182°5
148°0	19°273	8°280	7°718	5°382	9°947	212°0	178°0	18°312	0°571	0°534	0°366	0°658	182°0
148°5	19°245	8°175	7°621	5°311	9°809	211°5	178°5	18°311	0°428	0°401	0°274	0°493	181°5
149°0	19°217	8°069	7°523	5°240	9°671	211°0	179°0	18°309	0°286	0°267	0°183	0°329	181°0
149°5	19°189	7°962	7°424	5°168	9°532	210°5	179°5	18°309	0°143	0°133	0°091	0°165	180°5
150°0	19°162	7°854	7°324	5°096	9°392	210°0	180°0	18°308	0°000	0°000	0°000	0°000	180°0

TABLE 8.—Values of R for completing u for L_2 .

P	$I = \text{Inclination of moon's orbit.}$											
	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°
0	0	0	0	0	0	0	0	0	0	0	0	0
5	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00
10	1'76	2'00	2'27	2'57	2'90	3'27	3'67	4'14	4'65	5'22	5'88	6'57
15	3'43	3'90	4'42	5'00	5'63	6'32	7'09	7'94	8'89	9'94	11'12	12'43
20	4'94	5'61	6'39	7'15	8'03	8'99	10'04	11'20	12'47	13'86	15'40	17'10
25	6'24	7'06	7'97	8'94	10'00	11'16	12'40	13'76	15'23	16'82	18'56	20'43
30	7'28	8'21	9'22	10'32	11'49	12'76	14'12	15'57	17'14	18'81	20'60	22'50
35	8'02	9'02	10'10	11'26	12'48	13'80	15'19	16'68	18'25	19'90	21'66	23'50
40	8'48	9'51	10'60	11'77	13'00	14'31	15'68	17'13	18'64	20'23	21'89	23'61
45	8'66	9'69	10'75	11'89	13'08	14'34	15'65	17'02	18'44	19'92	21'44	23'02
50	8'56	9'54	10'57	11'64	12'77	13'95	15'17	16'43	17'73	19'08	20'46	21'87
55	8'22	9'13	10'11	11'09	12'12	13'20	14'30	15'44	16'61	17'81	19'03	20'28
60	7'66	8'49	9'36	10'26	11'18	12'14	13'12	14'13	15'16	16'20	17'27	18'34
65	6'91	7'64	8'40	9'19	10'00	10'83	11'68	12'55	13'43	14'32	15'23	16'14
70	6'00	6'62	7'27	7'94	8'62	9'32	10'03	10'75	11'49	12'23	12'98	13'73
75	4'96	5'46	5'99	6'53	7'08	7'64	8'21	8'79	9'38	9'97	10'56	11'16
80	3'80	4'19	4'59	5'00	5'36	5'84	6'26	6'70	7'14	7'58	8'02	8'47
85	2'58	2'84	3'11	3'38	3'66	3'94	4'23	4'52	4'81	5'10	5'40	5'69
90	1'30	1'43	1'57	1'70	1'84	1'98	2'13	2'27	2'42	2'56	2'71	2'86
95	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00	0'00
100	358'24	358'00	357'73	357'43	357'10	356'73	356'33	355'86	355'35	354'78	354'12	353'43
105	356'57	356'10	355'58	355'00	354'37	353'68	352'91	352'06	351'11	350'06	348'88	347'57
110	355'06	354'39	353'61	352'85	351'97	351'01	349'96	348'80	347'53	346'14	344'60	342'90
115	353'76	352'94	352'03	351'06	350'00	348'84	347'60	346'24	344'77	343'18	341'44	339'57
120	352'72	351'79	350'78	349'68	348'51	347'24	345'88	344'43	342'86	341'19	339'40	337'50
125	351'98	350'98	349'90	348'74	347'52	346'20	344'81	343'32	341'75	340'10	338'34	336'50
130	351'52	350'49	349'40	348'23	347'00	345'69	344'32	342'87	341'36	339'77	338'11	336'39
135	351'34	350'31	349'25	348'11	346'92	345'66	344'35	342'98	341'56	340'08	338'56	336'98
140	351'44	350'46	349'43	348'36	347'23	346'05	344'83	343'57	342'27	340'92	339'54	338'13
145	351'78	350'87	349'89	348'91	347'88	346'80	345'70	344'56	343'39	342'19	340'97	339'72
150	352'34	351'51	350'64	349'74	348'82	347'86	346'88	345'87	344'84	343'80	342'73	341'66
155	353'09	352'36	351'60	350'81	350'00	349'17	348'32	347'45	346'57	345'68	344'77	343'86
160	354'00	353'38	352'73	352'06	351'38	350'68	349'97	349'25	348'51	347'77	347'02	346'27
165	355'04	354'54	354'01	353'47	352'92	352'36	351'79	351'21	350'62	350'03	349'44	348'84
170	356'20	355'81	355'41	355'00	354'64	354'16	353'74	353'30	352'86	352'42	351'98	351'53
175	357'42	357'16	356'89	356'62	356'34	356'06	355'77	355'48	355'19	354'90	354'60	354'31
180	358'70	358'57	358'43	358'30	358'16	358'02	357'87	357'73	357'58	357'44	357'29	357'14
185	360'00	360'00	360'00	360'00	360'00	360'00	360'00	360'00	360'00	360'00	360'00	360'00

u for $L_2 = 2(\xi - \nu) - R$. The values of ξ and ν and to be obtained from Table 7; and the above values of R were computed from the equation $\tan R = \frac{\sin 2P}{\cot^2 \frac{1}{2} I - \cos 2P}$.

The values of I and P for the first day of every month are given in Table 6.

When P lies between 180° and 360° , subtract 180° from it and enter the table with the remainder.

TABLE 9.—Values of Q for completing u for M .

P	Q	P	Q	P	Q	P	Q	P	Q	P	Q
0	0°00	60	40°89	120	139°11	180	180°00	240	220°89	300	319°11
1	0°50	61	42°05	121	140°24	181	180°50	241	222°05	301	320°24
2	1°00	62	43°24	122	141°34	182	181°00	242	223°24	302	321°34
3	1°50	63	44°46	123	142°41	183	181°50	243	224°46	303	322°41
4	2°00	64	45°71	124	143°45	184	182°00	244	225°71	304	323°45
5	2°50	65	47°00	125	144°47	185	182°50	245	227°00	305	324°47
6	3°01	66	48°32	126	145°46	186	183°01	246	228°32	306	325°46
7	3°51	67	49°67	127	146°44	187	183°51	247	229°67	307	326°44
8	4°02	68	51°06	128	147°38	188	184°02	248	231°06	308	327°38
9	4°53	69	52°48	129	148°31	189	184°53	249	232°48	309	328°31
10	5°04	70	53°95	130	149°21	190	185°04	250	233°95	310	329°21
11	5°55	71	55°45	131	150°09	191	185°55	251	235°45	311	330°09
12	6°07	72	56°98	132	150°96	192	186°07	252	236°98	312	330°96
13	6°58	73	58°56	133	151°80	193	186°58	253	238°56	313	331°80
14	7°10	74	60°17	134	152°63	194	187°10	254	240°17	314	332°63
15	7°63	75	61°81	135	153°44	195	187°63	255	241°81	315	333°44
16	8°16	76	63°50	136	154°23	196	188°16	256	243°50	316	334°23
17	8°69	77	65°22	137	155°00	197	188°69	257	245°22	317	335°00
18	9°23	78	66°97	138	155°76	198	189°23	258	246°97	318	335°76
19	9°77	79	68°76	139	156°51	199	189°77	259	248°76	319	336°51
20	10°32	80	70°58	140	157°24	200	190°32	260	250°58	320	337°24
21	10°86	81	72°42	141	157°96	201	190°86	261	252°42	321	337°96
22	11°42	82	74°30	142	158°66	202	191°42	262	254°30	322	338°66
23	11°98	83	76°20	143	159°36	203	191°98	263	256°20	323	339°36
24	12°55	84	78°13	144	160°04	204	192°55	264	258°13	324	340°04
25	13°12	85	80°08	145	160°70	205	193°12	265	260°08	325	340°70
26	13°70	86	82°04	146	161°36	206	193°70	266	262°04	326	341°36
27	14°29	87	84°02	147	162°01	207	194°29	267	264°02	327	342°01
28	14°89	88	86°00	148	162°65	208	194°89	268	266°00	328	342°65
29	15°49	89	88°00	149	163°28	209	195°49	269	268°00	329	343°28
30	16°10	90	90°00	150	163°90	210	196°10	270	270°00	330	343°90
31	16°72	91	92°00	151	164°51	211	196°72	271	272°00	331	344°51
32	17°35	92	94°00	152	165°11	212	197°35	272	274°00	332	345°11
33	17°99	93	95°98	153	165°71	213	197°99	273	275°98	333	345°71
34	18°64	94	97°96	154	166°30	214	198°64	274	277°96	334	346°30
35	19°30	95	99°92	155	166°88	215	199°30	275	279°92	335	346°88
36	19°96	96	101°87	156	167°45	216	199°96	276	281°87	336	347°45
37	20°64	97	103°80	157	168°02	217	200°64	277	283°80	337	348°02
38	21°34	98	105°70	158	168°58	218	201°34	278	285°70	338	348°58
39	22°04	99	107°58	159	169°14	219	202°04	279	287°58	339	349°14
40	22°76	100	109°42	160	169°68	220	202°76	280	289°42	340	349°68
41	23°49	101	111°24	161	170°23	221	203°49	281	291°24	341	350°23
42	24°24	102	113°03	162	170°77	222	204°24	282	293°03	342	350°77
43	25°00	103	114°78	163	171°31	223	205°00	283	294°78	343	351°31
44	25°77	104	116°50	164	171°84	224	205°77	284	296°50	344	351°84
45	26°56	105	118°19	165	172°37	225	206°56	285	298°19	345	352°37
46	27°37	106	119°83	166	172°90	226	207°37	286	299°83	346	352°90
47	28°20	107	121°44	167	173°42	227	208°20	287	301°44	347	353°42
48	29°04	108	123°02	168	173°93	228	209°04	288	303°02	348	353°93
49	29°91	109	124°55	169	174°45	229	209°91	289	304°55	349	354°45
50	30°79	110	126°05	170	174°96	230	210°79	290	306°05	350	354°96
51	31°69	111	127°52	171	175°47	231	211°69	291	307°52	351	355°47
52	32°62	112	128°94	172	175°98	232	212°62	292	308°94	352	355°98
53	33°56	113	130°33	173	176°49	233	213°56	293	310°33	353	356°49
54	34°54	114	131°68	174	176°99	234	214°54	294	311°68	354	356°99
55	35°53	115	133°00	175	177°50	235	215°53	295	313°00	355	357°50
56	36°55	116	134°29	176	178°00	236	216°55	296	314°29	356	358°00
57	37°59	117	135°54	177	178°50	237	217°59	297	315°54	357	358°50
58	38°66	118	136°76	178	179°00	238	218°66	298	316°76	358	359°00
59	39°76	119	137°95	179	179°50	239	219°76	299	317°95	359	359°50

u for $M_1 = \xi - \nu + Q$. The values of ξ and ν are to be obtained from Table 7; the above values of Q were computed from the equation $\tan Q = \frac{1}{2} \tan P$.

The value of P for the first day of every month is given in Table 6.

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years.

Component.	1850		1851		1852		1853		1854		1855	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
$J_1, [M_1]^*$	1.1216 0.0498	0.8916 9.9502	1.0553 0.0234	0.9476 9.9766	0.9935 9.9972	1.0066 0.0028	0.9428 9.9744	1.0607 0.0256	0.9047 9.9565	1.1053 0.0435	0.8788 9.9439	1.1380 0.0561
K_1	1.0840 0.0350	0.9225 9.9650	1.0426 0.0181	0.9591 9.9819	1.0011 0.0005	0.9989 9.9995	0.9647 9.9844	1.0366 0.0156	0.9358 9.9712	1.0686 0.0288	0.9153 9.9616	1.0925 0.0384
K_2	1.2263 0.0886	0.8155 9.9114	1.1276 0.0521	0.8869 9.9479	1.0238 0.0102	0.9767 9.9898	0.9305 9.9687	1.0747 0.0313	0.8559 9.9324	1.1684 0.0676	0.8026 9.9045	1.2459 0.0955
L_2	0.8599 9.9344	1.1629 0.0656	1.1050 0.0434	0.9050 9.9566	1.1380 0.1399	0.7246 9.8601	0.9476 9.9766	1.0553 0.0234	0.7920 9.8987	1.2626 0.1013	1.0594 0.0251	0.9439 9.9749
$[L_2]^*$	0.9733 9.9882	1.0274 0.0118	0.9830 9.9925	1.0173 0.0075	0.9947 9.9977	1.0053 0.0023	1.0073 0.0032	0.9928 9.9968	1.0192 0.0083	0.9811 9.9917	1.0291 0.0124	0.9717 9.9876
M_1	0.9777 9.9902	1.0228 0.0098	0.5972 9.7761	1.6746 0.2239	0.5066 9.7046	1.9741 0.2954	0.6414 9.8072	1.5591 0.1929	0.8942 9.9514	1.1183 0.0486	0.5378 9.7306	1.8595 0.2694
$M_2, M S$	0.9733 9.9882	1.0274 0.0118	0.9830 9.9925	1.0173 0.0075	0.9947 9.9977	1.0053 0.0023	1.0073 0.0032	0.9928 9.9968	1.0192 0.0083	0.9811 9.9917	1.0291 0.0124	0.9717 9.9876
M_3	0.9602 9.9824	1.0415 0.0174	0.9746 9.9888	1.0261 0.0112	0.9921 9.9966	1.0080 0.0034	1.0110 0.0047	0.9891 9.9953	1.0289 0.0124	0.9719 9.9876	1.0440 0.0187	0.9579 9.9813
$M_4, M N$	0.9473 9.9765	1.0557 0.0235	0.9662 9.9851	1.0350 0.0149	0.9895 9.9954	1.0106 0.0046	1.0147 0.0063	0.9856 9.9937	1.0388 0.0165	0.9626 9.9835	1.0590 0.0249	0.9443 9.9751
M_6	0.9220 9.9647	1.0846 0.0353	0.9498 9.9776	1.0529 0.0224	0.9843 9.9931	1.0159 0.0069	1.0221 0.0095	0.9784 9.9905	1.0588 0.0248	0.9445 9.9752	1.0898 0.0374	0.9176 9.9626
M_8	0.8973 9.9530	1.1144 0.0470	0.9336 9.9702	1.0711 0.0298	0.9791 9.9908	1.0213 0.0092	1.0295 0.0126	0.9713 9.9874	1.0791 0.0331	0.9267 9.9669	1.1215 0.0498	0.8917 9.9502
$N_2, 2 N$	0.9733 9.9882	1.0274 0.0118	0.9830 9.9925	1.0173 0.0075	0.9947 9.9977	1.0053 0.0023	1.0073 0.0032	0.9928 9.9968	1.0192 0.0083	0.9811 9.9917	1.0291 0.0124	0.9717 9.9876
O_1, Q_1	1.1449 0.0588	0.8735 9.9412	1.0715 0.0300	0.9333 9.9700	1.0023 0.0010	0.9978 9.9990	0.9446 9.9752	1.0587 0.0248	0.9006 9.9545	1.1104 0.0455	0.8700 9.9395	1.1495 0.0605
OO	1.5841 0.1998	0.6313 9.8002	1.2732 0.1049	0.7855 9.8951	1.0175 0.0075	0.9829 9.9925	0.8305 9.9194	1.2041 0.0806	0.7031 9.8470	1.4223 0.1530	0.6218 9.7936	1.6084 0.2064
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 2, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	0.9733 9.9882	1.0274 0.0118	0.9830 9.9925	1.0173 0.0075	0.9947 9.9977	1.0053 0.0023	1.0073 0.0032	0.9928 9.9968	1.0192 0.0083	0.9811 9.9917	1.0291 0.0124	0.9717 9.9876
$M K$	1.0550 0.0233	0.9479 9.9767	1.0249 0.0107	0.9757 9.9893	0.9958 9.9982	1.0042 0.0018	0.9717 9.9875	1.0291 0.0125	0.9538 9.9795	1.0484 0.0205	0.9420 9.9740	1.0616 0.0260
$2 M K$	1.0268 0.0115	0.9739 9.9885	1.0074 0.0032	0.9926 9.9968	0.9906 9.9959	1.0095 0.0041	0.9788 9.9907	1.0217 0.0093	0.9722 9.9877	1.0286 0.0123	0.9693 9.9865	1.0316 0.0135
$2 M S$	0.9473 9.9765	1.0557 0.0235	0.9662 9.9851	1.0350 0.0149	0.9895 9.9954	1.0106 0.0046	1.0147 0.0063	0.9856 9.9937	1.0388 0.0165	0.9626 9.9835	1.0590 0.0249	0.9443 9.9751
$Msf, 2 SM$	0.9733 9.9882	1.0274 0.0118	0.9830 9.9925	1.0173 0.0075	0.9947 9.9977	1.0053 0.0023	1.0073 0.0032	0.9928 9.9968	1.0192 0.0083	0.9811 9.9917	1.0291 0.0124	0.9717 9.9876
Mf	1.3467 0.1293	0.7426 9.8707	1.1680 0.0674	0.8562 9.9326	1.0098 0.0042	0.9903 9.9958	0.8857 9.9473	1.1290 0.0527	0.7957 9.9008	1.2567 0.0992	0.7355 9.8666	1.3597 0.1334
Mm	0.9138 9.9608	1.0944 0.0392	0.9446 9.9752	1.0587 0.0248	0.9837 9.9929	1.0165 0.0071	1.0278 0.0119	0.9729 9.9881	1.0720 0.0302	0.9328 9.9698	1.1106 0.0455	0.9005 9.9545
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

* The unmarked L_2 and M_1 are the compound waves of the British forms, while $[L_2]$ and $[M_1]$ are the larger of their elements or simple components, corresponding to Ferrel's L_2 with the epoch changed by 180° , and to his m_1 or Q_1 , respectively.

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1856		1857		1858		1859		1860		1861	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
$J_1, [M_1]$	0.8636 9.9363	1.1580 0.0637	0.8583 9.9336	1.1651 0.0664	0.8625 9.9357	1.1595 0.0643	0.8764 9.9427	1.1411 0.0573	0.9010 9.9547	1.1098 0.0453	0.9376 9.9720	1.0665 0.0280
K_1	0.9030 9.9557	1.1074 0.0443	0.8987 9.9536	1.1128 0.0464	0.9021 9.9553	1.1085 0.0447	0.9134 9.9607	1.0948 0.0393	0.9330 9.9699	1.0718 0.0301	0.9609 9.9827	1.0407 0.0173
K_2	0.7707 9.8869	1.2976 0.1131	0.7593 9.8804	1.3170 0.1196	0.7683 9.8855	1.3017 0.1145	0.7977 9.9018	1.2537 0.0982	0.8484 9.9286	1.1786 0.0714	0.9207 9.9641	1.0862 0.0359
L_2	2.1324 0.3289	0.4690 9.6711	1.0393 0.0167	0.9622 9.9833	0.7796 9.8919	1.2827 0.1081	0.9992 9.9996	1.0008 0.0004	1.7598 0.2455	0.5683 9.7545	1.0822 0.0343	0.9241 9.9657
$[L_2]$	1.0357 0.0152	0.9655 9.9848	1.0382 0.0163	0.9632 9.9837	1.0362 0.0155	0.9650 9.9845	1.0301 0.0129	0.9708 9.9871	1.0205 0.0088	0.9799 9.9912	1.0088 0.0038	0.9913 9.9962
M_1	0.4260 9.6294	2.3475 0.3706	0.5343 9.7278	1.8716 0.2722	0.8499 9.9294	1.1766 0.0706	0.5631 9.7506	1.7759 0.2494	0.4490 9.6522	2.2274 0.3478	0.5581 9.7467	1.7919 0.2533
M_2, MS	1.0357 0.0152	0.9655 9.9848	1.0382 0.0163	0.9632 9.9837	1.0362 0.0155	0.9650 9.9845	1.0301 0.0129	0.9708 9.9871	1.0205 0.0088	0.9799 9.9912	1.0088 0.0038	0.9913 9.9962
M_3	1.0540 0.0229	0.9487 9.9771	1.0578 0.0244	0.9453 9.9756	1.0548 0.0232	0.9480 9.9768	1.0454 0.0193	0.9566 9.9807	1.0309 0.0132	0.9700 9.9868	1.0131 0.0057	0.9870 9.9943
M_4, MN	1.0727 0.0305	0.9322 9.9695	1.0779 0.0326	0.9278 9.9674	1.0738 0.0309	0.9313 9.9691	1.0611 0.0257	0.9425 9.9743	1.0415 0.0177	0.9602 9.9823	1.0176 0.0076	0.9827 9.9924
M_6	1.1110 0.0457	0.9001 9.9543	1.1190 0.0488	0.8936 9.9512	1.1127 0.0464	0.8987 9.9536	1.0930 0.0386	0.9149 9.9614	1.0629 0.0265	0.9408 9.9735	1.0265 0.0114	0.9742 9.9886
M_8	1.1507 0.0610	0.8690 9.9390	1.1618 0.0651	0.8608 9.9349	1.1530 0.0618	0.8673 9.9382	1.1258 0.0515	0.8882 9.9485	1.0847 0.0353	0.9219 9.9647	1.0355 0.0152	0.9657 9.9848
$N_2, 2 N$	1.0357 0.0152	0.9655 9.9848	1.0382 0.0163	0.9632 9.9837	1.0362 0.0155	0.9650 9.9845	1.0301 0.0129	0.9708 9.9871	1.0205 0.0088	0.9799 9.9912	1.0088 0.0038	0.9913 9.9962
O_1, Q_1	0.8519 9.9304	1.1739 0.0696	0.8455 9.9271	1.1827 0.0729	0.8505 9.9297	1.1758 0.0703	0.8672 9.9381	1.1532 0.0619	0.8962 9.9524	1.1158 0.0476	0.9387 9.9725	1.0654 0.0275
OO	0.5763 9.7606	1.7352 0.2394	0.5607 9.7488	1.7834 0.2512	0.5730 9.7581	1.7453 0.2419	0.6145 9.7885	1.6273 0.2115	0.6912 9.8396	1.4467 0.1604	0.8127 9.9099	1.2305 0.0901
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 2, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$\lambda_2 \mu_2 \nu_2$	1.0357 0.0152	0.9655 9.9848	1.0382 0.0163	0.9632 9.9837	1.0362 0.0155	0.9650 9.9845	1.0301 0.0129	0.9708 9.9871	1.0205 0.0088	0.9799 9.9912	1.0088 0.0038	0.9913 9.9962
MK	0.9353 9.9709	1.0692 0.0292	0.9330 9.9699	1.0718 0.0301	0.9348 9.9707	1.0697 0.0293	0.9409 9.9736	1.0628 0.0264	0.9521 9.9787	1.0503 0.0213	0.9693 9.9864	1.0317 0.0136
$2 MK$	0.9687 9.9862	1.0323 0.0138	0.9686 9.9862	1.0324 0.0138	0.9687 9.9862	1.0323 0.0138	0.9692 9.9864	1.0318 0.0136	0.9717 9.9875	1.0291 0.0125	0.9778 9.9902	1.0227 0.0098
$2 MS$	1.0727 0.0305	0.9322 9.9695	1.0779 0.0326	0.9278 9.9674	1.0738 0.0309	0.9313 9.9691	1.0611 0.0257	0.9425 9.9743	1.0415 0.0177	0.9602 9.9823	1.0176 0.0076	0.9827 9.9924
$Msf, 2 SM$	1.0357 0.0152	0.9655 9.9848	1.0382 0.0163	0.9632 9.9837	1.0362 0.0155	0.9650 9.9845	1.0301 0.0129	0.9708 9.9871	1.0205 0.0088	0.9799 9.9912	1.0088 0.0038	0.9913 9.9962
Mf	0.7007 9.8455	1.4272 0.1545	0.6886 9.8379	1.4523 0.1621	0.6981 9.8439	1.4325 0.1561	0.7300 9.8633	1.3699 0.1367	0.7871 9.8960	1.2705 0.1040	0.8734 9.9412	1.1449 0.0588
Mm	1.1374 0.0559	0.8792 9.9441	1.1476 0.0598	0.8713 9.9402	1.1395 0.0567	0.8776 9.9433	1.1145 0.0471	0.8973 9.9529	1.0770 0.0322	0.9285 9.9678	1.0331 0.0142	0.9679 9.9858
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component	1862		1863		1864		1865		1866		1867	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
$J_1, [M_1]$	0.9868 9.9942	1.0134 0.0058	1.0474 0.0201	0.9547 9.9799	1.1139 0.0468	0.8978 9.9532	1.1731 0.0693	0.8524 9.9307	1.2068 0.0816	0.8287 9.9184	1.2013 0.0796	0.8325 9.9204
K_1	0.9964 9.9984	1.0036 0.0016	1.0375 0.0160	0.9638 9.9840	1.0793 0.0331	0.9265 9.9669	1.1140 0.0469	0.8977 9.9531	1.1327 0.0541	0.8828 9.9459	1.1297 0.0530	0.8852 9.9470
K_2	1.0120 0.0052	0.9882 9.9948	1.1150 0.0473	0.8969 9.9527	1.2154 0.0847	0.8228 9.9153	1.2941 0.1120	0.7727 9.8880	1.3344 0.1253	0.7494 9.8747	1.3280 0.1232	0.7530 9.8768
L_2	0.8162 9.9118	1.2252 0.0882	0.8950 9.9518	1.1173 0.0482	1.1564 0.0631	0.8647 9.9369	1.1371 0.0558	0.8794 9.9442	0.9239 9.9656	1.0824 0.0344	0.8405 9.9245	1.1898 0.0755
$[L_2]$	0.9962 9.9984	1.0038 0.0016	0.9843 9.9931	1.0159 0.0069	0.9743 9.9887	1.0264 0.0113	0.9672 9.9855	1.0339 0.0145	0.9638 9.9840	1.0376 0.0160	0.9643 9.9842	1.0370 0.0158
M_1	0.9558 9.9804	1.0462 0.0196	0.7939 9.8998	1.2596 0.1002	0.5952 9.7746	1.6802 0.2254	0.6215 9.7934	1.6091 0.2066	0.8590 9.9340	1.1641 0.0660	1.2312 0.0903	0.8122 9.9097
M_2, MS	0.9962 9.9984	1.0038 0.0016	0.9843 9.9931	1.0159 0.0069	0.9743 9.9887	1.0264 0.0113	0.9672 9.9855	1.0339 0.0145	0.9638 9.9840	1.0376 0.0160	0.9643 9.9842	1.0370 0.0158
M_3	0.9944 9.9975	1.0057 0.0025	0.9766 9.9897	1.0240 0.0103	0.9617 9.9830	1.0398 0.0170	0.9513 9.9783	1.0512 0.0217	0.9462 9.9760	1.0569 0.0240	0.9470 9.9763	1.0560 0.0237
M_4, MN	0.9925 9.9967	1.0076 0.0033	0.9689 9.9863	1.0321 0.0137	0.9492 9.9774	1.0535 0.0226	0.9355 9.9711	1.0689 0.0289	0.9289 9.9680	1.0765 0.0320	0.9300 9.9685	1.0753 0.0315
M_6	0.9887 9.9951	1.0114 0.0049	0.9537 9.9794	1.0486 0.0206	0.9248 9.9661	1.0813 0.0339	0.9049 9.9566	1.1051 0.0434	0.8953 9.9520	1.1169 0.0480	0.8968 9.9527	1.1151 0.0473
M_8	0.9850 9.9934	1.0152 0.0066	0.9387 9.9725	1.0653 0.0275	0.9011 9.9548	1.1098 0.0452	0.8752 9.9421	1.1426 0.0579	0.8629 9.9360	1.1589 0.0640	0.8648 9.9369	1.1563 0.0631
$N_2, 2N$	0.9962 9.9984	1.0038 0.0016	0.9843 9.9931	1.0159 0.0069	0.9743 9.9887	1.0264 0.0113	0.9672 9.9855	1.0339 0.0145	0.9638 9.9840	1.0376 0.0160	0.9643 9.9842	1.0370 0.0158
O_1, Q_1	0.9947 9.9977	1.0053 0.0023	1.0627 0.0264	0.9410 9.9736	1.1363 0.0555	0.8800 9.9445	1.2014 0.0797	0.8324 9.9203	1.2382 0.0928	0.8076 9.9072	1.2322 0.0907	0.8116 9.9093
OO	0.9917 9.9964	1.0084 0.0036	1.2389 0.0930	0.8072 9.9070	1.5457 0.1891	0.6470 9.8109	1.8536 0.2680	0.5395 9.7320	2.0436 0.3104	0.4894 9.6896	2.0118 0.3036	0.4971 9.6964
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 2, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	0.9962 9.9984	1.0038 0.0016	0.9843 9.9931	1.0159 0.0069	0.9743 9.9887	1.0264 0.0113	0.9672 9.9855	1.0339 0.0145	0.9638 9.9840	1.0376 0.0160	0.9643 9.9842	1.0370 0.0158
MK	0.9927 9.9968	1.0074 0.0032	1.0212 0.0091	0.9792 9.9909	1.0516 0.0218	0.9510 9.9782	1.0775 0.0324	0.9281 9.9676	1.0917 0.0381	0.9160 9.9619	1.0894 0.0372	0.9179 9.9628
$2MK$	0.9889 9.9952	1.0112 0.0048	1.0052 0.0022	0.9948 9.9978	1.0245 0.0105	0.9761 9.9895	1.0422 0.0179	0.9595 9.9821	1.0522 0.0221	0.9504 9.9779	1.0506 0.0214	0.9519 9.9786
$2MS$	0.9925 9.9967	1.0076 0.0033	0.9689 9.9863	1.0321 0.0137	0.9492 9.9774	1.0535 0.0226	0.9355 9.9711	1.0689 0.0289	0.9289 9.9680	1.0765 0.0320	0.9300 9.9685	1.0753 0.0315
$MSf, 2SM$	0.9962 9.9984	1.0038 0.0016	0.9843 9.9931	1.0159 0.0069	0.9743 9.9887	1.0264 0.0113	0.9672 9.9855	1.0339 0.0145	0.9638 9.9840	1.0376 0.0160	0.9643 9.9842	1.0370 0.0158
Mf	0.9932 9.9970	1.0069 0.0030	1.1474 0.0597	0.8715 9.9403	1.3253 0.1223	0.7546 9.8777	1.4923 0.1738	0.6701 9.8262	1.5907 0.2016	0.6287 9.7984	1.5744 0.1971	0.6352 9.8029
Mm	0.9888 9.9951	1.0113 0.0049	0.9489 9.9772	1.0539 0.0228	0.9169 9.9623	1.0906 0.0377	0.8951 9.9519	1.1172 0.0481	0.8848 9.9468	1.1302 0.0532	0.8864 9.9476	1.1281 0.0524
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1868		1869		1870		1871		1872		1873	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
$J_1, [M_1]$	1.1589 0.0640	0.8629 9.9360	1.0961 0.0398	0.9124 9.9602	1.0303 0.0130	0.9706 9.9870	0.9724 9.9878	1.0284 0.0122	0.9266 9.9669	1.0793 0.0331	0.8933 9.9510	1.1195 0.0490
K_1	1.1059 0.0437	0.9043 9.9563	1.0684 0.0287	0.9360 9.9713	1.0262 0.0112	0.9745 9.9888	0.9862 9.9940	1.0140 0.0060	0.9526 9.9789	1.0498 0.0211	0.9269 9.9670	1.0789 0.0330
K_2	1.2762 0.1059	0.7836 9.8941	1.1898 0.0755	0.8405 9.9245	1.0869 0.0362	0.9201 9.9638	0.9859 9.9938	1.0143 0.0062	0.8992 9.9539	1.1121 0.0461	0.8327 9.9205	1.2010 0.0795
L_2	0.9164 9.9621	1.0912 0.0379	1.1901 0.0756	0.8403 9.9244	1.2073 0.0818	0.8283 9.9182	0.8708 9.9399	1.1484 0.0601	0.8172 9.9123	1.2237 0.0877	1.2260 0.0885	0.8157 9.9115
$[L_2]$	0.9688 9.9862	1.0322 0.0138	0.9767 9.9898	1.0238 0.0102	0.9874 9.9945	1.0128 0.0055	0.9996 9.9998	1.0004 0.0002	1.0121 0.0052	0.9881 9.9948	1.0234 0.0100	0.9772 9.9900
M_1	0.8409 9.9247	1.1892 0.0753	0.5776 9.7616	1.7313 0.2384	0.5521 9.7420	1.8113 0.2580	0.7693 9.8861	1.2999 0.1139	0.8438 9.9262	1.1851 0.0738	0.4990 9.6981	2.0040 0.3019
M_2, MS	0.9688 9.9862	1.0322 0.0138	0.9767 9.9898	1.0238 0.0102	0.9874 9.9945	1.0128 0.0055	0.9996 9.9998	1.0004 0.0002	1.0121 0.0052	0.9881 9.9948	1.0234 0.0100	0.9772 9.9900
M_3	0.9535 9.9793	1.0487 0.0207	0.9653 9.9847	1.0359 0.0153	0.9811 9.9917	1.0192 0.0083	0.9994 9.9997	1.0006 0.0003	1.0182 0.0078	0.9821 9.9922	1.0352 0.0150	0.9660 9.9850
M_4, MN	0.9386 9.9725	1.0655 0.0275	0.9540 9.9795	1.0482 0.0205	0.9749 9.9890	1.0257 0.0110	0.9992 9.9996	1.0008 0.0004	1.0243 0.0104	0.9763 9.9896	1.0473 0.0201	0.9549 9.9799
M_6	0.9093 9.9587	1.0998 0.0413	0.9318 9.9693	1.0732 0.0307	0.9626 9.9834	1.0389 0.0166	0.9988 9.9995	1.0012 0.0005	1.0366 0.0156	0.9647 9.9844	1.0717 0.0301	0.9331 9.9699
M_8	0.8809 9.9449	1.1352 0.0551	0.9101 9.9591	1.0988 0.0409	0.9504 9.9779	1.0522 0.0221	0.9983 9.9993	1.0017 0.0007	1.0492 0.0208	0.9531 9.9792	1.0968 0.0401	0.9118 9.9599
$N_2, 2N$	0.9688 9.9862	1.0322 0.0138	0.9767 9.9898	1.0238 0.0102	0.9874 9.9945	1.0128 0.0055	0.9996 9.9998	1.0004 0.0002	1.0121 0.0052	0.9881 9.9948	1.0234 0.0100	0.9772 9.9900
O_1, Q_1	1.1858 0.0740	0.8433 9.9260	1.1167 0.0479	0.8955 9.9521	1.0436 0.0185	0.9582 9.9815	0.9784 9.9905	1.0221 0.0095	0.9259 9.9666	1.0800 0.0334	0.8871 9.9480	1.1272 0.0520
OO	1.7768 0.2496	0.5628 9.7504	1.4596 0.1642	0.6852 9.8358	1.1658 0.0666	0.8578 9.9334	0.9373 9.9719	1.0669 0.0281	0.7750 9.8893	1.2904 0.1107	0.6667 9.8239	1.5000 0.1761
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, S_2, S_3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	0.9688 9.9862	1.0322 0.0138	0.9767 9.9898	1.0238 0.0102	0.9874 9.9945	1.0128 0.0055	0.9996 9.9998	1.0004 0.0002	1.0121 0.0052	0.9881 9.9948	1.0234 0.0100	0.9772 9.9900
MK	1.0714 0.0299	0.9334 9.9701	1.0435 0.0185	0.9583 9.9815	1.0132 0.0057	0.9870 9.9943	0.9858 9.9938	1.0144 0.0062	0.9641 9.9841	1.0373 0.0159	0.9485 9.9770	1.0543 0.0230
$2MK$	1.0379 0.0162	0.9635 9.9838	1.0192 0.0083	0.9811 9.9917	1.0004 0.0002	0.9996 9.9998	0.9854 9.9936	1.0148 0.0064	0.9757 9.9893	1.0249 0.0107	0.9707 9.9871	1.0302 0.0129
$2MS$	0.9386 9.9725	1.0655 0.0275	0.9540 9.9795	1.0482 0.0205	0.9749 9.9890	1.0257 0.0110	0.9992 9.9996	1.0008 0.0004	1.0243 0.0104	0.9763 9.9896	1.0473 0.0201	0.9549 9.9799
$MSI, 2SM$	0.9688 9.9862	1.0322 0.0138	0.9767 9.9898	1.0238 0.0102	0.9874 9.9945	1.0128 0.0055	0.9996 9.9998	1.0004 0.0002	1.0121 0.0052	0.9881 9.9948	1.0234 0.0100	0.9772 9.9900
Mf	1.4515 0.1618	0.6889 9.8382	1.2766 0.1061	0.7833 9.8939	1.1030 0.0426	0.9066 9.9574	0.9576 9.9812	1.0443 0.0188	0.8471 9.9279	1.1806 0.0721	0.7690 9.8860	1.3003 0.1140
Mm	0.8999 9.9542	1.1112 0.0458	0.9246 9.9659	1.0816 0.0341	0.9589 9.9818	1.0428 0.0182	1.0004 0.0002	0.9996 9.9998	1.0452 0.0192	0.9568 9.9808	1.0880 0.0366	0.9191 9.9634
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1874		1875		1876		1877		1878		1879	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
$J_1, [M_1]$	0.8717 9.9404	1.1472 0.0596	0.8604 9.9347	1.1622 0.0653	0.8588 9.9339	1.1644 0.0661	0.8667 9.9378	1.1538 0.0622	0.8846 9.9468	1.1305 0.0532	0.9137 9.9608	1.0945 0.0392
K_1	0.9096 9.9589	1.0994 0.0411	0.9004 9.9544	1.1106 0.0456	0.8991 9.9538	1.1122 0.0462	0.9056 9.9569	1.1043 0.0431	0.9200 9.9638	1.0869 0.0362	0.9428 9.9744	1.0607 0.0256
K_2	0.7878 9.8964	1.2694 0.1036	0.7638 9.8830	1.3092 0.1170	0.7604 9.8810	1.3151 0.1190	0.7772 9.8905	1.2867 0.1095	0.8148 9.9110	1.2273 0.0890	0.8738 9.9414	1.1444 0.0586
L_1	1.8337 0.2633	0.5453 9.7367	0.9196 9.9636	1.0874 0.0364	0.7874 9.8962	1.2700 0.1038	1.1657 0.0666	0.8578 9.9334	1.8420 0.2653	0.5429 9.7347	0.9649 9.9845	1.0364 0.0155
$[L_1]$	1.0321 0.0137	0.9689 9.9863	1.0372 0.0159	0.9641 9.9841	1.0380 0.0162	0.9634 9.9838	1.0343 0.0146	0.9668 9.9854	1.0267 0.0115	0.9740 9.9885	1.0162 0.0070	0.9841 9.9930
M_1	0.4374 9.6408	2.2864 0.3592	0.6038 9.7809	1.6562 0.2191	0.8150 9.9111	1.2270 0.0889	0.5005 9.6994	1.9981 0.3006	0.4408 9.6442	2.2688 0.3558	0.6079 9.7838	1.6450 0.2162
M_2, MS	1.0321 0.0137	0.9689 9.9863	1.0372 0.0159	0.9641 9.9841	1.0380 0.0162	0.9634 9.9838	1.0343 0.0146	0.9668 9.9854	1.0267 0.0115	0.9740 9.9885	1.0162 0.0070	0.9841 9.9930
M_3	1.0486 0.0206	0.9537 9.9794	1.0563 0.0238	0.9467 9.9762	1.0575 0.0243	0.9456 9.9757	1.0519 0.0220	0.9506 9.9780	1.0404 0.0172	0.9612 9.9828	1.0244 0.0105	0.9762 9.9895
M_4, MN	1.0653 0.0275	0.9387 9.9725	1.0758 0.0317	0.9296 9.9683	1.0774 0.0324	0.9282 9.9676	1.0698 0.0293	0.9348 9.9707	1.0542 0.0229	0.9486 9.9771	1.0326 0.0139	0.9684 9.9861
M_6	1.0995 0.0412	0.9095 9.9588	1.1158 0.0476	0.8962 9.9524	1.1183 0.0485	0.8943 9.9515	1.1065 0.0440	0.9038 9.9561	1.0824 0.0344	0.9239 9.9656	1.0493 0.0209	0.9530 9.9791
M_8	1.1348 0.0549	0.8812 9.9451	1.1573 0.0634	0.8641 9.9366	1.1607 0.0647	0.8615 9.9353	1.1445 0.0586	0.8738 9.9414	1.1113 0.0458	0.8998 9.9542	1.0663 0.0279	0.9378 9.9721
$N_2, 2 N$	1.0321 0.0137	0.9689 9.9863	1.0372 0.0159	0.9641 9.9841	1.0380 0.0162	0.9634 9.9838	1.0343 0.0146	0.9668 9.9854	1.0267 0.0115	0.9740 9.9885	1.0162 0.0070	0.9841 9.9930
O_1, Q_1	0.8615 9.9353	1.1607 0.0647	0.8480 9.9284	1.1792 0.0716	0.8461 9.9274	1.1819 0.0726	0.8556 9.9323	1.1688 0.0677	0.8769 9.9430	1.1404 0.0570	0.9110 9.9595	1.0977 0.0405
OO	0.6003 9.7784	1.6658 0.2216	0.5669 9.7535	1.7639 0.2465	0.5622 9.7499	1.7787 0.2501	0.5854 9.7675	1.7082 0.2325	0.6396 9.8059	1.5634 0.1941	0.7322 9.8646	1.3658 0.1354
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 2, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	1.0321 0.0137	0.9689 9.9863	1.0372 0.0159	0.9641 9.9841	1.0380 0.0162	0.9634 9.9838	1.0343 0.0146	0.9668 9.9854	1.0267 0.0115	0.9740 9.9885	1.0162 0.0070	0.9841 9.9930
MK	0.9388 9.9726	1.0651 0.0274	0.9339 9.9703	1.0708 0.0297	0.9332 9.9700	1.0716 0.0300	0.9366 9.9716	1.0677 0.0284	0.9446 9.9753	1.0586 0.0248	0.9580 9.9814	1.0438 0.0186
$2 MK$	0.9690 9.9863	1.0320 0.0137	0.9686 9.9862	1.0324 0.0138	0.9686 9.9862	1.0324 0.0138	0.9687 9.9862	1.0323 0.0138	0.9699 9.9867	1.0311 0.0133	0.9735 9.9883	1.0272 0.0117
$2 MS$	1.0653 0.0275	0.9387 9.9725	1.0758 0.0317	0.9296 9.9683	1.0774 0.0324	0.9282 9.9676	1.0698 0.0293	0.9348 9.9707	1.0542 0.0229	0.9486 9.9771	1.0326 0.0139	0.9684 9.9861
$MSf, 2 SM$	1.0321 0.0137	0.9689 9.9863	1.0372 0.0159	0.9641 9.9841	1.0380 0.0162	0.9634 9.9838	1.0343 0.0146	0.9668 9.9854	1.0267 0.0115	0.9740 9.9885	1.0162 0.0070	0.9841 9.9930
Mf	0.7192 9.8568	1.3905 0.1432	0.6934 9.8410	1.4422 0.1590	0.6897 9.8387	1.4499 0.1613	0.7077 9.8499	1.4130 0.1501	0.7489 9.8744	1.3352 0.1256	0.8167 9.9121	1.2244 0.0879
Mm	1.1226 0.0502	0.8908 9.9498	1.1435 0.0582	0.8745 9.9418	1.1466 0.0594	0.8721 9.9406	1.1316 0.0537	0.8837 9.9463	1.1012 0.0418	0.9081 9.9582	1.0605 0.0255	0.9430 9.9745
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1880		1881		1882		1883		1884		1885	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
$J_1, [M_1]$	0.9551 9.9801	1.0470 0.0199	1.0092 0.0040	0.9909 9.9960	1.0731 0.0306	0.9319 9.9694	1.1386 0.0564	0.8783 9.9436	1.1902 0.0756	0.8402 9.9244	1.2095 0.0826	0.8268 9.9174
K_1	0.9738 9.9884	1.0269 0.0116	1.0119 0.0052	0.9882 9.9948	1.0540 0.0229	0.9488 9.9772	1.0940 0.0390	0.9140 9.9610	1.1236 0.0506	0.8900 9.9494	1.1342 0.0547	0.8817 9.9453
K_2	0.9540 9.9795	1.0483 0.0205	1.0512 0.0217	0.9512 9.9783	1.1553 0.0627	0.8656 9.9373	1.2494 0.0967	0.8004 9.9033	1.3149 0.1189	0.7605 9.8811	1.3376 0.1263	0.7476 9.8737
L_2	0.8024 9.9044	1.2463 0.0956	0.9803 9.9914	1.0201 0.0086	1.2726 0.1047	0.7858 9.8953	1.0594 0.0250	0.9440 9.9750	0.8678 9.9384	1.1523 0.0616	0.8538 9.9314	1.1712 0.0686
$[L_2]$	1.0039 0.0017	0.9961 9.9983	0.9914 9.9963	1.0086 0.0037	0.9801 9.9913	1.0203 0.0087	0.9712 9.9873	1.0297 0.0127	0.9654 9.9847	1.0358 0.0153	0.9635 9.9839	1.0378 0.0161
M_1	0.9564 9.9807	1.0456 0.0193	0.6543 9.8158	1.5284 0.1842	0.5482 9.7389	1.8242 0.2611	0.6541 9.8156	1.5289 0.1844	1.0314 0.0134	0.9696 9.9866	1.1408 0.0572	0.8766 9.9428
M_2, MS	1.0039 0.0017	0.9961 9.9983	0.9914 9.9963	1.0086 0.0037	0.9801 9.9913	1.0203 0.0087	0.9712 9.9873	1.0297 0.0127	0.9654 9.9847	1.0358 0.0153	0.9635 9.9839	1.0378 0.0161
M_3	1.0059 0.0026	0.9941 9.9974	0.9872 9.9944	1.0130 0.0056	0.9703 9.9869	1.0306 0.0131	0.9571 9.9810	1.0448 0.0190	0.9487 9.9771	1.0541 0.0229	0.9458 9.9758	1.0573 0.0242
M_4, MN	1.0079 0.0034	0.9922 9.9966	0.9830 9.9925	1.0173 0.0075	0.9606 9.9826	1.0410 0.0174	0.9432 9.9746	1.0602 0.0254	0.9321 9.9695	1.0729 0.0305	0.9284 9.9677	1.0771 0.0323
M_6	1.0118 0.0051	0.9883 9.9949	0.9746 9.9888	1.0261 0.0112	0.9415 9.9738	1.0621 0.0262	0.9160 9.9619	1.0917 0.0381	0.8999 9.9542	1.1112 0.0458	0.8946 9.9516	1.1179 0.0484
M_8	1.0158 0.0068	0.9845 9.9932	0.9662 9.9851	1.0350 0.0149	0.9228 9.9651	1.0836 0.0349	0.8896 9.9492	1.1241 0.0508	0.8688 9.9389	1.1510 0.0611	0.8619 9.9355	1.1602 0.0645
$N_2, 2 N$	1.0039 0.0017	0.9961 9.9983	0.9914 9.9963	1.0086 0.0037	0.9801 9.9913	1.0203 0.0087	0.9712 9.9873	1.0297 0.0127	0.9654 9.9847	1.0358 0.0153	0.9635 9.9839	1.0378 0.0161
O_1, Q_1	0.9587 9.9817	1.0430 0.0183	1.0199 0.0086	0.9804 9.9914	1.0912 0.0379	0.9164 9.9621	1.1635 0.0658	0.8595 9.9342	1.2201 0.0864	0.8196 9.9136	1.2412 0.0938	0.8057 9.9062
OO	0.8744 9.9417	1.1437 0.0583	1.0794 0.0332	0.9264 9.9668	1.3526 0.1312	0.7393 9.8688	1.6700 0.2227	0.5988 9.7773	1.9486 0.2897	0.5132 9.7103	2.0597 0.3138	0.4855 9.6862
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, s, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	1.0039 0.0017	0.9961 9.9983	0.9914 9.9963	1.0086 0.0037	0.9801 9.9913	1.0203 0.0087	0.9712 9.9873	1.0297 0.0127	0.9654 9.9847	1.0358 0.0153	0.9635 9.9839	1.0378 0.0161
MK	0.9776 9.9902	1.0229 0.0098	1.0033 0.0014	0.9967 9.9986	1.0331 0.0141	0.9680 9.9859	1.0625 0.0263	0.9412 9.9737	1.0848 0.0353	0.9219 9.9647	1.0929 0.0386	0.9150 9.9614
$2 MK$	0.9814 9.9918	1.0189 0.0082	0.9947 9.9977	1.0053 0.0023	1.0125 0.0054	0.9876 9.9946	1.0319 0.0136	0.9691 9.9864	1.0473 0.0201	0.9549 9.9799	1.0530 0.0224	0.9496 9.9776
$2 MS$	1.0079 0.0034	0.9922 9.9966	0.9830 9.9925	1.0173 0.0075	0.9606 9.9826	1.0410 0.0174	0.9432 9.9746	1.0602 0.0254	0.9321 9.9695	1.0729 0.0305	0.9284 9.9677	1.0771 0.0323
$MSf, 2 SM$	1.0039 0.0017	0.9961 9.9983	0.9914 9.9963	1.0086 0.0037	0.9801 9.9913	1.0203 0.0087	0.9712 9.9873	1.0297 0.0127	0.9654 9.9847	1.0358 0.0153	0.9635 9.9839	1.0378 0.0161
Mf	0.9156 9.9617	1.0922 0.0383	1.0493 0.0209	0.9530 9.9791	1.2149 0.0845	0.8231 9.9155	1.3939 0.1442	0.7174 9.8558	1.5419 0.1880	0.6486 9.8120	1.5989 0.2038	0.6255 9.7962
Mm	1.0157 0.0068	0.9845 9.9932	0.9725 9.9879	1.0283 0.0121	0.9354 9.9710	1.0691 0.0290	0.9072 9.9577	1.1023 0.0423	0.8897 9.9493	1.1239 0.0507	0.8840 9.9465	1.1312 0.0535
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1886		1887		1888		1889		1890		1891	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
$J_1, [M_1]$	1.1885 0.0750	0.8414 9.9250	1.1360 0.0554	0.8803 9.9446	1.0702 0.0295	0.9344 9.9705	1.0066 0.0028	0.9935 9.9972	0.9531 9.9792	1.0492 0.0208	0.9123 9.9601	1.0962 0.0399
K_1	1.1226 0.0502	0.8908 9.9498	1.0925 0.0384	0.9153 9.9616	1.0522 0.0221	0.9504 9.9779	1.0101 0.0044	0.9900 9.9956	0.9723 9.9878	1.0285 0.0122	0.9417 9.9739	1.0620 0.0261
K_2	1.3129 0.1182	0.7617 9.8818	1.2460 0.0955	0.8026 9.9045	1.1508 0.0610	0.8689 9.9390	1.0467 0.0198	0.9554 9.9802	0.9502 9.9778	1.0524 0.0222	0.8710 9.9400	1.1481 0.0600
L_2	0.9943 9.9975	1.0058 0.0025	1.2143 0.0843	0.8235 9.9157	1.0581 0.0245	0.9451 9.9755	0.8301 9.9191	1.2047 0.0809	0.8671 9.9380	1.1533 0.0620	1.4101 0.1492	0.7092 9.8508
$[L_2]$	0.9656 9.9848	1.0356 0.0152	0.9715 9.9874	1.0294 0.0126	0.9806 9.9915	1.0198 0.0085	0.9920 9.9965	1.0081 0.0035	1.0045 0.0019	0.9956 9.9981	1.0166 0.0072	0.9836 9.9928
M_1	0.7330 9.8651	1.3643 0.1349	0.5810 9.7642	1.7211 0.2358	0.6278 9.7978	1.5929 0.2022	0.9302 9.9686	1.0750 0.0314	0.7560 9.8785	1.3228 0.1215	0.4783 9.6797	2.0907 0.3203
M_2, MS	0.9656 9.9848	1.0356 0.0152	0.9715 9.9874	1.0294 0.0126	0.9806 9.9915	1.0198 0.0085	0.9920 9.9965	1.0081 0.0035	1.0045 0.0019	0.9956 9.9981	1.0166 0.0072	0.9836 9.9928
M_3	0.9489 9.9772	1.0539 0.0228	0.9575 9.9811	1.0444 0.0189	0.9710 9.9872	1.0299 0.0128	0.9880 9.9948	1.0122 0.0052	1.0067 0.0029	0.9933 9.9971	1.0251 0.0108	0.9755 9.9892
M_4, MN	0.9324 9.9696	1.0725 0.0304	0.9438 9.9749	1.0596 0.0251	0.9615 9.9830	1.0400 0.0170	0.9840 9.9930	1.0162 0.0070	1.0089 0.0039	0.9912 9.9961	1.0336 0.0143	0.9675 9.9857
M_5	0.9003 9.9544	1.1107 0.0456	0.9168 9.9623	1.0907 0.0377	0.9428 9.9744	1.0606 0.0256	0.9761 9.9895	1.0245 0.0105	1.0134 0.0058	0.9868 9.9912	1.0508 0.0215	0.9517 9.9785
M_6	0.8694 9.9392	1.1503 0.0608	0.8907 9.9497	1.1227 0.0503	0.9245 9.9659	1.0816 0.0341	0.9683 9.9860	1.0328 0.0140	1.0179 0.0077	0.9824 9.9923	1.0683 0.0287	0.9361 9.9713
$N_2, 2N$	0.9656 9.9848	1.0356 0.0152	0.9715 9.9874	1.0294 0.0126	0.9806 9.9915	1.0198 0.0085	0.9920 9.9965	1.0081 0.0035	1.0045 0.0019	0.9956 9.9981	1.0166 0.0072	0.9836 9.9928
O_1, Q_1	1.2182 0.0857	0.8209 9.9143	1.1607 0.0647	0.8616 9.9353	1.0880 0.0366	0.9191 9.9634	1.0170 0.0073	0.9833 9.9927	0.9564 9.9806	1.0456 0.0194	0.9093 9.9587	1.0997 0.0413
OO	1.9390 0.2876	0.5157 9.7124	1.6568 0.2192	0.6036 9.7808	1.3395 0.1270	0.7465 9.8730	1.0689 0.0289	0.9356 9.9711	0.8671 9.9381	1.1532 0.0519	0.7275 9.8618	1.3746 0.1382
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 2, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	0.9656 9.9848	1.0356 0.0152	0.9715 9.9874	1.0294 0.0126	0.9806 9.9915	1.0198 0.0085	0.9920 9.9965	1.0081 0.0035	1.0045 0.0019	0.9956 9.9981	1.0166 0.0072	0.9836 9.9928
MK	1.0840 0.0350	0.9225 9.9650	1.0614 0.0259	0.9422 9.9741	1.0317 0.0136	0.9692 9.9864	1.0020 0.0009	0.9980 9.9991	0.9766 9.9897	1.0239 0.0103	0.9573 9.9811	1.0446 0.0189
$2MK$	1.0467 0.0198	0.9554 9.9802	1.0311 0.0133	0.9698 9.9867	1.0117 0.0050	0.9884 9.9950	0.9940 9.9974	1.0061 0.0026	0.9810 9.9917	1.0194 0.0083	0.9733 9.9882	1.0275 0.0118
$2MS$	0.9324 9.9696	1.0725 0.0304	0.9438 9.9749	1.0596 0.0251	0.9615 9.9830	1.0400 0.0170	0.9840 9.9930	1.0162 0.0070	1.0089 0.0039	0.9912 9.9961	1.0336 0.0143	0.9675 9.9857
$MSf, 2SM$	0.9656 9.9848	1.0356 0.0152	0.9715 9.9874	1.0294 0.0126	0.9806 9.9915	1.0198 0.0085	0.9920 9.9965	1.0081 0.0035	1.0045 0.0019	0.9956 9.9981	1.0166 0.0072	0.9836 9.9928
Mf	1.5369 0.1866	0.6506 9.8134	1.3867 0.1420	0.7212 9.8580	1.2072 0.0818	0.8283 9.9182	1.0426 0.0181	0.9591 9.9819	0.9107 9.9594	1.0981 0.0406	0.8133 9.9103	1.2295 0.0897
Mm	0.8903 9.9495	1.1233 0.0505	0.9082 9.9582	1.1011 0.0418	0.9368 9.9717	1.0674 0.0283	0.9744 9.9887	1.0263 0.0113	0.9827 9.9924	1.0623 0.0262	0.9414 9.9738	
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1892		1893		1894		1895		1896		1897	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
J_1 , $[M_1]$	0.8836 9.9463	1.1317 0.0537	0.8661 9.9376	1.1546 0.0624	0.8587 9.9338	1.1646 0.0662	0.8607 9.9348	1.1619 0.0652	0.8724 9.9407	1.1463 0.0593	0.8945 9.9516	1.1179 0.0484
K_1	0.9192 9.9634	1.0878 0.0366	0.9051 9.9567	1.1049 0.0433	0.8990 9.9538	1.1124 0.0462	0.9006 9.9546	1.1103 0.0454	0.9102 9.9591	1.0987 0.0409	0.9278 9.9675	1.0778 0.0325
K_2	0.8128 9.9100	1.2303 0.0900	0.7760 9.8899	1.2886 0.1101	0.7601 9.8809	1.3156 0.1191	0.7644 9.8833	1.3082 0.1167	0.7892 9.8972	1.2670 0.1028	0.8352 9.9218	1.1974 0.0782
L_2	1.4651 0.1659	0.6826 9.8341	0.8437 9.9262	1.1853 0.0738	0.8202 9.9139	1.2192 0.0861	1.4211 0.1526	0.7037 9.8474	1.6465 0.2166	0.6073 9.7834	0.8766 9.9428	1.1408 0.0572
$[L_2]$	1.0271 0.0116	0.9736 9.9884	1.0346 0.0148	0.9666 9.9852	1.0380 0.0162	0.9634 9.9838	1.0371 0.0158	0.9643 9.9842	1.0318 0.0136	0.9692 9.9864	1.0229 0.0098	0.9776 9.9902
M_1	0.4633 9.6659	2.1582 0.3341	0.6975 9.8435	1.4337 0.1565	0.7305 9.8636	1.3689 0.1364	0.4596 9.6624	2.1758 0.3376	0.4465 9.6498	2.2398 0.3502	0.6799 9.8324	1.4708 0.1676
M_2 , MS	1.0271 0.0116	0.9736 9.9884	1.0346 0.0148	0.9666 9.9852	1.0380 0.0162	0.9634 9.9838	1.0371 0.0158	0.9643 9.9842	1.0318 0.0136	0.9692 9.9864	1.0229 0.0098	0.9776 9.9902
M_3	1.0410 0.0174	0.9607 9.9826	1.0523 0.0221	0.9503 9.9779	1.0576 0.0243	0.9455 9.9757	1.0562 0.0237	0.9468 9.9763	1.0481 0.0204	0.9541 9.9796	1.0346 0.0148	0.9666 9.9852
M_4 , MN	1.0550 0.0232	0.9479 9.9768	1.0703 0.0295	0.9343 9.9705	1.0775 0.0324	0.9281 9.9676	1.0755 0.0316	0.9298 9.9684	1.0646 0.0272	0.9393 9.9728	1.0464 0.0197	0.9557 9.9803
M_6	1.0836 0.0349	0.9229 9.9651	1.1073 0.0443	0.9031 9.9557	1.1185 0.0486	0.8941 9.9514	1.1154 0.0474	0.8965 9.9526	1.0985 0.0408	0.9103 9.9592	1.0703 0.0295	0.9343 9.9705
M_8	1.1130 0.0465	0.8985 9.9535	1.1456 0.0590	0.8729 9.9410	1.1610 0.0648	0.8613 9.9352	1.1568 0.0632	0.8645 9.9368	1.1334 0.0544	0.8823 9.9456	1.0949 0.0394	0.9134 9.9606
N_2 , 2N	1.0271 0.0116	0.9736 9.9884	1.0346 0.0148	0.9666 9.9852	1.0380 0.0162	0.9634 9.9838	1.0371 0.0158	0.9643 9.9842	1.0318 0.0136	0.9692 9.9864	1.0229 0.0098	0.9776 9.9902
O_1 , Q_1	0.8758 9.9424	1.1419 0.0576	0.8549 9.9319	1.1697 0.0681	0.8459 9.9273	1.1821 0.0727	0.8484 9.9286	1.1787 0.0714	0.8624 9.9357	1.1596 0.0643	0.8886 9.9487	1.1254 0.0513
OO	0.6367 9.8039	1.5707 0.1961	0.5837 9.7662	1.7131 0.2338	0.5618 9.7496	1.7800 0.2504	0.5677 9.7541	1.7614 0.2459	0.6024 9.7799	1.6600 0.2201	0.6706 9.8264	1.4914 0.1736
P_1 , R_2 , T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_{1-2-3-4}$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2 , μ_2 , ν_2	1.0271 0.0116	0.9736 9.9884	1.0346 0.0148	0.9666 9.9852	1.0380 0.0162	0.9634 9.9838	1.0371 0.0158	0.9643 9.9842	1.0318 0.0136	0.9692 9.9864	1.0229 0.0098	0.9776 9.9902
MK	0.9442 9.9750	1.0591 0.0250	0.9364 9.9714	1.0679 0.0286	0.9332 9.9700	1.0716 0.0300	0.9340 9.9704	1.0706 0.0296	0.9391 9.9727	1.0648 0.0273	0.9491 9.9773	1.0536 0.0227
2 MK	0.9698 9.9867	1.0312 0.0133	0.9687 9.9862	1.0323 0.0138	0.9687 9.9862	1.0324 0.0138	0.9687 9.9862	1.0324 0.0138	0.9690 9.9863	1.0320 0.0137	0.9709 9.9872	1.0300 0.0128
2 MS	1.0550 0.0232	0.9479 9.9768	1.0703 0.0295	0.9343 9.9705	1.0775 0.0324	0.9281 9.9676	1.0755 0.0316	0.9298 9.9684	1.0646 0.0272	0.9393 9.9728	1.0464 0.0197	0.9557 9.9803
MSf, 2 SM	1.0271 0.0116	0.9736 9.9884	1.0346 0.0148	0.9666 9.9852	1.0380 0.0162	0.9634 9.9838	1.0371 0.0158	0.9643 9.9842	1.0318 0.0136	0.9692 9.9864	1.0229 0.0098	0.9776 9.9902
Mf	0.7467 9.8732	1.3392 0.1268	0.7064 9.8491	1.4156 0.1509	0.6894 9.8385	1.4506 0.1615	0.6940 9.8414	1.4409 0.1586	0.7208 9.8578	1.3874 0.1422	0.7719 9.8876	1.2955 0.1124
Mm	1.1027 0.0424	0.9069 9.9576	1.1326 0.0541	0.8829 9.9459	1.1469 0.0595	0.8719 9.9405	1.1430 0.0580	0.8749 9.9420	1.1214 0.0498	0.8918 9.9502	1.0862 0.0359	0.9206 9.9641
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1898		1899		1900		1901		1902		1903	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
$J_1, [M_1]$	0.9283 9.9677	1.0773 0.0323	0.9746 9.9888	1.0261 0.0112	1.0329 0.0141	0.9681 9.9859	1.0988 0.0409	0.9101 9.9591	1.1611 0.0649	0.8612 9.9351	1.2022 0.0800	0.8318 9.9200
K_1	0.9538 9.9795	1.0484 0.0205	0.9878 9.9947	1.0124 0.0053	1.0279 0.0120	0.9728 9.9880	1.0701 0.0294	0.9345 9.9706	1.1072 0.0442	0.9032 9.9558	1.1302 0.0532	0.8848 9.9468
K_2	0.9025 9.9555	1.1080 0.0445	0.9899 9.9956	1.0102 0.0044	1.0913 0.0379	0.9164 9.9621	1.1938 0.0769	0.8377 9.9231	1.2790 0.1069	0.7819 9.8931	1.3292 0.1236	0.7524 9.8764
L_2	0.8138 9.9105	1.2288 0.0895	1.1150 0.0473	0.8969 9.9527	1.3279 0.1232	0.7531 9.8768	0.9704 9.9870	1.0305 0.0130	0.8383 9.9234	1.1929 0.0766	0.8951 9.9518	1.1172 0.0482
$[L_2]$	1.0116 0.0050	0.9886 9.9950	0.9991 9.9996	1.0009 0.0004	0.9869 9.9943	1.0133 0.0057	0.9764 9.9896	1.0242 0.0104	0.9686 9.9861	1.0325 0.0139	0.9642 9.9842	1.0371 0.0158
M_1	0.8576 9.9333	1.1660 0.0667	0.5615 9.7494	1.7809 0.2506	0.5259 9.7209	1.9016 0.2791	0.7149 9.8542	1.3988 0.1458	1.1662 0.0668	0.8575 9.9332	0.9354 9.9710	1.0690 0.0290
M_2, MS	1.0116 0.0050	0.9886 9.9950	0.9991 9.9996	1.0009 0.0004	0.9869 9.9943	1.0133 0.0057	0.9764 9.9896	1.0242 0.0104	0.9686 9.9861	1.0325 0.0139	0.9642 9.9842	1.0371 0.0158
M_3	1.0173 0.0075	0.9829 9.9925	0.9986 9.9994	1.0015 0.0006	0.9804 9.9913	1.0200 0.0086	0.9648 9.9844	1.0365 0.0156	0.9532 9.9792	1.0491 0.0208	0.9468 9.9763	1.0562 0.0237
M_4, MN	1.0232 0.0100	0.9773 9.9900	0.9981 9.9992	1.0019 0.0008	0.9739 9.9885	1.0268 0.0115	0.9533 9.9792	1.0490 0.0208	0.9381 9.9722	1.0660 0.0278	0.9298 9.9684	1.0755 0.0316
M_5	1.0350 0.0149	0.9662 9.9851	0.9972 9.9988	1.0028 0.0012	0.9611 9.9828	1.0404 0.0172	0.9307 9.9688	1.0744 0.0312	0.9086 9.9584	1.1006 0.0416	0.8965 9.9526	1.1154 0.0474
M_6	1.0469 0.0199	0.9552 9.9801	0.9962 9.9984	1.0038 0.0016	0.9485 9.9770	1.0543 0.0230	0.9087 9.9584	1.1004 0.0416	0.8800 9.9445	1.1364 0.0555	0.8645 9.9368	1.1568 0.0632
$N_2, 2 N$	1.0116 0.0050	0.9886 9.9950	0.9991 9.9996	1.0009 0.0004	0.9869 9.9943	1.0133 0.0057	0.9764 9.9896	1.0242 0.0104	0.9686 9.9861	1.0325 0.0139	0.9642 9.9842	1.0371 0.0158
O_1, Q_1	0.9279 9.9675	1.0778 0.0325	0.9808 9.9916	1.0195 0.0084	1.0465 0.0198	0.9555 9.9802	1.1197 0.0491	0.8931 9.9509	1.1882 0.0749	0.8416 9.9251	1.2332 0.0910	0.8109 9.9090
OO	0.7807 9.8925	1.2809 0.1075	0.9454 9.9756	1.0577 0.0244	1.1769 0.0707	0.8497 9.9293	1.4725 0.1681	0.6791 9.8319	1.7884 0.2525	0.5592 9.7475	2.0172 0.3048	0.4957 9.6952
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_{1, 2, 3, 4}$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	1.0116 0.0050	0.9886 9.9950	0.9991 9.9996	1.0009 0.0004	0.9869 9.9943	1.0133 0.0057	0.9764 9.9896	1.0242 0.0104	0.9686 9.9861	1.0325 0.0139	0.9642 9.9842	1.0371 0.0158
MK	0.9648 9.9844	1.0365 0.0156	0.9868 9.9942	1.0133 0.0058	1.0144 0.0062	0.9858 9.9938	1.0448 0.0190	0.9571 9.9810	1.0723 0.0303	0.9326 9.9697	1.0898 0.0374	0.9176 9.9626
$2 MK$	0.9760 9.9894	1.0246 0.0106	0.9859 9.9938	1.0143 0.0062	1.0011 0.0005	0.9989 9.9995	1.0201 0.0086	0.9803 9.9914	1.0386 0.0164	0.9629 9.9836	1.0508 0.0215	0.9516 9.9785
$2 MS$	1.0232 0.0100	0.9773 9.9900	0.9981 9.9992	1.0019 0.0008	0.9739 9.9885	1.0268 0.0115	0.9533 9.9792	1.0490 0.0208	0.9381 9.9722	1.0660 0.0278	0.9298 9.9684	1.0755 0.0316
$MSf, 2 SM$	1.0116 0.0050	0.9886 9.9950	0.9991 9.9996	1.0009 0.0004	0.9869 9.9943	1.0133 0.0057	0.9764 9.9896	1.0242 0.0104	0.9686 9.9861	1.0325 0.0139	0.9642 9.9842	1.0371 0.0158
Mf	0.8511 9.9300	1.1750 0.0700	0.9630 9.9836	1.0384 0.0164	1.1098 0.0452	0.9011 9.9548	1.2840 0.1086	0.7788 9.8914	1.4578 0.1637	0.6860 9.8363	1.5772 0.1979	0.6340 9.8021
Mm	1.0433 0.0184	0.9585 9.9816	0.9986 9.9994	1.0014 0.0006	0.9573 9.9811	1.0446 0.0189	0.9234 9.9654	1.0830 0.0346	0.8991 9.9538	1.1122 0.0462	0.8861 9.9475	1.1285 0.0525
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1904		1905		1906		1907		1908		1909	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
$J_1, [M_1]$	1.2061 0.0814	0.8291 9.9186	1.1711 0.0686	0.8539 9.9314	1.1113 0.0458	0.8999 9.9542	1.0449 0.0191	0.9570 9.9809	0.9846 9.9933	1.0156 0.0067	0.9358 9.9712	1.0686 0.0288
K_1	1.1323 0.0540	0.8831 9.9460	1.1128 0.0464	0.8986 9.9536	1.0777 0.0325	0.9279 9.9675	1.0358 0.0153	0.9654 9.9847	0.9949 9.9978	1.0052 0.0023	0.9595 9.9821	1.0422 0.0179
K_2	1.3336 0.1250	0.7499 9.8750	1.2916 0.1111	0.7742 9.8889	1.2117 0.0834	0.8253 9.9166	1.1109 0.0457	0.9002 9.9543	1.0080 0.0035	0.9920 9.9965	0.9172 9.9625	1.0902 0.0375
L_2	1.0807 0.0337	0.9254 9.9663	1.1650 0.0663	0.8584 9.9337	0.9514 9.9783	1.0511 0.0217	0.8192 9.9134	1.2207 0.0866	0.9416 9.9739	1.0620 0.0261	1.5314 0.1851	0.6530 9.8149
$[L_2]$	0.9639 9.9840	1.0375 0.0160	0.9674 9.9856	1.0336 0.0144	0.9746 9.9888	1.0260 0.0112	0.9848 9.9933	1.0155 0.0067	0.9967 9.9986	1.0033 0.0014	1.0093 0.0040	0.9908 9.9960
M_1	0.6636 9.8219	1.5070 0.1781	0.6086 9.7843	1.6432 0.2157	0.7465 9.8730	1.3396 0.1270	1.0574 0.0242	0.9457 9.9758	0.6762 9.8300	1.4789 0.1700	0.4735 9.6753	2.1120 0.3247
M_2, MS	0.9639 9.9840	1.0375 0.0160	0.9674 9.9856	1.0336 0.0144	0.9746 9.9888	1.0260 0.0112	0.9848 9.9933	1.0155 0.0067	0.9967 9.9986	1.0033 0.0014	1.0093 0.0040	0.9908 9.9960
M_3	0.9463 9.9760	1.0567 0.0240	0.9515 9.9784	1.0509 0.0216	0.9623 9.9833	1.0392 0.0167	0.9773 9.9900	1.0233 0.0100	0.9951 9.9979	1.0049 0.0021	1.0140 0.0060	0.9862 9.9940
M_4, MN	0.9291 9.9680	1.0764 0.0320	0.9360 9.9713	1.0684 0.0287	0.9499 9.9777	1.0527 0.0223	0.9698 9.9867	1.0312 0.0133	0.9934 9.9971	1.0066 0.0029	1.0186 0.0080	0.9817 9.9920
M_6	0.8955 9.9521	1.1167 0.0479	0.9055 9.9569	1.1044 0.0431	0.9259 9.9666	1.0801 0.0334	0.9550 9.9800	1.0472 0.0200	0.9902 9.9957	1.0099 0.0043	1.0281 0.0120	0.9727 9.9880
M_8	0.8631 9.9361	1.1586 0.0639	0.8760 9.9425	1.1415 0.0575	0.9024 9.9554	1.1082 0.0446	0.9404 9.9733	1.0634 0.0267	0.9869 9.9943	1.0133 0.0057	1.0376 0.0160	0.9637 9.9840
$N_2, 2 N$	0.9639 9.9840	1.0375 0.0160	0.9674 9.9856	1.0336 0.0144	0.9746 9.9888	1.0260 0.0112	0.9848 9.9933	1.0155 0.0067	0.9967 9.9986	1.0033 0.0014	1.0093 0.0040	0.9908 9.9960
O_1, Q_1	1.2375 0.0925	0.8081 9.9075	1.1992 0.0789	0.8339 9.9211	1.1335 0.0544	0.8823 9.9456	1.0599 0.0253	0.9435 9.9747	0.9922 9.9966	1.0078 0.0034	0.9366 9.9716	1.0677 0.0284
OO	2.0396 0.3095	0.4903 9.6905	1.8424 0.2654	0.5428 9.7346	1.5329 0.1855	0.6524 9.8145	1.2279 0.0892	0.8144 9.9108	0.9833 9.9927	1.0170 0.0073	0.8066 9.9066	1.2398 0.0934
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 2, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	0.9639 9.9840	1.0375 0.0160	0.9674 9.9856	1.0336 0.0144	0.9746 9.9888	1.0260 0.0112	0.9848 9.9933	1.0155 0.0067	0.9967 9.9986	1.0033 0.0014	1.0093 0.0040	0.9908 9.9960
MK	1.0914 0.0380	0.9162 9.9620	1.0766 0.0321	0.9288 9.9679	1.0504 0.0214	0.9520 9.9786	1.0201 0.0086	0.9803 9.9914	0.9916 9.9963	1.0085 0.0037	0.9684 9.9861	1.0326 0.0139
$2 MK$	1.0520 0.0220	0.9506 9.9780	1.0416 0.0177	0.9601 9.9823	1.0238 0.0102	0.9768 9.9898	1.0045 0.0020	0.9955 9.9980	0.9883 9.9949	1.0118 0.0051	0.9774 9.9901	1.0231 0.0099
$2 MS$	0.9291 9.9680	1.0764 0.0320	0.9360 9.9713	1.0684 0.0287	0.9499 9.9777	1.0527 0.0223	0.9698 9.9867	1.0312 0.0133	0.9934 9.9971	1.0066 0.0029	1.0186 0.0080	0.9817 9.9920
$MSf, 2 SM$	0.9639 9.9840	1.0375 0.0160	0.9674 9.9856	1.0336 0.0144	0.9746 9.9888	1.0260 0.0112	0.9848 9.9933	1.0155 0.0067	0.9967 9.9986	1.0033 0.0014	1.0093 0.0040	0.9908 9.9960
Mf	1.5887 0.2010	0.6295 9.7990	1.4864 0.1721	0.6728 9.8279	1.3181 0.1200	0.7586 9.8800	1.1408 0.0572	0.8766 9.9428	0.9878 9.9947	1.0124 0.0053	0.8692 9.9391	1.1506 0.0609
Mm	0.8850 9.9470	1.1299 0.0530	0.8958 9.9522	1.1163 0.0478	0.9180 9.9628	1.0893 0.0372	0.9503 9.9779	1.0523 0.0221	0.9905 9.9958	1.0096 0.0042	1.0350 0.0149	0.9662 9.9851
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1910		1911		1912		1913		1914		1915	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
$J_1, [M_1]$	0.8998 9.9542	1.1113 0.0458	0.8757 9.9423	1.1420 0.0577	0.8621 9.9356	1.1599 0.0644	0.8583 9.9336	1.1650 0.0664	0.8640 9.9365	1.1574 0.0635	0.8796 9.9443	1.1369 0.0557
K_1	0.9320 9.9694	1.0729 0.0306	0.9128 9.9604	1.0955 0.0396	0.9018 9.9551	1.1089 0.0449	0.8987 9.9536	1.1127 0.0464	0.9034 9.9559	1.1070 0.0414	0.9160 9.9619	1.0917 0.0381
K_2	0.8460 9.9274	1.1820 0.0726	0.7962 9.9010	1.2560 0.0990	0.7675 9.8851	1.3030 0.1149	0.7594 9.8805	1.3168 0.1195	0.7715 9.8873	1.2962 0.1127	0.8043 9.9054	1.2433 0.0946
L_2	1.1998 0.0791	0.8335 9.9209	0.8024 9.9044	1.2463 0.0956	0.8811 9.9450	1.1349 0.0550	1.7834 0.2512	0.5607 9.7488	1.3712 0.1371	0.7293 9.8629	0.8193 9.9134	1.2206 0.0866
$[L_2]$	1.0210 0.0090	0.9795 9.9910	1.0304 0.0130	0.9705 9.9870	1.0364 0.0155	0.9649 9.9845	1.0382 0.0163	0.9632 9.9837	1.0355 0.0152	0.9657 9.9848	1.0288 0.0123	0.9720 9.9877
M_1	0.5070 9.7050	1.9723 0.2950	0.8010 9.9036	1.2484 0.0964	0.6424 9.8078	1.5567 0.1922	0.4354 9.6388	2.2969 0.3612	0.4661 9.6685	2.1455 0.3315	0.7636 9.8829	1.3096 0.1171
M_2, MS	1.0210 0.0090	0.9795 9.9910	1.0304 0.0130	0.9705 9.9870	1.0364 0.0155	0.9649 9.9845	1.0382 0.0163	0.9632 9.9837	1.0355 0.0152	0.9657 9.9848	1.0288 0.0123	0.9720 9.9877
M_3	1.0316 0.0135	0.9693 9.9864	1.0459 0.0195	0.9561 9.9805	1.0551 0.0233	0.9477 9.9767	1.0578 0.0244	0.9454 9.9756	1.0538 0.0227	0.9490 9.9773	1.0435 0.0185	0.9583 9.9815
M_4, MN	1.0424 0.0180	0.9594 9.9820	1.0617 0.0260	0.9419 9.9740	1.0741 0.0311	0.9310 9.9689	1.0778 0.0325	0.9278 9.9675	1.0723 0.0303	0.9326 9.9697	1.0584 0.0246	0.9448 9.9754
M_5	1.0642 0.0270	0.9397 9.9730	1.0940 0.0390	0.9141 9.9610	1.1132 0.0466	0.8983 9.9534	1.1189 0.0488	0.8937 9.9512	1.1104 0.0455	0.9006 9.9545	1.0888 0.0370	0.9184 9.9630
M_6	1.0865 0.0360	0.9204 9.9640	1.1272 0.0520	0.8872 9.9480	1.1538 0.0621	0.8667 9.9379	1.1617 0.0651	0.8608 9.9349	1.1498 0.0606	0.8697 9.9394	1.1202 0.0493	0.8927 9.9507
$N_2, 2N$	1.0210 0.0090	0.9795 9.9910	1.0304 0.0130	0.9705 9.9870	1.0364 0.0155	0.9649 9.9845	1.0382 0.0163	0.9632 9.9837	1.0355 0.0152	0.9657 9.9848	1.0288 0.0123	0.9720 9.9877
O_1, Q_1	0.8948 9.9517	1.1175 0.0483	0.8663 9.9377	1.1543 0.0623	0.8501 9.9295	1.1763 0.0705	0.8455 9.9271	1.1827 0.0729	0.8524 9.9306	1.1732 0.0694	0.8710 9.9400	1.1482 0.0600
OO	0.6874 9.8372	1.4548 0.1628	0.6124 9.7870	1.6330 0.2130	0.5719 9.7574	1.7484 0.2426	0.5609 9.7488	1.7830 0.2512	0.5775 9.7615	1.7317 0.2385	0.6242 9.7953	1.6020 0.2047
P_1, R_1, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
S_1, S_2, S_3, S_4	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	1.0210 0.0090	0.9795 9.9910	1.0304 0.0130	0.9705 9.9870	1.0364 0.0155	0.9649 9.9845	1.0382 0.0163	0.9632 9.9837	1.0355 0.0152	0.9657 9.9848	1.0288 0.0123	0.9720 9.9877
MK	0.9516 9.9784	1.0509 0.0216	0.9406 9.9734	1.0632 0.0266	0.9347 9.9706	1.0699 0.0294	0.9330 9.9699	1.0718 0.0301	0.9354 9.9710	1.0690 0.0290	0.9423 9.9742	1.0612 0.0258
$2MK$	0.9715 9.9874	1.0293 0.0126	0.9692 9.9864	1.0318 0.0136	0.9687 9.9862	1.0323 0.0138	0.9686 9.9862	1.0324 0.0138	0.9687 9.9862	1.0323 0.0138	0.9695 9.9865	1.0315 0.0135
$2MS$	1.0424 0.0180	0.9594 9.9820	1.0617 0.0260	0.9419 9.9740	1.0741 0.0311	0.9310 9.9689	1.0778 0.0325	0.9278 9.9675	1.0723 0.0303	0.9326 9.9697	1.0584 0.0246	0.9448 9.9754
$MSf, 2SM$	1.0210 0.0090	0.9795 9.9910	1.0304 0.0130	0.9705 9.9870	1.0364 0.0155	0.9649 9.9845	1.0382 0.0163	0.9632 9.9837	1.0355 0.0152	0.9657 9.9848	1.0288 0.0123	0.9720 9.9877
Mf	0.7843 9.8945	1.2750 0.1055	0.7284 9.8623	1.3730 0.1377	0.6973 9.8434	1.4341 0.1566	0.6886 9.8380	1.4521 0.1620	0.7016 9.8461	1.4254 0.1539	0.7373 9.8677	1.3562 0.1323
Mm	1.0787 0.0329	0.9270 9.9671	1.1157 0.0476	0.8963 9.9524	1.1402 0.0570	0.8770 9.9430	1.1476 0.0598	0.8714 9.9402	1.1366 0.0556	0.8798 9.9444	1.1092 0.0450	0.9015 9.9550
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1916		1917		1918		1919		1920		1921	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
$J_1, [M_1]$	0'9060 9'9571	1'1038 0'0429	0'9446 9'9752	1'0587 0'0248	0'9958 9'9982	1'0042 0'0018	1'0578 0'0244	0'9453 9'9756	1'1242 0'0508	0'8895 9'9492	1'1806 0'0721	0'8470 9'9279
K_1	0'9368 9'9717	1'0674 0'0283	0'9660 9'9850	1'0352 0'0150	1'0027 0'0012	0'9973 9'9988	1'0443 0'0188	0'9576 9'9812	1'0855 0'0356	0'9212 9'9644	1'1182 0'0485	0'8943 9'9515
K_2	0'8585 9'9337	1'1649 0'0663	0'9340 9'9704	1'0707 0'0296	1'0278 0'0119	0'9729 9'9881	1'1316 0'0537	0'8838 9'9463	1'2299 0'0899	0'8131 9'9101	1'3034 0'1151	0'7672 9'8849
L_2	0'8529 9'9309	1'1725 0'0691	1'3141 0'1186	0'7610 9'8814	1'2821 0'1079	0'7800 9'8921	0'8944 9'9515	1'1181 0'0485	0'8351 9'9218	1'1975 0'0782	0'9669 9'9854	1'0342 0'0146
$[L_2]$	1'0188 0'0081	0'9816 9'9919	1'0068 0'0029	0'9933 9'9971	0'9942 9'9975	1'0058 0'0025	0'9826 9'9924	1'0177 0'0076	0'9730 9'9881	1'0278 0'0119	0'9664 9'9852	1'0347 0'0148
M_1	0'7294 9'8630	1'3710 0'1370	0'5018 9'7005	1'9929 0'2995	0'5237 9'7191	1'9094 0'2809	0'8047 9'9056	1'2427 0'0944	1'1157 0'0476	0'8963 9'9524	0'7644 9'8833	1'3083 0'1167
M_2, MS	1'0188 0'0081	0'9816 9'9919	1'0068 0'0029	0'9933 9'9971	0'9942 9'9975	1'0058 0'0025	0'9826 9'9924	1'0177 0'0076	0'9730 9'9881	1'0278 0'0119	0'9664 9'9852	1'0347 0'0148
M_3	1'0283 0'0121	0'9725 9'9879	1'0102 0'0044	0'9899 9'9956	0'9914 9'9962	1'0087 0'0038	0'9740 9'9885	1'0267 0'0115	0'9597 9'9822	1'0420 0'0179	0'9501 9'9778	1'0525 0'0222
M_4, MN	1'0379 0'0162	0'9635 9'9838	1'0136 0'0059	0'9866 9'9941	0'9886 9'9950	1'0116 0'0050	0'9654 9'9847	1'0358 0'0153	0'9466 9'9762	1'0564 0'0238	0'9340 9'9703	1'0707 0'0297
M_5	1'0574 0'0242	0'9457 9'9758	1'0205 0'0088	0'9799 9'9912	0'9829 9'9925	1'0174 0'0075	0'9486 9'9771	1'0542 0'0229	0'9210 9'9643	1'0858 0'0357	0'9026 9'9555	1'1079 0'0445
M_6	1'0773 0'0323	0'9283 9'9677	1'0274 0'0118	0'9733 9'9882	0'9772 9'9900	1'0233 0'0100	0'9321 9'9694	1'0729 0'0306	0'8961 9'9524	1'1159 0'0476	0'8723 9'9407	1'1464 0'0593
$N_2, 2 N$	1'0188 0'0081	0'9816 9'9919	1'0068 0'0029	0'9933 9'9971	0'9942 9'9975	1'0058 0'0025	0'9826 9'9924	1'0177 0'0076	0'9730 9'9881	1'0278 0'0119	0'9664 9'9852	1'0347 0'0148
O_1, Q_1	0'9020 9'9552	1'1086 0'0448	0'9466 9'9762	1'0564 0'0238	1'0048 0'0021	0'9952 9'9979	1'0743 0'0311	0'9308 9'9689	1'1477 0'0598	0'8713 9'9402	1'2096 0'0826	0'8267 9'9174
OO	0'7072 9'8495	1'4141 0'1505	0'8369 9'9227	1'1948 0'0773	1'0263 0'0113	0'9744 9'9887	1'2843 0'1086	0'7787 9'8914	1'5969 0'2033	0'6262 9'7967	1'8950 0'2776	0'5277 9'7224
P_1, R_2, T_2	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000
$S_1, 2, 3, 4$	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000
λ_2, μ_2, ν_2	1'0188 0'0081	0'9816 9'9919	1'0068 0'0029	0'9933 9'9971	0'9942 9'9975	1'0058 0'0025	0'9826 9'9924	1'0177 0'0076	0'9730 9'9881	1'0278 0'0119	0'9664 9'9852	1'0347 0'0148
MK	0'9544 9'9797	1'0478 0'0203	0'9726 9'9879	1'0282 0'0121	0'9969 9'9987	1'0031 0'0013	1'0261 0'0112	0'9746 9'9888	1'0562 0'0237	0'9468 9'9763	1'0807 0'0337	0'9253 9'9663
$2 MK$	0'9723 9'9878	1'0284 0'0122	0'9792 9'9909	1'0213 0'0091	0'9912 9'9962	1'0089 0'0038	1'0082 0'0035	0'9919 9'9965	1'0276 0'0118	0'9732 9'9882	1'0444 0'0189	0'9575 9'9811
$2 MS$	1'0379 0'0162	0'9635 9'9838	1'0136 0'0059	0'9866 9'9941	0'9886 9'9950	1'0116 0'0050	0'9654 9'9847	1'0358 0'0153	0'9466 9'9762	1'0564 0'0238	0'9340 9'9703	1'0707 0'0297
$MSf, 2 SM$	1'0188 0'0081	0'9816 9'9919	1'0068 0'0029	0'9933 9'9971	0'9942 9'9975	1'0058 0'0025	0'9826 9'9924	1'0177 0'0076	0'9730 9'9881	1'0278 0'0119	0'9664 9'9852	1'0347 0'0148
Mf	0'7987 9'9024	1'2521 0'0976	0'8901 9'9494	1'1235 0'0506	1'0155 0'0067	0'9847 9'9933	1'1746 0'0699	0'8514 9'9301	1'3538 0'1316	0'7387 9'8684	1'5140 0'1801	0'6605 9'8199
Mm	1'0703 0'0295	0'9343 9'9705	1'0260 0'0111	0'9747 9'9889	0'9820 9'9921	1'0183 0'0079	0'9432 9'9746	1'0602 0'0254	0'9127 9'9603	1'0956 0'0397	0'8927 9'9507	1'1202 0'0493
Sa, Ssa	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1922		1923		1924		1925		1926		1927	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
$J_1, [M_1]$	1'2087 0'0823	0'8273 9'9177	1'1967 0'0780	0'8357 9'9220	1'1498 0'0606	0'8697 9'9394	1'0854 0'0356	0'9213 9'9644	1'0204 0'0088	0'9800 9'9912	0'9643 9'9842	1'0370 0'0158
K_1	1'1338 0'0545	0'8820 9'9455	1'1272 0'0520	0'8872 9'9480	1'1006 0'0416	0'9086 9'9584	1'0618 0'0260	0'9418 9'9740	1'0195 0'0084	0'9808 9'9916	0'9804 9'9914	1'0200 0'0086
K_2	1'3366 0'1260	0'7482 9'8740	1'3226 0'1214	0'7561 9'8786	1'2644 0'1019	0'7909 9'8981	1'1740 0'0697	0'8518 9'9303	1'0703 0'0295	0'9343 9'9705	0'9710 9'9872	1'0299 0'0128
L_2	1'1489 0'0603	0'8704 9'9397	1'0734 0'0308	0'9316 9'9692	0'8828 9'9459	1'1328 0'0541	0'8340 9'9212	1'1990 0'0788	1'0387 0'0165	0'9627 9'9835	1'4953 1'1747	0'6688 9'8253
$[L_2]$	0'9636 9'9839	1'0378 0'0161	0'9648 9'9844	1'0365 0'0156	0'9698 9'9867	1'0311 0'0133	0'9783 9'9905	1'0222 0'0095	0'9892 9'9953	1'0109 0'0047	1'0016 0'0007	0'9984 9'9993
M_1	0'6260 9'7966	1'5974 0'2034	0'6653 9'8230	1'5030 0'1770	0'9244 9'9659	1'0818 0'0341	1'0488 0'0207	0'9535 9'9793	0'6177 9'7908	1'6190 0'2092	0'4847 9'6855	2'0630 0'3145
M_2, MS	0'9636 9'9839	1'0378 0'0161	0'9648 9'9844	1'0365 0'0156	0'9698 9'9867	1'0311 0'0133	0'9783 9'9905	1'0222 0'0095	0'9892 9'9953	1'0109 0'0047	1'0016 0'0007	0'9984 9'9993
M_3	0'9459 9'9758	1'0572 0'0242	0'9476 9'9766	1'0553 0'0234	0'9551 9'9800	1'0470 0'0200	0'9676 9'9857	1'0335 0'0143	0'9839 9'9930	1'0164 0'0070	1'0024 0'0010	0'9976 9'9990
M_4, MN	0'9285 9'9678	1'0770 0'0322	0'9308 9'9689	1'0743 0'0311	0'9406 9'9734	1'0632 0'0266	0'9570 9'9809	1'0449 0'0191	0'9786 9'9906	1'0219 0'0094	1'0032 0'0014	0'9968 9'9986
M_5	0'8947 9'9517	1'1176 0'0483	0'8980 9'9533	1'1136 0'0467	0'9122 9'9601	1'0962 0'0399	0'9362 9'9714	1'0681 0'0286	0'9681 9'9859	1'0330 0'0141	1'0048 0'0021	0'9952 9'9979
M_6	0'8622 9'9356	1'1598 0'0644	0'8664 9'9377	1'1542 0'0623	0'8847 9'9468	1'1303 0'0532	0'9159 9'9618	1'0918 0'0382	0'9576 9'9812	1'0442 0'0188	1'0064 0'0028	0'9937 9'9972
$N_2, 2 N$	0'9636 9'9839	1'0378 0'0161	0'9648 9'9844	1'0365 0'0156	0'9698 9'9867	1'0311 0'0133	0'9783 9'9905	1'0222 0'0095	0'9892 9'9953	1'0109 0'0047	1'0016 0'0007	0'9984 9'9993
O_1, Q_1	1'2403 0'0935	0'8063 9'9065	1'2272 0'0889	0'8149 9'9111	1'1758 0'0704	0'8504 9'9296	1'1049 0'0433	0'9050 9'9567	1'0325 0'0139	0'9685 9'9861	0'9692 9'9864	1'0318 0'0136
OO	2'0547 0'3128	0'4867 9'6872	1'9853 0'2978	0'5037 9'7022	1'7284 0'2376	0'5786 9'7624	1'4095 0'1491	0'7095 9'8509	1'1248 0'0510	0'8891 9'9490	0'9074 9'9578	1'1020 0'0422
P_1, R_2, T_2	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000
$S_1, 2, 3, 4$	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000
λ, μ_2, ν_2	0'9636 9'9839	1'0378 0'0161	0'9648 9'9844	1'0365 0'0156	0'9698 9'9867	1'0311 0'0133	0'9783 9'9905	1'0222 0'0095	0'9892 9'9953	1'0109 0'0047	1'0016 0'0007	0'9984 9'9993
MK	1'0925 0'0384	0'9153 9'9616	1'0875 0'0364	0'9196 9'9636	1'0674 0'0283	0'9368 9'9717	1'0387 0'0165	0'9627 9'9835	1'0086 0'0037	0'9915 9'9963	0'9820 9'9921	1'0184 0'0079
$2 MK$	1'0527 0'0223	0'9499 9'9777	1'0492 0'0208	0'9531 9'9792	1'0352 0'0150	0'9660 9'9850	1'0162 0'0070	0'9841 9'9930	0'9977 9'9990	1'0023 0'0010	0'9835 9'9928	1'0168 0'0072
$2 MS$	0'9285 9'9678	1'0770 0'0322	0'9308 9'9689	1'0743 0'0311	0'9406 9'9734	1'0632 0'0266	0'9570 9'9809	1'0449 0'0191	0'9786 9'9906	1'0219 0'0094	1'0032 0'0014	0'9968 9'9986
$MSf, 2 SM$	0'9636 9'9839	1'0378 0'0161	0'9648 9'9844	1'0365 0'0156	0'9698 9'9867	1'0311 0'0133	0'9783 9'9905	1'0222 0'0095	0'9892 9'9953	1'0109 0'0047	1'0016 0'0007	0'9984 9'9993
Mf	1'5964 0'2031	0'6264 9'7969	1'5608 0'1933	0'6407 9'8066	1'4256 0'1540	0'7014 9'8460	1'2480 0'0962	0'8013 9'9038	1'0776 0'0325	0'9280 9'9675	0'9378 9'9721	1'0664 0'0279
Mm	0'8843 9'9466	1'1309 0'0534	0'8878 9'9483	1'1264 0'0517	0'9031 9'9557	1'1073 0'0443	0'9294 9'9682	1'0759 0'0318	0'9651 9'9846	1'0361 0'0154	1'0074 0'0032	0'9926 9'9968
Sa, Ssa	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1928		1929		1930		1931		1932		1933	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
$J_1, [M_1]$	0.9204 9.9640	1.0864 0.0360	0.8891 9.9490	1.1247 0.0510	0.8692 9.9391	1.1505 0.0609	0.8595 9.9342	1.1634 0.0658	0.8594 9.9342	1.1636 0.0658	0.8688 9.9389	1.1510 0.0611
K_1	0.9479 9.9768	1.0549 0.0232	0.9236 9.9655	1.0827 0.0345	0.9076 9.9579	1.1018 0.0421	0.8997 9.9541	1.1115 0.0459	0.8996 9.9540	1.1116 0.0460	0.9073 9.9578	1.1022 0.0422
K_2	0.8872 9.9480	1.1271 0.0520	0.8241 9.9160	1.2135 0.0840	0.7826 9.8935	1.2778 0.1065	0.7619 9.8819	1.3125 0.1181	0.7617 9.8818	1.3129 0.1182	0.7818 9.8931	1.2791 0.1069
L_2	1.0256 0.0110	0.9750 9.9890	0.7874 9.8962	1.2700 0.1038	0.9781 9.9904	1.0224 0.0096	2.1230 0.3270	0.4710 9.6730	1.1452 0.0589	0.8732 9.9411	0.7874 9.8962	1.2700 0.1038
$[L_2]$	1.0140 0.0060	0.9862 9.9940	1.0250 0.0107	0.9756 9.9893	1.0332 0.0142	0.9679 9.9858	1.0376 0.0160	0.9637 9.9840	1.0377 0.0161	0.9637 9.9839	1.0334 0.0142	0.9677 9.9858
M_1	0.5750 9.7597	1.7392 0.2403	0.8787 9.9438	1.1380 0.0562	0.7625 9.8823	1.3115 0.1117	0.4250 9.6284	2.3529 0.3716	0.5020 9.7007	1.9921 0.2993	0.8354 9.9219	1.1970 0.0761
M_2, MS	1.0140 0.0060	0.9862 9.9940	1.0250 0.0107	0.9756 9.9893	1.0332 0.0142	0.9679 9.9858	1.0376 0.0160	0.9637 9.9840	1.0377 0.0161	0.9637 9.9839	1.0334 0.0142	0.9677 9.9858
M_3	1.0210 0.0090	0.9794 9.9910	1.0376 0.0160	0.9637 9.9840	1.0502 0.0213	0.9522 9.9787	1.0570 0.0241	0.9461 9.9759	1.0570 0.0241	0.9460 9.9759	1.0505 0.0214	0.9519 9.9786
M_4, MN	1.0282 0.0121	0.9726 9.9879	1.0506 0.0214	0.9519 9.9786	1.0675 0.0284	0.9368 9.9716	1.0767 0.0321	0.9288 9.9679	1.0768 0.0321	0.9287 9.9679	1.0678 0.0285	0.9365 9.9715
M_6	1.0425 0.0181	0.9592 9.9819	1.0768 0.0321	0.9287 9.9679	1.1029 0.0425	0.9067 9.9575	1.1772 0.0481	0.8951 9.9519	1.1173 0.0482	0.8950 9.9518	1.1034 0.0428	0.9063 9.9572
M_8	1.0571 0.0241	0.9460 9.9759	1.1037 0.0428	0.9061 9.9572	1.1395 0.0567	0.8776 9.9433	1.1592 0.0642	0.8627 9.9358	1.1594 0.0642	0.8625 9.9358	1.1402 0.0570	0.8770 9.9430
$N_3, 2 N$	1.0140 0.0060	0.9862 9.9940	1.0250 0.0107	0.9756 9.9893	1.0332 0.0142	0.9679 9.9858	1.0376 0.0160	0.9637 9.9840	1.0377 0.0161	0.9637 9.9839	1.0334 0.0142	0.9677 9.9858
O_1, Q_1	0.9188 9.9632	1.0883 0.0368	0.8822 9.9456	1.1335 0.0544	0.8586 9.9338	1.1647 0.0662	0.8470 9.9279	1.1807 0.0721	0.8468 9.9278	1.1809 0.0722	0.8582 9.9336	1.1653 0.0664
OO	0.7545 9.8777	1.3254 0.1223	0.6536 9.8153	1.5300 0.1847	0.5930 9.7730	1.6865 0.2270	0.5643 9.7515	1.7721 0.2485	0.5640 9.7513	1.7732 0.2487	0.5919 9.7722	1.6896 0.2278
P_1, R_2, T_3	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 3, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	1.0140 0.0060	0.9862 9.9940	1.0250 0.0107	0.9756 9.9893	1.0332 0.0142	0.9679 9.9858	1.0376 0.0160	0.9637 9.9840	1.0377 0.0161	0.9637 9.9839	1.0334 0.0142	0.9677 9.9858
MK	0.9612 9.9828	1.0404 0.0172	0.9467 9.9762	1.0564 0.0238	0.9377 9.9721	1.0664 0.0279	0.9335 9.9701	1.0712 0.0299	0.9335 9.9701	1.0713 0.0299	0.9376 9.9720	1.0666 0.0280
$2 MK$	0.9746 9.9888	1.0260 0.0112	0.9703 9.9869	1.0306 0.0131	0.9689 9.9863	1.0321 0.0137	0.9686 9.9862	1.0324 0.0138	0.9686 9.9862	1.0324 0.0138	0.9689 9.9863	1.0321 0.0137
$2 MS$	1.0282 0.0121	0.9726 9.9879	1.0506 0.0214	0.9519 9.9786	1.0675 0.0284	0.9368 9.9716	1.0767 0.0321	0.9288 9.9679	1.0768 0.0321	0.9287 9.9679	1.0678 0.0285	0.9365 9.9715
$MSI, 2 SM$	1.0140 0.0060	0.9862 9.9940	1.0250 0.0107	0.9756 9.9893	1.0332 0.0142	0.9679 9.9858	1.0376 0.0160	0.9637 9.9840	1.0377 0.0161	0.9637 9.9839	1.0334 0.0142	0.9677 9.9858
Mf	0.8326 9.9204	1.2010 0.0796	0.7594 9.8804	1.3169 0.1196	0.7135 9.8534	1.4015 0.1466	0.6913 9.8397	1.4465 0.1603	0.6911 9.8395	1.4470 0.1605	0.7127 9.8529	1.4032 0.1471
Mm	1.0523 0.0221	0.9503 9.9779	1.0942 0.0391	0.9139 9.9609	1.1270 0.0519	0.8873 9.9481	1.1452 0.0589	0.8732 9.9411	1.1455 0.0590	0.8730 9.9410	1.1277 0.0522	0.8868 9.9478
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—*Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.*

Component.	1934		1935		1936		1937		1938		1939	
	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$	$\log F$	$\log f$
J ₁ , [M ₁]	0.8884 9.9486	1.1256 0.0514	0.9194 9.9635	1.0877 0.0365	0.9629 9.9836	1.0385 0.0164	1.0188 0.0081	0.9815 9.9919	1.0837 0.0349	0.9228 9.9651	1.1482 0.0600	0.8710 9.9400
K ₁	0.9231 9.9652	1.0834 0.0348	0.9471 9.9764	1.0558 0.0236	0.9794 9.9910	1.0210 0.0090	1.0185 0.0080	0.9819 9.9920	1.0607 0.0256	0.9428 9.9744	1.0997 0.0413	0.9094 9.9587
K ₂	0.8227 9.9152	1.2155 0.0848	0.8852 9.9470	1.1297 0.0530	0.9684 9.9861	1.0326 0.0139	1.0677 0.0284	0.9366 9.9716	1.1714 0.0687	0.8537 9.9313	1.2622 0.1011	0.7923 9.8989
L ₂	0.9274 9.9673	1.0783 0.0327	1.5720 0.1965	0.6361 9.8035	1.1645 0.0661	0.8588 9.9339	0.8401 9.9243	1.1903 0.0757	0.8602 9.9346	1.1625 0.0654	1.0681 0.0286	0.9363 9.9714
[L ₂]	1.0252 0.0108	0.9754 9.9892	1.0143 0.0062	0.9859 9.9938	1.0019 0.0008	0.9981 9.9992	0.9895 9.9954	1.0106 0.0046	0.9785 9.9906	1.0220 0.0094	0.9700 9.9868	1.0309 0.0132
M ₁	0.6196 9.7921	1.6139 0.2079	0.4655 9.6679	2.1482 0.3321	0.5407 9.7329	1.8495 0.2671	0.9109 9.9595	1.0978 0.0405	0.9269 9.9670	1.0789 0.0330	0.6518 9.8141	1.5342 0.1859
M ₂ , MS	1.0252 0.0108	0.9754 9.9892	1.0143 0.0062	0.9859 9.9938	1.0019 0.0008	0.9981 9.9992	0.9895 9.9954	1.0106 0.0046	0.9785 9.9906	1.0220 0.0094	0.9700 9.9868	1.0309 0.0132
M ₃	1.0381 0.0162	0.9633 9.9838	1.0216 0.0093	0.9789 9.9907	1.0028 0.0012	0.9972 9.9988	0.9843 9.9931	1.0159 0.0069	0.9680 9.9859	1.0331 0.0141	0.9554 9.9802	1.0467 0.0198
M ₄ , MN	1.0511 0.0216	0.9514 9.9784	1.0288 0.0123	0.9720 9.9877	1.0038 0.0017	0.9962 9.9983	0.9792 9.9909	1.0213 0.0091	0.9575 9.9811	1.0444 0.0189	0.9409 9.9736	1.0628 0.0264
M ₆	1.0776 0.0325	0.9280 9.9675	1.0435 0.0185	0.9513 9.9815	1.0057 0.0025	0.9943 9.9975	0.9689 9.9863	1.0321 0.0137	0.9369 9.9717	1.0673 0.0283	0.9127 9.9603	1.0956 0.0397
M ₈	1.1048 0.0433	0.9051 9.9567	1.0585 0.0247	0.9448 9.9753	1.0077 0.0033	0.9924 9.9967	0.9588 9.9817	1.0430 0.0183	0.9168 9.9623	1.0907 0.0377	0.8854 9.9471	1.1295 0.0529
N ₂ , 2 N	1.0252 0.0108	0.9754 9.9892	1.0143 0.0062	0.9859 9.9938	1.0019 0.0008	0.9981 9.9992	0.9895 9.9954	1.0106 0.0046	0.9785 9.9906	1.0220 0.0094	0.9700 9.9868	1.0309 0.0132
O ₁ , Q ₁	0.8814 9.9452	1.1345 0.0548	0.9176 9.9627	1.0898 0.0373	0.9676 9.9857	1.0335 0.0143	1.0307 0.0132	0.9702 9.9868	1.1030 0.0426	0.9066 9.9574	1.1740 0.0697	0.8518 9.9303
OO	0.6515 9.8139	1.5348 0.1861	0.7510 9.8757	1.3315 0.1243	0.9024 9.9554	1.1081 0.0446	1.1184 0.0486	0.8942 9.9514	1.4014 0.1466	0.7136 9.8534	1.7198 0.2355	0.5814 9.7645
P ₁ , R ₂ , T ₂	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
S ₁ , 2, 3, 4	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ ₂ , μ ₂ , ν ₂	1.0252 0.0108	0.9754 9.9892	1.0143 0.0062	0.9859 9.9938	1.0019 0.0008	0.9981 9.9992	0.9895 9.9954	1.0106 0.0046	0.9785 9.9906	1.0220 0.0094	0.9700 9.9868	1.0309 0.0132
MK	0.9463 9.9760	1.0567 0.0240	0.9607 9.9826	1.0409 0.0174	0.9813 9.9918	1.0191 0.0082	1.0078 0.0034	0.9922 9.9966	1.0379 0.0162	0.9635 9.9838	1.0667 0.0280	0.9375 9.9720
2 MK	0.9702 9.9869	1.0307 0.0131	0.9744 9.9888	1.0262 0.0112	0.9832 9.9926	1.0171 0.0074	0.9973 9.9988	1.0027 0.0012	1.0156 0.0067	0.9846 9.9933	1.0347 0.0148	0.9665 9.9852
2 MS	1.0511 0.0216	0.9514 9.9784	1.0288 0.0123	0.9720 9.9877	1.0038 0.0017	0.9962 9.9983	0.9792 9.9909	1.0213 0.0091	0.9575 9.9811	1.0444 0.0189	0.9409 9.9736	1.0628 0.0264
MSf, 2 SM	1.0252 0.0108	0.9754 9.9892	1.0143 0.0062	0.9859 9.9938	1.0019 0.0008	0.9981 9.9992	0.9895 9.9954	1.0106 0.0046	0.9785 9.9906	1.0220 0.0094	0.9700 9.9868	1.0309 0.0132
Mf	0.7578 9.8796	1.3196 0.1204	0.8302 9.9192	1.2046 0.0808	0.9344 9.9706	1.0702 0.0294	1.0736 0.0309	0.9314 9.9691	1.2433 0.0946	0.8043 9.9054	1.4210 0.1526	0.7038 9.8474
Mm	1.0952 0.0395	0.9131 9.9605	1.0535 0.0226	0.9492 9.9774	1.0086 0.0037	0.9914 9.9963	0.9662 9.9850	1.0350 0.0150	0.9302 9.9686	1.0750 0.0314	0.9037 9.9560	1.1066 0.0440
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1940		1941		1942		1943		1944		1945	
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f
$J_1, [M_1]$	1.1958 0.0776	0.8363 9.9224	1.2089 0.0824	0.8272 9.9176	1.1818 0.0726	0.8462 9.9274	1.1260 0.0515	0.8881 9.9485	1.0597 0.0252	0.9437 9.9748	0.9973 9.9988	1.0028 0.0012
K_1	1.1267 0.0518	0.8876 9.9482	1.1339 0.0546	0.8819 9.9454	1.1189 0.0488	0.8937 9.9512	1.0866 0.0361	0.9203 9.9639	1.0454 0.0193	0.9565 9.9807	1.0037 0.0016	0.9963 9.9984
K_2	1.3216 0.1211	0.7567 9.8789	1.3369 0.1261	0.7480 9.8739	1.3048 0.1156	0.7664 9.8844	1.2323 0.0907	0.8115 9.9093	1.1344 0.0548	0.8815 9.9452	1.0305 0.0130	0.9704 9.9870
L_2	1.1635 0.0658	0.8595 9.9342	0.9794 9.9909	1.0211 0.0091	0.8474 9.9281	1.1801 0.0719	0.8738 9.9414	1.1444 0.0586	1.1415 0.0575	0.8760 9.9425	1.3364 0.1259	0.7483 9.8741
$[L_2]$	0.9649 9.9845	1.0364 0.0155	0.9636 9.9839	1.0378 0.0161	0.9663 9.9851	1.0349 0.0149	0.9727 9.9880	1.0280 0.0120	0.9823 9.9922	1.0181 0.0078	0.9939 9.9974	1.0061 0.0026
M_1	0.6160 9.7896	1.6234 0.2104	0.7616 9.8817	1.3131 0.1183	1.1380 0.0561	0.8787 9.9439	0.9294 9.9682	1.0760 0.0318	0.5834 9.7660	1.7142 0.2340	0.5144 9.7113	1.9442 0.2887
M_2, MS	0.9649 9.9845	1.0364 0.0155	0.9636 9.9839	1.0378 0.0161	0.9663 9.9851	1.0349 0.0149	0.9727 9.9880	1.0280 0.0120	0.9823 9.9922	1.0181 0.0078	0.9939 9.9974	1.0061 0.0026
M_3	0.9477 9.9767	1.0551 0.0233	0.9459 9.9758	1.0572 0.0242	0.9499 9.9777	1.0527 0.0223	0.9593 9.9820	1.0424 0.0180	0.9735 9.9883	1.0272 0.0117	0.9909 9.9960	1.0092 0.0040
M_4, MN	0.9310 9.9689	1.0741 0.0311	0.9285 9.9678	1.0770 0.0322	0.9338 9.9702	1.0709 0.0298	0.9462 9.9760	1.0569 0.0240	0.9649 9.9845	1.0364 0.0155	0.9879 9.9947	1.0122 0.0053
M_6	0.8983 9.9534	1.1132 0.0466	0.8947 9.9517	1.1177 0.0483	0.9023 9.9554	1.11083 0.0446	0.9204 9.9640	1.0865 0.0360	0.9477 9.9767	1.0551 0.0233	0.9819 9.9921	1.0184 0.0079
M_8	0.8667 9.9379	1.1538 0.0621	0.8621 9.9356	1.1600 0.0644	0.8719 9.9405	1.1469 0.0595	0.8953 9.9520	1.1170 0.0480	0.9309 9.9689	1.0742 0.0311	0.9760 9.9894	1.0246 0.0106
$N_2, 2N$	0.9649 9.9845	1.0364 0.0155	0.9636 9.9839	1.0378 0.0161	0.9663 9.9851	1.0349 0.0149	0.9727 9.9880	1.0280 0.0120	0.9823 9.9922	1.0181 0.0078	0.9939 9.9974	1.0061 0.0026
O_1, Q_1	1.2262 0.0886	0.8155 9.9114	1.2406 0.0936	0.8061 9.9064	1.2109 0.0831	0.8258 9.9169	1.1497 0.0606	0.8698 9.9394	1.0763 0.0319	0.9291 9.9681	1.0065 0.0028	0.9935 9.9972
OO	1.9802 0.2907	0.5050 9.7033	2.0561 0.3130	0.4864 9.6870	1.9016 0.2791	0.5259 9.7209	1.6059 0.2057	0.6227 9.7943	1.2923 0.1114	0.7738 9.8886	1.0321 0.0137	0.9689 9.9863
P_1, R_2, T_2	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
$S_1, 2, 3, 4$	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000
λ_2, μ_2, ν_2	0.9649 9.9845	1.0364 0.0155	0.9636 9.9839	1.0378 0.0161	0.9663 9.9851	1.0349 0.0149	0.9727 9.9880	1.0280 0.0120	0.9823 9.9922	1.0181 0.0078	0.9939 9.9974	1.0061 0.0026
MK	1.0871 0.0363	0.9199 9.9637	1.0926 0.0385	0.9153 9.9615	1.0812 0.0339	0.9249 9.9661	1.0570 0.0241	0.9461 9.9759	1.0269 0.0115	0.9738 9.9885	0.9976 9.9990	1.0024 0.0010
$2MK$	1.0489 0.0207	0.9534 9.9793	1.0528 0.0224	0.9498 9.9776	1.0448 0.0190	0.9571 9.9810	1.0281 0.0120	0.9726 9.9880	1.0087 0.0038	0.9914 9.9962	0.9916 9.9963	1.0085 0.0037
$2MS$	0.9310 9.9689	1.0741 0.0311	0.9285 9.9678	1.0770 0.0322	0.9338 9.9702	1.0709 0.0298	0.9462 9.9760	1.0569 0.0240	0.9649 9.9845	1.0364 0.0155	0.9879 9.9947	1.0122 0.0053
$MSf, 2SM$	0.9649 9.9845	1.0364 0.0155	0.9636 9.9839	1.0378 0.0161	0.9663 9.9851	1.0349 0.0149	0.9727 9.9880	1.0280 0.0120	0.9823 9.9922	1.0181 0.0078	0.9939 9.9974	1.0061 0.0026
Mf	1.5582 0.1926	0.6417 9.8074	1.5971 0.2033	0.6262 9.7967	1.5174 0.1811	0.6590 9.8189	1.3588 0.1331	0.7360 9.8669	1.1794 0.0717	0.8479 9.9283	1.0192 0.0083	0.9811 9.9917
Mm	0.8330 9.9484	1.1261 0.0516	0.8842 9.9466	1.1310 0.0534	0.8923 9.9505	1.1206 0.0495	0.9120 9.9600	1.0965 0.0400	0.9422 9.9742	1.0613 0.0258	0.9810 9.9916	1.0194 0.0084
Sa, Ssa	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

TABLE 10.—Factors F and f for reduction and prediction of tides; computed for the middle of each year, or for July 2, at Greenwich mean noon for common years, and at preceding midnight for leap years—Continued.

Component.	1946		1947		1948		1949		1950		Mean value of f from the years 1850-1942.
	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	F log F	f log f	
$J_1, [M_1]$	0'9458 9'9758	1'0573 0'0242	0'9069 9'9576	1'1027 0'0424	0'8801 9'9446	1'1362 0'0554	0'8643 9'9367	1'1570 0'0633	0'8584 9'9337	1'1650 0'0663	1'00490
K_1	0'9669 9'9854	1'0342 0'0146	0'9375 9'9720	1'0666 0'0280	0'9164 9'9621	1'0912 0'0379	0'9036 9'9560	1'1067 0'0440	0'8987 9'9536	1'1127 0'0464	1'00609
K_2	0'9363 9'9714	1'0681 0'0286	0'8602 9'9346	1'1624 0'0654	0'8055 9'9061	1'2415 0'0939	0'7721 9'8877	1'2951 0'1123	0'7594 9'8805	1'3168 0'1195	1'02421
L_2	0'9165 9'9621	1'0911 0'0379	0'7970 9'9015	1'2547 0'0985	1'1182 0'0485	0'8943 9'9515	2'0764 0'3173	0'4816 9'6827	0'9888 9'9951	1'0113 0'0049	0'97803
$[L_2]$	1'0065 0'0028	0'9936 9'9972	1'0185 0'0080	0'9819 9'9920	1'0285 0'0122	0'9723 9'9878	1'0354 0'0151	0'9658 9'9849	1'0382 0'0163	0'9632 9'9837	1'00033
M_1	0'6755 9'8296	1'4804 0'1704	0'8786 9'9438	1'1382 0'0562	0'5190 9'7151	1'9269 0'2849	0'4275 9'6310	2'3390 0'3690	0'5572 9'7460	1'7947 0'2540	1'55050
M_2, MS	1'0065 0'0028	0'9936 9'9972	1'0185 0'0080	0'9819 9'9920	1'0285 0'0122	0'9723 9'9878	1'0354 0'0151	0'9658 9'9849	1'0382 0'0163	0'9632 9'9837	1'00033
M_3	1'0096 0'0042	0'9904 9'9958	1'0278 0'0119	0'9729 9'9881	1'0431 0'0183	0'9587 9'9817	1'0535 0'0226	0'9492 9'9774	1'0578 0'0244	0'9454 9'9756	1'00074
M_4, MN	1'0129 0'0056	0'9872 9'9944	1'0373 0'0159	0'9641 9'9841	1'0579 0'0244	0'9453 9'9756	1'0721 0'0302	0'9328 9'9698	1'0778 0'0325	0'9278 9'9675	1'00137
M_6	1'0195 0'0084	0'9809 9'9916	1'0565 0'0238	0'9466 9'9762	1'0881 0'0367	0'9191 9'9633	1'1100 0'0453	0'9009 9'9547	1'1189 0'0488	0'8937 9'9512	1'00307
M_8	1'0260 0'0112	0'9746 9'9888	1'0760 0'0318	0'9294 9'9682	1'1191 0'0489	0'8936 9'9511	1'1493 0'0604	0'8701 9'9396	1'1617 0'0651	0'8608 9'9349	1'00552
$N_2, 2 N$	1'0065 0'0028	0'9936 9'9972	1'0185 0'0080	0'9819 9'9920	1'0285 0'0122	0'9723 9'9878	1'0354 0'0151	0'9658 9'9849	1'0382 0'0163	0'9632 9'9837	1'00033
O_1, Q_1	0'9480 9'9768	1'0548 0'0232	0'9031 9'9557	1'1073 0'0443	0'8716 9'9403	1'1473 0'0597	0'8527 9'9308	1'1727 0'0692	0'8456 9'9272	1'1826 0'0728	1'00905
OO	0'8411 9'9249	1'1889 0'0751	0'7100 9'8513	1'4084 0'1487	0'6259 9'7965	1'5976 0'2035	0'5784 9'7622	1'7290 0'2378	0'5609 9'7489	1'7828 0'2511	1.10310
P_1, R_2, T_2	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'00000
$S_1, 2, 3, 4$	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0060	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'00000
λ_2, μ_2, ν_2	1'0065 0'0028	0'9936 9'9972	1'0185 0'0080	0'9819 9'9920	1'0285 0'0122	0'9723 9'9878	1'0354 0'0151	0'9658 9'9849	1'0382 0'0163	0'9632 9'9837	1'00033
MK	0'9731 9'9882	1'0276 0'0118	0'9548 9'9799	1'0473 0'0201	0'9426 9'9743	1'0609 0'0257	0'9356 9'9711	1'0688 0'0289	0'9330 9'9699	1'0718 0'0301	1'00428
$2 MK$	0'9794 9'9910	1'0210 0'0090	0'9725 9'9879	1'0283 0'0121	0'9695 9'9865	1'0315 0'0135	0'9687 9'9862	1'0323 0'0138	0'9686 9'9862	1'0324 0'0138	1'00315
$2 MS$	1'0129 0'0056	0'9872 9'9944	1'0373 0'0159	0'9641 9'9841	1'0579 0'0244	0'9453 9'9756	1'0721 0'0302	0'9328 9'9698	1'0778 0'0325	0'9278 9'9675	1'00137
$MSf, 2 SM$	1'0065 0'0028	0'9936 9'9972	1'0185 0'0080	0'9819 9'9920	1'0285 0'0122	0'9723 9'9878	1'0354 0'0151	0'9658 9'9849	1'0382 0'0163	0'9632 9'9837	1'00033
Mf	0'8930 9'9508	1'1198 0'0492	0'8008 9'9035	1'2488 0'0965	0'7386 9'8684	1'3539 0'1316	0'7023 9'8465	1'4240 0'1535	0'6887 9'8380	1'4520 0'1620	1'04317
Mm	1'0248 0'0106	0'9758 9'9894	1'0692 0'0290	0'9353 9'9710	1'1083 0'0447	0'9023 9'9553	1'1360 0'0554	0'8802 9'9446	1'1475 0'0598	0'8714 9'9,02	0'99992
Sa, Ssa	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'00000

TABLE 11.—Values of log R' for obtaining the factor F of L_2 from that of M_1 .

P	$I = \text{inclination of moon's orbit.}$											
	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°
0												
5	0.0778	0.0889	0.1014	0.1154	0.1312	0.1491	0.1696	0.1935	0.2216	0.2555	0.2976	0.3523
10	0.0764	0.0873	0.0994	0.1131	0.1284	0.1459	0.1658	0.1888	0.2158	0.2482	0.2881	0.3393
15	0.0722	0.0824	0.0938	0.1064	0.1206	0.1365	0.1547	0.1754	0.1995	0.2279	0.2622	0.3047
20	0.0656	0.0747	0.0847	0.0959	0.1083	0.1221	0.1377	0.1552	0.1753	0.1984	0.2255	0.2579
25	0.0569	0.0646	0.0731	0.0824	0.0926	0.1040	0.1165	0.1305	0.1462	0.1639	0.1840	0.2072
30	0.0467	0.0528	0.0595	0.0668	0.0748	0.0835	0.0930	0.1035	0.1150	0.1277	0.1418	0.1575
35	0.0354	0.0400	0.0448	0.0501	0.0558	0.0620	0.0687	0.0759	0.0837	0.0922	0.1014	0.1114
40	0.0236	0.0265	0.0296	0.0330	0.0366	0.0404	0.0445	0.0489	0.0536	0.0586	0.0640	0.0697
45	0.0117	0.0130	0.0146	0.0161	0.0178	0.0196	0.0215	0.0235	0.0256	0.0278	0.0301	0.0326
50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
55	9.9889	9.9877	9.9864	9.9850	9.9836	9.9820	9.9805	9.9788	9.9771	9.9754	9.9736	9.9717
60	9.9787	9.9764	9.9739	9.9714	9.9687	9.9659	9.9631	9.9601	9.9570	9.9539	9.9506	9.9473
65	9.9696	9.9663	9.9628	9.9593	9.9556	9.9518	9.9479	9.9439	9.9398	9.9355	9.9312	9.9267
70	9.9616	9.9575	9.9533	9.9490	9.9445	9.9398	9.9351	9.9302	9.9252	9.9201	9.9149	9.9097
75	9.9549	9.9503	9.9454	9.9404	9.9353	9.9300	9.9246	9.9190	9.9134	9.9077	9.9018	9.8959
80	9.9497	9.9445	9.9392	9.9337	9.9281	9.9223	9.9164	9.9104	9.9043	9.8981	9.8918	9.8854
85	9.9459	9.9404	9.9347	9.9289	9.9229	9.9168	9.9106	9.9042	9.8978	9.8913	9.8846	9.8780
90	9.9436	9.9379	9.9320	9.9260	9.9198	9.9135	9.9071	9.9006	9.8939	9.8872	9.8804	9.8735
95	9.9429	9.9371	9.9312	9.9250	9.9188	9.9124	9.9059	9.8993	9.8926	9.8858	9.8790	9.8720
100	9.9436	9.9379	9.9320	9.9260	9.9198	9.9135	9.9071	9.9006	9.8939	9.8872	9.8804	9.8735
105	9.9459	9.9404	9.9347	9.9289	9.9229	9.9168	9.9106	9.9042	9.8978	9.8913	9.8846	9.8780
110	9.9497	9.9445	9.9392	9.9337	9.9281	9.9223	9.9164	9.9104	9.9043	9.8981	9.8918	9.8854
115	9.9549	9.9503	9.9454	9.9404	9.9353	9.9300	9.9246	9.9190	9.9134	9.9077	9.9018	9.8959
120	9.9616	9.9575	9.9533	9.9490	9.9445	9.9398	9.9351	9.9302	9.9252	9.9201	9.9149	9.9097
125	9.9696	9.9663	9.9628	9.9593	9.9556	9.9518	9.9479	9.9439	9.9398	9.9355	9.9312	9.9267
130	9.9787	9.9764	9.9739	9.9714	9.9687	9.9659	9.9631	9.9601	9.9570	9.9539	9.9506	9.9473
135	9.9889	9.9877	9.9864	9.9850	9.9836	9.9820	9.9805	9.9788	9.9771	9.9754	9.9736	9.9717
140	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
145	0.0117	0.0130	0.0146	0.0161	0.0178	0.0196	0.0215	0.0235	0.0256	0.0278	0.0301	0.0326
150	0.0236	0.0265	0.0296	0.0330	0.0366	0.0404	0.0445	0.0489	0.0536	0.0586	0.0640	0.0697
155	0.0354	0.0400	0.0448	0.0501	0.0558	0.0620	0.0687	0.0759	0.0837	0.0922	0.1014	0.1114
160	0.0467	0.0528	0.0595	0.0668	0.0748	0.0835	0.0930	0.1035	0.1150	0.1277	0.1418	0.1575
165	0.0569	0.0646	0.0731	0.0824	0.0926	0.1040	0.1165	0.1305	0.1462	0.1639	0.1840	0.2072
170	0.0656	0.0747	0.0847	0.0959	0.1083	0.1221	0.1377	0.1552	0.1753	0.1984	0.2255	0.2579
175	0.0722	0.0824	0.0938	0.1064	0.1206	0.1365	0.1547	0.1754	0.1995	0.2279	0.2622	0.3047
180	0.0764	0.0873	0.0994	0.1131	0.1284	0.1459	0.1658	0.1888	0.2158	0.2482	0.2881	0.3393
185	0.0778	0.0889	0.1014	0.1154	0.1312	0.1491	0.1696	0.1935	0.2216	0.2555	0.2976	0.3523

$\log F(I_2) = \log F(M_1) + \log R'$, where $R' = \left(\frac{1}{1 - 12 \tan^2 \frac{1}{2} I \cos 2 P} \right)^{\frac{1}{2}}$. The values of I and P for the first day of every month are given in Table 6.

When P lies between 180° and 360° subtract 180° from it, and enter the table with the remainder.

TABLE 12.—Values of $\log Q'$ for obtaining the factor F of M_1 from that of O_1 .

P	$\log Q'$	P	$\log Q'$	P	$\log Q'$	P	$\log Q'$	P	$\log Q'$	P	$\log Q'$
0	.	0	.	0	.	0	.	0	.	0	.
0	9.6990	60	9.8785	120	9.8785	180	9.6990	240	9.8785	300	9.8785
1	9.6990	61	9.8841	121	9.8729	181	9.6990	241	9.8841	301	9.8729
2	9.6992	62	9.8898	122	9.8673	182	9.6992	242	9.8898	302	9.8673
3	9.6994	63	9.8955	123	9.8618	183	9.6994	243	9.8955	303	9.8618
4	9.6998	64	9.9012	124	9.8563	184	9.6998	244	9.9012	304	9.8563
5	9.7002	65	9.9068	125	9.8509	185	9.7002	245	9.9068	305	9.8509
6	9.7008	66	9.9125	126	9.8456	186	9.7008	246	9.9125	306	9.8456
7	9.7014	67	9.9181	127	9.8403	187	9.7014	247	9.9181	307	9.8403
8	9.7022	68	9.9237	128	9.8351	188	9.7022	248	9.9237	308	9.8351
9	9.7030	69	9.9292	129	9.8300	189	9.7030	249	9.9292	309	9.8300
10	9.7039	70	9.9347	130	9.8249	190	9.7039	250	9.9347	310	9.8249
11	9.7050	71	9.9400	131	9.8200	191	9.7050	251	9.9400	311	9.8200
12	9.7061	72	9.9453	132	9.8151	192	9.7061	252	9.9453	312	9.8151
13	9.7074	73	9.9504	133	9.8103	193	9.7074	253	9.9504	313	9.8103
14	9.7087	74	9.9554	134	9.8056	194	9.7087	254	9.9554	314	9.8056
15	9.7102	75	9.9602	135	9.8010	195	9.7102	255	9.9602	315	9.8010
16	9.7117	76	9.9649	136	9.7965	196	9.7117	256	9.9649	316	9.7965
17	9.7134	77	9.9693	137	9.7921	197	9.7134	257	9.9693	317	9.7921
18	9.7151	78	9.9735	138	9.7878	198	9.7151	258	9.9735	318	9.7878
19	9.7170	79	9.9775	139	9.7836	199	9.7170	259	9.9775	319	9.7836
20	9.7190	80	9.9812	140	9.7795	200	9.7190	260	9.9812	320	9.7795
21	9.7210	81	9.9846	141	9.7755	201	9.7210	261	9.9846	321	9.7755
22	9.7231	82	9.9877	142	9.7716	202	9.7231	262	9.9877	322	9.7716
23	9.7254	83	9.9905	143	9.7678	203	9.7254	263	9.9905	323	9.7678
24	9.7277	84	9.9930	144	9.7641	204	9.7277	264	9.9930	324	9.7641
25	9.7302	85	9.9951	145	9.7605	205	9.7302	265	9.9951	325	9.7605
26	9.7328	86	9.9968	146	9.7570	206	9.7328	266	9.9968	326	9.7570
27	9.7354	87	9.9982	147	9.7536	207	9.7354	267	9.9982	327	9.7536
28	9.7382	88	9.9992	148	9.7503	208	9.7382	268	9.9992	328	9.7503
29	9.7411	89	9.9998	149	9.7471	209	9.7411	269	9.9998	329	9.7471
30	9.7441	90	10.0000	150	9.7441	210	9.7441	270	10.0000	330	9.7441
31	9.7471	91	9.9998	151	9.7411	211	9.7471	271	9.9998	331	9.7411
32	9.7503	92	9.9992	152	9.7382	212	9.7503	272	9.9992	332	9.7382
33	9.7536	93	9.9982	153	9.7354	213	9.7536	273	9.9982	333	9.7354
34	9.7570	94	9.9968	154	9.7328	214	9.7570	274	9.9968	334	9.7328
35	9.7605	95	9.9951	155	9.7302	215	9.7605	275	9.9951	335	9.7302
36	9.7641	96	9.9930	156	9.7277	216	9.7641	276	9.9930	336	9.7277
37	9.7678	97	9.9905	157	9.7254	217	9.7678	277	9.9905	337	9.7254
38	9.7716	98	9.9877	158	9.7231	218	9.7716	278	9.9877	338	9.7231
39	9.7755	99	9.9846	159	9.7210	219	9.7755	279	9.9846	339	9.7210
40	9.7795	100	9.9812	160	9.7190	220	9.7795	280	9.9812	340	9.7190
41	9.7836	101	9.9775	161	9.7170	221	9.7836	281	9.9775	341	9.7170
42	9.7878	102	9.9735	162	9.7151	222	9.7878	282	9.9735	342	9.7151
43	9.7921	103	9.9693	163	9.7134	223	9.7921	283	9.9693	343	9.7134
44	9.7965	104	9.9649	164	9.7117	224	9.7965	284	9.9649	344	9.7117
45	9.8010	105	9.9602	165	9.7102	225	9.8010	285	9.9602	345	9.7102
46	9.8056	106	9.9554	166	9.7087	226	9.8056	286	9.9554	346	9.7087
47	9.8103	107	9.9504	167	9.7074	227	9.8103	287	9.9504	347	9.7074
48	9.8151	108	9.9453	168	9.7061	228	9.8151	288	9.9453	348	9.7061
49	9.8200	109	9.9400	169	9.7050	229	9.8200	289	9.9400	349	9.7050
50	9.8249	110	9.9347	170	9.7039	230	9.8249	290	9.9347	350	9.7039
51	9.8300	111	9.9292	171	9.7030	231	9.8300	291	9.9292	351	9.7030
52	9.8351	112	9.9237	172	9.7022	232	9.8351	292	9.9237	352	9.7022
53	9.8403	113	9.9181	173	9.7014	233	9.8403	293	9.9181	353	9.7014
54	9.8456	114	9.9125	174	9.7008	234	9.8456	294	9.9125	354	9.7008
55	9.8509	115	9.9068	175	9.7002	235	9.8509	295	9.9068	355	9.7002
56	9.8563	116	9.9012	176	9.6998	236	9.8563	296	9.9012	356	9.6998
57	9.8618	117	9.8955	177	9.6994	237	9.8618	297	9.8955	357	9.6994
58	9.8673	118	9.8898	178	9.6992	238	9.8673	298	9.8898	358	9.6992
59	9.8729	119	9.8841	179	9.6990	239	9.8729	299	9.8841	359	9.6990

$\log F(M_1) = \log F(O_1) + \log Q'$, where $Q' = \frac{1}{(2.5 + 1.5 \cos 2P)^{\frac{1}{2}}}$.

The value of P for the first day of every month is given in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides.

I	$F(J_1)$	Diff.	$f(J_1)$	Diff.	I	$F(J_1)$	Diff.	$f(J_1)$	Diff.
0					0				
18.3	1.21005	565	0.82640	389	23.3	0.99297	326	1.00708	332
4	20440	558	.83029	387	4	.98971	323	.01040	330
5	19882	552	.83416	385	5	.98648	319	.01370	330
6	19330	546	.83801	385	6	.98329	317	.01700	328
7	18784	539	.84186	384	7	.98012	313	.02028	327
8	18245	533	.84570	383	8	.97699	310	.02355	326
18.9	17712	527	.84953	382	23.9	.97389	306	.02681	324
19.0	17185	520	.85335	381	24.0	.97083	304	.03005	323
1	16665	514	.85716	379	1	.96779	300	.03328	322
2	16151	509	.86095	379	2	.96479	298	.03650	320
3	15642	503	.86474	377	3	.96181	295	.03970	320
4	15139	497	.86851	377	4	.95886	291	.04290	318
19.5	14642	491	.87228	375	24.5	.95595	288	.04608	316
6	14151	486	.87603	375	6	.95307	286	.04924	316
7	13665	480	.87978	373	7	.95021	283	.05240	314
8	13185	475	.88351	372	8	.94738	280	.05554	313
19.9	12710	470	.88723	372	24.9	.94458	277	.05867	312
20.0	12240	464	.89095	370	25.0	.94181	275	.06179	310
1	11776	459	.89465	369	1	.93906	272	.06489	309
2	11317	454	.89834	368	2	.93634	269	.06798	308
3	10863	449	.90202	366	3	.93365	266	.07106	306
4	10414	444	.90568	366	4	.93099	264	.07412	305
20.5	.99970	439	.90934	365	25.5	.92835	261	.07717	304
6	.99531	435	.91299	363	6	.92574	258	.08021	303
7	.99096	429	.91662	362	7	.92316	256	.08324	301
8	.98667	425	.92024	362	8	.92060	254	.08625	300
20.9	.98242	420	.92386	360	25.9	.91806	251	.08925	298
21.0	.97822	416	.92746	359	26.0	.91555	248	.09223	298
1	.97406	411	.93105	358	1	.91307	246	.09521	296
2	.96995	407	.93463	356	2	.91061	244	.09817	294
3	.96588	403	.93819	356	3	.90817	241	.10111	293
4	.96185	398	.94175	354	4	.90576	239	.10404	292
21.5	.95787	394	.94529	354	26.5	.90337	236	.10696	291
6	.95393	389	.94883	352	6	.90101	234	.10987	289
7	.95004	386	.95235	351	7	.89867	232	.11276	288
8	.94618	381	.95586	350	8	.89635	230	.11564	286
21.9	.94237	378	.95936	348	26.9	.89405	227	.11850	285
22.0	.93859	373	.96284	348	27.0	.89178	225	.12135	284
1	.93486	370	.96632	346	1	.88953	223	.12419	282
2	.93116	365	.96978	345	2	.88730	220	.12701	281
3	.92751	362	.97323	344	3	.88510	219	.12982	280
4	.92389	358	.97667	343	4	.88291	216	.13262	278
22.5	.92031	354	.98010	341	27.5	.88075	214	.13540	277
6	.91677	351	.98351	341	6	.87861	213	.13817	275
7	.91326	347	.98692	339	7	.87648	210	.14092	274
8	.90979	343	.99031	338	8	.87438	208	.14366	273
22.9	.90636	340	.99369	336	27.9	.87230	205	.14639	271
23.0	1.00296	337	0.99705	336	28.0	.87025	204	.14910	270
1	0.99959	333	1.00041	334	1	.86821	202	.15180	268
2	.99626	329	.00375	333	2	.86619	200	.15448	267
23.3	0.99297	326	1.00708	332	3	.86419	198	.15715	266
					4	.86221	196	.15981	264
					5	.86025	194	.16245	263
					28.6	0.85831		1.16508	

$$F = 1/f = \frac{\sin \omega \cos \omega (1 - \frac{1}{2} \sin^2 i)}{\sin I \cos I} = \frac{0.72147}{\sin 2I}$$

$F(J_1) = F$ for lunar K_1 .

$f(J_1) = f$ for lunar K_1 .

I is given for the first of each month in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued.

I	$F(K_1)$	Diff.	$f(K_1)$	Diff.	I	$F(K_1)$	Diff.	$f(K_1)$	Diff.
0					0				
18.3	1.13450	308	0.88145	240	23.3	1.00073	228	0.99927	228
4	1.13142	307	.88385	241	4	0.99845	226	1.00155	228
5	1.12835	305	.88626	240	5	.99619	224	.00383	227
6	1.12530	303	.88866	240	6	.99395	223	.00610	226
7	1.12227	301	.89106	239	7	.99172	222	.00836	226
8	1.11926	299	.89345	240	8	.98950	221	.01062	225
18.9	1.11627	298	.89585	239	23.9	.98729	219	.01287	225
19.0	1.11329	296	.89824	240	24.0	.98510	217	.01512	225
1	1.11033	294	.90064	240	1	.98293	216	.01737	224
2	1.10739	292	.90304	339	2	.98077	215	.01961	224
3	1.10447	291	.90543	238	3	.97862	214	.02185	224
4	1.10156	289	.90781	238	4	.97648	213	.02409	223
19.5	.09867	288	.91019	239	24.5	.97435	210	.02632	222
6	.09579	286	.91258	239	6	.97225	209	.02854	222
7	.09293	284	.91497	238	7	.97016	208	.03076	221
8	.09009	282	.91735	239	8	.96808	207	.03297	221
19.9	.08727	281	.91974	238	24.9	.96601	205	.03518	220
20.0	.08446	280	.92212	239	25.0	.96396	203	.03738	220
1	.08166	277	.92451	238	1	.96193	202	.03958	219
2	.07889	276	.92689	238	2	.95991	201	.04177	219
3	.07613	274	.92927	237	3	.95790	200	.04396	218
4	.07339	272	.93164	236	4	.95590	199	.04614	218
20.5	.07067	271	.93400	237	25.5	.95391	197	.04832	217
6	.06796	269	.93637	237	6	.95194	195	.05049	216
7	.06527	268	.93874	237	7	.94999	194	.05265	216
8	.06259	267	.94111	236	8	.94805	193	.05481	215
20.9	.05992	265	.94347	236	25.9	.94612	193	.05696	215
21.0	.05727	263	.94583	236	26.0	.94419	191	.05911	214
1	.05464	261	.94819	235	1	.94228	189	.06125	213
2	.05203	260	.95054	235	2	.94039	188	.06338	213
3	.04943	258	.95289	235	3	.93851	187	.06551	213
4	.04685	256	.95524	235	4	.93664	185	.06764	212
21.5	.04429	255	.95759	235	26.5	.93479	184	.06976	211
6	.04174	254	.95994	234	6	.93295	183	.07187	211
7	.03920	252	.96228	234	7	.93112	182	.07398	210
8	.03668	251	.96462	234	8	.92930	181	.07608	209
21.9	.03417	249	.96696	233	26.9	.92749	179	.07817	209
22.0	.03168	247	.96929	233	27.0	.92570	178	.08026	208
1	.02921	245	.97162	232	1	.92392	176	.08234	207
2	.02676	244	.97394	232	2	.92216	175	.08441	206
3	.02432	243	.97626	232	3	.92041	174	.08647	206
4	.02189	241	.97858	231	4	.91867	174	.08853	206
22.5	.01948	239	.98089	231	27.5	.91693	172	.09059	205
6	.01709	238	.98320	231	6	.91521	170	.09264	204
7	.01471	237	.98551	230	7	.91351	170	.09468	204
8	.01234	236	.98781	230	8	.91181	168	.09672	203
22.9	.00998	234	.99011	230	27.9	.91013	167	.09875	202
23.0	.00764	232	.99241	229	28.0	.90846	166	.10077	201
1	.00532	230	.99470	228	1	.90680	165	.10278	200
2	.00302	229	.99698	229	2	.90515	163	.10478	200
23.3	1.00073	228	0.99927	228	3	.90352	162	.10678	200
					4	.90190	162	.10878	199
					5	.90028	160	.11077	198
					28.6	0.89868		1.11275	

$$F = 1/f = \frac{1.05628}{(\sin^2 2I + 0.66962 \cos 2I + 0.11210)^{1/2}}$$

I is given for the first of each month in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued.

I	$F(K_2)$	Diff.	$f(K_2)$	Diff.	I	$F(K_2)$	Diff.	$f(K_2)$	Diff.
0					0				
18.3	1.33821	652	0.74732	363	23.3	1.02291	578	0.97760	556
4	.33169	651	.75095	367	4	.01713	576	.98316	560
5	.32518	652	.75462	372	5	.01137	572	.98876	563
6	.31866	652	.75834	377	6	.00565	569	.99439	566
7	.31214	652	.76211	381	7	0.99996	567	1.00005	570
8	.30562	652	.76592	385	8	.99429	564	.00575	573
18.9	.29910	653	.76977	388	23.9	.98865	561	.01148	577
19.0	.29257	653	.77365	392	24.0	.98304	558	.01725	580
1	.28604	653	.77757	397	1	.97746	554	.02305	583
2	.27951	653	.78154	401	2	.97192	551	.02888	587
3	.27298	653	.78555	405	3	.96641	548	.03475	590
4	.26645	652	.78960	409	4	.96093	545	.04065	594
19.5	.25993	652	.79369	412	24.5	.95548	542	.04659	597
6	.25341	651	.79781	417	6	.95006	539	.05256	600
7	.24690	651	.80198	421	7	.94467	535	.05856	604
8	.24039	650	.80619	425	8	.93932	532	.06460	607
19.9	.23389	649	.81044	429	24.9	.93400	529	.07067	610
20.0	.22740	648	.81473	432	25.0	.92871	526	.07677	613
1	.22092	647	.81905	436	1	.92345	523	.08290	616
2	.21445	646	.82341	440	2	.91822	520	.08906	620
3	.20799	645	.82781	445	3	.91302	516	.09526	623
4	.20154	644	.83226	449	4	.90786	513	.10149	626
20.5	.19510	642	.83675	452	25.5	.90273	510	.10775	629
6	.18868	641	.84127	456	6	.89763	507	.11404	633
7	.18227	639	.84583	460	7	.89256	504	.12037	636
8	.17588	638	.85043	464	8	.88752	500	.12673	639
20.9	.16950	637	.85507	468	25.9	.88252	497	.13312	642
21.0	.16313	634	.85975	471	26.0	.87755	494	.13954	645
1	.15679	632	.86446	475	1	.87261	491	.14599	648
2	.15047	630	.86921	479	2	.86770	488	.15247	651
3	.14417	629	.87400	483	3	.86282	484	.15898	654
4	.13788	627	.87883	487	4	.85798	481	.16552	658
21.5	.13161	625	.88370	490	26.5	.85317	478	.17210	660
6	.12536	622	.88860	494	6	.84839	475	.17870	663
7	.11914	620	.89354	498	7	.84364	472	.18533	666
8	.11294	618	.89852	502	8	.83892	468	.19199	670
21.9	.10676	616	.90354	506	26.9	.83424	465	.19869	673
22.0	.10060	613	.90860	509	27.0	.82959	462	.20542	675
1	.09447	610	.91369	513	1	.82497	459	.21217	678
2	.08837	608	.91882	516	2	.82038	456	.21895	681
3	.08229	606	.92398	520	3	.81582	453	.22576	684
4	.07623	604	.92918	524	4	.81129	450	.23260	688
22.5	.07019	601	.93442	527	27.5	.80679	447	.23948	690
6	.06418	598	.93969	531	6	.80232	444	.24638	693
7	.05820	595	.94500	534	7	.79788	440	.25331	695
8	.05225	592	.95034	538	8	.79348	437	.26026	698
22.9	.04633	590	.95572	542	27.9	.78911	434	.26724	701
23.0	.04043	587	.96114	545	28.0	.78477	431	.27425	704
1	.03456	584	.96659	549	1	.78046	428	.28129	707
2	.02872	581	.97208	552	2	.77618	425	.28836	710
23.3	1.02291	578	0.97760	556	3	.77193	422	.29546	713
					4	.76771	420	.30259	715
					5	.76351	417	.30974	718
					28.6	0.75934		1.31692	

$$F = 1/f = \frac{0.22915}{(\sin^4 I + 0.14527 \cos 2I \sin^2 I + 0.00528)^\frac{1}{2}}$$

I is given for the first of each month in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued.

I	F for lunar K_2	Diff.	f for lunar K_2	Diff.	I	F for lunar K_2	Diff.	f for lunar K_2	Diff.
0					0				
18.3	1.58749	1661	0.62992	666	23.3	1.00037	805	0.99963	811
4	.57088	1634	.63658	670	4	0.99232	796	1.00774	814
5	.55454	1609	.64328	672	5	.98436	785	.01588	817
6	.53845	1583	.65000	676	6	.97651	775	.02405	820
7	.52262	1557	.65676	679	7	.96876	766	.03225	822
8	.50705	1533	.66355	682	8	.96110	756	.04047	825
18.9	.49172	1509	.67037	685	23.9	.95354	746	.04872	827
19.0	.47663		.67722		24.0	.94608		.05699	
1	.46178	1485	.68410	688	1	.93871	737	.06529	830
2	.44716	1462	.69101	691	2	.93143	728	.07362	833
3	.43276	1440	.69795	694	3	.92424	719	.08197	835
4	.41859	1417	.70493	698	4	.91714	710	.09035	838
		1395		700			701		840
19.5	.40464		.71193		24.5	.91013		.09875	
6	.39090	1374	.71896	703	6	.90320	693	.10718	843
7	.37737	1353	.72602	706	7	.89635	685	.11563	845
8	.36404	1333	.73312	710	8	.88959	676	.12411	848
19.9	.35091	1313	.74024	712	24.9	.88291	668	.13262	851
		1292		715			660		853
20.0	.33799		.74739		25.0	.87631		.14115	
1	.32525	1274	.75457	718	1	.86979	652	.14970	855
2	.31270	1255	.76179	722	2	.86335	644	.15828	858
3	.30034	1236	.76903	724	3	.85698	637	.16689	861
4	.28816	1218	.77630	727	4	.85069	629	.17552	863
		1200		730			622		865
20.5	.27616		.78360		25.5	.84447		.18417	
6	.26433	1183	.79093	733	6	.83833	614	.19285	868
7	.25267	1166	.79829	736	7	.83226	607	.20155	870
8	.24118	1149	.80598	739	8	.82626	600	.21028	873
20.9	.22986	1132	.81310	742	25.9	.82032	594	.21903	875
		1116		745			586		878
21.0	.21870		.82055		26.0	.81446		.22781	
1	.20770	1100	.82802	747	1	.80867	579	.23661	880
2	.19685	1085	.83553	751	2	.80294	573	.24543	882
3	.18615	1070	.84306	753	3	.79727	567	.25427	884
4	.17561	1054	.85062	756	4	.79167	560	.26314	887
		1040		759			555		890
21.5	.16521		.85821		26.5	.78614		.27204	
6	.15496	1025	.86582	762	6	.78067	547	.28096	892
7	.14484	1012	.87348	765	7	.77526	541	.28990	894
8	.13487	997	.88116	768	8	.76991	535	.29886	896
21.9	.12503	984	.88886	770	26.9	.76462	529	.30785	899
		970		773			524		901
22.0	.11533		.89659		27.0	.75938		.31686	
1	.10576	957	.90435	776	1	.75421	517	.32589	903
2	.09632	944	.91214	779	2	.74909	512	.33495	906
3	.08701	931	.91996	782	3	.74403	506	.34403	908
4	.07782	919	.92780	784	4	.73903	500	.35313	910
		907		787			495		912
22.5	.06875		.93567		27.5	.73408		.36225	
6	.05980	895	.94357	790	6	.72918	490	.37140	915
7	.05097	883	.95150	793	7	.72434	484	.38056	916
8	.04226	871	.95945	795	8	.71955	479	.38975	919
22.9	.03366	860	.96743	798	27.9	.71481	474	.39896	921
		848		801			468		924
23.0	.02518		.97544		28.0	.71013		.40820	
1	.01680	838	.98348	804	1	.70549	464	.41746	926
2	.00853	827	.99154	806	2	.70090	459	.42673	927
23.3	1.00037	816	0.99963	809	3	.69636	454	.43603	930
		805		811	4	.69187	449	.44535	932
					5	.68743	444	.45469	934
					28.6	0.68303	440	1.46406	937

$$F = 1/f = \frac{\sin^2 \omega (1 - \frac{1}{2} \sin^2 i)}{\sin^2 I} = \frac{0.15651}{\sin^2 I}$$

I is given for the first of each month in Table 6.

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TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued.

I	$F(M_2)$	Diff.	$f(M_2)$	Diff.	I	$F(M_2)$	Diff.	$f(M_2)$	Diff.
0					0				
18'3	0'96349		1'03789		23'3	0'99486		1'00517	
4	'96403	54	'03731	58	4	'99558	72	'00444	73
5	'96458	55	'03672	59	5	'99630	72	'00372	72
6	'96513	55	'03613	59	6	'99702	72	'00299	73
7	'96568	55	'03554	59	7	'99775	73	'00225	74
8	'96624	56	'03494	60	8	'99848	73	'00152	73
18'9	'96680	56	'03434	60	23'9	'99922	74	'00078	74
19'0	'96736		'03374		24'0	0'99996		1'00004	
1	'96793	57	'03313	61	1	1'00070	74	0'99930	74
2	'96850	57	'03252	61	2	'00145	75	'99855	75
3	'96907	57	'03191	61	3	'00220	75	'99780	75
4	'96965	58	'03130	62	4	'00296	76	'99705	75
19'5	'97023		'03068		24'5	'00372		'99630	
6	'97081	58	'03006	62	6	'00448	76	'99554	76
7	'97140	59	'02944	62	7	'00525	77	'99478	76
8	'97199	59	'02881	63	8	'00602	77	'99402	76
19'9	'97259	59	'02819	63	24'9	'00679	78	'99326	77
20'0	'97318		'02756		25'0	'00757		'99249	
1	'97378	60	'02692	64	1	'00835	78	'99172	77
2	'97439	61	'02629	63	2	'00914	79	'99095	77
3	'97500	61	'02565	64	3	'00992	78	'99017	78
4	'97561	61	'02500	65	4	'01072	80	'98940	77
20'5	'97622		'02436		25'5	'01152		'98862	
6	'97684	62	'02371	65	6	'01232	80	'98783	79
7	'97746	62	'02306	65	7	'01312	80	'98705	78
8	'97809	63	'02241	65	8	'01393	81	'98626	79
20'9	'97871	62	'02175	66	25'9	'01474	82	'98547	79
21'0	'97935		'02109		26'0	'01556		'98468	
1	'97998	63	'02043	66	1	'01638	82	'98389	79
2	'98062	64	'01976	67	2	'01720	82	'98309	80
3	'98126	64	'01910	66	3	'01803	83	'98229	80
4	'98191	65	'01843	67	4	'01886	83	'98148	81
21'5	'98256		'01775		26'5	'01970		'98068	
6	'98321	65	'01708	67	6	'02054	84	'97987	81
7	'98387	66	'01640	68	7	'02138	84	'97906	81
8	'98453	66	'01572	68	8	'02223	85	'97825	81
21'9	'98519	66	'01503	69	26'9	'02308	85	'97744	82
22'0	'98586		'01435		27'0	'02394		'97662	
1	'98653	67	'01366	69	1	'02480	86	'97580	82
2	'98720	67	'01296	70	2	'02566	86	'97498	82
3	'98788	68	'01227	69	3	'02653	87	'97415	83
4	'98856	68	'01157	70	4	'02740	87	'97333	82
22'5	'98925		'01087		27'5	'02828		'97250	
6	'98994	69	'01017	70	6	'02916	88	'97167	83
7	'99063	69	'00946	71	7	'03005	89	'97083	84
8	'99132	69	'00875	71	8	'03093	88	'96999	84
22'9	'99202	70	'00804	71	27'9	'03183	90	'96915	84
23'0	'99273		'00733		28'0	'03272		'96831	
1	'99343	70	'00661	72	1	'03363	91	'96747	84
2	'99414	71	'00589	72	2	'03453	90	'96662	85
23'3	0'99486	72	1'00517	73	3	'03544	91	'96577	85
					4	'03635	91	'96492	85
					5	'03727	92	'96407	85
					28'6	1'03819	92	0'96321	86

$$F = 1/f = \frac{\cos^4 \frac{1}{2} \omega \cos^4 \frac{1}{2} i}{\cos^4 \frac{1}{2} I} = \frac{0.91538}{\cos^4 \frac{1}{2} I}$$

$$F(M_2) = F(N_2) = F(2N) = F(MS) = F(2SM) = F(MSf) = F(\lambda_2) = F(\mu_2) = F(\nu_2).$$

$$f(M_2) = f(N_2) = f(2N) = f(MS) = f(2SM) = f(MSf) = f(\lambda_2) = f(\mu_2) = f(\nu_2).$$

I is given for the first of each month in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued.

I	$F(O_1)$	Diff.	$f(O_1)$	Diff.	I	$F(O_1)$	Diff.	$f(O_1)$	Diff.
0					0				
18.3	1.24178	617	0.80529	403	23.3	1.00166	368	0.99834	368
4	.23561	610	.80932	402	4	0.99798	364	1.00202	368
5	.22951	603	.81334	400	5	.99434	362	.00570	367
6	.22348	596	.81734	400	6	.99072	358	.00937	366
7	.21752	590	.82134	400	7	.98714	354	.01303	365
8	.21162	583	.82534	399	8	.98360	352	.01668	365
18.9	.20579	576	.82933	398	23.9	.98008	348	.02033	363
19.0	.20003		.83331		24.0	.97660		.02396	
1	.19433	570	.83729	398	1	.97315	345	.02759	363
2	.18869	564	.84126	397	2	.96973	342	.03122	363
3	.18311	558	.84523	397	3	.96634	339	.03483	361
4	.17759	552	.84919	396	4	.96298	336	.03844	361
		545		395			332		360
19.5	.17214		.85314		24.5	.95966		.04204	
6	.16674	540	.85708	394	6	.95636	330	.04563	359
7	.16141	533	.86102	394	7	.95309	327	.04921	358
8	.15613	528	.86496	394	8	.94986	323	.05279	358
19.9	.15090	523	.86888	392	24.9	.94665	321	.05636	357
		517		392			318		356
20.0	.14573		.87280		25.0	.94347		.05992	
1	.14062	511	.87672	392	1	.94031	316	.06347	355
2	.13556	506	.88062	390	2	.93719	312	.06702	355
3	.13055	501	.88452	390	3	.93409	310	.07056	354
4	.12560	495	.88842	390	4	.93102	307	.07409	353
		491		388			304		352
20.5	.12069		.89230		25.5	.92798		.07761	
6	.11584	485	.89618	388	6	.92497	301	.08112	351
7	.11104	480	.90006	388	7	.92198	299	.08463	351
8	.10629	475	.90393	387	8	.91902	296	.08812	349
20.9	.10158	471	.90779	386	25.9	.91608	294	.09161	349
		465		385			292		348
21.0	.09693		.91164		26.0	.91316		.09509	
1	.09232	461	.91549	385	1	.91027	289	.09857	348
2	.08775	457	.91933	384	2	.90741	286	.10203	346
3	.08324	451	.92316	383	3	.90458	283	.10549	346
4	.07877	447	.92699	383	4	.90176	282	.10894	345
		443		382			279		344
21.5	.07434		.93081		26.5	.89897		.11238	
6	.06996	438	.93462	381	6	.89621	276	.11581	343
7	.06562	434	.93842	380	7	.89347	274	.11924	343
8	.06132	430	.94222	380	8	.89075	272	.12266	342
21.9	.05707	425	.94601	379	26.9	.88805	270	.12606	340
		421		379			267		340
22.0	.05286		.94980		27.0	.88538		.12946	
1	.04869	417	.95357	377	1	.88273	265	.13285	339
2	.04456	413	.95734	377	2	.88010	263	.13624	339
3	.04047	409	.96111	377	3	.87749	261	.13961	337
4	.03642	405	.96486	375	4	.87491	258	.14298	337
		401		375			256		336
22.5	.03241		.96861		27.5	.87235		.14634	
6	.02843	398	.97235	374	6	.86981	254	.14969	335
7	.02450	393	.97609	374	7	.86728	253	.15303	334
8	.02060	390	.97981	372	8	.86478	250	.15636	333
22.9	.01674	386	.98353	372	27.9	.86230	248	.15968	332
		382		371			245		332
23.0	.01292		.98724		28.0	.85985		.16300	
1	.00913	379	.99095	371	1	.85741	244	.16631	331
2	.00538	375	.99465	370	2	.85499	242	.16961	330
23.3	1.00166	372	0.99834	369	3	.85259	240	.17290	329
		368		368	4	.85021	238	.17618	328
					5	.84785	236	.17945	327
					28.6	0.84551	234	1.18271	326

$$F = 1/f = \frac{\sin \omega \cos^2 \frac{1}{2} \omega \cos^2 \frac{1}{2} i}{\sin I \cos^2 \frac{1}{2} I} = \frac{0.38005}{\sin I \cos^2 \frac{1}{2} I}$$

$$F(O_1) = F(Q_1) = F(M_1) \div Q'$$

$$f(O_1) = f(Q_1) = f(M_1) \times Q'$$

I is given for the first of each month in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued.

I	$F(00)$	Diff.	$f(00)$	Diff.	I	$F(00)$	Diff.	$f(00)$	Diff.
0					0				
18.3	2.06270	3289	0.48482	784	23.3	1.01542	1261	0.98482	1237
4	2.02981	3218	.49226	793	4	1.00281	1239	0.99719	1248
5	1.99763	3149	.50059	802	5	0.99042	1218	1.00967	1258
6	.96614	3082	.50861	810	6	.97824	1198	.02225	1267
7	.93532	3017	.51671	818	7	.96626	1178	.03492	1277
8	.90515	2954	.52489	827	8	.95448	1159	.04769	1288
18.9	.87561	2892	.53316	835	23.9	.94289	1139	.06057	1297
19.0	.84669		.54151		24.0	.93150		.07354	
1	.81837	2832	.54994	843	1	.92029	1121	.08661	1307
2	.79063	2774	.55846	852	2	.90927	1102	.09979	1318
3	.76346	2717	.56707	861	3	.89842	1085	.11306	1327
4	.73684	2662	.57576	869	4	.88775	1067	.12643	1337
		2607		877			1049		1348
19.5	.71077		.58453		24.5	.87726		.13991	
6	.68522	2555	.59339	886	6	.86693	1033	.15350	1359
7	.66019	2503	.60234	895	7	.85677	1016	.16718	1368
8	.63565	2454	.61138	904	8	.84677	1000	.18096	1378
19.9	.61161	2404	.62050	912	24.9	.83693	984	.19485	1389
		2357		921			969		1399
20.0	.58804		.62971		25.0	.82724		.20884	
1	.56494	2310	.63900	929	1	.81771	953	.22293	1409
2	.54229	2265	.64839	939	2	.80832	939	.23713	1420
3	.52008	2221	.65786	947	3	.79909	923	.25143	1430
4	.49830	2178	.66742	956	4	.78999	910	.26583	1440
		2135		965			895		1451
20.5	.47695		.67707		25.5	.78104		.28034	
6	.45600	2095	.68681	974	6	.77223	881	.29495	1461
7	.43545	2055	.69664	983	7	.76355	868	.30967	1472
8	.41530	2015	.70656	992	8	.75501	854	.32449	1482
20.9	.39553	1977	.71658	1002	25.9	.74659	842	.33942	1493
		1940		1010			829		1503
21.0	.37613		.72668		26.0	.73830		.35445	
1	.35709	1904	.73687	1019	1	.73014	816	.36959	1514
2	.33841	1868	.74715	1028	2	.72210	804	.38484	1525
3	.32008	1833	.75753	1038	3	.71419	791	.40019	1535
4	.30209	1799	.76800	1047	4	.70639	780	.41565	1546
		1766		1056			768		1556
21.5	.28443		.77856		26.5	.69871		.43121	
6	.26709	1734	.78921	1065	6	.69114	757	.44688	1567
7	.25007	1702	.79996	1075	7	.68368	746	.46266	1578
8	.23335	1672	.81080	1084	8	.67634	734	.47855	1589
21.9	.21694	1641	.82173	1093	26.9	.66910	724	.49455	1600
		1611		1103			713		1610
22.0	.20083		.83276		27.0	.66197		.51065	
1	.18500	1583	.84388	1112	1	.65494	703	.52686	1621
2	.16946	1554	.85510	1122	2	.64801	693	.54318	1632
3	.15419	1527	.86641	1131	3	.64119	682	.55961	1643
4	.13919	1500	.87782	1141	4	.63446	673	.57615	1654
		1473		1150			663		1664
22.5	.12446		.88932		27.5	.62783		.59279	
6	.10998	1448	.90092	1160	6	.62129	654	.60955	1676
7	.09576	1422	.91261	1169	7	.61485	644	.62641	1686
8	.08178	1398	.92440	1179	8	.60850	635	.64339	1698
22.9	.06804	1374	.93629	1189	27.9	.60224	626	.66047	1708
		1350		1199			617		1720
23.0	.05454		.94828		28.0	.59607		.67767	
1	.04128	1326	.96036	1208	1	.58998	609	.69497	1730
2	.02823	1305	.97254	1218	2	.58398	600	.71239	1742
23.3	1.01542	1281	0.98482	1228	3	.57806	592	.72992	1753
		1261		1237	4	.57223	583	.74756	1764
					5	.56647	576	.76531	1775
					28.6	0.56080	567	1.78317	1786

$$F = 1/f = \frac{\sin \omega \sin^2 \frac{1}{2} \omega \cos^2 \frac{1}{2} i}{\sin I \sin^2 \frac{1}{2} I} = \frac{0.01638}{\sin I \sin^2 \frac{1}{2} I}$$

I is given for the first of each month in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued.

I	F (Mf)	Diff.	f (Mf)	Diff.	I	F (Mf)	Diff.	f (Mf)	Diff.
0					23°3	1°00852	812	0°99156	805
18°3	1°60043	1675	0°62486	658	23°4	1°00040	802	0°99961	807
4	58368	1648	63144	663	5	0°99238	792	1°00768	810
5	56720	1622	63807	668	6	0°98446	782	0°1578	813
6	55098	1596	64475	671	7	0°97664	771	0°2391	816
7	53502	1571	65146	673	8	0°96893	762	0°3207	818
8	51931	1545	65819	677	23°9	0°96131	753	0°4025	821
18°9	50386	1521	66496	679	24°0	0°95378	743	0°4846	823
19°0	48865	1497	67175	683	1	0°94635	734	0°5669	826
1	47368	1474	67858	685	2	0°93901	725	0°6495	828
2	45894	1451	68543	689	3	0°93176	716	0°7323	831
3	44443	1429	69232	691	4	0°92460	707	0°8154	834
4	43014	1407	69923	695	24°5	0°91753	698	0°8988	836
19°5	41607	1385	70618	697	6	0°91055	690	0°9824	839
6	40222	1364	71315	701	7	0°90365	682	1°0663	841
7	38858	1343	72016	704	8	0°89683	673	1°1504	843
8	37515	1324	72720	707	24°9	0°89010	666	1°2347	846
19°9	36191	1303	73427	709	25°0	0°88344	657	1°3193	849
20°0	34888	1284	74136	712	1	0°87687	649	1°4042	851
1	33604	1265	74848	716	2	0°87038	642	1°4893	853
2	32339	1246	75564	718	3	0°86396	634	1°5746	856
3	31093	1228	76282	721	4	0°85762	627	1°6602	859
4	29865	1210	77003	724	25°5	0°85135	620	1°7461	861
20°5	28655	1193	77727	728	6	0°84515	612	1°8322	863
6	27462	1175	78455	730	7	0°83903	605	1°9185	865
7	26287	1158	79185	733	8	0°83298	598	2°0050	868
8	25129	1142	79918	736	25°9	0°82700	591	2°0918	871
20°9	23987	1125	80654	738	26°0	0°82109	584	2°1789	873
21°0	22862	1109	81392	742	1	0°81525	578	2°2662	875
1	21753	1094	82134	744	2	0°80947	570	2°3537	877
2	20659	1078	82878	747	3	0°80377	565	2°4414	880
3	19581	1063	83625	750	4	0°79812	558	2°5294	883
4	18518	1048	84375	753	26°5	0°79254	552	2°6177	884
21°5	17470	1034	85128	756	6	0°78702	545	2°7061	887
6	16436	1020	85884	759	7	0°78157	540	2°7984	889
7	15416	1005	86643	761	8	0°77617	533	2°8837	892
8	14411	992	87404	764	26°9	0°77084	527	2°9729	893
21°9	13419	978	88168	767	27°0	0°76557	522	3°0622	896
22°0	12441	965	88935	770	1	0°76035	516	3°1518	899
1	11476	952	89705	773	2	0°75519	510	3°2417	900
2	10524	939	90478	775	3	0°75009	504	3°3317	903
3	09585	926	91253	778	4	0°74505	499	3°4220	905
4	08659	914	92031	781	27°5	0°74006	494	3°5125	907
22°5	07745	902	92812	783	6	0°73512	488	3°6032	909
6	06843	890	93595	786	7	0°73024	483	3°6941	912
7	05953	878	94381	789	8	0°72541	478	3°7853	914
8	05075	867	95170	792	27°9	0°72063	472	3°8767	916
22°9	04208	856	95962	794	28°0	0°71591	468	3°9683	918
23°0	03352	844	96756	797	1	0°71123	462	4°0601	920
1	02508	834	97553	800	2	0°70661	458	4°1521	922
2	01674	822	98353	803	3	0°70203	452	4°2443	925
23°3	1°00852	812	0°99156	805	4	0°69751	448	4°3368	927
					5	0°69303	443	4°4295	928
					28°6	0°68860		1°45223	

$$F = 1/f = \frac{\sin^2 \omega \cos^4 \frac{1}{2} i}{\sin^2 I} = \frac{0.15779}{\sin^2 I}$$

I is given for the first of each month in Table 6.

TABLE 13.—Factors F and f , corresponding to every tenth of a degree of I , for reduction and prediction of tides—Continued

I	F (Mm)	Diff.	f (Mm)	Diff.	I	F (Mm)	Diff.	f (Mm)	Diff.
0					23° 3'	0.98411	246	1.01614	253
18° 3'	0.88387	163	1.13139	208	23° 4'	0.98557	248	1.01361	253
4	0.88550	164	1.12931	209	23° 5'	0.98905	249	1.01108	255
5	0.88714	165	1.12722	209	23° 6'	0.99154	252	1.00853	256
6	0.88879	167	1.12513	211	23° 7'	0.99406	254	1.00597	256
7	0.89046	168	1.12302	212	23° 8'	0.99660	256	1.00341	257
8	0.89214	169	1.12090	212	23° 9'	0.99916	258	1.00084	258
18° 9'	0.89383	171	1.11878	214	24° 0'	1.00174	260	0.99826	259
19° 0'	0.89554	172	1.11664	214	24° 1'	1.00434	263	0.99567	259
1	0.89726	174	1.11450	216	24° 2'	1.00697	265	0.99308	260
2	0.89900	175	1.11234	216	24° 3'	1.00962	267	0.99048	262
3	0.90075	177	1.11018	218	24° 4'	1.01229	269	0.98786	262
4	0.90252	178	1.10800	218	24° 5'	1.01498	271	0.98524	262
19° 5'	0.90430	180	1.10582	219	24° 6'	1.01769	274	0.98262	264
6	0.90610	181	1.10363	220	24° 7'	1.02043	276	0.97998	264
7	0.90791	183	1.10143	221	24° 8'	1.02319	278	0.97734	265
8	0.90974	184	1.09922	222	24° 9'	1.02597	281	0.97469	266
19° 9'	0.91158	185	1.09700	223	25° 0'	1.02878	283	0.97203	267
20° 0'	0.91343	187	1.09477	224	25° 1'	1.03161	285	0.96936	267
1	0.91530	189	1.09253	225	25° 2'	1.03446	288	0.96669	268
2	0.91719	191	1.09028	226	25° 3'	1.03734	290	0.96401	269
3	0.91910	192	1.08802	226	25° 4'	1.04024	293	0.96132	270
4	0.92102	193	1.08576	228	25° 5'	1.04317	295	0.95862	271
20° 5'	0.92295	195	1.08348	228	25° 6'	1.04612	298	0.95591	271
6	0.92490	197	1.08120	230	25° 7'	1.04910	300	0.95320	272
7	0.92687	198	1.07890	230	25° 8'	1.05210	303	0.95048	273
8	0.92885	200	1.07660	231	25° 9'	1.05513	305	0.94775	273
20° 9'	0.93085	201	1.07429	232	26° 0'	1.05818	308	0.94502	275
21° 0'	0.93286	204	1.07197	233	26° 1'	1.06126	311	0.94227	275
1	0.93490	205	1.06964	234	26° 2'	1.06437	313	0.93952	275
2	0.93695	206	1.06730	235	26° 3'	1.06750	316	0.93677	277
3	0.93901	209	1.06495	236	26° 4'	1.07066	319	0.93400	277
4	0.94110	210	1.06259	237	26° 5'	1.07385	322	0.93123	278
21° 5'	0.94320	211	1.06022	237	26° 6'	1.07707	324	0.92845	279
6	0.94531	214	1.05785	238	26° 7'	1.08031	327	0.92566	279
7	0.94745	215	1.05547	240	26° 8'	1.08358	330	0.92287	280
8	0.94960	217	1.05307	240	26° 9'	1.08688	333	0.92007	281
21° 9'	0.95177	219	1.05067	241	27° 0'	1.09021	335	0.91726	282
22° 0'	0.95396	221	1.04826	242	27° 1'	1.09356	339	0.91444	282
1	0.95617	222	1.04584	243	27° 2'	1.09695	342	0.91162	283
2	0.95839	224	1.04341	243	27° 3'	1.10037	344	0.90879	284
3	0.96063	227	1.04098	245	27° 4'	1.10381	348	0.90595	284
4	0.96290	228	1.03853	245	27° 5'	1.10729	350	0.90311	285
22° 5'	0.96518	230	1.03608	246	27° 6'	1.11079	354	0.90026	286
6	0.96748	232	1.03362	247	27° 7'	1.11433	357	0.89740	287
7	0.96980	233	1.03115	248	27° 8'	1.11790	360	0.89453	287
8	0.97213	236	1.02867	249	27° 9'	1.12150	363	0.89166	288
22° 9'	0.97449	238	1.02618	250	28° 0'	1.12513	367	0.88878	288
23° 0'	0.97687	239	1.02368	250	28° 1'	1.12880	369	0.88590	289
1	0.97926	242	1.02118	251	28° 2'	1.13249	373	0.88301	290
2	0.98168	243	1.01867	253	28° 3'	1.13622	377	0.88011	291
23° 3'	0.98411	246	1.01614	253	28° 4'	1.13999	379	0.87720	291
					28° 5'	1.14378	383	0.87429	292
					28° 6'	1.14761		0.87137	292

$$F = 1/f = \frac{(1 - \frac{1}{2} \sin^2 \omega)(1 - \frac{1}{2} \sin^2 i)}{1 - \frac{1}{2} \sin^2 I} = \frac{0.75316}{1 - \frac{1}{2} \sin^2 I}$$

I is given for the first of each month in Table 6.

TABLE 14.—Factors (F) for reducing Mn , K_1+O_1 , and X to mean values.

		I , or inclination of orbit to equator.											
		$18\frac{1}{2}^\circ$	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	$28\frac{1}{2}^\circ$
$F(Mn)$													
$\frac{K_1+O_1}{M_1}=0.0$	0.970	0.972	0.977	0.982	0.988	0.994	1.000	1.006	1.012	1.019	1.026	1.029	
0.2	0.971	0.973	0.978	0.982	0.988	0.994	1.000	1.006	1.012	1.019	1.026	1.029	
0.4	0.972	0.974	0.979	0.983	0.988	0.994	1.000	1.006	1.012	1.018	1.025	1.028	
0.6	0.974	0.976	0.980	0.984	0.989	0.994	1.000	1.006	1.011	1.017	1.023	1.026	
0.8	0.977	0.979	0.982	0.986	0.990	0.995	1.000	1.005	1.010	1.015	1.020	1.022	
1.0	0.980	0.982	0.985	0.989	0.992	0.996	1.000	1.004	1.008	1.013	1.017	1.019	
1.2	0.984	0.985	0.988	0.991	0.994	0.997	1.000	1.003	1.007	1.010	1.014	1.016	
1.4	0.988	0.989	0.991	0.994	0.996	0.998	1.000	1.002	1.005	1.007	1.010	1.012	
1.6	0.994	0.994	0.995	0.997	0.998	0.999	1.000	1.001	1.003	1.004	1.005	1.006	
1.8	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	
2.0	1.008	1.007	1.006	1.004	1.003	1.001	1.000	0.998	0.997	0.995	0.994	0.993	
2.5	1.025	1.023	1.019	1.014	1.009	1.004	0.999	0.994	0.989	0.984	0.979	0.977	
$F(K_1+O_1)$													
	1.168	1.148	1.109	1.073	1.040	1.010	0.982	0.955	0.931	0.909	0.888	0.878	
$F(X)$													
$X'=0.1$	0.54	0.61	0.73	0.83	0.91	0.98	1.04	1.09	1.14	1.19	1.23	1.24	
0.2	0.86	0.88	0.91	0.94	0.97	0.99	1.01	1.04	1.05	1.07	1.08	1.09	
0.3	0.96	0.96	0.97	0.98	0.99	1.00	1.01	1.01	1.02	1.02	1.03	1.03	
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

$$F(K_1+O_1) = \frac{2.4066}{1.4066 f(K_1) + f(O_1)}, \quad F(X) = \frac{X}{X'}, \quad \cos(X \cdot 180^\circ) = F(K_1+O_1) \cos(X' \cdot 180^\circ).$$

This table is based upon Tables 1, 13, 21, and §§ 3, 21, 50.

TABLE 15.—Acceleration in HW and LW of *a*

[The amplitude of the principal wave is taken as unity.]

HW phase. LW phase.*	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
Amplitude of subordinate wave.	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /
0.0	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
0.1	0 00	0 54	1 47	2 38	3 25	4 07	4 43	5 12	5 32	5 42
0.2	0 00	1 40	3 18	4 52	6 22	7 44	8 57	9 59	10 47	11 19
0.3	0 00	2 18	4 35	6 47	8 55	10 54	12 44	14 20	15 41	16 42
0.4	0 00	2 51	5 41	8 27	11 08	13 42	16 06	18 18	20 13	21 48
0.5	0 00	3 20	6 38	9 54	13 05	16 10	19 06	21 52	24 22	26 34
0.6	0 00	3 45	7 28	11 10	14 48	18 21	21 47	25 04	28 09	30 58
0.7	0 00	4 07	8 13	12 18	16 19	20 18	24 11	27 57	31 35	35 00
0.8	0 00	4 27	8 53	13 18	17 41	22 02	26 20	30 33	34 40	38 40
0.9	0 00	4 44	9 28	14 12	18 54	23 36	28 16	32 53	37 28	41 59
1.0	0 00	5 00	10 00	15 00	20 00	25 00	30 00	35 00	40 00	45 00
HW phase. LW phase.	360° 180	350° 170	340° 160	330° 150	320° 140	310° 130	300° 120	290° 110	280° 100	270° 90

* I. e. the argument, or phase, of the subordinate component (*B*) at the time of HW and LW, respectively, of the principal component (*A*). By § 2

$$\tan \text{ acceleration in } \frac{\text{HW}}{\text{LW}} = \frac{a}{b} \frac{B \delta^2 \sin \text{HW phase}}{A \delta^2 \cos \text{LW phase}} \pm 1 + \frac{B \delta^2 \cos \text{HW phase}}{A \delta^2 \sin \text{LW phase}}$$

(When $\delta = a$, this formula is exact.) If t denote the time after the conspiring of *A* and *B*, § 17, then

$$\begin{aligned} \text{phase} &= (b - a) t; \\ &= \frac{b}{a} 2\pi n \text{ for HW,} \\ &= \frac{b}{a} 2\pi n + \pi \text{ for LW,} \end{aligned}$$

n being an integer denoting the number of high waters of *A* since the coincidence of the maxima of *A* and *B*.

TABLE 16.—Height of HW and LW for *a*

[The amplitude of the principal wave is taken as unity.]

HW phase. LW phase.	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
Amplitude of subordinate wave.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0000	1.0986	1.0945	1.0877	1.0785	1.0670	1.0536	1.0385	1.0221	1.0050
0.2	1.2000	1.1975	1.1899	1.1775	1.1603	1.1389	1.1135	1.0849	1.0532	1.0198
0.3	1.3000	1.2965	1.2860	1.2687	1.2448	1.2148	1.1790	1.1381	1.0928	1.0440
0.4	1.4000	1.3957	1.3827	1.3611	1.3315	1.2939	1.2490	1.1973	1.1397	1.0770
0.5	1.5000	1.4949	1.4798	1.4546	1.4198	1.3758	1.3228	1.2618	1.1931	1.1180
0.6	1.6000	1.5943	1.5772	1.5490	1.5097	1.4599	1.4000	1.3306	1.2523	1.1662
0.7	1.7000	1.6937	1.6749	1.6439	1.6007	1.5459	1.4798	1.4032	1.3165	1.2207
0.8	1.8000	1.7932	1.7730	1.7395	1.6928	1.6336	1.5620	1.4789	1.3849	1.2806
0.9	1.9000	1.8928	1.8712	1.8354	1.7858	1.7225	1.6463	1.5575	1.4569	1.3453
1.0	2.0000	1.9924	1.9696	1.9318	1.8794	1.8126	1.7320	1.6384	1.5320	1.4142
HW phase. LW phase.	360° 180	350° 170	340° 160	330° 150	320° 140	310° 130	300° 120	290° 110	280° 100	270° 90

For high waters use the tabular values as given; but for low waters alter their signs.

wave due to another wave of equal speed.

[The amplitude of the principal wave is taken as unity.]

HW phase. LW phase.	100° 280	110° 290	120° 300	130° 310	140° 320	150° 330	160° 340	170° 350	180° 360	
Amplitude of subordinate wave.	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	
0°0	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	
0°1	5 43	5 33	5 13	4 41	3 59	3 08	2 10	1 06	0 00	
0°2	11 32	11 24	10 54	9 58	8 38	6 54	4 49	2 29	0 00	
0°3	17 19	17 26	17 00	15 54	14 03	11 27	8 08	4 14	0 00	
0°4	22 57	23 32	23 25	22 25	20 20	17 01	12 22	6 32	0 00	
0°5	28 20	29 33	30 00	29 27	27 31	23 48	17 53	9 42	0 00	
0°6	33 25	35 21	36 35	36 48	35 31	31 59	25 12	14 17	0 00	
0°7	38 07	40 51	43 00	44 16	44 08	41 38	34 59	21 22	0 00	
0°8	42 27	45 59	49 06	51 36	53 01	52 29	47 49	33 13	0 00	
0°9	46 25	50 42	54 47	58 34	61 46	63 53	63 23	53 59	0 00	
1°0	50 00	55 00	60 00	65 00	70 00	75 00	80 00	85 00	0 00	
HW phase. LW phase.	260° 80	250° 70	240° 60	230° 50	220° 40	210° 30	200° 20	190° 10	180° 0	

When the top argument is used the tabular values are positive; when the bottom argument, they are negative.
To express the acceleration in time, divide by a , the speed of the greater component.

To find the acceleration when b is not exactly equal to a , multiply the tabular values by $\frac{b}{a}$.

This acceleration is directly expressed in time by multiplying the tabular values by $\frac{b}{a^2}$.

tide composed of two waves of equal speed.

[The amplitude of the principal wave is taken as unity.]

HW phase. LW phase.	100° 280	110° 290	120° 300	130° 310	140° 320	150° 330	160° 340	170° 350	180° 360	Mean value.†
Amplitude of subordinate wave.	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000
0°0	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000
0°1	0°9875	0°9703	0°9540	0°9389	0°9256	0°9148	0°9067	0°9017	0°9000	1°0025
0°2	0°9852	0°9504	0°9166	0°8848	0°8565	0°8329	0°8150	0°8038	0°8000	1°0100
0°3	0°9929	0°9407	0°8888	0°8392	0°7940	0°7552	0°7253	0°7065	0°7000	1°0226
0°4	1°0104	0°9415	0°8717	0°8036	0°7397	0°6835	0°6389	0°6100	0°6000	1°0404
0°5	1°0375	0°9529	0°8660	0°7792	0°6957	0°6197	0°5571	0°5150	0°5000	1°0635
0°6	1°0731	0°9744	0°8718	0°7672	0°6639	0°5664	0°4820	0°4222	0°4000	1°0922
0°7	1°1167	1°0055	0°8889	0°7682	0°6462	0°5269	0°4176	0°3336	0°3000	1°1268
0°8	1°1672	1°0454	0°9165	0°7819	0°6437	0°5044	0°3694	0°2536	0°2000	1°1678
0°9	1°2237	1°0929	0°9539	0°8081	0°6566	0°5009	0°3443	0°1933	0°1000	1°2160
1°0	1°2856	1°1472	1°0000	0°8452	0°6840	0°5176	0°3472	0°1744	0°0000	1°2724
HW phase. LW phase.	260° 80	250° 70	240° 60	230° 50	220° 40	210° 30	200° 20	190° 10	180° 0	

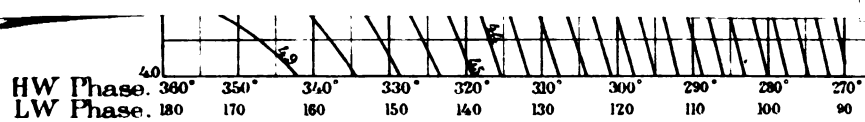
† When b is not exactly to a , mean value = $1 + (\text{tabular value} - 1) \frac{b^2}{a^2}$.

When the top argument is used, the tabular values are

** Note. is, the argument, or phase, of the diurnal wave (A*

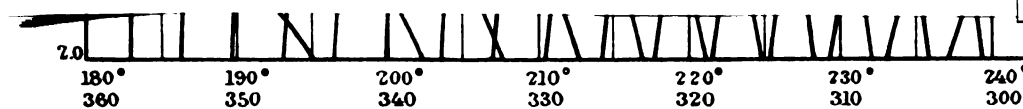
HW Phase-- $n\pi + \frac{1}{2}A^ - B^*$*

n being an int



For high waters, use the tabular values as given

** See footnote.*



HW Phase.

High water inequality curves terminate in the left marg

TABLE 20.—Great tropic range and its duration.

[The amplitude of the semidiurnal wave is taken as unity.]

Elevation of great tropic HW > depression of great tropic LW.						Depression of gt. tropic LW > elevation of gt. tropic HW.				
HW phase. {	0° 180	10° 170	20° 160	30° 150	40° 140	50° 130	60° 120	70° 110	80° 100	90° 90
Amplitude of diurnal wave.	1·6 { 3·921 7 50	4·149 7 54	4·326 7 56	4·444 7 57	4·504 7 57	4·504 7 57	4·444 7 57	4·326 7 56	4·149 7 54	3·921 7 50
	1·7 { 4·062 7 57	4·291 7 59	4·487 8 01	4·610 8 02	4·674 8 03	4·674 8 03	4·610 8 02	4·487 8 01	4·291 7 59	4·062 7 57
	1·8 { 4·205 8 03	4·456 8 05	4·649 8 06	4·777 8 07	4·844 8 08	4·844 8 08	4·777 8 07	4·649 8 06	4·456 8 05	4·205 8 03
	1·9 { 4·351 8 10	4·612 8 11	4·813 8 12	4·947 8 12	5·015 8 12	5·015 8 12	4·947 8 12	4·813 8 12	4·612 8 11	4·351 8 10
	2·0 { 4·500 8 17	4·772 8 17	4·978 8 17	5·117 8 17	5·187 8 17	5·187 8 17	5·117 8 17	4·978 8 17	4·772 8 17	4·500 8 17
	2·1 { 4·651 8 24	4·932 8 23	5·144 8 22	5·288 8 21	5·360 8 21	5·360 8 21	5·288 8 21	5·144 8 22	4·932 8 23	4·651 8 24
	2·2 { 4·805 8 31	5·093 8 28	5·311 8 27	5·460 8 26	5·535 8 26	5·535 8 26	5·460 8 26	5·311 8 27	5·093 8 28	4·805 8 31
	2·3 { 4·962 8 38	5·256 8 34	5·487 8 31	5·634 8 31	5·710 8 30	5·710 8 30	5·634 8 31	5·487 8 31	5·256 8 34	4·962 8 38
	2·4 { 5·121 8 45	5·421 8 40	5·652 8 37	5·808 8 35	5·886 8 34	5·886 8 34	5·808 8 35	5·652 8 37	5·421 8 40	5·121 8 45
	2·5 { 5·282 8 53	5·589 8 45	5·824 8 41	5·984 8 39	6·063 8 38	6·063 8 38	5·984 8 39	5·824 8 41	5·589 8 45	5·282 8 53
	2·6 { 5·445 9 01	5·757 8 51	5·997 8 46	6·160 8 44	6·241 8 42	6·241 8 42	6·160 8 44	5·997 8 46	5·757 8 51	5·445 9 01
	2·7 { 5·611 9 08	5·927 8 57	6·170 8 51	6·337 8 48	6·420 8 46	6·420 8 46	6·337 8 48	6·170 8 51	5·927 8 57	5·611 9 08
	2·8 { 5·780 9 17	6·097 9 02	6·345 8 55	6·514 8 52	6·600 8 50	6·600 8 50	6·514 8 52	6·345 8 55	6·097 9 02	5·780 9 17
	2·9 { 5·951 9 25	6·274 9 07	6·522 8 59	6·692 8 55	6·779 8 54	6·779 8 54	6·692 8 55	6·522 8 59	6·274 9 07	5·951 9 25
	3·0 { 6·125 9 34	6·446 9 13	6·700 9 03	6·875 8 59	6·959 8 57	6·959 8 57	6·875 8 59	6·700 9 03	6·446 9 13	6·125 9 34
	4·0 { 8·000 12 25	8·261 10 03	8·517 9 42	8·701 9 32	8·795 9 28	8·795 9 28	8·701 9 32	8·517 9 42	8·261 10 03	8·000 12 25
	5·0 { 10·000 12 25	10·165 10 43	10·399 10 11	10·579 9 58	10·675 9 52	10·675 9 52	10·579 9 58	10·399 10 11	10·165 10 43	10 00 12 25
	10·0 { 20·000 12 25	20·037 11 48	20·146 11 18	20·289 11 01	20·387 10 53	20·387 10 53	20·289 11 01	20·146 11 18	20·037 11 48	20 00 12 25
HW phase. {	180° 360	190° 350	200° 340	210° 330	220° 320	230° 310	240° 300	250° 290	260° 280	270° 270

The first value of each pair is the value of the great tropic range; the second, its duration in hours and minutes.

This table assumes that $d_1 = m_1$.

See §§ 25, 37, and 53.

TABLE 21.—Effects of various tidal components upon the mean semirange of tide. *

[The amplitude of M_2 is taken as unity.]

Ampli- tude of subordi- nate com- ponent.	Semidiurnal components.								Ampli- tude of subordi- nate com- ponent.	Diurnal components.				
	K_2	L_2	N_2	S_2	λ_2	μ_2	ν_2	$\frac{E^2}{4}$		K_1	O_1	P_1	Q_1	$\frac{E^2}{16}$
0.02	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	0.04	.0001	.0001	.0001	.0001	.0001
0.04	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004	0.08	.0004	.0004	.0004	.0003	.0004
0.06	.0010	.0009	.0009	.0010	.0009	.0008	.0009	.0009	0.12	.0010	.0008	.0010	.0008	.0009
0.08	.0017	.0017	.0015	.0017	.0017	.0015	.0015	.0016	0.16	.0017	.0015	.0017	.0014	.0016
0.10	.0027	.0026	.0024	.0027	.0026	.0023	.0024	.0025	0.20	.0027	.0023	.0027	.0021	.0025
0.12	.0039	.0037	.0035	.0039				.0036	0.24	.0039	.0033	.0038	.0030	.0036
0.14	.0053	.0051	.0047	.0052				.0049	0.28	.0053	.0045	.0052	.0041	.0049
0.16	.0069		.0061	.0069				.0064	0.32	.0069	.0059	.0068		.0064
0.18	.0087		.0078	.0087				.0081	0.36	.0087	.0075	.0086		.0081
0.20	.0108		.0096	.0107				.0100	0.40	.0108	.0093	.0106		.0100
0.22	.0130		.0116	.0130				.0121	0.44	.0130	.0112	.0127		.0121
0.24	.0155		.0138	.0154				.0144	0.48	.0155	.0133	.0150		.0144
0.26			.0162	.0181				.0169	0.52	.0182	.0156	.0176		.0169
0.28			.0188	.0210				.0196	0.56	.0211	.0181			.0196
0.30			.0216	.0241				.0225	0.60	.0242	.0208			.0225
0.32				.0274				.0256	0.64	.0276	.0237			.0256
0.34				.0310				.0289	0.68	.0311	.0268			.0289
0.36				.0347				.0324	0.72	.0349	.0300			.0324
0.38				.0387				.0361	0.76	.0389	.0334			.0361
0.40				.0428				.0400	0.80	.0431	.0370			.0400
0.42				.0472				.0441	0.84	.0475	.0407			.0441
0.44				.0518				.0484	0.88	.0521	.0447			.0484
0.46				.0567				.0529	0.92	.0570	.0488			.0529
0.48				.0617				.0576	0.96	.0620	.0532			.0576
0.50				.0669				.0625	1.00	.0673	.0577			.0625
									1.04	.0728	.0624			.0676
0.60				.0964				.0900	1.08	.0785	.0673			.0729
0.70				.1312				.1225	1.12	.0845	.0723			.0784
0.80				.1715				.1600	1.16	.0906	.0776			.0841
0.90				.2169				.2025	1.20	.0969	.0831			.0900
1.00				.2678				.2500	1.24	.1035	.0887			.0961
									1.28	.1103	.0945			.1024
									1.32	.1173				.1089
									1.36	.1245				.1156
									1.40	.1320				.1225
									1.44	.1396				.1296
									1.48	.1475				.1369
									1.52	.1555				.1444
									1.56	.1637				.1521
									1.60	.1720				.1600
									1.64	.1805				.1681
									1.68	.1892				.1764

* Tabular value for component $C = \frac{C^2}{4 M_2} \frac{c^2}{m_2^3}$, § 13.

TABLE 22.—*Value of $\frac{1}{2}$ Mn when $M_2=1$.*

$\frac{K_1+O_1}{M_2}$	S_2/M_2							
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.0	1.0127	1.0213	1.0357	1.0558	1.0817	1.1134	1.1508	1.1940
0.2	1.0142	1.0228	1.0372	1.0573	1.0832	1.1149	1.1523	1.1955
0.4	1.0185	1.0271	1.0415	1.0616	1.0875	1.1192	1.1566	1.1998
0.6	1.0257	1.0343	1.0487	1.0688	1.0947	1.1264	1.1638	1.2070
0.8	1.0357	1.0443	1.0587	1.0788	1.1047	1.1364	1.1738	1.2170
1.0	1.0486	1.0572	1.0716	1.0917	1.1176	1.1493	1.1867	1.2299
1.2	1.0643	1.0729	1.0873	1.1074	1.1333	1.1650	1.2024	1.2456
1.4	1.0830	1.0916	1.1060	1.1261	1.1520	1.1837	1.2211	1.2643
1.6	1.1045	1.1131	1.1275	1.1476	1.1735	1.2052	1.2426	1.2858
1.8	1.1289	1.1375	1.1519	1.1720	1.1979	1.2296	1.2670	1.3102
2.0	1.1559	1.1645	1.1789	1.1990	1.2249	1.2566	1.2940	1.3372
2.5	1.2360	1.2446	1.2590	1.2791	1.3050	1.3367	1.3741	1.4173

This table, based upon Tables 1 and 21, is computed upon the assumption that the ratios between the diurnal components, the pure lunar semidiurnals, the solar semidiurnals (including luni-solar K_2), are respectively constant for all stations; also that shallow water tides do not occur.

On account of nonpredictable inequalities, the tabular values should be multiplied by about 1.02.

TABLE 23.—*Value of M_2 when $\frac{1}{2}$ Mn = 1.*

$\frac{K_1+O_1}{\frac{1}{2} Mn}$	$S_2/\frac{1}{2} Mn$							
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.0	0.9873	0.9785	0.9634	0.9414	0.9116	0.8725	0.8221	0.7574
0.2	0.9858	0.9770	0.9618	0.9396	0.9096	0.8702	0.8194	0.7542
0.4	0.9814	0.9725	0.9572	0.9348	0.9045	0.8648	0.8136	0.7479
0.6	0.9740	0.9650	0.9495	0.9269	0.8963	0.8562	0.8045	0.7382
0.8	0.9635	0.9543	0.9386	0.9157	0.8847	0.8442	0.7919	0.7249
1.0	0.9497	0.9403	0.9243	0.9011	0.8696	0.8286	0.7756	0.7077
1.2	0.9322	0.9225	0.9062	0.8826	0.8506	0.8090	0.7552	0.6863
1.4	0.9106	0.9006	0.8839	0.8598	0.8272	0.7849	0.7302	0.6602
1.6	0.8844	0.8740	0.8568	0.8321	0.7988	0.7557	0.7000	0.6288
1.8	0.8529	0.8421	0.8243	0.7989	0.7648	0.7207	0.6639	0.5913
2.0	0.8151	0.8038	0.7854	0.7592	0.7242	0.6789	0.6208	0.5465
2.5	0.7175	0.7049	0.6848	0.6566	0.6193	0.5710	0.5093	0.4304

This table is Table 22 reverted.

On account of nonpredictable inequalities, the tabular values should be divided by about 1.02.

TABLE 24.—Variation in lunital interval and mean semirange of tide, due to the phase wave composed of S_2 and μ_2

Time.		Increase in lunital interval due to S_2 .	Increase in mean semirange of tide due to S_2 and μ_2 . Length of half group.					
			0 tides.	4 tides.	6 tides.	8 tides.	10 tides.	
After spring tides.	<i>d. h.</i>	<i>m.</i>	S_2/M_2					
	0 00	0	"	+0.93 S_2	+0.90 S_2	+0.88 S_2	+0.84 S_2	+0.79 S_2
	0 06	— 9	"	+0.92 "	+0.90 "	+0.87 "	+0.83 "	+0.78 "
	0 12	— 18	"	+0.91 "	+0.89 "	+0.86 "	+0.82 "	+0.77 "
	0 18	— 27	"	+0.89 "	+0.87 "	+0.84 "	+0.81 "	+0.76 "
	1 00	— 35	"	+0.86 "	+0.84 "	+0.81 "	+0.78 "	+0.73 "
	1 06	— 44	"	+0.82 "	+0.80 "	+0.77 "	+0.74 "	+0.69 "
	1 12	— 52	"	+0.78 "	+0.76 "	+0.73 "	+0.70 "	+0.66 "
	1 18	— 61	"	+0.73 "	+0.71 "	+0.69 "	+0.65 "	+0.62 "
	2 00	— 69	"	+0.67 "	+0.65 "	+0.63 "	+0.59 "	+0.56 "
	2 06	— 77	"	+0.60 "	+0.58 "	+0.56 "	+0.53 "	+0.50 "
	2 12	— 84	"	+0.53 "	+0.51 "	+0.50 "	+0.47 "	+0.44 "
	2 18	— 92	"	+0.45 "	+0.44 "	+0.43 "	+0.40 "	+0.37 "
	3 00	— 98	"	+0.36 "	+0.35 "	+0.34 "	+0.32 "	+0.29 "
	3 06	— 105	"	+0.27 "	+0.26 "	+0.25 "	+0.23 "	+0.21 "
	3 12	— 111	"	+0.18 "	+0.18 "	+0.17 "	+0.15 "	+0.13 "
	3 18	— 116	"	+0.07 "	+0.07 "	+0.07 "	+0.05 "	+0.03 "
4 00	— 120	"	— 0.03 "	— 0.04 "	— 0.04 "	— 0.06 "	— 0.08 "	
Before neap tides.	4 00	— 108	"	+0.22 "	+0.20 "	+0.19 "	+0.19 "	+0.18 "
	3 18	— 113	"	+0.12 "	+0.10 "	+0.09 "	+0.08 "	+0.07 "
	3 12	— 118	"	+0.02 "	0.00 "	— 0.01 "	— 0.02 "	— 0.03 "
	3 06	— 122	"	— 0.09 "	— 0.10 "	— 0.11 "	— 0.12 "	— 0.13 "
	3 00	— 125	"	— 0.20 "	— 0.20 "	— 0.22 "	— 0.23 "	— 0.23 "
	2 18	— 127	"	— 0.31 "	— 0.30 "	— 0.32 "	— 0.32 "	— 0.32 "
	2 12	— 127	"	— 0.42 "	— 0.41 "	— 0.42 "	— 0.42 "	— 0.40 "
	2 06	— 126	"	— 0.53 "	— 0.52 "	— 0.52 "	— 0.51 "	— 0.49 "
	2 00	— 123	"	— 0.64 "	— 0.63 "	— 0.61 "	— 0.60 "	— 0.57 "
	1 18	— 117	"	— 0.74 "	— 0.73 "	— 0.70 "	— 0.68 "	— 0.64 "
	1 12	— 109	"	— 0.84 "	— 0.82 "	— 0.78 "	— 0.76 "	— 0.70 "
	1 06	— 99	"	— 0.93 "	— 0.90 "	— 0.86 "	— 0.83 "	— 0.75 "
	1 00	— 84	"	— 1.01 "	— 0.98 "	— 0.93 "	— 0.87 "	— 0.80 "
	0 18	— 67	"	— 1.08 "	— 1.05 "	— 0.99 "	— 0.93 "	— 0.85 "
	0 12	— 47	"	— 1.13 "	— 1.09 "	— 1.03 "	— 0.96 "	— 0.88 "
	0 06	— 24	"	— 1.16 "	— 1.11 "	— 1.05 "	— 0.97 "	— 0.89 "
	0 00	0	"	— 1.18 "	— 1.12 "	— 1.06 "	— 0.98 "	— 0.89 "
After neap tides.	0 00	0	"	— 1.18 "	— 1.12 "	— 1.06 "	— 0.98 "	— 0.89 "
	0 06	+ 24	"	— 1.16 "	— 1.11 "	— 1.05 "	— 0.97 "	— 0.89 "
	0 12	+ 47	"	— 1.13 "	— 1.09 "	— 1.03 "	— 0.96 "	— 0.88 "
	0 18	+ 67	"	— 1.08 "	— 1.05 "	— 0.99 "	— 0.93 "	— 0.85 "
	1 00	+ 84	"	— 1.01 "	— 0.98 "	— 0.93 "	— 0.87 "	— 0.80 "
	1 06	+ 99	"	— 0.93 "	— 0.90 "	— 0.86 "	— 0.83 "	— 0.75 "
	1 12	+ 109	"	— 0.84 "	— 0.82 "	— 0.78 "	— 0.76 "	— 0.70 "
	1 18	+ 117	"	— 0.74 "	— 0.73 "	— 0.70 "	— 0.68 "	— 0.64 "
	2 00	+ 123	"	— 0.64 "	— 0.63 "	— 0.61 "	— 0.59 "	— 0.56 "
	2 06	+ 126	"	— 0.53 "	— 0.52 "	— 0.52 "	— 0.50 "	— 0.48 "
	2 12	+ 127	"	— 0.42 "	— 0.41 "	— 0.42 "	— 0.42 "	— 0.40 "
	2 18	+ 127	"	— 0.31 "	— 0.30 "	— 0.32 "	— 0.32 "	— 0.31 "
	3 00	+ 125	"	— 0.20 "	— 0.20 "	— 0.21 "	— 0.22 "	— 0.22 "
	3 06	+ 122	"	— 0.09 "	— 0.09 "	— 0.11 "	— 0.11 "	— 0.11 "
	3 12	+ 118	"	+ 0.02 "	+ 0.01 "	+ 0.01 "	0.00 "	0.00 "
	3 18	+ 113	"	+ 0.12 "	+ 0.11 "	+ 0.11 "	+ 0.10 "	+ 0.10 "
	4 00	+ 108	"	+ 0.22 "	+ 0.21 "	+ 0.21 "	+ 0.20 "	+ 0.19 "
Before spring tides.	4 00	+ 120	"	— 0.03 "	— 0.04 "	— 0.05 "	— 0.06 "	— 0.07 "
	3 18	+ 116	"	+ 0.07 "	+ 0.07 "	+ 0.06 "	+ 0.04 "	+ 0.03 "
	3 12	+ 111	"	+ 0.18 "	+ 0.18 "	+ 0.17 "	+ 0.15 "	+ 0.13 "
	3 06	+ 105	"	+ 0.27 "	+ 0.27 "	+ 0.26 "	+ 0.24 "	+ 0.22 "
	3 00	+ 98	"	+ 0.36 "	+ 0.36 "	+ 0.35 "	+ 0.32 "	+ 0.30 "
	2 18	+ 92	"	+ 0.45 "	+ 0.45 "	+ 0.43 "	+ 0.40 "	+ 0.38 "
	2 12	+ 84	"	+ 0.53 "	+ 0.52 "	+ 0.50 "	+ 0.47 "	+ 0.45 "
	2 06	+ 77	"	+ 0.60 "	+ 0.58 "	+ 0.56 "	+ 0.53 "	+ 0.50 "
	2 00	+ 69	"	+ 0.67 "	+ 0.65 "	+ 0.63 "	+ 0.60 "	+ 0.56 "
	1 18	+ 61	"	+ 0.73 "	+ 0.71 "	+ 0.69 "	+ 0.66 "	+ 0.62 "
	1 12	+ 52	"	+ 0.78 "	+ 0.76 "	+ 0.73 "	+ 0.71 "	+ 0.66 "
	1 06	+ 44	"	+ 0.82 "	+ 0.80 "	+ 0.77 "	+ 0.74 "	+ 0.69 "
	1 00	+ 35	"	+ 0.86 "	+ 0.84 "	+ 0.81 "	+ 0.78 "	+ 0.73 "
	0 18	+ 27	"	+ 0.89 "	+ 0.87 "	+ 0.84 "	+ 0.81 "	+ 0.76 "
	0 12	+ 18	"	+ 0.91 "	+ 0.89 "	+ 0.86 "	+ 0.82 "	+ 0.77 "
	0 06	+ 9	"	+ 0.92 "	+ 0.90 "	+ 0.87 "	+ 0.83 "	+ 0.78 "
	0 00	0	"	+ 0.93 "	+ 0.90 "	+ 0.88 "	+ 0.84 "	+ 0.79 "

In clearing tides of phase or semimenstrual height inequality, apply the tabulated values as they stand to the low waters, but alter their signs for the high waters.

Spring and neap tides occur $\frac{S_2^\circ - M_2^\circ}{1.0159}$ hours after syzygy and quadrature.

This table is based upon Tables 1, 15, 16, and 21.

TABLE 25.—Variation in mean semirange of tide due to the parallax wave composed of N_1 , L_2 , and $2N_2$.

Time.		Increase in mean semirange of tide due to N_1 , L_2 , and $2N_2$. Length of half group.				Time.		Increase in mean semirange of tide due to N_1 , L_2 , and $2N_2$. Length of half group.			
		0 tides.	4 tides.	8 tides.	12 tides.			0 tides.	4 tides.	8 tides.	12 tides.
After perigean tides.	d. h.					d. h.					
	0 00	+0.94 N_2	+0.93 N_2	+0.90 N_2	+0.85 N_2	0 00	-0.77 N_2	-0.77 N_2	-0.76 N_2	-0.73 N_2	
	0 06	+0.94 "	+0.93 "	+0.90 "	+0.85 "	0 06	-0.77 "	-0.77 "	-0.76 "	-0.73 "	
	0 12	+0.93 "	+0.93 "	+0.89 "	+0.84 "	0 12	-0.77 "	-0.77 "	-0.76 "	-0.73 "	
	0 18	+0.92 "	+0.91 "	+0.88 "	+0.83 "	0 18	-0.76 "	-0.76 "	-0.75 "	-0.73 "	
	1 00	+0.91 "	+0.91 "	+0.87 "	+0.82 "	1 00	-0.75 "	-0.75 "	-0.74 "	-0.72 "	
	1 06	+0.89 "	+0.89 "	+0.85 "	+0.80 "	1 06	-0.75 "	-0.75 "	-0.74 "	-0.72 "	
	1 12	+0.87 "	+0.87 "	+0.84 "	+0.79 "	1 12	-0.74 "	-0.74 "	-0.73 "	-0.71 "	
	1 18	+0.85 "	+0.85 "	+0.82 "	+0.77 "	1 18	-0.73 "	-0.72 "	-0.72 "	-0.70 "	
	2 00	+0.82 "	+0.82 "	+0.80 "	+0.76 "	2 00	-0.71 "	-0.70 "	-0.70 "	-0.67 "	
	2 06	+0.79 "	+0.79 "	+0.77 "	+0.73 "	2 06	-0.69 "	-0.69 "	-0.68 "	-0.65 "	
	2 12	+0.75 "	+0.75 "	+0.74 "	+0.70 "	2 12	-0.68 "	-0.68 "	-0.65 "	-0.64 "	
Before midtime tides.	2 18	+0.72 "	+0.72 "	+0.72 "	+0.67 "	2 18	-0.66 "	-0.66 "	-0.64 "	-0.63 "	
	3 00	+0.68 "	+0.68 "	+0.67 "	+0.64 "	3 00	-0.63 "	-0.63 "	-0.62 "	-0.60 "	
	3 06	+0.64 "	+0.64 "	+0.63 "	+0.61 "	3 06	-0.61 "	-0.61 "	-0.60 "	-0.57 "	
	3 12	+0.60 "	+0.60 "	+0.59 "	+0.57 "	3 12	-0.58 "	-0.58 "	-0.57 "	-0.54 "	
	3 12	+0.61 "	+0.61 "	+0.59 "	+0.57 "	3 12	-0.60 "	-0.60 "	-0.58 "	-0.56 "	
	3 06	+0.57 "	+0.57 "	+0.55 "	+0.53 "	3 06	-0.57 "	-0.57 "	-0.55 "	-0.53 "	
	3 00	+0.52 "	+0.51 "	+0.50 "	+0.49 "	3 00	-0.55 "	-0.54 "	-0.52 "	-0.50 "	
	2 18	+0.48 "	+0.47 "	+0.45 "	+0.45 "	2 18	-0.51 "	-0.51 "	-0.50 "	-0.46 "	
	2 12	+0.43 "	+0.43 "	+0.40 "	+0.40 "	2 12	-0.47 "	-0.47 "	-0.46 "	-0.43 "	
	2 06	+0.38 "	+0.38 "	+0.36 "	+0.36 "	2 06	-0.45 "	-0.45 "	-0.44 "	-0.41 "	
	2 00	+0.33 "	+0.33 "	+0.31 "	+0.31 "	2 00	-0.41 "	-0.41 "	-0.40 "	-0.37 "	
	1 18	+0.28 "	+0.28 "	+0.27 "	+0.28 "	1 18	-0.37 "	-0.37 "	-0.35 "	-0.32 "	
After midtime tides.	1 12	+0.23 "	+0.23 "	+0.23 "	+0.24 "	1 12	-0.33 "	-0.33 "	-0.32 "	-0.29 "	
	1 06	+0.18 "	+0.18 "	+0.19 "	+0.20 "	1 06	-0.29 "	-0.29 "	-0.27 "	-0.24 "	
	1 00	+0.12 "	+0.12 "	+0.13 "	+0.14 "	1 00	-0.26 "	-0.26 "	-0.24 "	-0.21 "	
	0 18	+0.08 "	+0.08 "	+0.09 "	+0.10 "	0 18	-0.21 "	-0.21 "	-0.19 "	-0.16 "	
	0 12	+0.02 "	+0.02 "	+0.04 "	+0.04 "	0 12	-0.17 "	-0.17 "	-0.15 "	-0.13 "	
	0 06	-0.02 "	-0.02 "	0.00 "	+0.01 "	0 06	-0.12 "	-0.12 "	-0.10 "	-0.09 "	
	0 00	-0.07 "	-0.07 "	-0.06 "	-0.04 "	0 00	-0.07 "	-0.07 "	-0.06 "	-0.04 "	
	0 00	-0.07 "	-0.07 "	-0.06 "	-0.04 "	0 00	-0.07 "	-0.07 "	-0.06 "	-0.04 "	
	0 06	-0.12 "	-0.12 "	-0.10 "	-0.09 "	0 06	-0.02 "	-0.02 "	0.00 "	+0.01 "	
	0 12	-0.17 "	-0.17 "	-0.15 "	-0.13 "	0 12	+0.02 "	+0.02 "	+0.04 "	+0.04 "	
	0 18	-0.21 "	-0.21 "	-0.19 "	-0.16 "	0 18	+0.08 "	+0.08 "	+0.09 "	+0.10 "	
	1 00	-0.26 "	-0.26 "	-0.24 "	-0.21 "	1 00	+0.12 "	+0.12 "	+0.13 "	+0.14 "	
Before apogean tides.	1 06	-0.29 "	-0.29 "	-0.27 "	-0.24 "	1 06	+0.18 "	+0.18 "	+0.19 "	+0.20 "	
	1 12	-0.33 "	-0.33 "	-0.32 "	-0.29 "	1 12	+0.23 "	+0.23 "	+0.23 "	+0.24 "	
	1 18	-0.37 "	-0.37 "	-0.35 "	-0.32 "	1 18	+0.28 "	+0.28 "	+0.27 "	+0.28 "	
	2 00	-0.41 "	-0.41 "	-0.40 "	-0.37 "	2 00	+0.33 "	+0.33 "	+0.31 "	+0.31 "	
	2 06	-0.45 "	-0.45 "	-0.44 "	-0.41 "	2 06	+0.38 "	+0.38 "	+0.36 "	+0.36 "	
	2 12	-0.47 "	-0.47 "	-0.46 "	-0.43 "	2 12	+0.43 "	+0.43 "	+0.40 "	+0.40 "	
	2 18	-0.51 "	-0.51 "	-0.50 "	-0.46 "	2 18	+0.48 "	+0.47 "	+0.45 "	+0.45 "	
	3 00	-0.55 "	-0.54 "	-0.52 "	-0.50 "	3 00	+0.52 "	+0.51 "	+0.50 "	+0.49 "	
	3 06	-0.57 "	-0.57 "	-0.55 "	-0.53 "	3 06	+0.57 "	+0.57 "	+0.55 "	+0.53 "	
	3 12	-0.60 "	-0.60 "	-0.58 "	-0.56 "	3 12	+0.61 "	+0.61 "	+0.59 "	+0.57 "	
	3 12	-0.58 "	-0.58 "	-0.57 "	-0.54 "	3 12	+0.60 "	+0.60 "	+0.59 "	+0.57 "	
	3 06	-0.61 "	-0.61 "	-0.60 "	-0.57 "	3 06	+0.64 "	+0.64 "	+0.63 "	+0.61 "	
After apogean tides.	3 00	-0.63 "	-0.63 "	-0.62 "	-0.60 "	3 00	+0.68 "	+0.68 "	+0.67 "	+0.64 "	
	2 18	-0.66 "	-0.66 "	-0.64 "	-0.63 "	2 18	+0.72 "	+0.72 "	+0.72 "	+0.67 "	
	2 12	-0.68 "	-0.68 "	-0.65 "	-0.64 "	2 12	+0.75 "	+0.75 "	+0.74 "	+0.70 "	
	2 06	-0.69 "	-0.69 "	-0.67 "	-0.65 "	2 06	+0.79 "	+0.79 "	+0.77 "	+0.73 "	
	2 00	-0.71 "	-0.70 "	-0.68 "	-0.67 "	2 00	+0.82 "	+0.82 "	+0.80 "	+0.76 "	
	1 18	-0.73 "	-0.72 "	-0.72 "	-0.70 "	1 18	+0.85 "	+0.85 "	+0.82 "	+0.77 "	
	1 12	-0.74 "	-0.74 "	-0.73 "	-0.71 "	1 12	+0.87 "	+0.87 "	+0.84 "	+0.79 "	
	1 06	-0.75 "	-0.75 "	-0.74 "	-0.72 "	1 06	+0.89 "	+0.89 "	+0.85 "	+0.80 "	
	1 00	-0.75 "	-0.75 "	-0.74 "	-0.72 "	1 00	+0.91 "	+0.91 "	+0.87 "	+0.82 "	
	0 18	-0.76 "	-0.76 "	-0.75 "	-0.73 "	0 18	+0.92 "	+0.91 "	+0.88 "	+0.83 "	
	0 12	-0.77 "	-0.77 "	-0.76 "	-0.73 "	0 12	+0.93 "	+0.93 "	+0.89 "	+0.84 "	
	0 06	-0.77 "	-0.77 "	-0.76 "	-0.73 "	0 06	+0.94 "	+0.93 "	+0.90 "	+0.85 "	
	0 00	-0.77 "	-0.77 "	-0.76 "	-0.73 "	0 00	+0.94 "	+0.93 "	+0.90 "	+0.85 "	

In clearing tides of this inequality, apply the tabular values as they stand to the low waters, but alter their signs for the high waters.

Perigean tides, apogean tides, etc., occur $\frac{M_2^\circ - N_2^\circ}{0.5444}$ hours after perigee, apogee, etc.

This table is based upon Tables 1, 16, and 21.

TABLE 26.—Effect of Q_1 upon the amplitude of O_1 .

Time.	Resultant amp. O_1 and Q_1 , $O_1 = 1$.	Time.	Resultant amp. O_1 and Q_1 , $O_1 = 1$.
After perigee. $\left. \begin{matrix} d. \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right\}$	$\left. \begin{matrix} 1'19 \\ 1'19 \\ 1'18 \\ 1'16 \end{matrix} \right\}$	After apogee. $\left. \begin{matrix} d. \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right\}$	$\left. \begin{matrix} 0'81 \\ 0'81 \\ 0'83 \\ 0'86 \end{matrix} \right\}$
Before midtime. $\left. \begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right\}$	$\left. \begin{matrix} 1'13 \\ 1'10 \\ 1'06 \\ 1'02 \end{matrix} \right\}$	Before midtime. $\left. \begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right\}$	$\left. \begin{matrix} 0'89 \\ 0'93 \\ 0'98 \\ 0'02 \end{matrix} \right\}$
After midtime. $\left. \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right\}$	$\left. \begin{matrix} 1'02 \\ 0'98 \\ 0'93 \\ 0'89 \end{matrix} \right\}$	After midtime. $\left. \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right\}$	$\left. \begin{matrix} 1'02 \\ 1'06 \\ 1'10 \\ 1'13 \end{matrix} \right\}$
Before apogee. $\left. \begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right\}$	$\left. \begin{matrix} 0'86 \\ 0'83 \\ 0'81 \\ 0'81 \end{matrix} \right\}$	Before perigee. $\left. \begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right\}$	$\left. \begin{matrix} 1'16 \\ 1'18 \\ 1'19 \\ 1'19 \end{matrix} \right\}$

This table is based upon Tables 1 and 16.

TABLE 27.—Perturbations in K_1 due to O_1 .

Time.	Increase in K_1 tidal interval due to O_1 .	Increase in semi- range of K_1 tide due to O_1 .	Time.	Increase in K_1 tidal interval due to O_1 .	Increase in semi- range of K_1 tide due to O_1 .
After maximum declinational tides. $\left. \begin{matrix} d. & h. \\ 0 & 00 \\ 0 & 06 \\ 0 & 12 \\ 0 & 18 \\ 1 & 00 \\ 1 & 06 \\ 1 & 12 \\ 1 & 18 \\ 2 & 00 \\ 2 & 06 \\ 2 & 12 \\ 2 & 18 \\ 3 & 00 \\ 3 & 06 \\ 3 & 12 \end{matrix} \right\}$	$\left. \begin{matrix} h. & m. \\ +0 & 00 \\ +0 & 14 \\ +0 & 28 \\ +0 & 43 \\ +0 & 57 \\ +1 & 11 \\ +1 & 25 \\ +1 & 39 \\ +1 & 53 \\ +2 & 06 \\ +2 & 19 \\ +2 & 32 \\ +2 & 45 \\ +2 & 57 \\ +3 & 09 \end{matrix} \right\} O_1/K_1$	$\left. \begin{matrix} +1'00 \\ +0'99 \\ +0'98 \\ +0'96 \\ +0'94 \\ +0'90 \\ +0'86 \\ +0'81 \\ +0'75 \\ +0'69 \\ +0'62 \\ +0'55 \\ +0'46 \\ +0'37 \\ +0'28 \end{matrix} \right\} O_1$	After minimum declinational tides. $\left. \begin{matrix} d. & h. \\ 0 & 00 \\ 0 & 06 \\ 0 & 12 \\ 0 & 18 \\ 1 & 00 \\ 1 & 06 \\ 1 & 12 \\ 1 & 18 \\ 2 & 00 \\ 2 & 06 \\ 2 & 12 \\ 2 & 18 \\ 3 & 00 \\ 3 & 06 \\ 3 & 12 \end{matrix} \right\}$	$\left. \begin{matrix} h. & m. \\ 0 & 00 \\ -1 & 14 \\ -2 & 16 \\ -3 & 03 \\ -3 & 27 \\ -3 & 44 \\ -3 & 53 \\ -3 & 54 \\ -3 & 52 \\ -3 & 48 \\ -3 & 40 \\ -3 & 32 \\ -3 & 23 \\ -3 & 12 \\ -3 & 01 \end{matrix} \right\} O_1/K_1$	$\left. \begin{matrix} -1'00 \\ -0'97 \\ -0'91 \\ -0'83 \\ -0'73 \\ -0'63 \\ -0'51 \\ -0'40 \\ -0'28 \\ -0'17 \\ -0'06 \\ +0'05 \\ +0'15 \\ +0'25 \\ +0'35 \end{matrix} \right\} O_1$
Before minimum declinational tides. $\left. \begin{matrix} 3 & 12 \\ 3 & 06 \\ 3 & 00 \\ 2 & 18 \\ 2 & 12 \\ 2 & 06 \\ 2 & 00 \\ 1 & 18 \\ 1 & 12 \\ 1 & 06 \\ 1 & 00 \\ 0 & 18 \\ 0 & 12 \\ 0 & 06 \\ 0 & 00 \end{matrix} \right\}$	$\left. \begin{matrix} +3 & 01 \\ +3 & 12 \\ +3 & 23 \\ +3 & 32 \\ +3 & 40 \\ +3 & 48 \\ +3 & 52 \\ +3 & 54 \\ +3 & 53 \\ +3 & 44 \\ +3 & 27 \\ +3 & 03 \\ +2 & 16 \\ +1 & 14 \\ +0 & 00 \end{matrix} \right\}$	$\left. \begin{matrix} +0'35 \\ +0'25 \\ +0'15 \\ +0'05 \\ -0'06 \\ -0'17 \\ -0'28 \\ -0'40 \\ -0'51 \\ -0'63 \\ -0'73 \\ -0'83 \\ -0'91 \\ -0'97 \\ -1'00 \end{matrix} \right\}$	Before maximum declinational tides. $\left. \begin{matrix} 3 & 12 \\ 3 & 06 \\ 3 & 00 \\ 2 & 18 \\ 2 & 12 \\ 2 & 06 \\ 2 & 00 \\ 1 & 18 \\ 1 & 12 \\ 1 & 06 \\ 1 & 00 \\ 0 & 18 \\ 0 & 12 \\ 0 & 06 \\ 0 & 00 \end{matrix} \right\}$	$\left. \begin{matrix} -3 & 09 \\ -2 & 57 \\ -2 & 45 \\ -2 & 32 \\ -2 & 19 \\ -2 & 06 \\ -1 & 53 \\ -1 & 39 \\ -1 & 25 \\ -1 & 11 \\ -0 & 57 \\ -0 & 43 \\ -0 & 28 \\ -0 & 14 \\ -0 & 00 \end{matrix} \right\}$	$\left. \begin{matrix} +0'28 \\ +0'37 \\ +0'46 \\ +0'55 \\ +0'62 \\ +0'69 \\ +0'75 \\ +0'81 \\ +0'86 \\ +0'90 \\ +0'94 \\ +0'96 \\ +0'98 \\ +0'99 \\ +1'00 \end{matrix} \right\}$

Maximum and minimum declinational tides occur $\frac{K_1^0 - O_1^0}{1'0980}$ hours after extreme and zero declination, provided K_1^0 is decreased by the acceleration in K_1 due to P_1 , Table 31. See § 67.

This table is based upon Tables 1, 15, and 16.

TABLE 28.—The speed which corresponds to a period of given length.

Length of half period.			Speed.		Time equal to one degree.
			Per hour.	Per minute.	
<i>h.</i>	<i>m.</i>	<i>m.</i>	<i>°</i>	<i>°</i>	<i>m.</i>
10	00	600	18°000	0°3000	3°333
	10	610	17°705	0°2951	3°389
	20	620	17°419	0°2903	3°445
	30	630	17°143	0°2857	3°500
	40	640	16°875	0°2813	3°556
	50	650	16°615	0°2769	3°611
11	00	660	16°364	0°2727	3°667
	10	670	16°119	0°2687	3°722
	20	680	15°882	0°2647	3°778
	30	690	15°652	0°2609	3°833
	40	700	15°429	0°2571	3°889
	50	710	15°211	0°2535	3°945
12	00	720	15°000	0°2500	4°000
	10	730	14°795	0°2466	4°055
	20	740	14°595	0°2433	4°111
	30	750	14°400	0°2400	4°167
	40	760	14°211	0°2369	4°222
	50	770	14°026	0°2338	4°278
13	00	780	13°846	0°2308	4°333
	10	790	13°671	0°2279	4°389
	20	800	13°500	0°2250	4°444
	30	810	13°333	0°2222	4°500
	40	820	13°171	0°2195	4°555
	50	830	13°012	0°2169	4°611
14	00	840	12°857	0°2143	4°667

TABLE 29.—The sun's mean longitude at Greenwich mean noon.

Day.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	280°6	311°2	338°7	9°3	38°9	69°4	99°0	129°6	160°1	189°7	220°2	249°8
2	281°6	312°1	339°7	10°3	39°9	70°4	100°0	130°6	161°1	190°7	221°2	250°8
3	282°6	313°1	340°7	11°3	40°9	71°4	101°0	131°5	162°1	191°7	222°2	251°8
4	283°6	314°1	341°7	12°3	41°8	72°4	102°0	132°5	163°1	192°6	223°2	252°8
5	284°5	315°1	342°7	13°3	42°8	73°4	102°9	133°5	164°1	193°6	224°2	253°7
6	285°5	316°1	343°7	14°2	43°8	74°4	103°9	134°5	165°0	194°6	225°2	254°7
7	286°5	317°1	344°7	15°2	44°8	75°3	104°9	135°5	166°0	195°6	226°1	255°7
8	287°5	318°0	345°7	16°2	45°8	76°3	105°9	136°5	167°0	196°6	227°1	256°7
9	288°5	319°0	346°6	17°2	46°8	77°3	106°9	137°4	168°0	197°6	228°1	257°7
10	289°5	320°0	347°6	18°2	47°8	78°3	107°9	138°4	169°0	198°5	229°1	258°7
11	290°5	321°0	348°6	19°2	48°7	79°3	108°9	139°4	170°0	199°5	230°1	259°7
12	291°4	322°0	349°6	20°2	49°7	80°3	109°8	140°4	170°9	200°5	231°1	260°7
13	292°4	323°0	350°6	21°1	50°7	81°3	110°8	141°4	171°9	201°5	232°1	261°6
14	293°4	324°0	351°6	22°1	51°7	82°2	111°8	142°4	172°9	202°5	233°1	262°6
15	294°4	324°9	352°5	23°1	52°7	83°2	112°8	143°4	173°9	203°5	234°0	263°6
16	295°4	325°9	353°5	24°1	53°7	84°2	113°8	144°3	174°9	204°5	235°0	264°6
17	296°4	326°9	354°5	25°1	54°7	85°2	114°8	145°3	175°9	205°5	236°0	265°6
18	297°4	327°9	355°5	26°1	55°7	86°2	115°8	146°3	176°9	206°4	237°0	266°6
19	298°3	328°9	356°5	27°1	56°6	87°2	116°7	147°3	177°9	207°4	238°0	267°5
20	299°3	329°9	357°5	28°0	57°6	88°2	117°7	148°3	178°8	208°4	239°0	268°5
21	300°3	330°9	358°5	29°0	58°6	89°1	118°7	149°3	179°8	209°4	240°0	269°5
22	301°3	331°8	359°4	30°0	59°6	90°1	119°7	150°3	180°8	210°4	240°9	270°5
23	302°3	332°8	0°4	31°0	60°6	91°1	120°7	151°2	181°8	211°4	241°9	271°5
24	303°3	333°8	1°4	32°0	61°6	92°1	121°7	152°2	182°8	212°3	242°9	272°5
25	304°3	334°8	2°4	33°0	62°5	93°1	122°7	153°2	183°8	213°3	243°9	273°5
26	305°2	335°8	3°4	33°9	63°5	94°1	123°6	154°2	184°7	214°3	244°9	274°5
27	306°2	336°8	4°4	34°9	64°5	95°1	124°6	155°2	185°7	215°3	245°9	275°4
28	307°2	337°8	5°4	35°9	65°5	96°0	125°6	156°2	186°7	216°3	246°9	276°4
29	308°2	338°7	6°3	36°9	66°5	97°0	126°6	157°2	187°7	217°3	247°8	277°4
30	309°2		7°3	37°9	67°5	98°0	127°6	158°1	188°7	218°3	248°8	278°4
31	310°2		8°3		68°5		128°6	159°1		219°3		279°4

This table assumes that the value for January 1 is 280°6.

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TABLE 30.—*Approximate equation of time at Greenwich mean noon.*

[To change apparent to mean time.]

Day.	Jan.	Feb.	Mar.	Apr.	May.	June.	July	Aug.	Sept.	Oct.	Nov.	Dec.
	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>	<i>m.</i>
1	+ 3'8	+13'8	+12'5	+3'9	-3'0	-2'4	+3'5	+6'1	0'0	-10'3	-16'3	-10'8
2	+ 4'2	+13'9	+12'3	+3'6	-3'1	-2'3	+3'7	+6'0	- 0'4	-10'6	-16'3	-10'5
3	+ 4'7	+14'0	+12'1	+3'3	-3'2	-2'1	+3'9	+6'0	- 0'7	-10'9	-16'3	-10'0
4	+ 5'2	+14'1	+11'9	+3'0	-3'3	-2'0	+4'1	+5'9	- 1'1	-11'2	-16'3	- 9'7
5	+ 5'6	+14'2	+11'7	+2'8	-3'4	-1'8	+4'3	+5'8	- 1'3	-11'5	-16'3	- 9'3
6	+ 6'1	+14'3	+11'4	+2'5	-3'5	-1'6	+4'5	+5'7	- 1'7	-11'8	-16'3	- 8'8
7	+ 6'5	+14'3	+11'2	+2'2	-3'6	-1'4	+4'6	+5'6	- 2'0	-12'1	-16'2	- 8'4
8	+ 6'9	+14'4	+11'0	+1'9	-3'7	-1'3	+4'8	+5'5	- 2'4	-12'4	-16'2	- 8'0
9	+ 7'3	+14'4	+10'7	+1'6	-3'7	-1'0	+4'9	+5'3	- 2'7	-12'7	-16'0	- 7'5
10	+ 7'7	+14'4	+10'4	+1'3	-3'8	-0'9	+5'0	+5'2	- 3'1	-12'9	-16'0	- 7'1
11	+ 8'1	+14'4	+10'2	+1'0	-3'8	-0'7	+5'2	+5'1	- 3'4	-13'2	-15'9	- 6'6
12	+ 8'5	+14'4	+ 9'9	+0'8	-3'8	-0'5	+5'3	+4'9	- 3'7	-13'5	-15'7	- 6'1
13	+ 8'9	+14'4	+ 9'6	+0'5	-3'9	-0'3	+5'5	+4'7	- 4'1	-13'7	-15'6	- 5'7
14	+ 9'3	+14'4	+ 9'3	+0'3	-3'9	-0'1	+5'6	+4'5	- 4'4	-13'9	-15'3	- 5'2
15	+ 9'6	+14'3	+ 9'1	+0'1	-3'9	+0'2	+5'7	+4'3	- 4'8	-14'1	-15'3	- 4'7
16	+10'0	+14'3	+ 8'7	-0'2	-3'9	+0'4	+5'8	+4'1	- 5'2	-14'4	-15'1	- 4'2
17	+10'3	+14'2	+ 8'5	-0'5	-3'8	+0'6	+5'9	+3'9	- 5'5	-14'6	-14'9	- 3'7
18	+10'6	+14'1	+ 8'2	-0'7	-3'8	+0'8	+6'0	+3'7	- 5'9	-14'8	-14'7	- 3'2
19	+10'9	+14'0	+ 7'9	-0'9	-3'8	+1'0	+6'1	+3'5	- 6'2	-14'9	-14'5	- 2'7
20	+11'2	+13'9	+ 7'6	-1'1	-3'7	+1'2	+6'1	+3'3	- 6'6	-15'1	-14'3	- 2'2
21	+11'5	+13'8	+ 7'3	-1'3	-3'6	+1'5	+6'2	+3'0	- 6'9	-15'3	-14'0	- 1'7
22	+11'8	+13'7	+ 7'0	-1'5	-3'6	+1'7	+6'2	+2'8	- 7'3	-15'4	-13'7	- 1'2
23	+12'1	+13'5	+ 6'7	-1'7	-3'5	+1'9	+6'2	+2'5	- 7'6	-15'6	-13'5	- 0'7
24	+12'3	+13'4	+ 6'4	-1'9	-3'4	+2'1	+6'3	+2'3	- 8'0	-15'7	-13'2	- 0'2
25	+12'5	+13'2	+ 6'0	-2'1	-3'3	+2'3	+6'3	+2'0	- 8'3	-15'8	-12'9	+ 0'3
26	+12'8	+13'0	+ 5'8	-2'3	-3'2	+2'5	+6'3	+1'7	- 8'6	-15'9	-12'6	+ 0'7
27	+13'0	+12'9	+ 5'5	-2'4	-3'1	+2'7	+6'3	+1'4	- 9'0	-16'0	-12'2	+ 1'2
28	+13'2	+12'7	+ 5'2	-2'6	-3'0	+3'0	+6'3	+1'2	- 9'3	-16'1	-11'9	+ 1'7
29	+13'3	+12'5	+ 4'9	-2'7	-2'9	+3'2	+6'3	+0'9	- 9'6	-16'2	-11'6	+ 2'2
30	+13'5		+ 4'6	-2'9	-2'7	+3'4	+6'2	+0'6	-10'0	-16'2	-11'2	+ 2'7
31	+13'7		+ 4'3		-2'6		+6'2	+0'3		-16'3		+ 3'2

Sun's mean longitude
15 = right ascension of mean sun = sidereal time of mean noon.

Sun's right ascension = right ascension of mean sun + equation of time.

TABLE 31.—*Perturbations in K_1 , S_2 , due to the components P_1 , K_2 , and T_2 .*

Date. Greenwich mean noon.	Acceleration				Resultant amplitude.			
	In K_1 due to P_1 .	In S_2 due to K_2 .	In S_2 due to solar K_2 .	In S_2 due to T_2 .	K_1 and P_1 $K_1=1$.*	S_2 and K_2 $S_2=1$.	S_2 and solar K_2 $S_2=1$.	S_2 and T_2 $S_2=1$.
	°	°	°	c				
Jan. 1	— 5.2	— 7.6	— 2.4	+ 0.1	1.314	.76	0.92	1.06
11	— 9.9	— 12.5	— 3.9	— 0.5	1.267	.82	0.94	1.06
21	— 13.8	— 15.2	— 4.8	— 1.0	1.197	.90	0.96	1.06
31	— 17.0	— 15.6	— 4.9	— 1.5	1.105	.99	0.99	1.05
Feb. 10	— 18.8	— 14.1	— 4.5	— 1.9	0.993	1.08	1.02	1.05
20	— 18.7	— 11.6	— 3.7	— 2.3	0.885	1.16	1.05	1.04
Mar. 2	— 15.8	— 8.2	— 2.6	— 2.7	0.782	1.22	1.07	1.03
12	— 9.4	— 4.4	— 1.4	— 3.0	0.706	1.25	1.08	1.02
22	— 0.5	— 0.3	— 0.1	— 3.2	0.676	1.27	1.09	1.01
Apr. 1	+ 8.5	+ 3.9	+ 1.2	— 3.4	0.701	1.26	1.08	1.00
11	+ 15.4	+ 7.8	+ 2.5	— 3.4	0.772	1.23	1.07	0.99
21	+ 18.6	+ 11.2	+ 3.6	— 3.3	0.873	1.16	1.04	0.98
May 1	+ 18.9	+ 14.0	+ 4.4	— 3.1	0.985	1.09	1.02	0.97
11	+ 17.3	+ 15.4	+ 4.9	— 2.9	1.092	1.00	0.99	0.96
21	+ 14.2	+ 15.4	+ 4.9	— 2.5	1.188	.91	0.96	0.96
31	+ 10.2	+ 13.0	+ 4.1	— 2.0	1.261	.82	0.94	0.95
June 10	+ 5.7	+ 8.2	+ 2.6	— 1.4	1.309	.76	0.92	0.95
20	+ 0.9	+ 1.3	+ 0.4	— 0.8	1.329	.73	0.91	0.94
30	— 4.0	— 5.6	— 1.8	— 0.3	1.321	.74	0.92	0.94
July 10	— 8.7	— 11.5	— 3.7	+ 0.4	1.282	.80	0.93	0.94
20	— 12.8	— 14.6	— 4.6	+ 1.0	1.217	.88	0.95	0.94
30	— 16.3	— 15.6	— 5.0	+ 1.6	1.131	.97	0.98	0.95
Aug. 9	— 18.6	— 14.6	— 4.7	+ 2.1	1.028	1.06	1.01	0.95
19	— 19.1	— 12.3	— 3.9	+ 2.6	0.913	1.14	1.04	0.96
29	— 17.0	— 9.2	— 2.9	+ 2.9	0.807	1.21	1.06	0.97
Sept. 8	— 11.7	— 5.5	— 1.8	+ 3.2	0.721	1.25	1.08	0.98
18	— 3.1	— 1.3	— 0.4	+ 3.3	0.677	1.27	1.08	0.99
28	+ 6.4	+ 2.8	+ 0.9	+ 3.4	0.687	1.26	1.08	1.00
Oct. 8	+ 14.0	+ 6.8	+ 2.2	+ 3.3	0.748	1.23	1.08	1.01
18	+ 18.2	+ 10.4	+ 3.3	+ 3.2	0.844	1.18	1.06	1.02
28	+ 19.2	+ 13.3	+ 4.2	+ 3.0	0.956	1.11	1.03	1.03
Nov. 7	+ 18.0	+ 15.2	+ 4.8	+ 2.7	1.065	1.03	1.00	1.03
17	+ 15.1	+ 15.4	+ 4.9	+ 2.3	1.165	.93	0.97	1.04
27	+ 11.3	+ 13.7	+ 4.4	+ 1.9	1.244	.84	0.95	1.05
Dec. 7	+ 7.0	+ 9.7	+ 3.1	+ 1.4	1.299	.78	0.93	1.05
17	+ 2.2	+ 3.3	+ 1.0	+ 0.7	1.327	.73	0.92	1.05
27	— 2.7	— 4.0	— 1.2	+ 0.3	1.325	.74	0.92	1.06
Jan. 6	— 7.4	— 9.9	— 3.3	— 0.2	1.295	.79	0.92	1.06
Modifica- tion of tabular value for long. of moon's node.	Tabular value \times $F(K_1)$.	Tabular value \times $f(K_2)$.	Tabular value.	Tabular value.	(Tab. — 1) $\times F(K_1)$ + 1 = ϵ_1 or ϵ_{11} .	(Tab. — 1) $\times f(K_2)$ + 1.	Tabular value.	Tabular value.

* i. e. K_1' or the K_1 for the given year.For the acceleration in S_2 due to lunar K_2 , use the tabular value multiplied by $[f(K_2) - 0.317]$.

This table is based upon Tables 1, 15, 16, and 29.

TABLE 32.—Factor F_1 for clearing D_1 of the effects of the longitude of the moon's node and of P_1 .

Date.	I , or inclination of orbit to equator.											
	18½°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	28½°
Jan. 1	0.963	0.949	0.923	0.898	0.874	0.853	0.833	0.814	0.796	0.779	0.764	0.757
11	0.988	0.974	0.946	0.919	0.895	0.872	0.851	0.831	0.813	0.795	0.779	0.772
21	1.030	1.014	0.983	0.955	0.929	0.905	0.882	0.861	0.841	0.822	0.805	0.797
30	1.091	1.073	1.038	1.008	0.978	0.951	0.926	0.903	0.881	0.861	0.842	0.833
Feb. 10	1.172	1.151	1.112	1.077	1.043	1.012	0.984	0.958	0.933	0.911	0.889	0.879
20	1.270	1.245	1.200	1.158	1.119	1.084	1.052	1.022	0.994	0.968	0.945	0.933
Mar. 2	1.373	1.345	1.293	1.245	1.200	1.159	1.122	1.088	1.057	1.028	1.001	0.989
12	1.463	1.431	1.371	1.317	1.268	1.223	1.182	1.144	1.109	1.077	1.047	1.034
22	1.499	1.466	1.404	1.347	1.296	1.248	1.205	1.166	1.130	1.097	1.067	1.052
Apr. 1	1.468	1.436	1.376	1.321	1.272	1.226	1.185	1.147	1.112	1.080	1.050	1.037
11	1.384	1.356	1.302	1.253	1.208	1.167	1.130	1.095	1.063	1.034	1.007	0.994
21	1.280	1.256	1.209	1.167	1.128	1.093	1.059	1.029	1.000	0.974	0.951	0.939
May 1	1.181	1.161	1.121	1.084	1.051	1.020	0.991	0.964	0.940	0.916	0.895	0.885
11	1.100	1.082	1.048	1.015	0.986	0.959	0.933	0.909	0.887	0.866	0.848	0.839
21	1.036	1.019	0.989	0.961	0.934	0.910	0.886	0.865	0.845	0.826	0.809	0.801
31	0.991	0.977	0.949	0.922	0.898	0.875	0.854	0.833	0.815	0.798	0.782	0.774
June 10	0.965	0.951	0.925	0.900	0.876	0.854	0.834	0.815	0.797	0.780	0.765	0.758
20	0.954	0.941	0.915	0.890	0.867	0.846	0.826	0.808	0.790	0.773	0.758	0.751
30	0.958	0.944	0.918	0.893	0.871	0.849	0.829	0.810	0.793	0.776	0.761	0.754
July 10	0.980	0.965	0.938	0.912	0.888	0.866	0.845	0.825	0.807	0.790	0.774	0.767
20	1.017	1.002	0.972	0.945	0.920	0.895	0.873	0.852	0.833	0.815	0.798	0.790
30	1.072	1.055	1.022	0.992	0.964	0.937	0.913	0.890	0.869	0.850	0.832	0.823
Aug. 9	1.146	1.127	1.090	1.055	1.024	0.994	0.966	0.941	0.917	0.895	0.875	0.866
19	1.243	1.219	1.176	1.135	1.099	1.065	1.033	1.005	0.977	0.953	0.930	0.919
29	1.346	1.319	1.268	1.222	1.179	1.140	1.104	1.071	1.040	1.013	0.987	0.974
Sept. 8	1.443	1.412	1.354	1.301	1.253	1.209	1.169	1.132	1.098	1.066	1.038	1.024
18	1.499	1.465	1.403	1.346	1.295	1.248	1.205	1.166	1.130	1.097	1.066	1.052
28	1.487	1.454	1.392	1.336	1.286	1.240	1.197	1.159	1.123	1.091	1.060	1.046
Oct. 8	1.412	1.382	1.326	1.275	1.229	1.187	1.148	1.112	1.079	1.050	1.021	1.008
18	1.309	1.283	1.235	1.191	1.150	1.113	1.079	1.047	1.018	0.991	0.967	0.955
28	1.205	1.184	1.142	1.105	1.070	1.037	1.008	0.980	0.954	0.931	0.909	0.898
Nov. 7	1.119	1.100	1.065	1.031	1.001	0.973	0.946	0.922	0.899	0.878	0.858	0.849
17	1.050	1.033	1.002	0.973	0.946	0.920	0.896	0.874	0.855	0.836	0.818	0.809
27	1.001	0.986	0.957	0.930	0.906	0.883	0.861	0.841	0.822	0.804	0.788	0.780
Dec. 7	0.970	0.956	0.929	0.904	0.880	0.858	0.838	0.818	0.801	0.784	0.768	0.761
17	0.955	0.941	0.915	0.891	0.868	0.846	0.826	0.808	0.790	0.774	0.759	0.752
27	0.956	0.942	0.916	0.892	0.869	0.847	0.827	0.809	0.791	0.775	0.760	0.752
Jan. 6	0.976	0.962	0.934	0.908	0.884	0.862	0.842	0.822	0.804	0.787	0.772	0.764

$$F_1 = \frac{2.4066}{c_{11} 1.4066 f(K_1) + f(O_1)} \cdot F_1 \times \text{observed } D_1 = K_1 + O_1. \quad 1.02 F_1 \times \text{observed } D_1 = D_1.$$

This table is based on Tables 13 and 31.

TABLE 33.—Factor F_2 for clearing S_2 of K_2 and T_2 .

Date.	i , or inclination of orbit to equator.											
	$18\frac{1}{2}^\circ$	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	$28\frac{1}{2}^\circ$
Jan. 1	1.14	1.15	1.16	1.18	1.19	1.20	1.22	1.24	1.27	1.30	1.33	1.34
11	1.08	1.09	1.10	1.10	1.11	1.12	1.14	1.15	1.17	1.19	1.21	1.22
21	1.02	1.02	1.02	1.03	1.03	1.04	1.04	1.05	1.06	1.06	1.07	1.08
31	.96	.96	.96	.96	.96	.96	.96	.96	.96	.96	.96	.96
Feb. 10	.90	.90	.90	.89	.89	.89	.88	.88	.88	.87	.87	.86
20	.86	.86	.85	.85	.84	.84	.83	.83	.82	.81	.81	.80
Mar. 2	.83	.83	.83	.82	.82	.81	.80	.79	.78	.77	.76	.76
12	.83	.82	.82	.81	.80	.79	.78	.77	.76	.75	.74	.74
22	.83	.82	.81	.80	.79	.79	.78	.77	.76	.75	.74	.73
Apr. 1	.83	.83	.83	.82	.81	.80	.79	.78	.77	.76	.75	.75
11	.86	.86	.85	.84	.83	.82	.82	.81	.80	.79	.78	.78
21	.91	.91	.90	.89	.89	.88	.88	.87	.86	.85	.85	.84
May 1	.96	.96	.96	.95	.95	.95	.94	.94	.93	.93	.92	.92
11	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04
21	1.12	1.12	1.13	1.13	1.14	1.14	1.15	1.16	1.16	1.17	1.18	1.19
31	1.23	1.24	1.25	1.26	1.27	1.28	1.30	1.32	1.34	1.37	1.39	1.40
June 10	1.30	1.31	1.33	1.35	1.37	1.39	1.41	1.44	1.47	1.51	1.55	1.57
20	1.35	1.37	1.39	1.41	1.44	1.47	1.50	1.54	1.58	1.63	1.68	1.70
30	1.34	1.35	1.37	1.39	1.42	1.45	1.48	1.51	1.55	1.59	1.64	1.67
July 10	1.26	1.27	1.28	1.30	1.32	1.34	1.36	1.38	1.41	1.43	1.46	1.47
20	1.17	1.18	1.19	1.19	1.20	1.22	1.23	1.24	1.25	1.26	1.27	1.28
30	1.07	1.07	1.08	1.08	1.08	1.09	1.09	1.09	1.09	1.10	1.10	1.10
Aug. 9	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.98	.98	.97	.97
19	.94	.93	.93	.93	.92	.92	.91	.90	.89	.88	.88	.87
29	.88	.88	.88	.87	.86	.85	.84	.83	.83	.82	.81	.80
Sept. 8	.85	.85	.84	.84	.83	.82	.81	.80	.79	.78	.77	.76
18	.84	.83	.83	.82	.81	.80	.79	.78	.77	.76	.75	.75
28	.83	.83	.83	.82	.81	.80	.79	.78	.77	.76	.75	.74
Oct. 8	.85	.84	.83	.83	.82	.81	.80	.79	.78	.77	.77	.76
18	.86	.86	.86	.85	.85	.84	.83	.82	.81	.80	.79	.79
28	.90	.89	.89	.89	.88	.88	.88	.87	.86	.86	.85	.85
Nov. 7	.95	.95	.95	.95	.95	.94	.94	.94	.94	.94	.93	.93
17	1.01	1.01	1.02	1.02	1.02	1.03	1.03	1.04	1.04	1.05	1.05	1.06
27	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.14	1.15	1.16	1.18	1.19
Dec. 7	1.13	1.14	1.15	1.16	1.18	1.19	1.21	1.23	1.25	1.28	1.31	1.32
17	1.16	1.18	1.19	1.21	1.23	1.25	1.27	1.30	1.33	1.36	1.39	1.41
27	1.16	1.17	1.18	1.19	1.21	1.23	1.25	1.28	1.31	1.34	1.37	1.39
Jan. 6	1.11	1.11	1.12	1.14	1.15	1.16	1.18	1.20	1.22	1.24	1.27	1.28

This table is based upon Tables 13 and 31.

TABLE 34.—Effect of v_2 upon the amplitude of N_2 .

Apparent time of moon's upper or lower transit. Δ in perigee or apogee.			Resultant amplitude N_2 and v_2 . $N_2=1$.	Apparent time of moon's upper or lower transit. Δ in perigee or apogee.			Resultant amplitude N_2 and v_2 . $N_2=1$.
$h.$	$h.$	$m.$		$h.$	$h.$	$m.$	
0,	12	00	1.19	6,	18	00	0.81
		20	1.19			20	0.81
		40	1.18			40	0.82
1,	13	00	1.17	7,	19	00	0.84
		20	1.16			20	0.86
		40	1.14			40	0.89
2,	14	00	1.11	8,	20	00	0.92
		20	1.08			20	0.95
		40	1.05			40	0.99
3,	15	00	1.02	9,	21	00	1.02
		20	0.99			20	1.05
		40	0.95			40	1.08
4,	16	00	0.92	10,	22	00	1.11
		20	0.89			20	1.14
		40	0.86			40	1.16
5,	17	00	0.84	11,	23	00	1.17
		20	0.82			20	1.18
		40	0.81			40	1.19

This table is based upon Tables 1 and 16.

TABLE 35.—Group factors.

For phase reduction.

Number of tides before springs or neaps.	Number of tides after springs or neaps.							
	0	2	4	6	8	10	12	
$S_2/M_3=0.2$	0	1.00	1.01	1.03	1.07	1.13	1.22	1.34
	2	1.01	1.01	1.02	1.06	1.11	1.18	1.29
	4	1.03	1.02	1.03	1.05	1.10	1.16	1.25
	6	1.07	1.06	1.05	1.07	1.11	1.16	1.24
	8	1.13	1.11	1.10	1.11	1.13	1.18	1.25
	10	1.22	1.18	1.16	1.16	1.18	1.22	1.28
	12	1.34	1.29	1.25	1.24	1.25	1.28	1.34
$S_2/M_3=0.3$	0	1.00	1.01	1.03	1.07	1.14	1.23	1.35
	2	1.01	1.01	1.03	1.06	1.11	1.19	1.30
	4	1.03	1.03	1.03	1.06	1.10	1.17	1.26
	6	1.07	1.06	1.06	1.08	1.11	1.17	1.25
	8	1.14	1.11	1.10	1.11	1.14	1.19	1.26
	10	1.23	1.19	1.17	1.17	1.19	1.23	1.29
	12	1.35	1.30	1.26	1.25	1.26	1.29	1.35
$S_2/M_3=0.4$	0	1.00	1.01	1.03	1.08	1.14	1.24	1.37
	2	1.01	1.01	1.03	1.06	1.12	1.20	1.31
	4	1.03	1.03	1.03	1.06	1.11	1.18	1.27
	6	1.08	1.06	1.06	1.08	1.12	1.17	1.26
	8	1.14	1.12	1.11	1.12	1.15	1.19	1.27
	10	1.24	1.20	1.18	1.17	1.19	1.24	1.30
	12	1.37	1.31	1.27	1.26	1.27	1.30	1.37
$S_2/M_3=0.5$	0	1.00	1.01	1.04	1.09	1.16	1.25	1.39
	2	1.01	1.01	1.03	1.07	1.13	1.21	1.33
	4	1.04	1.03	1.04	1.07	1.12	1.19	1.29
	6	1.09	1.07	1.07	1.09	1.13	1.19	1.27
	8	1.16	1.13	1.12	1.13	1.16	1.21	1.29
	10	1.25	1.21	1.19	1.19	1.21	1.25	1.32
	12	1.39	1.33	1.29	1.27	1.29	1.32	1.39

For parallax reduction.

Number of tides before or after greatest and least parallax effects.	Factor.
0	1.00
4	1.01
6	1.02
8	1.04
10	1.06

For declinational reduction.

Number of tides before or after moon's extreme declination.	Factor (for diurnal wave).
0	1.000
2	1.005
4	1.012
6	1.024
8	1.040

This table is based upon Tables 1 and 16.

TABLE 36.—Shallow-water components.

[Terms from y'^2 .]

SEMIDIURNAL COMPONENTS.

Designation of component.		Primitive amplitude.	Speed.		Argument.	Primitive epoch.
(K, K ₁) (K, O ₁) (K, P ₁)	K ₁ M ₂ S ₂	$\frac{1}{2}K_1^2$ K ₁ O ₁ K ₁ P ₁	$k_1 + k_1 = k_2$	30°0821372	2 arg K ₁ arg K ₁ + arg O ₁ arg K ₁ + arg P ₁	$2K_1^0$ K ₁ ⁰ + O ₁ ⁰ K ₁ ⁰ + P ₁ ⁰
			$k_1 + o_1 = m_2$	28°9841042		
			$k_1 + p_1 = s_2$	30°0000000		
(O, O ₁) (O, P ₁)	O ₁	$\frac{1}{2}O_1^2$ O ₁ P ₁	$o_1 + o_1 = o_2$	27°8860712	2 arg O ₁ arg O ₁ + arg P ₁	$2O_1^0$ O ₁ ⁰ + P ₁ ⁰
			$o_1 + p_1$	28°9019670		
(P, P ₁)	P ₁	$\frac{1}{2}P_1^2$	$p_1 + p_1 = p_2$	29°9178628	2 arg P ₁	2P ₁ ⁰

COMPONENTS OF LONG PERIOD.

(K ₁ ~K ₁) (K ₁ ~O ₁) (K ₁ ~P ₁)	Mf Ssa	$\frac{1}{2}K_1^2$ K ₁ O ₁ K ₁ P ₁	$k_1 - k_1 = o$	o	arg K ₁ - arg O ₁ arg K ₁ - arg P ₁	$K_1^0 - O_1^0$ K ₁ ⁰ - P ₁ ⁰
			$k_1 - o_1 = mf$	1°0980330		
			$k_1 - p_1 = ssa$	0°0821372		
(O ₁ ~O ₁) (O ₁ ~P ₁)	MSf	$\frac{1}{2}O_1^2$ O ₁ P ₁	$o_1 - o_1 = o$	o	arg P ₁ - arg O ₁	$P_1^0 - O_1^0$
			$p_1 - o_1 = msf$	1°0158958		
(P ₁ ~P ₁)		$\frac{1}{2}P_1^2$	$p_1 - p_1 = o$	o	o	o

TERDIURNAL COMPONENTS.

(M ₂ K ₁) (M ₂ O ₁) (M ₂ P ₁)	MK 2MK	M ₂ K ₁ M ₂ O ₁ M ₂ P ₁	$m_2 + k_1 = mk$	44°0251728	arg M ₂ + arg K ₁ arg M ₂ + arg O ₁ arg M ₂ + arg P ₁	$M_2^0 + K_1^0$ M ₂ ⁰ + O ₁ ⁰ M ₂ ⁰ + P ₁ ⁰
			$m_2 + o_1$	42°9271398		
			$m_2 + p_1$	43°9430356		
(S ₂ K ₁) (S ₂ O ₁) (S ₂ P ₁)		S ₂ K ₁ S ₂ O ₁ S ₂ P ₁	$s_2 + k_1$	45°0410686	arg S ₂ + arg K ₁ arg S ₂ + arg O ₁ arg S ₂ + arg P ₁	$S_2^0 + K_1^0$ S ₂ ⁰ + O ₁ ⁰ S ₂ ⁰ + P ₁ ⁰
			$s_2 + o_1$	43°9430356		
			$s_2 + p_1$	44°9589314		
(N ₂ K ₁) (N ₂ O ₁) (N ₂ P ₁)		N ₂ K ₁ N ₂ O ₁ N ₂ P ₁	$n_2 + k_1$	43°4807982	arg N ₂ + arg K ₁ arg N ₂ + arg O ₁ arg N ₂ + arg P ₁	$N_2^0 + K_1^0$ N ₂ ⁰ + O ₁ ⁰ N ₂ ⁰ + P ₁ ⁰
			$n_2 + o_1$	42°3827652		
			$n_2 + p_1$	43°3986610		
(K ₂ K ₁) (K ₂ O ₁) (K ₂ P ₁)	K ₂ MK	K ₂ K ₁ K ₂ O ₁ K ₂ P ₁	$3k_1$	45°1232058	3 arg K ₁ arg K ₂ + arg O ₁ arg K ₂ + arg P ₁	$3K_1^0$ K ₂ ⁰ + O ₁ ⁰ K ₂ ⁰ + P ₁ ⁰
			$k_2 + o_1 = mk$	44°0251728		
			$k_2 + p_1$	45°0410686		
(L ₂ K ₁) (L ₂ O ₁) (L ₂ P ₁)		L ₂ K ₁ L ₂ O ₁ L ₂ P ₁	$l_2 + k_1$	44°5695474	arg L ₂ + arg K ₁ arg L ₂ + arg O ₁ arg L ₂ + arg P ₁	$L_2^0 + K_1^0$ L ₂ ⁰ + O ₁ ⁰ L ₂ ⁰ + P ₁ ⁰
			$l_2 + o_1$	43°4715144		
			$l_2 + p_1$	44°4874102		

DIURNAL COMPONENTS.

(M ₂ ~K ₁) (M ₂ ~O ₁) (M ₂ ~P ₁)	O ₁ K ₁	M ₂ K ₁ M ₂ O ₁ M ₂ P ₁	$m_2 - k_1 = o$	13°9430356	arg M ₂ - arg K ₁ arg M ₂ - arg O ₁ arg M ₂ - arg P ₁	$M_2^0 - K_1^0$ M ₂ ⁰ - O ₁ ⁰ M ₂ ⁰ - P ₁ ⁰
			$m_2 - o_1 = k_1$	15°0410686		
			$m_2 - p_1$	14°0251728		
(S ₂ ~K ₁) (S ₂ ~O ₁) (S ₂ ~P ₁)	P ₁ K ₁	S ₂ K ₁ S ₂ O ₁ S ₂ P ₁	$s_2 - k_1 = p_1$	14°9589314	arg S ₂ - arg K ₁ arg S ₂ - arg O ₁ arg S ₂ - arg P ₁	$S_2^0 - K_1^0$ S ₂ ⁰ - O ₁ ⁰ S ₂ ⁰ - P ₁ ⁰
			$s_2 - o_1$	16°0569644		
			$s_2 - p_1 = k_1$	15°0410686		
(N ₂ ~K ₁) (N ₂ ~O ₁) (N ₂ ~P ₁)	Q ₁ [M ₁]	N ₂ K ₁ N ₂ O ₁ N ₂ P ₁	$n_2 - k_1 = q_1$	13°3986610	arg N ₂ - arg K ₁ arg N ₂ - arg O ₁ arg N ₂ - arg P ₁	$N_2^0 - K_1^0$ N ₂ ⁰ - O ₁ ⁰ N ₂ ⁰ - P ₁ ⁰
			$n_2 - o_1 = [m_1]$	14°4966940		
			$n_2 - p_1$	13°4807982		
(K ₂ ~K ₁) (K ₂ ~O ₁) (K ₂ ~P ₁)	K ₁	K ₂ K ₁ K ₂ O ₁ K ₂ P ₁	$k_2 - k_1 = k_1$	15°0410686	arg K ₂ - arg K ₁ arg K ₂ - arg O ₁ arg K ₂ - arg P ₁	$K_2^0 - K_1^0$ K ₂ ⁰ - O ₁ ⁰ K ₂ ⁰ - P ₁ ⁰
			$k_2 - o_1$	16°1391016		
			$k_2 - p_1$	15°1232058		
(L ₂ ~K ₁) (L ₂ ~O ₁) (L ₂ ~P ₁)	J ₁	L ₂ K ₁ L ₂ O ₁ L ₂ P ₁	$l_2 - k_1$	14°4874102	arg L ₂ - arg K ₁ arg L ₂ - arg O ₁ arg L ₂ - arg P ₁	$L_2^0 - K_1^0$ L ₂ ⁰ - O ₁ ⁰ L ₂ ⁰ - P ₁ ⁰
			$l_2 - o_1 = j_1$	15°5854432		
			$l_2 - p_1$	14°5695474		

For a description of this table, see § 48, Part II.

For sake of clearness we have supposed (AB) to denote a component whose speed is $a + b$, and $(A \sim B)$ a component whose speed is $a \sim b$.

TABLE 36.—*Shallow-water components—Continued.*[Terms from y'^2 .]

QUARTER-DIURNAL COMPONENTS.

Designation of component.		Primitive amplitude.	Speed.		Argument.	Primitive epoch.
(M_2M_2) (M_2S_2) (M_2N_2) (M_2K_2) (M_2L_2)	M_4	$\frac{1}{2} M_2^2$	$m_2 + m_2 = m_4$	57°9682084	$2 \arg M_2$	$2 M_2^\circ$
	MS	M_2S_2	$m_2 + s_2$	58°9841042	$\arg M_2 + \arg S_2$	$M_2^\circ + S_2^\circ$
	MN	M_2N_2	$m_2 + n_2 = mn$	57°4238338	$\arg M_2 + \arg N_2$	$M_2^\circ + N_2^\circ$
		M_2K_2	$m_2 + k_2$	59°0662414	$\arg M_2 + \arg K_2$	$M_2^\circ + K_2^\circ$
		M_2L_2	$m_2 + l_2$	58°5125830	$\arg M_2 + \arg L_2$	$M_2^\circ + L_2^\circ$
(S_2S_2) (S_2N_2) (S_2K_2) (S_2L_2)	S_4	$\frac{1}{2} S_2^2$	$s_2 + s_2 = s_4$	60°0000000	$2 \arg S_2$	$2 S_2^\circ$
		S_2N_2	$s_2 + n_2$	58°4397296	$\arg S_2 + \arg N_2$	$S_2^\circ + N_2^\circ$
	R_4	S_2K_2	$s_2 + k_2 = r_4$	60°0821372	$\arg S_2 + \arg K_2$	$S_2^\circ + K_2^\circ$
		S_2L_2	$s_2 + l_2$	59°5284788	$\arg S_2 + \arg L_2$	$S_2^\circ + L_2^\circ$
(N_2N_2) (N_2K_2) (N_2L_2)	N_4	$\frac{1}{2} N_2^2$	$n_2 + n_2 = n_4$	56°8794592	$2 \arg N_2$	$2 N_2^\circ$
		N_2K_2	$n_2 + k_2$	58°5218668	$\arg N_2 + \arg K_2$	$N_2^\circ + K_2^\circ$
		N_2L_2	$n_2 + l_2$	57°9682084	$\arg N_2 + \arg L_2$	$N_2^\circ + L_2^\circ$
(K_2K_2) (K_2L_2)	K_4	$\frac{1}{2} K_2^2$	$k_2 + k_2 = k_4$	60°1642744	$2 \arg K_2$	$2 K_2^\circ$
		K_2L_2	$k_2 + l_2$	59°6106160	$\arg K_2 + \arg L_2$	$K_2^\circ + L_2^\circ$
(L_2L_2)	L_4	$\frac{1}{2} L_2^2$	$l_2 + l_2 = l_4$	59°0569576	$2 \arg L_2$	$2 L_2^\circ$

COMPONENTS OF LONG PERIOD.

$(M_2 \sim M_2)$ $(M_2 \sim S_2)$ $(M_2 \sim N_2)$ $(M_2 \sim K_2)$ $(M_2 \sim L_2)$	MSf	$\frac{1}{2} M_2^2$	$m_2 - m_2 = 0$	0	$\arg S_2 - \arg M_2$	$S_2^\circ - M_2^\circ$
	Mm	M_2S_2	$s_2 - m_2 = msf$	1°0158958	$\arg M_2 - \arg N_2$	$M_2^\circ - N_2^\circ$
	Mf	M_2N_2	$m_2 - n_2 = mm$	0°5443746	$\arg M_2 - \arg K_2$	$M_2^\circ - K_2^\circ$
	Mm	M_2K_2	$m_2 - k_2 = mf$	1°0980330	$\arg L_2 - \arg M_2$	$L_2^\circ - M_2^\circ$
		M_2L_2	$l_2 - m_2 = mm$	0°5443746		
$(S_2 \sim S_2)$ $(S_2 \sim N_2)$ $(S_2 \sim K_2)$ $(S_2 \sim L_2)$	Ssa	$\frac{1}{2} S_2^2$	$s_2 - s_2 = 0$	0	$\arg S_2 - \arg N_2$	$S_2^\circ - N_2^\circ$
		S_2N_2	$s_2 - n_2$	1°5602704	$\arg K_2 - \arg S_2$	$K_2^\circ - S_2^\circ$
		S_2K_2	$k_2 - s_2 = ssa$	0°0821372	$\arg S_2 - \arg L_2$	$S_2^\circ - L_2^\circ$
		S_2L_2	$s_2 - l_2$	0°4715212		
$(N_2 \sim N_2)$ $(N_2 \sim K_2)$ $(N_2 \sim L_2)$		$\frac{1}{2} N_2^2$	$n_2 - n_2 = 0$	0	$\arg K_2 - \arg N_2$	$K_2^\circ - N_2^\circ$
		N_2K_2	$k_2 - n_2$	1°6424076	$\arg L_2 - \arg N_2$	$L_2^\circ - N_2^\circ$
		K_2L_2	$l_2 - n_2$	1°0887492		
$(K_2 \sim K_2)$ $(K_2 \sim L_2)$		$\frac{1}{2} K_2^2$	$k_2 - k_2 = 0$	0	$\arg K_2 - \arg L_2$	$K_2^\circ - L_2^\circ$
		K_2L_2	$k_2 - l_2$	0°5536584		
$(L_2 \sim L_2)$		$\frac{1}{2} L_2^2$	$l_2 - l_2 = 0$	0		0

TABLE 36.—*Shallow-water components*—Continued.[Terms from y'^3 or $y' \times y'^2$.]

ONE-SIXTH-DIURNAL COMPONENTS.

Designation of component.		Primitive amplitude.	Speed.		Argument.	Primitive epoch.
$(M_2 M_2 M_2)$ $(M_2 M_2 S_2)$ $(M_2 M_2 N_2)$ $(M_2 M_2 K_2)$ $(M_2 M_2 L_2)$	M_6	$\frac{1}{2} M_2^3$ $M_2^2 S_2$ $M_2^2 N_2$ $M_2^2 K_2$ $M_2^2 L_2$	$3 m_2 = m_6$ $2 m_2 + s_2$ $2 m_2 + n_2$ $2 m_2 + k_2$ $2 m_2 + l_2$	86°9523126 87°9682084 86°4079380 88°0503456 87°4966872	$3 \arg M_2$ $2 \arg M_2 + \arg S_2$ $2 \arg M_2 + \arg N_2$ $2 \arg M_2 + \arg K_2$ $2 \arg M_2 + \arg L_2$	$3 M_2^\circ$ $2 M_2^\circ + S_2^\circ$ $2 M_2^\circ + N_2^\circ$ $2 M_2^\circ + K_2^\circ$ $2 M_2^\circ + L_2^\circ$
$(M_2 S_2 S_2)$ $(M_2 S_2 N_2)$ $(M_2 S_2 K_2)$ $(M_2 S_2 L_2)$		$\frac{1}{2} M_2 S_2^2$ $M_2 S_2 N_2$ $M_2 S_2 K_2$ $M_2 S_2 L_2$	$2 s_2 + m_2$ $m_2 + s_2 + n_2$ $m_2 + s_2 + k_2$ $m_2 + s_2 + l_2$	88°9841042 87°4238338 89°0662414 88°5125830	$2 \arg S_2 + \arg M_2$ $\arg M_2 + \arg S_2 + \arg N_2$ $\arg M_2 + \arg S_2 + \arg K_2$ $\arg M_2 + \arg S_2 + \arg L_2$	$M_2' + 2 S_2^\circ$ $M_2^\circ + S_2^\circ + N_2^\circ$ $M_2^\circ + S_2^\circ + K_2^\circ$ $M_2^\circ + S_2^\circ + L_2^\circ$
$(S_2 M_2 M_2)$ $(S_2 M_2 S_2)$ $(S_2 M_2 N_2)$ $(S_2 M_2 K_2)$ $(S_2 M_2 L_2)$		$\frac{1}{2} S_2 M_2^2$ $M_2 S_2^2$ $M_2 S_2 N_2$ $M_2 S_2 K_2$ $M_2 S_2 L_2$	$2 m_2 + s_2$ $m_2 + 2 s_2$ $m_2 + s_2 + n_2$ $m_2 + s_2 + k_2$ $m_2 + s_2 + l_2$	87°9682084 88°9841042 87°4238338 89°0662414 88°5125830	$2 \arg M_2 + \arg S_2$ $\arg M_2 + 2 \arg S_2$ $\arg M_2 + \arg S_2 + \arg N_2$ $\arg M_2 + \arg S_2 + \arg K_2$ $\arg M_2 + \arg S_2 + \arg L_2$	$2 M_2^\circ + S_2^\circ$ $M_2^\circ + 2 S_2^\circ$ $M_2^\circ + S_2^\circ + N_2^\circ$ $M_2^\circ + S_2^\circ + K_2^\circ$ $M_2^\circ + S_2^\circ + L_2^\circ$
$(S_2 S_2 S_2)$ $(S_2 S_2 N_2)$ $(S_2 S_2 K_2)$ $(S_2 S_2 L_2)$	S_6	$\frac{1}{2} S_2^3$ $S_2^2 N_2$ $S_2^2 K_2$ $S_2^2 L_2$	$3 s_2 = s_6$ $2 s_2 + n_2$ $2 s_2 + k_2$ $2 s_2 + l_2$	90°0000000 88°4397296 90°0821372 89°5284788	$3 \arg S_2$ $2 \arg S_2 + \arg N_2$ $2 \arg S_2 + \arg K_2$ $2 \arg S_2 + \arg L_2$	$3 S_2^\circ$ $2 S_2^\circ + N_2^\circ$ $2 S_2^\circ + K_2^\circ$ $2 S_2^\circ + L_2^\circ$

SEMIDIURNAL COMPONENTS.

$(M_2 \sim M_2 M_2)$ $(M_2 \sim M_2 S_2)$ $(M_2 \sim M_2 N_2)$ $(M_2 \sim M_2 K_2)$ $(M_2 \sim M_2 L_2)$	M_2 S_2 N_2 K_2 L_2	$\frac{1}{2} M_2^3$ $M_2^2 S_2$ $M_2^2 N_2$ $M_2^2 K_2$ $M_2^2 L_2$	m_2 s_2 n_2 k_2 l_2	28°9841042 30°0000000 28°4397296 30°0821372 29°5284788	$\arg M_2$ $\arg S_2$ $\arg N_2$ $\arg K_2$ $\arg L_2$	M_2° S_2° N_2° K_2° L_2°
$(M_2 \sim S_2 S_2)$ $(M_2 \sim S_2 N_2)$ $(M_2 \sim S_2 K_2)$ $(M_2 \sim S_2 L_2)$	$2 SM$ λ_2	$\frac{1}{2} M_2 S_2^2$ $M_2 S_2 N_2$ $M_2 S_2 K_2$ $M_2 S_2 L_2$	$2 s_2 - m_2$ $s_2 + n_2 - m_2 = \lambda_2$ $s_2 + k_2 - m_2$ $s_2 + l_2 - m_2$	31°0158958 29°4556254 31°0980330 30°5443746	$2 \arg S_2 - \arg M_2$ $\arg S_2 + \arg N_2 - \arg M_2$ $\arg S_2 + \arg K_2 - \arg M_2$ $\arg S_2 + \arg L_2 - \arg M_2$	$2 S_2^\circ - M_2^\circ$ $S_2^\circ + N_2^\circ - M_2^\circ$ $S_2^\circ + K_2^\circ - M_2^\circ$ $S_2^\circ + L_2^\circ - M_2^\circ$
$(S_2 \sim M_2 M_2)$ $(S_2 \sim M_2 S_2)$ $(S_2 \sim M_2 N_2)$ $(S_2 \sim M_2 K_2)$ $(S_2 \sim M_2 L_2)$	$2 MS$ μ_2	$\frac{1}{2} S_2 M_2^2$ $M_2 S_2^2$ $M_2 S_2 N_2$ $M_2 S_2 K_2$ $M_2 S_2 L_2$	$2 m_2 - s_2 = \mu_2$ m_2 $m_2 + n_2 - s_2$ $m_2 + k_2 - s_2$ $m_2 + l_2 - s_2 = \nu_2$	27°9682084 28°9841042 27°4238338 29°0662414 28°5125830	$2 \arg M_2 - \arg S_2$ $\arg M_2$ $\arg M_2 + \arg N_2 - \arg S_2$ $\arg M_2 + \arg K_2 - \arg S_2$ $\arg M_2 + \arg L_2 - \arg S_2$	$2 M_2^\circ - S_2^\circ$ M_2° $M_2^\circ + N_2^\circ - S_2^\circ$ $M_2^\circ + K_2^\circ - S_2^\circ$ $M_2^\circ + L_2^\circ - S_2^\circ$
$(S_2 \sim S_2 S_2)$ $(S_2 \sim S_2 N_2)$ $(S_2 \sim S_2 K_2)$ $(S_2 \sim S_2 L_2)$	S_2 N_2 K_2 L_2	$\frac{1}{2} S_2^3$ $S_2^2 N_2$ $S_2^2 K_2$ $S_2^2 L_2$	s_2 n_2 k_2 l_2	30°0000000 28°4397296 30°0821372 29°5284788	$\arg S_2$ $\arg N_2$ $\arg K_2$ $\arg L_2$	S_2° N_2° K_2° L_2°

[Terms from y'^4 or $y' \times y'^3$.]

ONE-EIGHTH-DIURNAL COMPONENTS.

$(M_2 M_6)$ $(M_2 S_6)$ $(S_2 M_6)$ $(S_2 S_6)$	M_8 S_8	$\frac{1}{2} M_2^4$ $\frac{1}{2} M_2 S_2^3$ $\frac{1}{2} S_2 M_2^3$ $\frac{1}{2} S_2^4$	$4 m_2 = m_8$ $3 s_2 + m_2$ $3 m_2 + s_2$ $4 s_2 = s_8$	115°9364168 118°9841042 116°9523126 120°0000000	$4 \arg M_2$ $3 \arg S_2 + \arg M_2$ $3 \arg M_2 + \arg S_2$ $4 \arg S_2$	$4 M_2^\circ$ $3 S_2^\circ + M_2^\circ$ $3 M_2^\circ + S_2^\circ$ $4 S_2^\circ$
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QUARTER-DIURNAL COMPONENTS.

$(M_2 \sim M_6)$ $(M_2 \sim S_6)$ $(S_2 \sim M_6)$ $(S_2 \sim S_6)$	M_4 S_4	$\frac{1}{2} M_2^4$ $\frac{1}{2} M_2 S_2^3$ $\frac{1}{2} S_2 M_2^3$ $\frac{1}{2} S_2^4$	$2 m_2 = m_4$ $3 s_2 - m_2$ $3 m_2 - s_2$ $2 s_2 = s_4$	57°9682084 61°0158958 56°9523126 60°0000000	$2 \arg M_2$ $3 \arg S_2 - \arg M_2$ $3 \arg M_2 - \arg S_2$ $2 \arg S_2$	$2 M_2^\circ$ $3 S_2^\circ - M_2^\circ$ $3 M_2^\circ - S_2^\circ$ $2 S_2^\circ$
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TABLE 37.—The theoretical amplitudes of some of the more important components for every 5 degrees of latitude.

λ	$\cos^2 \lambda$	$\sin 2 \lambda$	$\frac{1}{2} - \frac{1}{2} \sin^2 \lambda$	M_2	N_2	S_2	K_1	O_1	P_1	Mf
°				Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
+90	0.0000	0.0000	-1.0000	0.000	0.000	0.000	0.000	0.000	0.000	-0.138
+85	0.0076	0.1736	-0.9886	0.006	0.001	0.003	+0.081	+0.058	+0.027	-0.136
+80	0.0301	0.3420	-0.9548	0.024	0.005	0.011	+0.160	+0.114	+0.053	-0.132
+75	0.0670	0.5000	-0.8995	0.054	0.010	0.025	+0.233	+0.166	+0.077	-0.124
+70	0.1170	0.6428	-0.8245	0.094	0.018	0.044	+0.300	+0.213	+0.099	-0.114
+65	0.1786	0.7660	-0.7321	0.143	0.028	0.066	+0.358	+0.254	+0.118	-0.101
+60	0.2500	0.8660	-0.6250	0.200	0.039	0.093	+0.404	+0.287	+0.134	-0.086
+55	0.3290	0.9397	-0.5065	0.263	0.051	0.122	+0.439	+0.312	+0.145	-0.070
+50	0.4132	0.9848	-0.3802	0.330	0.064	0.154	+0.460	+0.327	+0.152	-0.052
+45	0.5000	1.0000	-0.2500	0.400	0.077	0.186	+0.467	+0.332	+0.154	-0.034
+40	0.5868	0.9848	-0.1198	0.469	0.091	0.218	+0.460	+0.327	+0.152	-0.017
+35	0.6711	0.9397	+0.0066	0.537	0.104	0.250	+0.439	+0.312	+0.145	+0.001
+30	0.7500	0.8660	+0.1250	0.600	0.116	0.279	+0.404	+0.287	+0.134	+0.017
+25	0.8214	0.7660	+0.2321	0.657	0.127	0.306	+0.358	+0.254	+0.118	+0.032
+20	0.8830	0.6428	+0.3245	0.706	0.137	0.329	+0.300	+0.213	+0.099	+0.045
+15	0.9330	0.5000	+0.3995	0.746	0.145	0.347	+0.233	+0.166	+0.077	+0.055
+10	0.9698	0.3420	+0.4547	0.776	0.150	0.361	+0.160	+0.114	+0.053	+0.063
+5	0.9924	0.1736	+0.4886	0.794	0.154	0.369	+0.081	+0.058	+0.027	+0.067
0	1.0000	0.0000	+0.5000	0.800	0.155	0.372	+0.000	+0.000	+0.000	+0.069
-5	0.9924	-0.1736	+0.4886	0.794	0.154	0.369	-0.081	-0.058	-0.027	+0.067
-10	0.9698	-0.3420	+0.4547	0.776	0.150	0.361	-0.160	-0.114	-0.053	+0.063
-15	0.9330	-0.5000	+0.3995	0.746	0.145	0.347	-0.233	-0.166	-0.077	+0.055
-20	0.8830	-0.6428	+0.3245	0.706	0.137	0.329	-0.300	-0.213	-0.099	+0.045
-25	0.8214	-0.7660	+0.2321	0.657	0.127	0.306	-0.358	-0.254	-0.118	+0.032
-30	0.7500	-0.8660	+0.1250	0.600	0.116	0.279	-0.404	-0.287	-0.134	+0.017
-35	0.6711	-0.9397	+0.0066	0.537	0.104	0.250	-0.439	-0.312	-0.145	+0.001
-40	0.5868	-0.9848	-0.1198	0.469	0.091	0.218	-0.460	-0.327	-0.152	-0.017
-45	0.5000	-1.0000	-0.2500	0.400	0.077	0.186	-0.467	-0.332	-0.154	-0.034
-50	0.4132	-0.9848	-0.3802	0.330	0.064	0.154	-0.460	-0.327	-0.152	-0.052
-55	0.3290	-0.9397	-0.5065	0.263	0.051	0.122	-0.439	-0.312	-0.145	-0.070
-60	0.2500	-0.8660	-0.6250	0.200	0.039	0.093	-0.404	-0.287	-0.134	-0.086
-65	0.1786	-0.7660	-0.7321	0.143	0.028	0.066	-0.358	-0.254	-0.118	-0.101
-70	0.1170	-0.6428	-0.8245	0.094	0.018	0.044	-0.300	-0.213	-0.099	-0.114
-75	0.0670	-0.5000	-0.8995	0.054	0.010	0.025	-0.233	-0.166	-0.077	-0.124
-80	0.0301	-0.3420	-0.9548	0.024	0.005	0.011	-0.160	-0.114	-0.053	-0.132
-85	0.0076	-0.1736	-0.9886	0.006	0.001	0.003	-0.081	-0.058	-0.027	-0.136
-90	0.0000	0.0000	-1.0000	0.000	0.000	0.000	0.000	0.000	0.000	-0.138

Tabular value = $\left[\frac{1}{2} \frac{M}{E} \left(\frac{a}{c} \right)^3 a = 1.760 \right] \times \text{latitude factor} \times \text{coefficient}$. The latitude factor is given in column 2, 3, or 4; the coefficient in Table 1. For this table it is assumed that $\frac{M}{E} = \frac{1}{81.07}$, $\frac{a}{c} = \frac{1}{60.34}$, $a = 20\,902\,000$ feet, according to Harkness, Solar Parallax, pages 138, 140, using a mean radius of the earth instead of the equatorial radius. The negative amplitude signifies that the phase of the tide is altered by 180°. The north latitude is +, the south —.

TABLE 38.—*Augmenting factors.*

Components.		Subscript.							
		1	2	3	4	5	6	7	8
S		1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000	1'0000 0'0000
J	2 SM	1'00307 0'001331	Group covers one solar hour.						
K	P, R, T	1'00287 0'001246	1'01158 0'004998	1'02632 0'011281	1'04746 0'020138				
L	λ, MS	1'00273 0'001196	1'01116 0'004819	1'02534 0'010868	1'04568 0'019400				
M		1'00266 0'001153	1'01075 0'004644	1'02440 0'010470	1'04396 0'018683	1'06989 0'029339	1'10283 0'042507	1'14363 0'058286	1'19343 0'076797
N	ν	1'00256 0'001111	1'01033 0'004464						
O	2 N, μ	1'00249 0'001081	1'00994 0'004295	1'02256 0'009691	1'04300 0'018285				
OO		1'00333 0'001442							
Q	ρ	1'00227 0'000983							
2 Q		1'00209 0'000906							
MN	2 MK	1'00261 0'001132	1'01055 0'004557	1'02394 0'010274	1'04311 0'018331				
MK		1'00274 0'001189	1'01102 0'004760	1'02503 0'010739					
All		1'00286 0'001240	1'01152 0'004974	Group covers one component hour.				1'15496 0'062571	1'20920 0'082498

The tabular value for any component other than S is

$$\frac{\text{arc } c \tau}{\text{chord } c \tau}$$

where τ = the length of the group. It is a solar hour when each component hour receives one, and only one, hourly height; it may be regarded as a component hour when all hourly heights are used in the summation. (See § 57, Part II.)

Tides of long period.

When all daily means are used, the factors given under the heading "Group covers one component hour" are to be applied to the long-period tides, the subscripts referring to the year or month, instead of the day as in the case of tides of short period. When attention is paid to the arrows, Table 43, in making the summations, the augmenting factors due to using solar instead of component time, are given by the above formula by putting τ = one solar day, and c = mf, msf, mm. The results are: 1.00887 (log. = 0.003835), 1.00759 (log. = 0.003282), 1.00217 (log. = 0.000941). In case of any long-period tide there is, besides the augmenting factor proper, what might be called a group factor, due to using the mean of 24 heights each day. The numerical values just given are also the group factors for Mf, MSf, and Mm.

TABLE 39.—Values of $b-a$ and of $24 \times (b-a)$.

DIURNALS.

A	B									
	J ₁	K ₁	M ₁	O ₁	OO	P ₁	Q ₁	2Q	S ₁	p ₁
J ₁	0	- 0°5443747	- 1°0933912	- 1°6424077	+ 0°5536583	- 0°6265119	- 2°1867824	- 2°7311571	- 0°5854433	2°1139289
	0	-13°064993	-26°241389	-39°417785	+13°287799	-15°036286	-52°482778	-65°547770	-14°050639	-50°734294
K ₁	+ 0°5443747	0	- 0°5490165	- 1°0980330	+ 1°0980330	- 0°0821372	- 1°6424077	- 2°1867824	- 0°0410686	- 1°5695542
	+13°064993	0	-13°176396	-26°352792	+26°352792	- 1°971293	-39°417785	-52°482778	- 0°985646	-37°669301
M ₁	+ 1°0933912	+ 0°5490165	0	- 0°5490165	+ 1°6470495	+ 0°4668793	- 1°0933912	- 1°6377659	+ 0°5079479	- 1°0205377
	+26°241389	+13°176396	0	-13°176396	+39°529188	+11°205103	-26°241389	-39°306382	+12°190750	-24°492905
O ₁	+ 1°6424077	+ 1°0980330	+ 0°5490165	0	+ 2°1960660	+ 1°0158958	- 0°5443747	- 1°0887494	+ 1°0569644	- 0°4715212
	+39°417785	+26°352792	+13°176396	0	+52°705584	+24°381499	-13°064993	-26°129986	+25°367146	-11°316509
OO	- 0°5536583	- 1°0980330	- 1°6470495	- 2°1960660	0	- 1°1801702	- 2°7404407	- 3°2848154	- 1°1391016	- 2°6675872
	-13°287799	-26°352792	-39°529188	-52°705584	0	-28°324085	-65°770577	-78°835570	-27°338438	-64°022093
P ₁	+ 0°6265119	+ 0°0821372	- 0°4668793	- 1°0158958	+ 1°1801702	0	- 1°5602705	- 2°1046452	+ 0°0410686	- 1°4874170
	+15°036286	+ 1°971293	-11°205103	-24°381499	+28°324085	0	-37°446492	-50°511485	+ 0°985646	-35°698008
Q ₁	+ 2°1867824	+ 1°6424077	+ 1°0933912	+ 0°5443747	+ 2°7404407	+ 1°5602705	0	- 0°5443747	+ 1°6013391	+ 0°0728535
	+52°482778	+39°417785	+26°241389	+13°064993	+65°770577	+37°446492	0	-13°064993	+38°432138	+ 1°748484
2Q	+ 2°7311571	+ 2°1867824	+ 1°6377659	+ 1°0887494	+ 3°2848154	+ 2°1046452	+ 0°5443747	0	+ 2°1457138	+ 0°6172282
	+65°547770	+52°482778	+39°306382	+26°129986	+78°835570	+50°511485	+13°064993	0	+51°497131	+14°813477
S ₁	+ 0°5854433	+ 0°0410686	- 0°5079479	- 1°0569644	+ 1°1391016	- 0°0410686	- 1°6013391	- 2°1457138	0	- 1°5284856
	+14°050639	+ 0°985646	-12°190750	-25°367146	+27°338438	- 0°985646	-38°432138	-51°497131	0	-36°683654
p ₁	+ 2°1139289	+ 1°5695542	+ 1°0205377	+ 0°4715212	+ 2°6675872	+ 1°4874170	- 0°0728535	- 0°6172282	+ 1°5284856	0
	+50°734294	+37°669301	+24°492905	+11°316509	+64°022093	+35°698008	- 1°748484	-14°813477	+36°683654	0

TABLE 39.—Values of $b-a$ and of $24 \times (b-a)$ —Continued.

SEMIJOURNALS.

A	B										
	K_2	L_2	M_2	N_2	$2N$	R_2	S_2	T_2	λ_2	μ_2	ν_2
K_2	0 0	-0'5536584 -13'287802	-1'0980330 -26'352792	-1'6424076 -39'417782	-2'1867824 -52'482778	-0'0410686 -0'985646	-0'0821372 -1'971293	-0'1232058 -2'956939	-0'6265118 -15'036283	-2'1139288 -50'734291	-1'5955542 -37'669301
L_2	+0'5536584 +13'287802	0 0	-0'5443746 -13'064990	-1'0887492 -26'129981	-1'6331240 -39'194976	+0'5125598 +12'302155	+0'4715212 +11'316509	+0'4304526 +10'330862	-0'0728534 -1'748482	-1'5602704 -37'446490	-1'0158958 -24'381499
M_2	+1'0980330 +26'352792	+0'5443746 +13'064990	0 0	-0'5443746 -13'064990	-1'0887492 -26'129981	+1'0565644 +25'367146	+1'0158958 +24'381499	+0'9748272 +23'395553	+0'4715212 +11'316509	-1'0158958 -24'381499	-0'4715212 -11'316509
N_2	+1'6424076 +39'417782	+1'0887492 +26'129981	+0'5443746 +13'064990	0 0	-0'5443748 -13'064995	+1'6013390 +38'432136	+1'5602704 +37'446490	+1'5192018 +36'460843	+1'0158958 +24'381499	-0'4715212 -11'316509	+0'0728534 +1'748482
$2N$	+2'1867824 +52'482778	+1'6331240 +39'194976	+1'0887492 +26'129981	+0'5443748 +13'064995	0 0	+2'1457138 +51'497131	+2'1046452 +50'511485	+2'0635766 +49'525338	+1'5602706 +37'446494	+0'0728536 +1'748486	+0'6172282 +14'813477
R_2	+0'0410686 +0'985646	-0'5125598 -12'302155	-1'0565644 -25'367146	-1'6013390 -38'432136	-2'1457138 -51'497131	0 0	-0'0410686 -0'985646	-0'0821372 -1'971293	-0'5854432 -14'056637	-2'0728602 -49'748645	-1'5284856 -36'683654
S_2	+0'0821372 +1'971293	-0'4715212 -11'316509	-1'0158958 -24'381499	-1'5602704 -37'446490	-2'1046452 -50'511485	+0'0410686 +0'985646	0 0	-0'0410686 -0'985646	-0'5443746 -13'064990	-2'0317916 -48'762998	-1'4874170 -35'680008
T_2	+0'1232058 +2'956939	-0'4304526 -10'330862	-0'9748272 -23'395853	-1'5192018 -36'460843	-2'0635766 -49'525338	+0'0821372 +1'971293	+0'0410686 +0'985646	0 0	-0'5033060 -12'079344	-1'9907230 -47'777352	-1'4463484 -34'712362
λ_2	+0'6265118 +15'036283	+0'0728534 +1'748482	-0'4715212 -11'316509	-1'0158958 -24'381499	-1'5602706 -37'446494	+0'5854432 +14'056637	+0'5443746 +13'064990	+0'5033060 +12'079344	0 0	-1'4874170 -35'680008	-0'9430424 -22'633018
μ_2	+2'1139288 +50'734291	+1'5602704 +37'446490	+1'0158958 +24'381499	+0'4715212 +11'316509	-0'0728536 -1'748486	+2'0728602 +49'748645	+2'0317916 +48'762998	+1'9907230 +47'777352	+1'4874170 +35'680008	0 0	+0'5443746 +13'064990
ν_2	+1'5955542 +37'669301	+1'0158958 +24'381499	+0'4715212 +11'316509	-0'0728534 -1'748482	-0'6172282 -14'813477	+1'5284856 +36'683654	+1'4874170 +35'680008	+1'4463484 +34'712362	+0'9430424 +22'633018	-0'5443746 -13'064990	0 0
$2SM$	-0'9337586 -22'410206	-1'4874170 -35'680008	-2'0317916 -48'762998	-2'5761662 -61'827989	-3'1205410 -74'892984	-0'9748272 -23'395853	-1'0158958 -24'381499	-1'0565644 -25'367146	-1'5602704 -37'446490	-3'0476874 -73'144498	-2'5033128 -60'079507

In this table a , b denote the hourly speeds of the components A , B .

TABLE 40.—*Synodic periods in days and hours.*
DIURNALS.

A	B									
	J ₁	K ₁	M ₁	O ₁	OO	P ₁	Q ₁	2 Q	S ₁	ρ ₁
J ₁	∞ ∞									
K ₁	27°55455 661°3092	∞ ∞								
M ₁	13°71879 329°2509	27°32158 655°7180	∞ ∞							
O ₁	9°13293 219°1904	13°66079 327°8590	27°32158 655°7180	∞ ∞						
OO	27°09252 650°2205	13°66079 327°8590	9°10719 218°5727	6°83040 163°9295	∞ ∞					
P ₁	23°94208 574°6100	182°62127 4382°9105	32°12822 771°0772	14°76529 354°3671	12°71003 305°0407	∞ ∞				
Q ₁	6°85939 164°6254	9°13293 219°1904	13°71879 329°2509	27°55455 661°3092	5°47357 131°3657	9°61372 230°7292	∞ ∞			
2 Q	5°49218 131°8123	6°85939 164°6254	9°15882 219°8116	13°77728 330°6546	4°56647 109°5952	7°12709 171°0502	27°55455 661°3092	∞ ∞		
S ₁	25°62161 614°9186	365°24255 8765°8211	29°53059 708°7341	14°19158 340°5980	13°16827 316°0385	365°24255 8765°8211	9°36716 224°8118	6°99068 167°7763	∞ ∞	
ρ ₁	7°09579 170°2990	9°55685 229°3645	14°69813 352°7552	31°81193 763°4863	5°62306 134°9534	10°08460 242°0304	205°89265 4941°4235	24°30219 583°2527	9°81364 235°5272	∞ ∞

SEMI-DIURNALS.

A	B											
	K ₂	L ₂	M ₂	N ₂	2 N	R ₂	S ₂	T ₂	λ ₂	μ ₂	ν ₂	2 SM
K ₂	∞ ∞											
L ₂	27°09252 650°2204	∞ ∞										
M ₂	13°66079 327°8590	27°55456 661°3094	∞ ∞									
N ₂	9°13293 219°1904	13°77728 330°6547	27°55456 661°3094	∞ ∞								
2 N	6°85939 164°6254	9°18485 220°4364	13°77728 330°6546	27°55455 661°3092	∞ ∞							
R ₂	365°24255 8765°8211	29°26317 702°3160	14°19158 340°5980	9°36716 224°8118	6°99068 167°7763	∞ ∞						
S ₂	182°62127 4382°9105	31°81193 763°4863	14°76529 354°3671	9°61372 230°7292	7°12709 171°0502	365°24255 8765°8211	∞ ∞					
T ₂	121°74751 2921°9403	34°84704 836°3290	15°38734 369°2962	9°87361 236°9665	7°26893 174°4544	182°62127 4382°9105	365°24255 8765°8211	∞ ∞				
λ ₂	23°94208 574°6100	205°89265 4941°4235	31°81193 763°4863	14°76529 354°3671	9°61372 230°7292	25°62161 614°9186	27°55456 661°3094	29°80294 715°2706	∞ ∞			
μ ₂	7°09579 170°2990	9°61372 230°7292	14°76529 354°3671	31°81193 763°4863	205°89236 4941°4165	7°23638 173°6731	7°38265 177°1835	7°53495 180°8388	10°08460 242°0304	∞ ∞		
ν ₂	9°55685 229°3645	14°76529 354°3671	31°81193 763°4863	205°89265 4941°4235	24°30216 583°2526	9°81364 235°5272	10°08460 242°0304	10°37095 248°9027	15°90597 381°7432	27°55456 661°3094	∞ ∞	
2 SM	16°06411 385°5386	10°08460 242°0304	7°38265 177°1835	5°82261 139°7425	4°80686 115°3646	15°38734 369°2962	14°76529 354°3671	14°19158 340°5980	9°61372 230°7292	4°92176 118°1224	5°99206 143°8094	∞ ∞

Synodic period = $\frac{15}{b-a}$ days or $\frac{360}{b-a}$ hours.

TABLE 41.—For clearing one component of the effects of others.
[Length of series, 29 days.]

Component sought. (A)	Disturbing components (B, C, etc.).									
	J ₁	K ₁	M ₁	O ₁	OO	P ₁	Q ₁	2 Q	S ₁	ρ ₁
J ₁		.0497 351 π	.053 339 π	.0525 328 π	.065 13 π	.162 322 π	.049 319 π	.046 310 π	.113 336 π	.021 344 π
K ₁	.050 9 π		.057 349 π	.0565 338 π	.056 22 π	.959 331 π	.052 328 π	.049 319 π	.990 346 π	.011 354 π
M ₁	.053 21 π	.0575 11 π		.0575 349 π	.055 33 π	.106 162 π	.053 339 π	.050 330 π	.018 177 π	.014 185 π
O ₁	.052 32 π	.0565 22 π	.057 11 π		.052 44 π	.018 174 π	.050 351 π	.049 341 π	.021 8 π	.096 196 π
OO	.065 347 π	.0565 338 π	.055 327 π	.0523 316 π		.108 309 π	.048 306 π	.045 297 π	.086 324 π	.029 332 π
P ₁	.162 38 π	.9590 29 π	.106 198 π	.0182 186 π	.108 51 π		.005 3 π	.017 348 π	.990 14 π	.042 202 π
Q ₁	.049 41 π	.0525 32 π	.053 21 π	.0497 9 π	.048 54 π	.005 3 π		.050 351 π	.031 17 π	.968 25 π
2 Q	.046 50 π	.0494 41 π	.050 30 π	.0489 19 π	.045 63 π	.017 12 π	.050 9 π		.034 27 π	.152 35 π
S ₁	.113 24 π	.9902 14 π	.018 183 π	.0212 352 π	.086 36 π	.990 346 π	.031 343 π	.034 333 π		.015 188 π
ρ ₁	.021 16 π	.0113 6 π	.014 175 π	.0958 164 π	.029 28 π	.042 158 π	.968 335 π	.152 325 π	.015 172 π	

Component sought. (A)	Disturbing components (B, C, etc.).											
	K ₂	L ₂	M ₂	N ₂	2 N	R ₂	S ₂	T ₂	λ ₂	μ ₂	ν ₂	2 SM
K ₂		.065 347 π	.0565 338 π	.052 328 π	.049 319 π	.990 346 π	.9590 331 π	.909 317 π	.162 322 π	.021 344 π	.011 354 π	.101 145 π
L ₂	.065 13 π		.0497 351 π	.049 341 π	.048 332 π	.009 178 π	.0958 164 π	.192 150 π	.968 335 π	.005 357 π	.018 186 π	.042 158 π
M ₂	.056 22 π	.050 9 π		.050 351 π	.049 341 π	.021 8 π	.0182 174 π	.060 159 π	.096 164 π	.018 186 π	.096 196 π	.018 67 π
N ₂	.052 32 π	.049 19 π	.0497 9 π		.050 351 π	.031 17 π	.0055 3 π	.021 169 π	.018 174 π	.096 196 π	.968 25 π	.004 177 π
2 N	.049 41 π	.048 28 π	.0489 19 π	.050 9 π		.034 27 π	.0168 12 π	.003 178 π	.005 3 π	.968 25 π	.152 35 π	.005 6 π
R ₂	.990 14 π	.009 182 π	.0212 352 π	.031 343 π	.034 333 π		.9902 346 π	.959 331 π	.113 336 π	.002 359 π	.015 188 π	.060 159 π
S ₂	.959 29 π	.096 196 π	.0182 186 π	.005 357 π	.017 348 π	.990 14 π		.990 346 π	.050 351 π	.018 293 π	.042 202 π	.018 174 π
T ₂	.909 43 π	.192 210 π	.0599 201 π	.021 191 π	.003 182 π	.959 29 π	.9902 14 π		.028 185 π	.038 207 π	.068 217 π	.021 8 π
λ ₂	.162 38 π	.968 25 π	.0958 196 π	.018 186 π	.005 357 π	.113 24 π	.0497 9 π	.028 175 π		.042 202 π	.092 212 π	.005 3 π
μ ₂	.021 16 π	.005 3 π	.0182 174 π	.096 164 π	.968 335 π	.002 1 π	.0181 67 π	.038 153 π	.042 158 π		.050 9 π	.018 161 π
ν ₂	.011 6 π	.018 174 π	.0958 164 π	.968 335 π	.152 325 π	.015 172 π	.0422 158 π	.068 143 π	.092 148 π	.050 351 π		.032 151 π
2 SM	.101 215 π	.042 202 π	.0181 293 π	.004 183 π	.005 354 π	.060 201 π	.0182 186 π	.021 352 π	.005 357 π	.018 199 π	.032 209 π	

TABLE 41—For clearing one component of the effects of others—Continued.
[Length of series, 369 days.]

Component sought. (A)	Disturbing components (B, C, etc.).									
	J ₁	K ₁	M ₁	O ₁	OO	P ₁	Q ₁	2 Q	S ₁	ρ ₁
J ₁		'0224 290 π	'004 198	'0075 287	'022 112 π	'020 286 π	'004 217 π	'003 326 π	'021 288	'000 180 π
K ₁	'022 70 π		'024 269 π	'0004 358 π	'000 2 π	'010 356 π	'007 287 π	'004 217 π	'010 358 π	'008 250 π
M ₁	'004 162	'0236 91 π		'0236 269 π	'008 93 π	'028 87 π	'004 198 π	'006 308 π	'025 89 π	'004 341 π
O ₁	'008 73	'0004 2 π	'024 91 π		'000 4 π	'000 178 π	'022 290 π	'007 219 π	'000 180 π	'026 252 π
OO	'022 248 π	'0004 358 π	'008 267 π	'0004 356 π		'001 354 π	'005 285 π	'002 215 π	'001 356 π	'004 248 π
P ₁	'020 74 π	'0102 4 π	'028 273 π	'0004 182 π	'001 6 π		'008 291 π	'004 221 π	'010 2 π	'008 254 π
Q ₁	'004 143 π	'0075 73 π	'004 162 π	'0224 70 π	'005 75 π	'008 69 π		'022 290 π	'008 71 π	'108 143 π
2 Q	'003 34 π	'0036 143 π	'006 52 π	'0075 141 π	'002 145 π	'004 139 π	'022 70 π		'004 141 π	'011 33 π
S ₁	'021 72 π	'0102 2 π	'025 271 π	'0000 180 π	'001 4 π	'010 358 π	'008 289 π	'004 219 π		'008 252 π
ρ ₁	'000 180 π	'0078 110 π	'004 19 π	'0261 108 π	'004 112 π	'008 106 π	'108 217 π	'011 327 π	'008 108 π	

Component sought. (A)	Disturbing components (B, C, etc.).											
	K ₂	L ₂	M ₂	N ₂	2 N	R ₂	S ₂	T ₂	λ ₂	μ ₂	ν ₂	2 SM
K ₂		'022 248 π	'0000 358 π	'008 287 π	'004 217 π	'010 358 π	'0102 356 π	'010 354 π	'020 286 π	'000 180 π	'008 250 π	'001 175 π
L ₂	'022 112 π		'0224 290 π	'007 219 π	'004 329 π	'024 110 π	'0261 108 π	'029 106 π	'108 217 π	'008 291 π	'000 182 π	'008 106 π
M ₂	'000 2 π	'022 70 π		'022 290 π	'007 219 π	'000 180 π	'0004 178 π	'001 177 π	'026 108 π	'000 182 π	'026 252 π	'000 177 π
N ₂	'008 73 π	'007 141 π	'0224 70 π		'022 290 π	'008 71 π	'0077 69 π	'008 67 π	'000 178 π	'026 252 π	'108 143 π	'005 67 π
2 N	'004 143 π	'004 31 π	'0075 141 π	'022 70 π		'004 141 π	'0040 139 π	'004 138 π	'008 69 π	'108 143 π	'011 33 π	'003 138 π
R ₂	'010 2 π	'024 250 π	'0000 180 π	'008 289 π	'004 219 π		'0102 358 π	'010 356 π	'021 288 π	'000 181 π	'008 252 π	'001 177 π
S ₂	'010 4 π	'026 252 π	'0004 182 π	'008 291 π	'004 221 π	'010 2 π		'010 358 π	'022 290 π	'000 183 π	'008 254 π	'000 178 π
T ₂	'010 6 π	'029 254 π	'0008 183 π	'008 293 π	'004 222 π	'010 4 π	'0102 2 π		'024 291 π	'001 185 π	'009 256 π	'000 180 π
λ ₂	'020 74 π	'108 143 π	'0261 252 π	'000 182 π	'008 291 π	'021 72 π	'0224 70 π	'024 69 π		'008 254 π	'008 324 π	'008 69 π
μ ₂	'000 180 π	'008 69 π	'0004 178 π	'026 108 π	'108 217 π	'000 179 π	'0004 177 π	'001 175 π	'008 106 π		'022 70 π	'000 175 π
ν ₂	'008 110 π	'000 178 π	'0261 108 π	'108 217 π	'011 327 π	'008 108 π	'0084 106 π	'009 104 π	'008 36 π	'022 290 π		'005 105 π
2 SM	'001 185 π	'008 254 π	'0004 183 π	'005 293 π	'003 222 π	'001 183 π	'0004 182 π	'000 180 π	'008 291 π	'000 185 π	'005 255 π	

TABLE 42.—Component hours derived from solar hours.

Day of series.	J	K	L	M	N	2N	O	OO	P
1	13+1 ←			15-1 ↑	10-1 ↑	8-1 ←	22-2 ←	7+1 ←	20+2 ←
2	15+2 ↑		8-1 ↑	21-2 ←	5-2 ↑	12-3 ↑	12-3 ←	9+3 ←	23+4 ↑
3	17+3 ↑				1-3 ←	2-4 ↑	17-5 ←	12+5 ↑	
4	18+4 ↑			2-3 ↑	15-5 ↑	7-6 ↑	21-7 ↑	1+6 ↑	14+7 ←
5	20+5 ↑		0-2 ←	8-4 ←	10-6 ↑	11-8 ↑	11-8 ←	3+8 ←	16+9 ←
6	21+6 ←			13-5 ↑	5-7 ↑	2-9 ←	16-10 ←	6+10 ↑	19+11 ↑
7	23+7 ←		16-3 ←	19-6 ↑	1-8 ←	6-11 ↑	20-12 ↑	8+12 ↑	21-11 ←
8		15+1 ←			15-10 ↑	11+11 ←	10+11 ←	10-10 ←	23-9 ←
9	1+8 ↑			0-7 ↑	10-11 ↑	1+10 ↑	15+9 ↑	13-8 ↑	15-1 ↑
10	2+9 ←		7-4 ↑	6-8 ←	6-12 ←	5+8 ↑	20+7 ←	2-7 ↑	15-6 ↑
11	4+10 ↑			12-9 ←	1+11 ←	10+6 ←	9+6 ←	4-5 ←	17-4 ←
12	6+11 ↑		23-5 ←	17-10 ↑	15+9 ↑	0+5 ↑	14+4 ↑	6-3 ←	20-2 ←
13	7+12 ↑			23-11 ←	11+8 ←	5+3 ←	19+2 ←	9-1 ↑	22-0 ↑
14	9-11 ↑				6+7 ↑	9+1 ↑	23+0 ↑	11+1 ↑	
15	10-10 ←		14-6 ↑	4-12 ↑	1+6 ←	14-1 ←	12-1 ↑	0+2 ←	13+3 ←
16	12-9 ←			10+11 ←	15+4 ↑	4-2 ←	18-3 ↑	3+4 ↑	16+5 ↑
17	14-8 ↑			15+10 ↑	11+3 ←	8-4 ↑	23-5 ↑	5+6 ↑	18+7 ↑
18	15-7 ↑		6-7 ↑	21+9 ←	6+2 ←	13-6 ←	11-6 ←	7+8 ↑	20+9 ↑
19	17-6 ↑				1+1 ↑	3-7 ↑	18-8 ←	10+10 ↑	23+11 ↑
20	18-5 ←		22-8 ←	2+8 ↑	16-1 ←	8-9 ←	22-10 ↑	12+12 ↑	
21	20-4 ←			8+7 ←	11-2 ←	12-11 ↑	10-11 ↑	1-11 ←	14-10 ←
22	22-3 ↑			13+6 ↑	6-3 ↑	3-12 ↑	17+11 ←	3-9 ←	17-8 ↑
23	23-2 ←	20+2 ←	13-9 ↑	19+5 ←	1-4 ↑	7+10 ↑	21+9 ↑	6-7 ↑	19-6 ↑
24					16-6 ←	12+8 ←	9+8 ↑	8-5 ←	21-4 ←
25	1-1 ↑			0+4 ↑	11-7 ←	2+7 ←	16+6 ↑	10-3 ←	
26	3-0 ↑		5-10 ←	6+3 ←	6-8 ↑	6+5 ↑	21+4 ←	0-2 ↑	13-1 ↑
27	4+1 ↑			11+2 ↑	1-9 ↑	11+3 ↑	22+2 ↑	2-0 ↑	15+1 ↑
28	6+2 ↑		21-11 ←	17+1 ←	16-11 ←	1+2 ↑	15+1 ↑	4+2 ↑	17+3 ↑
29	7+3 ←			22+0 ↑	11-12 ↑	6+0 ←	20-1 ↑	7+4 ↑	20+5 ↑
30	9+4 ←				6+11 ↑	10-2 ↑	7-2 ←	9+6 ←	22+7 ↑
31	11+5 ↑		12-12 ↑	4-1 ←	2+10 ←	0-3 ↑	15-4 ←	11+8 ↑	
32	12+6 ↑			10-2 ←	16+8 ↑	5-5 ←	19-6 ↑	1+9 ↑	14+10 ↑
33	14+7 ↑			15-3 ↑	11+7 ↑	9-7 ↑	20-8 ↑	3+11 ↑	16+12 ↑
34	16+8 ↑		4+11 ←	21-4 ←	6+6 ↑	0-8 ↑	14-9 ←	10-9 ↑	18-10 ←
35	17+9 ←				2+5 ←	4-10 ↑	18-11 ↑	8-9 ↑	21-8 ↑
36	19+10 ↑		19+10 ↑	2-5 ↑	16+3 ↑	9-12 ←	23+11 ←	10-7 ↑	23-6 ←
37	20+11 ↑			8-6 ←	11+2 ↑	13+10 ↑	23+9 ↑	12-5 ←	
38	22+12 ←			13-7 ↑	7+1 ↑	3+9 ↑	18+8 ←	1-4 ←	15-3 ←
39		2+3 ↑	11+9 ↑	19-8 ←	2+0 ←	8+7 ↑	22+6 ↑	4-2 ↑	17-1 ↑
40	0-11 ↑				16-2 ↑	12+5 ↑	8+5 ↑	6-0 ←	19+1 ←
41	1-10 ←			0-9 ↑	11-3 ↑	3+4 ↑	17+3 ↑	8+2 ↑	22+3 ↑
42	3-9 ↑		3+8 ←	6-10 ←	7-4 ←	7+2 ↑	21+1 ↑	11+4 ↑	
43	5-8 ↑			11-11 ↑	2-5 ←	12+0 ←	16-2 ↑	0+5 ↑	13+6 ↑
44	6-7 ↑		18+7 ↑	17-12 ←	16-7 ↑	2-1 ↑	16-2 ↑	2+7 ↑	15+8 ↑
45	8-6 ↑			22+11 ↑	12-8 ←	6-3 ↑	21-4 ←	5+9 ↑	18+10 ↑
46	9-5 ←				7-9 ↑	11-5 ↑	15-7 ↑	6-5 ↑	20-6 ↑
47	11-4 ←		10+6 ←	4+10 ←	2-10 ↑	1-6 ↑	15-7 ↑	11-7 ↑	22-10 ←
48	13-3 ↑			9+9 ↑	17-12 ←	6-8 ←	20-9 ←	1-8 ←	15-9 ←
49	14-2 ↑			15+8 ↑	12+11 ←	10-10 ↑	19-11 ↑	5-10 ↑	19-11 ↑
50	16-1 ↑		2+5 ←	20+7 ↑	7+10 ←	0-11 ↑	15-12 ←	9-12 ↑	3-6 ←
51	18-0 ↑				2+9 ↑	5+11 ←	19+10 ↑	0+11 ←	14+10 ←
52	19+1 ↑		17+4 ↑	2+6 ↑	17+7 ↑	9+9 ↑	18+8 ↑	4+9 ↑	18+8 ↑
53	21+2 ↑			8+5 ←	12+6 ←	0+8 ←	14+7 ←	8+7 ↑	23+6 ↑
54	22+3 ←	7+4 ↑		13+4 ↑	7+5 ↑	4+6 ↑	19+5 ↑	13+5 ↑	22+2 ↑
55			9+3 ←	19+3 ←	2+4 ↑	9+4 ↑	23+3 ↑	3+4 ↑	17+3 ↑
56	0+4 ←				17+2 ↑	13+2 ↑	18+0 ←	7+2 ↑	22+1 ←
57	2+5 ↑			0+2 ↑	12+1 ↑	4+1 ↑	22-2 ↑	12+0 ←	19+8 ↑
58	3+6 ↑		0+2 ↑	6+1 ←	7+0 ↑	8-1 ↑	22-2 ↑	2-1 ←	16-2 ↑
59	5+7 ↑			11+0 ↑	2-1 ↑	13-3 ←	21-4 ←	6-3 ↑	21-4 ←
60	6+8 ←		16+1 ↑	17-1 ←	17-3 ←	3-4 ←	17-5 ↑	11-5 ←	0+12 ←
61	8+9 ←			22-2 ↑	12-4 ↑	7-6 ↑	22-7 ←	1-6 ←	15-7 ↑
62	10+10 ↑				7-5 ↑	12-8 ↑	20-9 ←	5-8 ↑	18-7 ↑
63	11+11 ←		8+0 ←	4-3 ←	3-6 ←	2-9 ↑	16-10 ↑	10-10 ←	7-6 ←
64	13+12 ↑			9-4 ↑	17-8 ↑	7-11 ←	21-12 ←	0-11 ←	14-12 ↑
65	15-11 ↑		23-1 ↑	15-5 ←	12-9 ↑	11+11 ↑		4+11 ↑	18+10 ↑
66	16-10 ←			20-6 ↑	8-10 ←	1+10 ↑	16+9 ↑	9+9 ↑	23+8 ←
67	18-9 ↑				3-11 ←	6+8 ↑	20+7 ↑	13+7 ↑	16+2 ↑
68	19-8 ↑		15-2 ←	2-7 ←	17+11 ↑	10+6 ↑	14+7 ←	3+6 ↑	17+5 ↑
69	21-7 ←	12+5 ←		7-8 ↑	12+10 ↑	1+5 ←	15+4 ←	8+4 ↑	22+3 ↑
70	23-6 ↑			13-9 ←	8+9 ←	5+3 ↑	19+2 ↑	12+2 ↑	23+8 ←
71			7-3 ←	19-10 ←	3+8 ←	10+1 ←		2+1 ↑	16+0 ↑
72	0-5 ←				17+6 ↑	0+0 ↑	14-1 ↑	7-1 ↑	21-2 ←
73	2-4 ↑		22-4 ↑	0-11 ↑	13+5 ←	4-2 ↑	19-3 ↑	11-3 ←	15-5 ↑
74	4-3 ↑			6-12 ←	8+4 ↑	9-4 ↑	23-5 ↑	1-4 ↑	15-5 ↑
75	5-2 ←			11+11 ↑	3+3 ↑	13-6 ↑		6-6 ←	20-7 ←
76	7-1 ↑		14-5 ←	17+10 ←	17+1 ↑	4-7 ↑	18-8 ←	10-8 ↑	11-6 ←
77	8-0 ←			22+9 ↑	13+0 ↑	8-9 ↑	22-10 ↑	0-9 ↑	14-10 ↑
78	10+1 ↑				8-1 ←	13-11 ←		5-11 ←	19-12 ←
79	12+2 ↑		5-6 ↑	4+8 ←	3-2 ↑	3-12 ←	17+11 ↑	9+11 ↑	23+10 ↑
80	13+3 ←			9+7 ↑	18-4 ←	7+10 ↑	22+9 ←	13+9 ↑	7+1 ←

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series	J	K	L	M	N	2N	O	OO	P
81	15+4 ↑		21-7 ↑	15+6 ←	13-5 ←	12+8 ←	3+8 ↑	18+7 ↑	10+3 ↑
82	17+5 ↑			20+5 ↑	8-6 ↑	2+7 ↑	8+6 ←	22+5 ↑	12+5 ↑
83	18+6 ←				3-7 ↑	7+5 ←	12+4 ↑		1+6 ←
84	20+7 ↑	17+6 ←	13-8 ←	2+4 ←	18-9 ←	11+3 ↑	2+3 ↑	17+2 ←	3+8 ←
85	21+8 ←			7+3 ↑	13-10 ←	1+2 ↑	7+1 ←	21+0 ←	6+10 ↑
86	23+9 ←			13+2 ←	8-11 ↑	6+0 ←	11-1 ↑		8+12 ←
87			4-9 ↑	18+1 ↑	3-12 ↑	10-2 ↑	1-2 ↑	16-3 ←	11-10 ↑
88	1+10 ↑				18+10 ←	1-3 ←	6-4 ←	20-5 ←	0-9 ↑
89	2+11 ←		20-10 ←	0+0 ←	13+9 ←	5-5 ←	10-6 ↑		2-7 ←
90	4+12 ↑			5-1 ↑	8+8 ↑	10-7 ←	0-7 ↑	15-8 ←	4-5 ←
91	6-11 ↑			11-2 ←	4+7 ←	0-8 ↑	5-9 ←	19-10 ←	7-3 ↑
92	7-10 ←		12-11 ←	17-3 ←	18+5 ←	5-10 ←	9-11 ↑	23-12 ↑	9-1 ↑
93	9-9 ↑			22-4 ↑	13+4 ↑	9-12 ↑	13+11 ↑		11+1 ↑
94	10-8 ←			8+3 ↑	18+3 ↑	14+10 ←	4+10 ←	18+9 ←	1+2 ↑
95	12-7 ←		3-12 ↑	4-5 ←	4+2 ←	4+9 ←	8+8 ↑	22+7 ↑	3+4 ↑
96	14-6 ↑			9-6 ↑	18+0 ↑	8+7 ↑	12+6 ↑		5+6 ←
97	15-5 ←		19+11 ←	15-7 ←	13-1 ↑	13+5 ←	3+5 ←	17+4 ←	8+8 ↑
98	17-4 ↑			20-8 ↑	9-2 ←	3+4 ↑	7+3 ↑	21+2 ↑	10+10 ↑
99	18-3 ←	23+7 ↑		4-3 ←	23-4 ↑	8+2 ↑	11+1 ↑		12+12 ←
100	20-2 ←		10+10 ↑	2-9 ←	18-5 ↑	12+0 ↑	2+0 ←	16-1 ←	1-11 ←
101	22-1 ↑			7-10 ↑	14-6 ←	2-1 ↑	6-2 ←	20-3 ↑	4-9 ↑
102	23-0 ←			13-11 ←	9-7 ←	7-3 ←	10-4 ↑		6-7 ↑
103			2+9 ↑	18-12 ↑	4-8 ←	11-5 ↑	1-5 ←	15-6 ←	8-5 ←
104	1+1 ↑			18-10 ↑	18-10 ↑	2-6 ←	5-7 ←	19-8 ↑	11-3 ↑
105	3+2 ↑		18+8 ←	0+11 ←	14-11 ←	6-8 ↑	9-9 ↑		0-2 ↑
106	4+3 ←			5+10 ↑	9-12 ←	11-10 ←	0-10 ←	14-11 ←	2-0 ←
107	6+4 ↑			11+9 ←	4+11 ↑	1-11 ←	4-12 ←	18+11 ↑	5+2 ↑
108	7+5 ←		9+7 ↑	16+8 ↑	19+9 ←	5+11 ↑	8+10 ↑	22+9 ↑	7+4 ↑
109	9+6 ←			14+8 ←	14+8 ←	10+9 ←	13+8 ←		9+6 ←
110	11+7 ↑			9+7 ↑	9+7 ↑	0+8 ↑	3+7 ←	17+6 ↑	12+8 ↑
111	12+8 ←		1+6 ←	3+6 ↑	4+6 ↑	5+6 ←	7+5 ↑	21+4 ↑	1+9 ↑
112	14+9 ↑			9+5 ↑	19+4 ↑	9+4 ↑	12+3 ←		3+11 ←
113	16+10 ↑		17+5 ←	15+4 ↑	14+3 ←	14+2 ↑	2+2 ←	16+1 ←	5-11 ←
114	17+11 ←			20+3 ↑	9+2 ←	4+1 ←	6+0 ↑	20-1 ↑	8-9 ↑
115	19+12 ↑	4+8 ↑		4+1 ↑	4+1 ↑	8-1 ↑	11-2 ←		10-7 ←
116	20-11 ←		8+4 ↑	2+2 ←	0+0 ←	13-3 ←	1-3 ←	15-4 ←	12-5 ←
117	22-10 ←			7+1 ↑	14-2 ↑	3-4 ↑	5-5 ↑	19-6 ↑	2-4 ↑
118				13+0 ←	9-3 ↑	8-6 ←	10-7 ←		4-2 ↑
119	0-9 ↑		0+3 ←	18-1 ↑	5-4 ←	12-8 ↑	0-8 ←	14-9 ←	6-0 ←
120	1-8 ←			0-5 ←	19-6 ↑	2-9 ↑	4-10 ↑	18-11 ↑	9+2 ↑
121	3-7 ↑		15+2 ↑	0-2 ←	14-7 ↑	7-11 ←	8-12 ↑	23+11 ←	11+4 ↑
122	5-6 ↑			5-3 ↑	9-8 ↑	11+11 ↑	13+10 ←		0+5 ←
123	6-5 ←			11-4 ←	5-9 ←	2+10 ←	3+9 ↑	17+8 ↑	2+7 ↑
124	8-4 ↑		7+1 ↑	16-5 ↑	0-10 ←	6+8 ↑	7+7 ↑	22+6 ←	5+9 ↑
125	9-3 ←			22-6 ←	14-12 ↑	11+6 ←	12+5 ←		7+11 ←
126	11-2 ←		23+0 ←		10+11 ←	1+5 ↑	2+4 ↑	16+3 ↑	9-11 ←
127	13-1 ↑			3-7 ↑	5+10 ←	6+3 ←	6+2 ↑	21+1 ↑	12-8 ↑
128	14-0 ←			9-8 ←	0+9 ↑	10+1 ↑	11+0 ←		1-8 ←
129	16+1 ↑		14-1 ↑	14-9 ↑	15+7 ←	0+0 ↑	1-1 ←	15-2 ↑	3-6 ←
130	18+2 ↑	9+9 ←		20-10 ←	10+6 ←	5-2 ←	5-3 ↑	20-4 ←	6-4 ↑
131	19+3 ←		6-2 ←	2-11 ←	5+5 ←	9-4 ↑	10-5 ←		8-2 ↑
132	21+4 ↑			7-12 ↑	15+2 ↑	0-5 ←	14-6 ←	14-7 ↑	10-0 ←
133	22+5 ←		22-3 ←	13+11 ←	10+1 ←	4-7 ↑	4-8 ↑	19-9 ←	13+2 ↑
134				18+10 ↑	5+0 ←	9-9 ←	9-10 ←	23-11 ←	2+3 ↑
135	0+6 ←				0-1 ↑	13-11 ↑	13-12 ↑		4+5 ←
136	2+7 ↑		13-4 ↑	0+9 ←	15-3 ←	3-12 ↑	3+11 ↑	17+10 ↑	6+7 ↑
137	3+8 ←			5+8 ↑	10-4 ↑	8+10 ←	8+9 ←	22+8 ←	9+9 ↑
138	5+9 ↑			11+7 ←	5-5 ↑	12+8 ↑	12+7 ↑		11+11 ←
139	6+10 ←		5-5 ←	16+6 ↑	0-6 ↑	3+7 ↑	2+6 ↑	16+5 ↑	0+12 ←
140	8+11 ←				20-7 ←	7+5 ↑	7+4 ↑	21+3 ←	3-10 ↑
141	10+12 ↑		22+5 ←	15-8 ←	10-9 ↑	12+3 ←	11+2 ↑		5-8 ↑
142	11-11 ←		20-6 ↑	10-9 ↑	5-10 ↑	2+2 ←	1+1 ↑	15+0 ↑	7-6 ←
143	13-10 ↑			3+4 ↑	1-11 ←	6+0 ↑	6-1 ↑	20-2 ←	10-4 ←
144	15-9 ↑			9+3 ↑	15+11 ↑	11-2 ←	10-3 ←		12-4 ←
145	16-8 ←	14+10 ←	12-7 ↑	14+2 ↑	15+11 ↑	1-3 ↑	0-4 ↑	14-5 ↑	1-1 ←
146	18-7 ↑			20+1 ←	10+10 ↑	6-5 ←	5-6 ←	19-7 ←	4+1 ↑
147	19-6 ←			1+0 ↑	6+9 ←	10-8 ↑	9-8 ←	23-9 ↑	6+3 ↑
148	21-5 ←		4-8 ←	7-1 ↑	15+6 ↑	0-8 ↑	13-10 ↑		8+5 ↑
149	23-4 ↑			12-2 ↑	10+5 ↑	5-10 ←	3-11 ↑	18-12 ←	11+7 ↑
150			19-9 ↑			9-12 ↑	8+11 ←	22+10 ↑	0+8 ↑
151	0-3 ←			18-3 ←	6+4 ←	0+11 ←	12+9 ↑		2+10 ←
152	2-2 ↑			1+3 ↑	20+2 ↑	4+9 ↑	2+8 ↑	17+7 ←	4+12 ←
153	4-1 ↑		11-10 ←	0-4 ←	15+1 ↑	9+7 ↑	7+6 ←	21+5 ↑	7-10 ↑
154	5-0 ←			5-5 ↑	11+0 ←	13+5 ↑	11+4 ↑		9-8 ←
155	7+1 ↑			11-6 ←	6-1 ↑	3+4 ↑	1+3 ↑	16+2 ←	11-6 ←
156	8+2 ←		2-11 ↑	16-7 ↑	1-2 ↑	8+2 ↑	6+1 ↑	20+0 ←	1-5 ↑
157	10+3 ←			22-8 ←	15-4 ↑	12+0 ↑	10-1 ↑		3-3 ↑
158	12+4 ↑		18-12 ↑		6-6 ←	3-1 ←	0-2 ↑	15-3 ←	5-1 ↑
159	13+5 ↑			3-9 ↑	1-7 ↑	7-3 ↑	5-4 ←	19-5 ←	8+1 ↑
160	15+6 ↑	20+11 ↑		9-10 ←	20-8 ↑	12-5 ←	9-6 ↑	23-7 ↑	10+3 ↑

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	J	K	L	M	N	2N	O	OO	P
161	17+7 ↑		10+11←	14-11↑	16-9 ←	2-6 ↑	16-7 ↑	12+5 ←	
162	18+8 ←			20-12←	11-10←	7-8 ←	21-9 ←	1+6 ←	15+7 ↑
163	20+9 ↑				6-11↑	11-10↑		4+8 ↑	17+9 ↑
164	21+10←		1+10↑	1+11↑	1-12↑	1-11↑	16-12←	6+10←	19+11←
165	23+11←			7+10←	16+10←	6+11←	20+10↑	8+12←	22-11↑
166			17+9 ←	12+9 ↑	11+9 ←	10+9 ↑	7+8 ↑	11-10↑	
167	1+12↑			18+8 ←	6+8 ↑	1+8 ←	15+7 ←	0-9 ↑	13-8 ←
168	2-11←			23+7 ↑	1+7 ↑	5+6 ↑	19+5 ↑	2-7 ←	15-6 ←
169	4-10↑		9+8 ←		16+5 ←	10+4 ←	16+4 ↑	5-5 ↑	18-4 ↑
170	6-9 ↑			5+6 ←	11+4 ↑	0+3 ←	14+2 ↑	7-3 ↑	20-2 ←
171	7-8 ←			10+5 ↑	6+3 ↑	4+1 ↑	19+0 ←	1+0 ←	15-1 ←
172	9-7 ↑		0+7 ↑	16+4 ↑	2+2 ←	2+2 ←	23-2 ↑	5-2 ↑	19-3 ↑
173	10-6 ←			22+3 ←	16+0 ↑	13-3 ↑		9-4 ↑	14+3 ↑
174	12-5 ←		16+6 ←		11-1 ↑	4-4 ←	18-5 ↑	0-5 ←	14-6 ←
175	14-4 ↑			3+2 ↑	6-2 ↑	8-6 ↑	22-7 ↑	4-7 ←	18-8 ↑
176	15-3 ←	1+12↑		9+1 ←	2-3 ←	13-8 ←		8-9 ↑	21+9 ↑
177	17-2 ↑		7+5 ↑	14+0 ↑	16-5 ↑	3-9 ←	17-10↑	13-11←	10+10←
178	18-1 ←			20-1 ←	11-6 ↑	7-11↑	22-12←	3-12←	17+11↑
179	20-0 ←		23+4 ↑		7-7 ←	12+11↑		7+10↑	21+9 ↑
180	22+1 ↑			1-2 ↑	2-8 ←	2+10↑	16+9 ↑	12+8 ←	4-9 ↑
181	23+2 ←			7-3 ←	16-10↑	7+8 ←	21+7 ←	6-7 ←	19-6 ←
182			15+3 ←	12-4 ↑	12-11↑	11+6 ↑		6+5 ↑	20+4 ↑
183	1+3 ↑			18-5 ←	7-12←	1+5 ←	16+4 ←	11+3 ←	11-3 ←
184	3+4 ↑			23-6 ↑	2+11↑	6+3 ←	20+2 ↑	1+2 ←	15+1 ↑
185	4+5 ←		6+2 ↑		16+9 ↑	10+1 ↑		5+0 ↑	19-1 ↑
186	6+6 ↑			5-7 ←	12+8 ←	1+0 ←	15-1 ←	10-2 ←	18+3 ↑
187	7+7 ↑		22+1 ←	10-8 ↑	7+7 ←	5-2 ↑	19-3 ↑	0-3 ←	14-4 ←
188	9+8 ↑			16-9 ↑	2+6 ↑	10-4 ←		4-5 ↑	18-6 ↑
189	11+9 ↑			21-10↑	17+4 ↑	0-5 ←	14-6 ↑	9-7 ←	23-8 ↑
190	12+10←		14+0 ←		12+3 ←	4-7 ↑	19-8 ←	13-9 ←	12+8 ↑
191	14+11↑	6-11←		3-11←	7+2 ↑	9-9 ←	23-10↑	3-10↑	17-11↑
192	16+12↑			9-12←	2+1 ↑	13-11↑		7-12↑	22+11←
193	17-11←		5-1 ↑	14+11↑	17-1 ←	4-12←	18+11←	12+10←	
194	19-10↑		20+10←		12-2 ←	8+10↑	22+9 ↑	2+9 ↑	16+8 ↑
195	20-9 ←		21-2 ←		7-3 ↑	13+8 ←		6+7 ↑	21+6 ←
196	22-8 ←			1+9 ↑	2-4 ↑	3+7 ←	17+6 ↑	11+5 ←	
197				7+8 ←	17-6 ←	8+5 ←	22+4 ↑	1+4 ←	15+3 ↑
198	0-7 ↑		12-3 ↑	12+7 ↑	12-7 ↑	12+3 ↑		5+2 ↑	20+1 ↑
199	1-6 ↑			18+6 ←	7-8 ↑	2+2 ↑	17+1 ↑	10+0 ←	9+2 ↑
200	3-5 ↑		23+5 ↑		3-9 ←	7+0 ←	21-1 ↑	0-1 ←	14-2 ↑
201	5-4 ↑		4-4 ↑		17-11←	11-2 ↑		4-3 ↑	19-4 ←
202	6-3 ←			5+4 ←	12-12↑	2-3 ←	16-4 ←	9-5 ←	23-6 ←
203	8-2 ↑		20-5 ←	10+3 ↑	7+11↑	6-5 ↑	20-6 ↑	13-7 ↑	18+10←
204	9-1 ↑			16+2 ←	3+10←	11-7 ←		8-8 ↑	18-9 ↑
205	11-0 ←		21+1 ↑		17+8 ↑	1-8 ←	15-9 ↑	8-10←	22-11←
206	13+1 ↑	11-10←	11-6 ↑		12+7 ↑	5-10↑	20-11←	12-12↑	
207	14+2 ↑			3+0 ←	8+6 ←	10-12←		2+11↑	16+10↑
208	16+3 ↑			8-1 ↑	3+5 ←	0+11↑	14+10↑	7+9 ↑	21+8 ←
209	18+4 ↑		3-7 ←	14-2 ←	17+3 ↑	5+9 ↑	19+8 ←	11+7 ↑	
210	19+5 ←			19-3 ↑	12+2 ↑	9+7 ↑	23+6 ↑	1+6 ↑	15+5 ↑
211	21+6 ↑		19-8 ←		8+1 ←	14+5 ←		6+4 ←	20+3 ←
212	22+7 ↑			1-4 ←	3+0 ←	4+4 ←	18+3 ↑	10+2 ←	
213				7-5 ←	17-2 ↑	8-2 ↑	23+1 ↑	0+1 ↑	14+0 ↑
214	0+8 ←		10-9 ↑		13-3 ←	13+0 ←		5-1 ←	19-2 ←
215	2+9 ↑			18-7 ←	8-4 ←	3-1 ↑	17-2 ↑	9-3 ←	23-4 ↑
216	3+10←			23-8 ↑	8-5 ↑	8-3 ←	22-4 ←	13-5 ↑	
217	5+11↑		2-10←		18-7 ↑	12-5 ↑		4-6 ←	18-7 ←
218	6+12←			5-9 ←	13-8 ←	2-6 ↑	17-7 ←	3-8 ←	22-9 ↑
219	8-11←		17-11↑		10-10↑	7-8 ←	21-9 ↑	12-10↑	
220	10-10↑			16-11←	3-10↑	11-10↑		2-11↑	17-12←
221	11-9 ↑	17-9 ↑		21-12↑	18-12←	2-11←	16-12←	7+11←	21+10↑
222	13-8 ↑		9-12↑		13+11←	6+11↑	20+10↑	11+9 ↑	
223	15-7 ↑			3+11←	8+10↑	11+9 ↑		1+8 ↑	16+7 ←
224	16-6 ↑			8+10↑	3+9 ↑	1+8 ←	15+7 ↑	6+6 ←	20+5 ↑
225	18-5 ↑		1+11←	14+9 ←	18+7 ←	5+6 ↑	20+5 ←	10+4 ↑	
226	19-4 ←			19+8 ↑	13+6 ↑	10+4 ←		0+3 ↑	15+2 ←
227	21-3 ←		16+10↑		8+5 ↑	0+3 ↑	14+2 ↑	5+1 ↑	19+0 ←
228	23-2 ↑			1+7 ←	13+4 ↑	5+1 ↑	19+0 ←	3-1 ↑	23-2 ↑
229				6+6 ↑	18+2 ↑	9-1 ↑	23-2 ↑	14-3 ←	
230	0-1 ←		8+9 ←	12+5 ←	13+1 ↑	14-3 ←		4-4 ←	18-5 ←
231	2-0 ↑			17+4 ↑	8+0 ↑	4-4 ←	18-5 ↑	8-6 ↑	22-7 ↑
232	4-1 ↑			23+3 ↑	4-1 ↑	9-6 ←	23-7 ←	13-8 ←	
233	5+2 ↑		0+8 ←		18-3 ↑	13-8 ↑		3-9 ←	17-10←
234	7+3 ↑			5+2 ←	13-4 ↑	8-9 ↑	18-10←	7-11↑	21-12↑
235	8+4 ←		15+7 ↑	10+1 ↑	9-5 ←	8-11←	22-12↑	11+11↑	
236	10+5 ←	22-8 ↑		16+0 ←	4-6 ←	12+11↑		2+10←	16+9 ←
237	12+6 ↑			21-1 ↑	18-8 ↑	3+10←	17+9 ↑	6+8 ↑	20+7 ↑
238	13+7 ↑		7+6 ←		13-9 ↑	7+8 ↑	21+7 ↑	10+6 ↑	
239	15+8 ↑			3-2 ←	9-10←	12+6 ←		1+5 ←	15+4 ←
240	17+9 ↑		22+5 ↑	8-3 ↑	4-11←	2+5 ←	16+4 ↑	5+3 ←	19+2 ↑

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	J	K	L	M	N	2N	O	OO	P
241	18+10←			14-4←	18+11↑	6+3↑	21+2←	9+1↑	2+6↑
242	20+11↑			19-5↑	14+10←	11+1←	14-1←	0+0←	15+7↑
243	21+12←				9+9←	1+0↑	15-1↑	4-2←	4+8↑
244	23-11←		14+4↑	1-6←	4+8↑	6-2←	20-3←	8-4↑	6+10←
245				6-7↑	18+6↑	10-4↑	13-6←	11-10↑	9+12↑
246	1-10↑		6+3←	12-8←	14+5←	0-5↑	15-6←	3-7←	13-8←
247	2-9←			17-9↑	9+4←	5-7←	19-8↑	7-9↑	16-6↑
248	4-8↑			23-10←	4+3↑	9-9↑	12-11←	5-5↑	18-4↑
249	6-7↑		21+2↑	23-10←	19+1←	0-10←	14-11←	2-12←	20-2↑
250	7-6←			4-11↑	14+0←	4-12↑	15+11↑	6+10↑	23-0↑
251	9-5↑		13+1←	10-12←	9-1↑	9+10←	23+9←	11+8←	12+1↑
252	10-4←	3-7←		16+11←	4-2↑	13+8↑	15+6↑	1+7←	14+3←
253	12-3←			21+10↑	0-3←	3+7↑	18+6←	5+5↑	16+5←
254	14-2↑		5+0←		14-5←	8+5←	22+4↑	10+3←	19+7↑
255	15-1←			3+9←	9-6↑	12+3↑		0+2←	21+9←
256	17-0↑		20-1↑	8+8↑	4-7↑	3+2←	17+1←	4+0↑	10+10←
257	18+1←			14+7↑	0-8←	7+0↑	21-1↑	9-2←	23+11←
258	20+2←			19+6↑	14-10↑	12-2←		13-4←	13+12↑
259	22+3↑		12-2←		9-11↑	2-3←	16-4↑	3-5↑	2-11↑
260	23+4←			1+5←	5-12←	6-5↑	21-6←	8-7←	15-10←
261				6+4↑	19+10↑	11-7←		12-9←	4-9←
262	1+5↑		3-3↑	12+3←	14+9↑	1-8←	15-9↑	2-10↑	7-8←
263	3+6↑			17+2↑	9+8↑	6-10←	20-11←	6-12↑	22-8←
264	4+7←		19-4↑	23+1←	5+7←	10-12↑		11+10←	9-5←
265	6+8↑				0+6←	0+11↑	15+10←	1+9↑	11-3←
266	7+9←			4+0↑	14+4↑	5+9←	19+8↑	5+7↑	0-2←
267	9+10←	8-6←	11-5←	10-1←	10+3←	10+7↑		10+5←	14-1↑
268	11+11↑			15-2↑	5+2←	0+6←	14+5↑	0+4↑	16+1↑
269	12+12↑			21-3←	0+1↑	4+4↑	19+3←	4+2↑	18+3←
270	14-11↑		2-6↑		15-1←	9+2↑	23+1↑	9+0←	5+2←
271	16-10↑			2-4↑	10-2←	13+0↑		12-9←	22-4←
272	17-9↑		18-7←	8-5←	5-3←	4-1←	18-2←	3-3↑	11-3←
273	19-8↑			14-6←	0-4↑	8-3↑	22-4↑	8-5←	14-1↑
274	20-7←			19-7↑	15-6←	13-5←		12-7↑	16+1↑
275	22-6←		10-8←		10-7←	3-6←	17-7↑	2-8↑	18+3←
276				1-8←	5-8↑	7-8↑	22-9←	7-10←	15-5←
277	0-5↑			6-9↑	0-9↑	12-10←		11-12↑	4-2←
278	1-4←		1-9↑	12-10←	15-11←	2-11↑	16-12↑	1+11↑	18-1↑
279	3-3↑			17-11↑	10-12↑	7+11↑	21+10←	6+9←	20+1↑
280	5-2↑		17-10←	23-12←	5+11↑	11+9↑		10+7↑	22+3←
281	6-1←				0+10↑	1+8↑	16+7←	0+6↑	14+6↑
282	8-0↑	13-5←		4+11↑	15+8←	6+6←	20+5↑	5+4←	16+8←
283	9+1←		8-11↑	10+10←	10+7↑	10+4↑		9+2←	18+10←
284	11+2←			15+9↑	5+6↑	1+3←	15+2←	13+0←	21+12↑
285	13+3↑			21+8←	1+5←	5+1↑	19+0↑	4-1←	23-10←
286	14+4←		0-12↑		15+3↑	10-1←		8-3←	12-9←
287	16+5↑			2+7↑	10+2↑	0-2←	14-3↑	12-5↑	1-8←
288	18+6↑		16+11←	8+6←	6+1←	4-4↑	19-5←	3-6←	15-7↑
289	19+7←			13+5↑	1+0←	9-6←	23-7↑	7-8←	17-5↑
290	21+8↑			19+4←	15-2↑	13-8↑		11-10↑	19-3←
291	22+9←		7+10↑		10-3↑	4-9←	18-10←	1-11↑	22-1↑
292				0+3↑	6-4←	8-11↑	22-12↑	6+11←	11-0↑
293	0+10←		23+9←	6+2↑	1-5←	13+11←		10+9←	0+1↑
294	2+11↑			12+1←	15-7↑	3+10←	17+9↑	0+8↑	13+2←
295	3+12←			17+0↑	11-8←	7+8↑	22+7←	5+6←	15+4←
296	5-11↑		15+8←	23-1←	6-9←	12+6←		9+4↑	12+11↑
297	6-10←	19-4↑			1-10↑	2+5↑	16+4↑	14+2←	1+12↑
298	8-9←			4-2↑	15-12↑	7+3←	21+2←	4+1←	14-11↑
299	10-8↑		6+7↑	10-3←	11+11←	11+1↑		8-1↑	16-9←
300	11-7←			15-4↑	6+10←	1+0↑	16-1←	13-3←	19-7↑
301	13-6↑		22+6←	21-5←	1+9↑	6-2←	20-3↑	3-4←	22-5↑
302	15-5↑				16+7←	10-4↑		7-6↑	10-4←
303	16-4←			2-6↑	11+6←	1-5←	15-6↑	11-8↑	12-2←
304	18-3↑		13+5↑	8-7←	6+5↑	5-7↑	20-8←	2-9←	15-0↑
305	19-2←			13-8↑	1+4↑	10-9↑		6-11↑	17+2←
306	21-1←			19-9←	16+2←	0-10↑	14-11↑	10+11↑	4+1↑
307	23-0↑		5+4↑		11+1←	5-12←	19+11←	1+10←	6+3←
308				0-10↑	6+0↑	9+10↑	23+9↑	5+8↑	9+5↑
309	0+1←		21+3←	6-11←	1-1←	14+8←		9+6↑	13+9←
310	2+2↑			11-12↑	16-3←	4+7←	18+6↑	0+5←	16+11↑
311	4+3↑			17+11←	11-4↑	8+5↑	23+4←	4+3←	18-11↑
312	5+4←		12+2↑	23+10←	6-5←	13+3←		8+1↑	20-9←
313	7+5↑	0-3↑			2-6←	3+2←	17+1↑	13-1↑	23-7↑
314	8+6←			4+9↑	16-8↑	8+0←	22-1←	3-2←	15-0↑
315	10+7←		4+1←	10+8←	11-9↑	12-2←		7-4↑	14-4←
316	12+8↑			15+7↑	6-10↑	2-3↑	17-4←	12-6←	17-2←
317	13+9←		20+0←	21+6←	2-11←	7-5←	21-6↑	2-7←	19-0↑
318	15+10↑				16+11↑	11-7↑		6-9↑	21+2←
319	17+11↑			2+5↑	11+10↑	2-8←	16-9←	11-11←	10+3←
320	18+12←		11-1↑	8+4←	7+9←	6-10↑	20-11↑	1-12←	0+4↑

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	J	K	L	M	N	2N	O	OO	P
321	20-11 ↑			13+3 ↑	2+8 ← 21+7 ↑	11-12 ←	5+10 ↑ 19+9 ↑	4+8 ← 17+9 ←	
322	21-10 ←			19+2 ←	16+6 ↑	1+11 ← 15+10 ↑	10+8 ←	7+10 ↑ 20+11 ↑	
323	23-9 ←		3-2 ←		12+5 ←	5+9 ↑ 20+8 ←	0+7 ← 14+6 ↑	9+12 ↑ 22-11 ←	
324				0+1 ↑	7+4 ←	10+7 ←	4+5 ↑ 18+4 ↑	11-10 ←	
325	1-8 ↑		18-3 ↑	6+0 ←	2+3 ↑ 21+2 ↑	0+6 ↑ 14+5 ↑	9+3 ← 23+2 ←	0-9 ← 14-8 ↑	
326	2-7 ←			11-1 ↑	16+1 ↑	5+4 ← 19+3 ←	13+1 ←	3-7 ↑ 16-6 ↑	
327	4-6 ↑			17-2 ←	12+0 ←	9+2 ↑ 23+1 ↑	3+0 ↑ 17-1 ↑	5-5 ← 18-4 ↑	
328	6-5 ↑	5-2 ←	10-4 ↑	22-3 ↑	7-1 ←	14+0 ←	8-2 ← 22-3 ←	7-3 ← 21-2 ↑	5+2 ↑
329	7-4 ←			2-2 ↑	21-3 ↑	4-1 ← 18-2 ↑	12-4 ←	10-1 ↑ 23-0 ↑	
330	9-3 ↑			4-4 ←	17-4 ←	8-3 ↑ 23-4 ←	2-5 ↑ 16-6 ↑	12+1 ←	
331	10-2 ←		2-5 ←	9-5 ↑	12-5 ←	13-5 ←	7-7 ← 21-8 ←	1+2 ← 14+3 ←	
332	12-1 ←			15-6 ←	7-6 ←	3-6 ↑ 17-7 ↑	4+1 ← 17-5 ↑	4+4 ↑ 17+5 ↑	
333	14-0 ↑		17-6 ↑	21-7 ←	2-7 ↑ 21-8 ↑	8-8 ← 22-9 ←	1-10 ↑ 15-11 ↑	6+6 ↑ 19+7 ←	
334	15+1 ←			17-9 ←	12-10 ←	12-10 ↑	5-12 ↑ 20+11 ←	8-8 ← 21-9 ←	
335	17+2 ↑			2-8 ↑	12-10 ←	2-11 ↑ 17-12 ←	10+10 ←	11+10 ↑	
336	18+3 ←		9-7 ←	8-9 ←	7-11 ↑	7+11 ← 21+10 ↑	0+9 ↑ 14+8 ↑	0+11 ↑ 13+12 ↑	
337	20+4 ←			13-10 ↑	2-12 ↑ 22+11 ←	11+9 ↑	4+7 ↑ 19+6 ↑	2-11 ← 15-10 ←	
338	22+5 ↑			19-11 ←	17+10 ←	2+8 ← 16+7 ↑	9+5 ← 23+4 ←	4-9 ← 18-8 ↑	
339	23+6 ←		1-8 ←		12+9 ↑	6+6 ↑ 21+5 ←	13+3 ↑	7-7 ↑ 20-6 ↑	
340				0-12 ↑	7+8 ↑	11+4 ←	3+2 ↑ 18+1 ←	9-5 ← 22-4 ←	
341	1+7 ↑		16-9 ↑	6+11 ←	3+7 ← 22+6 ←	1+3 ↑ 15+2 ↑	8+0 ← 22-1 ←	11-3 ←	
342	3+8 ↑			17+5 ↑	17+5 ↑	6+1 ↑ 20+0 ←	12-2 ↑	1-2 ↑ 14-1 ↑	
343	4+9 ↑	10-1 ←		17+9 ↑	12+4 ↑	10-1 ↑	2-3 ↑ 17-4 ←	3-0 ↑ 16+1 ←	10+1 ↑
344	6+10 ↑		8-10 ←	22+8 ↑	7+3 ↑	0-2 ↑ 15-3 ←	7-5 ← 21-6 ←	5-2 ← 18+3 ←	
345	7+11 ←				3+2 ← 22+1 ←	5-4 ← 19-5 ↑	11-7 ↑	8+4 ↑ 21+5 ↑	
346	9+12 ←		23-11 ↑	4+7 ←	17+0 ↑	9-6 ↑	1-8 ↑ 15-9 ↑	10+6 ↑ 23+7 ←	
347	11-11 ↑			9+6 ↑	12-1 ↑	0-7 ↑ 14-8 ←	6-10 ← 20-11 ←	12+8 ←	
348	12-10 ←			15+5 ←	8-2 ←	4-9 ↑ 18-10 ↑	10-12 ↑	1-9 ← 15+10 ↑	
349	14-9 ↑		15-12 ↑	20+4 ↑	3-3 ← 22-4 ↑	9-11 ← 23-12 ←	0+11 ↑ 14+10 ↑	4+11 ↑ 17+12 ↑	
350	16-8 ↑				17-5 ↑	13+11 ↑	5+9 ← 19+8 ←	6-11 ← 19-10 ←	
351	17-7 ←			2+3 ←	13-6 ←	3+10 ↑ 18+9 ←	9+7 ↑ 23+6 ↑	8-9 ← 22-8 ↑	
352	19-6 ↑		7+11 ←	7+2 ↑	8-7 ←	8+8 ↑ 22+7 ↑	13+5 ↑	11-7 ↑	
353	20-5 ←			13+1 ↑	3-8 ← 22-9 ↑	12+6 ↑	4+4 ← 18+3 ←	0-6 ↑ 13-5 ←	
354	22-4 ←		22+10 ↑	17-10 ↑	19+0 ←	3+5 ← 17+4 ←	8+2 ← 22+1 ↑	2-4 ← 15-3 ←	
355				13-11 ←		7+3 ↑ 21+2 ↑	12+0 ↑	5-2 ↑ 18-1 ↑	
356	0-3 ↑			0-1 ↑	8-12 ←	12+1 ←	3-1 ← 17-2 ←	7-0 ↑ 20+1 ←	
357	1-2 ↑		14+9 ←	6-2 ←	3+11 ↑ 22+10 ↑	2+0 ← 16-1 ↑	7-3 ← 21-4 ↑	9+2 ← 22+3 ←	
358	3-1 ↑	16-0 ↑		11-3 ↑	18+9 ←	6-2 ↑ 21-3 ←	11-5 ↑	12+4 ↑	16+0 ←
359	5-0 ↑			17-4 ←	13+8 ←	11-4 ←	2-6 ← 16-7 ←	1+5 ↑ 14+6 ↑	
360	6+1 ←		6+8 ←	22-5 ↑	8+7 ↑	1-5 ↑ 15-6 ↑	6-8 ← 20-9 ↑	3+7 ← 16+8 ←	
361	8+2 ↑				3+6 ↑ 22+5 ↑	6-7 ← 20-8 ←	10-10 ↑	5+9 ← 19+10 ↑	
362	9+3 ↑		21+7 ↑	4-6 ←	18+4 ←	10-9 ↑	0-11 ↑ 15-12 ←	8+11 ↑ 21+12 ↑	
363	11+4 ↑			9-7 ↑	13+3 ←	0-10 ↑ 15-11 ←	5+11 ← 19+10 ↑	10-11 ← 23-10 ←	
364	13+5 ↑			15-8 ←	8+2 ↑	5-12 ← 19+11 ↑	9+9 ↑ 23+8 ↑	12-9 ←	
365	14+6 ←		13+6 ←	20-9 ↑	3+1 ↑ 23+0 ←	9+10 ↑	14+7 ←	2-8 ↑ 15-7 ↑	
366	16+7 ↑				18-1 ←	0+9 ← 14+8 ←	4+6 ← 18+5 ←	4-6 ↑ 17-5 ←	
367	18+8 ↑			2-10 ←	13-2 ↑	4+7 ↑ 18+6 ↑	8+4 ↑ 22+3 ↑	6-4 ← 19-3 ←	
368	19+9 ←		4+5 ↑	7-11 ↑	8-3 ↑	9+5 ← 23+4 ←	13+2 ←	9-2 ↑ 22-1 ↑	
369	21+10 ↑			13-12 ←	4-4 ← 23-5 ←	13+3 ↑	3+1 ← 17+0 ←	11-0 ↑	
370	22+11 ←		20+4 ↑	18+11 ↑	18-6 ←	3+2 ↑ 18+1 ←	7-1 ↑ 21-2 ↑	0+1 ← 13+2 ←	
371					13-7 ↑	8+0 ← 22-1 ↑	12-3 ←	2+3 ← 16+4 ↑	

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series	Q			2 Q			R	T	λ	μ OF 2 MS	
1	5-1 ↑	15-2 ←		4-1 ←	11-2 ←	18-3 ←				8-1 ←	23-2 ←
2	0-3 ←	9-4 ↑	19-5 ←	1-4 ←	8-5 ←	15-6 ←	22-7 ←		4-1 ↑	13-3 ↑	
3	4-6 ↑	13-7 ↑	23-8 ←	5-8 ←	12-9 ←	19-10 ←				4-4 ↑	19-5 ←
4	8-9 ↑	17-10 ↑		2-11 ←	9-12 ←	16+11 ←	23+10 ←		11-2 ↑	10-6 ←	
5	3-11 ←	12-12 ↑	22+11 ←	6+9 ←	13+8 ←	20+7 ←				0-7 ↑	15-8 ↑
6	7+10 ←	16+9 ↑		3+6 ←	10+5 ←	17+4 ←			18-3 ↑	6-9 ↑	21-10 ←
7	2+8 ←	11+7 ↑	20+6 ↑	0+3 ←	7+2 ←	14+1 ←	21+0 ←			12-11 ←	
8	6+5 ←	15+4 ↑		4-1 ←	11-2 ←	18-3 ←				2-12 ↑	17+11 ↑
9	1+3 ←	10+2 ←	19+1 ↑	1-4 ←	8-5 ←	15-6 ←	22-7 ←		1-4 ↑	8+10 ←	23+9 ←
10	5+0 ←	14-1 ←	23-2 ↑	5-8 ←	12-9 ←	19-10 ←				13+8 ↑	
11	9-3 ←	18-4 ↑		2-11 ←	9-12 ←	16+11 ←	23+10 ←		8-5 ↑	4+7 ↑	19+6 ←
12	3-5 ←	13-6 ↑	22-7 ↑	6+9 ←	13+8 ←	20+7 ←				10+5 ←	
13	8-8 ←	17-9 ↑		3+6 ←	10+5 ←	17+4 ←			16-6 ←	0+4 ↑	15+3 ↑
14	2-10 ↑	12-11 ←	21-12 ↑	0+3 ←	7+2 ←	14+1 ←	21+0 ←			6+2 ←	21+1 ↑
15	6+11 ↑	16+10 ←		4-1 ←	11-2 ←	18-3 ←			23-7 ←	11+0 ↑	
16	1+9 ↑	11+8 ↑	20+7 ←	1-4 ←	8-5 ←	15-6 ←	21-7 ↑	6+1 ↑	6-1 ←	2-1 ↑	17-2 ↑
17	5+6 ↑	15+5 ←		4-8 ↑	11-9 ↑	18-10 ↑				8-3 ←	23-4 ←
18	0+4 ←	9+3 ↑	19+2 ←	1-11 ↑	8-12 ↑	15+11 ↑	22+10 ↑		6-8 ←	13-5 ↑	
19	4+1 ↑	13+0 ↑	23-1 ←	5+9 ↑	12+8 ↑	19+7 ↑				4-6 ↑	19-7 ←
20	8-2 ↑	18-3 ←		2+6 ↑	9+5 ↑	16+4 ↑	23+3 ↑		13-9 ←	10-8 ←	
21	3-4 ←	12-5 ↑	22-6 ←	6+2 ↑	13+1 ↑	20+0 ↑				0-9 ↑	15-10 ↑
22	7-7 ↑	16-8 ↑		3-1 ↑	10-2 ↑	17-3 ↑			20-10 ↑	6-11 ←	21-12 ←
23	2-9 ↑	11-10 ↑	20-11 ↑	0-4 ↑	7-5 ↑	14-6 ↑	21-7 ↑			11+11 ↑	
24	6-12 ←	15+11 ↑		4-8 ↑	11-9 ↑	18-10 ↑				2+10 ↑	17+9 ←
25	1+10 ←	10+9 ↑	19+8 ↑	1-11 ↑	8-12 ↑	15+11 ↑	22+10 ↑		3-11 ↑	8+8 ←	22+7 ↑
26	5+7 ↑	14+6 ↑	23+5 ↑	5+9 ↑	12+8 ↑	19+7 ↑				13+6 ↑	
27	0+4 ←	9+3 ↑	19+2 ←	2+6 ↑	9+5 ↑	16+4 ↑	23+3 ↑		10-12 ↑	4+5 ↑	19+4 ←
28	4+2 ←	13+1 ↑	22+0 ↑	6+2 ↑	13+1 ↑	20+0 ↑				10+3 ↑	
29	8-1 ↑	17-2 ←		3-1 ↑	10-2 ↑	17-3 ↑			17+11 ↑	0+2 ↑	15+1 ↑
30	2-3 ↑	12-4 ←	21-5 ↑	0-4 ↑	7-5 ↑	14-6 ↑	21-7 ↑			6+0 ←	21-1 ←
31	6-6 ↑	16-7 ←		4-8 ↑	11-9 ↑	18-10 ↑				11-2 ↑	
32	1-8 ↑	11-9 ↑	20-10 ←	1-11 ↑	8-12 ↑	15+11 ↑	22+10 ←		0+10 ↑	2-3 ↑	17-4 ←
33	5-11 ↑	15-12 ←		5+9 ↑	12+8 ↑	19+7 ↑				8-5 ←	22-6 ↑
34	0+11 ↑	9+10 ↑	19+9 ←	2+6 ↑	9+5 ↑	16+4 ↑	23+3 ←		8+9 ←	13-7 ↑	
35	4+8 ↑	13+7 ↑	23+6 ←	6+2 ↑	13+1 ↑	20+0 ←				4-8 ←	19-9 ←
36	8+5 ↑	18+4 ↑		3-1 ↑	10-2 ↑	17-3 ←			15+8 ←	10-10 ←	
37	3+3 ↑	12+2 ↑	22+1 ←	0-4 ↑	7-5 ←	14-6 ←	21-7 ←			0-11 ↑	15-12 ↑
38	7+0 ↑	16-1 ↑		4-8 ↑	11-9 ↑	18-10 ←			22+7 ↑	6+11 ←	21+10 ←
39	2-2 ←	11-3 ↑	21-4 ←	1-11 ←	8-12 ←	15+11 ←	22+10 ←			11+9 ↑	
40	6-5 ←	15-6 ↑		5+9 ↑	12+8 ↑	19+7 ↑				2+8 ↑	17+7 ←
41	1-7 ←	10-8 ↑	19-9 ↑	2+6 ←	9+5 ←	16+4 ↑	23+3 ←			5+6 ←	
42	5-10 ←	14-11 ↑	23-12 ↑	6+2 ↑	13+1 ↑	20+0 ←				13+4 ↑	22+5 ↑
43	9+11 ←	18+10 ↑		3-1 ↑	10-2 ↑	17-3 ←				4+3 ↑	19+2 ←
44	4+9 ←	13+8 ←	22+7 ↑	0-4 ↑	7-5 ←	14-6 ←	21-7 ←			9+1 ↑	
45	8+6 ←	17+5 ↑		4-8 ↑	11-9 ↑	18-10 ←			19+4 ↑	0+0 ↑	15-1 ←
46	2+4 ↑	12+3 ←	21+2 ↑	1-11 ↑	8-12 ←	15+11 ←	22+10 ←	16+2 ←	16-2 ↑	6-2 ↑	21-3 ←
47	7+1 ↑	16+0 ←		5+9 ↑	12+8 ↑	19+7 ↑				11-4 ↑	
48	1-1 ↑	11-2 ←	20-3 ←	1+6 ↑	8+5 ↑	15+4 ↑	22+3 ↑			11-6 ↑	17-6 ←
49	5-4 ↑	15-5 ←		5+2 ↑	12+1 ↑	19+0 ↑			2+3 ↑	8-7 ↑	22-8 ↑
50	0-6 ↑	9-7 ↑	19-8 ←	2-1 ↑	9-2 ↑	16-3 ↑	23-4 ↑		9+2 ↑	13-9 ↑	
51	4-9 ↑	14-10 ←	23-11 ←	6-5 ↑	13-6 ↑	20-7 ↑				4-10 ←	19-11 ←
52	8-12 ↑	18+11 ←		3-8 ↑	10-9 ↑	17-10 ↑			16+1 ↑	9-12 ↑	
53	3+10 ↑	12+9 ↑	22+8 ←	0-11 ↑	7-12 ↑	14+11 ↑	21+10 ↑			0+11 ↑	15+10 ←
54	7+7 ↑	16+6 ↑		4+9 ↑	11+8 ↑	18+7 ↑				6+9 ↑	20+8 ↑
55	2+5 ←	11+4 ↑	21+3 ←	1+6 ↑	8+5 ↑	15+4 ↑	22+3 ↑		0+0 ←	11+7 ↑	
56	6+2 ←	15+1 ↑		5+2 ↑	12+1 ↑	19+0 ↑				2+6 ←	17+5 ←
57	1+0 ←	10-1 ↑	19-2 ↑	2-1 ↑	9-2 ↑	16-3 ↑	23-4 ↑		7-1 ←	8+4 ↑	22+3 ↑
58	5-3 ←	14-4 ↑		6-5 ↑	13-6 ↑	20-7 ↑				13+2 ↑	
59	0-5 ←	9-6 ↑	18-7 ↑	3-8 ↑	10-9 ↑	17-10 ↑			14-2 ←	4+1 ↑	19+0 ←
60	4-8 ←	13-9 ↑	22-10 ↑	0-11 ↑	7-12 ↑	14+11 ↑	21+10 ↑			9-1 ↑	
61	8-11 ←	17-12 ↑		4+9 ↑	11+8 ↑	18+7 ↑			21-3 ←	0-2 ↑	15-3 ←
62	2+11 ↑	12+10 ←	21+9 ↑	1+6 ↑	8+5 ↑	15+4 ↑	22+3 ↑			6-4 ↑	20-5 ↑
63	7+8 ↑	16+7 ↑		5+2 ↑	12+1 ↑	19+0 ↑				11-6 ↑	
64	1+6 ↑	11+5 ←	20+4 ↑	2-1 ↑	9-2 ↑	16-3 ↑	23-4 ←		4-4 ↑	2-7 ↑	17-8 ↑
65	5+3 ↑	15+2 ←		6-5 ↑	13-6 ↑	20-7 ↑				7-9 ↑	22-10 ↑
66	0+1 ↑	9+0 ↑	19-1 ←	3-8 ↑	10-9 ↑	17-10 ←			11-5 ↑	13-11 ↑	
67	4-2 ↑	14-3 ↑	23-4 ←	0-11 ↑	7-12 ←	14+11 ←	21+10 ←			4-12 ←	19+11 ←
68	8-5 ↑	18-6 ↑		4+9 ↑	11+8 ↑	18+7 ↑			18-6 ↑	9+10 ↑	
69	3-7 ↑	12-8 ↑	22-9 ←	1+6 ↑	8+5 ↑	15+4 ↑	22+3 ←			0+9 ↑	15+8 ↑
70	7-10 ↑	17-11 ↑		5+2 ↑	12+1 ↑	19+0 ↑				6+7 ↑	20+6 ↑
71	2-12 ←	11+11 ↑	21+10 ←	2-1 ↑	9-2 ↑	16-3 ↑	23-4 ←		1-7 ↑	11+5 ↑	
72	6+9 ↑	15+8 ↑		6-5 ↑	13-6 ↑	20-7 ↑				2+4 ↑	17+3 ↑
73	1+7 ↑	10+6 ↑	19+5 ↑	3-8 ↑	10-9 ↑	17-10 ←			8-8 ↑	7+2 ↑	22+1 ↑
74	5+4 ←	14+3 ↑		0-11 ↑	7-12 ←	14+11 ←	21+10 ←			13+0 ↑	
75	0+2 ←	9+1 ↑	18+0 ↑	4+9 ↑	11+8 ↑	18+7 ↑			16-9 ←	4-1 ↑	18-2 ↑
76	4-1 ←	13-2 ↑	22-3 ↑	1+6 ↑	8+5 ↑	15+4 ↑	22+3 ←			9-3 ↑	
77	8-4 ←	17-5 ↑		5+2 ↑	12+1 ↑	19+0 ↑		3+3 ↑	3-3 ←	0-4 ↑	15-5 ←
78	3-6 ←	12-7 ↑	21-8 ↑	2-1 ↑	9-2 ↑	16-3 ↑	23-4 ←			6-6 ←	20-7 ↑
79	7-9 ↑	16-10 ←		5-5 ↑	12-6 ↑	19-7 ↑				11-8 ↑	
80	1-11 ↑	11-12 ←	20+11 ↑	2-8 ↑	9-9 ↑	16-10 ↑	23-11 ↑		6-11 ←	2-9 ↑	17-10 ←

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series	Q			2 Q			R	T	λ	μ or 2 MS
81	5+10↑	15+9↑	6-12↑	13+11↑	20+10↑	7-11↑ 22-12↑
82	0+8↑	10+7↑	19+6↑	3+9↑	10+8↑	17+7↑	13-12↑	13+11↑
83	4+5↑	14+4↑	23+3↑	0+6↑	7+5↑	14+4↑	21+3↑	4+10↑ 18+9↑
84	8+2↑	18+1↑	4+2↑	11+1↑	18+0↑	20+11↑	9+8↑
85	3+0↑	12-1↑	22-2↑	1-1↑	8-2↑	15-3↑	22-4↑	0+7↑ 15+6↑
86	7-3↑	17-4↑	5-5↑	12-6↑	19-7↑	5+5↑ 20+4↑
87	2-5↑	11-6↑	21-7↑	2-8↑	9-9↑	16-10↑	23-11↑	3+10↑	11+3↑
88	6-8↑	15-9↑	6-12↑	13+11↑	20+10↑	2+2↑ 17+1↑
89	1-10↑	10-11↑	20-12↑	3+9↑	10+8↑	17+7↑	10+9↑	7+0↑ 22-1↑
90	5+11↑	14+10↑	0+6↑	7+5↑	14+4↑	21+3↑	13-2↑
91	0+9↑	9+8↑	18+7↑	4+2↑	11+1↑	18+0↑	17+8↑	4-3↑ 18-4↑
92	4+6↑	13+5↑	22+4↑	1-1↑	8-2↑	15-3↑	22-4↑	9-5↑
93	8+3↑	17+2↑	5-5↑	12-6↑	19-7↑	0-6↑ 15-7↑
94	3+1↑	12+0↑	21-1↑	2-8↑	9-9↑	16-10↑	23-11↑	0+7↑	5-8↑ 20-9↑
95	7-2↑	16-3↑	6-12↑	13+11↑	20+10↑	11-10↑
96	1-4↑	11-5↑	20-6↑	3+9↑	10+8↑	17+7↑	8+6↑	2-11↑ 17-12↑
97	6-7↑	15-8↑	0+6↑	7+5↑	14+4↑	21+3↑	7+11↑ 22+10↑
98	0-9↑	10-10↑	9-11↑	4+2↑	11+1↑	18+0↑	15+5↑	13+9↑
99	4-12↑	14+11↑	3+10↑	1-1↑	8-2↑	15-3↑	22-4↑	4+8↑ 18+7↑
100	8+9↑	18+8↑	5-5↑	12-6↑	19-7↑	22+4↑	9+6↑
101	3+7↑	13+6↑	22+5↑	2-8↑	9-9↑	16-10↑	23-11↑	0+5↑ 15+4↑
102	7+4↑	17+3↑	6-12↑	13+11↑	20+10↑	5+3↑ 20+2↑
103	2+2↑	11+1↑	21+0↑	3+9↑	10+8↑	17+7↑	5+3↑	11+1↑
104	6-1↑	15-2↑	0+6↑	7+5↑	14+4↑	21+3↑	2+0↑ 16-1↑
105	1-3↑	10-4↑	20-5↑	4+2↑	11+1↑	18+0↑	12+2↑	7-2↑ 22-3↑
106	5-6↑	14-7↑	1-1↑	8-2↑	15-3↑	22-4↑	13-4↑
107	0-8↑	9-9↑	18-10↑	5-5↑	12-6↑	19-7↑	4-5↑ 18-6↑
108	4-11↑	13-12↑	23+11↑	2-8↑	9-9↑	16-10↑	23-11↑	9-7↑
109	8+10↑	17+9↑	6-12↑	13+11↑	20+10↑	0-8↑ 15-9↑
110	3+8↑	12+7↑	21+6↑	3+9↑	10+8↑	16+7↑	23+6↑	2+0↑	5-10↑ 20-11↑
111	7+5↑	16+4↑	6+5↑	13+4↑	20+3↑	11-12↑
112	1+3↑	11+2↑	20+1↑	3+2↑	10+1↑	17+0↑	9-1↑	2+11↑ 16+10↑
113	6+0↑	15-1↑	0-1↑	7-2↑	14-3↑	21-4↑	7+9↑ 22+8↑
114	0-2↑	10-3↑	19-4↑	4-5↑	11-6↑	18-7↑	16-2↑	13+7↑
115	4-5↑	14-6↑	23-7↑	1-8↑	8-9↑	15-10↑	22-11↑	3+6↑ 18+5↑
116	8-8↑	18-9↑	5-12↑	12+11↑	19+10↑	9+4↑
117	3-10↑	13-11↑	22-12↑	2+9↑	9+8↑	16+7↑	23+6↑	0-3↑	0+3↑ 15+2↑
118	7+11↑	17+10↑	6+5↑	13+4↑	20+3↑	5+1↑ 20+0↑
119	2+9↑	11+8↑	21+7↑	3+2↑	10+1↑	17+0↑	7-4↑	11-1↑
120	6+6↑	16+5↑	0-1↑	7-2↑	14-3↑	21-4↑	2-2↑ 16-3↑
121	1+4↑	10+3↑	20+2↑	4-5↑	11-6↑	18-7↑	14-5↑	7-4↑ 22-5↑
122	5+1↑	14+0↑	1-8↑	8-9↑	15-10↑	22-11↑	13-6↑
123	0-1↑	9-2↑	18-3↑	5-12↑	12+11↑	19+10↑	3-7↑ 18-8↑
124	4-4↑	13-5↑	23-6↑	2+9↑	9+8↑	16+7↑	23+6↑	21-6↑	9-9↑
125	8-7↑	17-8↑	6+5↑	13+4↑	20+3↑	0-10↑ 14-11↑
126	3-9↑	12-10↑	21-11↑	3+2↑	10+1↑	17+0↑	4-7↑	5-12↑ 20+11↑
127	7-12↑	16+11↑	0-1↑	7-2↑	14-3↑	21-4↑	11+10↑
128	2+10↑	11+9↑	20+8↑	4-5↑	11-6↑	18-7↑	11-8↑	2+9↑ 16+8↑
129	6+7↑	15+6↑	1-8↑	8-9↑	15-10↑	22-11↑	7+7↑ 22+6↑
130	0+5↑	10+4↑	19+3↑	5-12↑	12+11↑	19+10↑	18-9↑	13+5↑
131	4+2↑	14+1↑	23+0↑	2+9↑	9+8↑	16+7↑	23+6↑	3+4↑ 18+3↑
132	9-1↑	18-2↑	6+5↑	13+4↑	20+3↑	9+2↑
133	3-3↑	13-4↑	22-5↑	3+2↑	10+1↑	17+0↑	11-10↑	0+1↑ 14+0↑
134	7-6↑	17-7↑	0-1↑	7-2↑	14-3↑	21-4↑	5-1↑ 20-2↑
135	2-8↑	11-9↑	21-10↑	4-5↑	11-6↑	18-7↑	8-11↑	11-3↑
136	6-11↑	16-12↑	1-8↑	8-9↑	15-10↑	22-11↑	1-4↑ 16-5↑
137	1+11↑	10+10↑	20+9↑	5-12↑	12+11↑	19+10↑	15-12↑	7-6↑ 22-7↑
138	5+8↑	14+7↑	2+9↑	9+8↑	16+7↑	23+6↑	0+5↑	0-5↑	13-8↑
139	0+6↑	9+5↑	19+4↑	6+5↑	13+4↑	20+3↑	23+11↑	3-9↑ 18-10↑
140	4+3↑	13+2↑	23+1↑	3+2↑	10+1↑	17+0↑	9-11↑
141	8+0↑	17-1↑	0-1↑	7-2↑	14-3↑	20-4↑	0-12↑ 14+11↑
142	3-2↑	12-3↑	21-4↑	3-5↑	10-6↑	17-7↑	6+10↑	5+10↑ 20+9↑
143	7-5↑	16-6↑	0-8↑	7-9↑	14-10↑	21-11↑	11+8↑
144	2-7↑	11-8↑	20-9↑	4-12↑	11+11↑	18+10↑	13+9↑	1+7↑ 16+6↑
145	6-10↑	15-11↑	1+9↑	8+8↑	15+7↑	22+6↑	7+5↑ 22+4↑
146	0-12↑	10+11↑	19+10↑	5+5↑	12+4↑	19+3↑	20+8↑	12+3↑
147	5+9↑	14+8↑	23+7↑	2+2↑	9+1↑	16+0↑	23-1↑	3+2↑ 18+1↑
148	9+6↑	18+5↑	6-2↑	13-3↑	20-4↑	9+0↑
149	3+4↑	13+3↑	22+2↑	3-5↑	10-6↑	17-7↑	3+7↑	0-1↑ 14-2↑
150	7+1↑	17+0↑	0-8↑	7-9↑	14-10↑	21-11↑	5-3↑ 20-4↑
151	2-1↑	12-2↑	21-3↑	4-12↑	11+11↑	18+10↑	10+6↑	11-5↑
152	6-4↑	16-5↑	1+9↑	8+8↑	15+7↑	22+6↑	1-6↑ 16-7↑
153	1-6↑	10-7↑	20-8↑	5+5↑	12+4↑	19+3↑	17+5↑	7-8↑ 22-9↑
154	5-9↑	14-10↑	2+2↑	9+1↑	16+0↑	23-1↑	12-10↑
155	0-11↑	9-12↑	19+11↑	6-2↑	13-3↑	20-4↑	3-11↑ 18-12↑
156	4+10↑	13+9↑	23+8↑	3-5↑	10-6↑	17-7↑	0+4↑	9+11↑
157	8+7↑	17+6↑	0-8↑	7-9↑	14-10↑	21-11↑	0+10↑ 14+9↑
158	3+5↑	12+4↑	22+3↑	4-12↑	11+11↑	18+10↑	7+3↑	5+8↑ 20+7↑
159	7+2↑	16+1↑	1+9↑	8+8↑	15+7↑	22+6↑	11+6↑
160	2+0↑	11-1↑	20-2↑	5+5↑	12+4↑	19+3↑	15+2↑	1+5↑ 16+4↑

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	Q			2 Q				R	T	λ	μ or 2 MS	
161	6-3	15-4	19-7	2+2	9+1	16+0	23-1				7+3	22+2
162	0-5	10-6	19-7	6-2	13-3	20-4				22+1	12+1	18-1
163	5-8	14-9	23-10	3-5	10-6	17-7					3+0	23-3
164	9-11	18-12		0-8	7-9	14-10	21-11				9-2	23-3
165	3+11	13+10	22+9	4-12	11+11	18+10				5+0	14-4	
166	7+8	17+7		1+9	8+8	15+7	22+6				5-5	20-6
167	2+6	12+5	21+4	5+5	12+4	19+3				12-1	11-7	16-9
168	6+3	16+2		2+2	9+1	16+0	23-1	10+6	10-6		1-8	22-11
169	1+1	10+0	20-1	6-2	13-3	20-4				19-2	7-10	22-11
170	5-2	15-3		3-5	10-6	17-7					12-12	
171	0-4	9-5	19-6	0-8	7-9	14-10	21-11				3+11	18+10
172	4-7	13-8	23-9	4-12	11+11	18+10				2-3	9+9	23+8
173	8-10	17-11		0+9	7+8	14+7	21+6				14+7	
174	3-12	12+11	22+10	4+5	11+4	18+3				9-4	5+6	20+5
175	7+9	16+8		1+2	8+1	15+0	22-1				10+4	
176	2+7	11+6	20+5	5-2	12-3	19-4				16-5	1+3	16+2
177	6+4	15+3		2-5	9-0	16-7	23-8				7+1	22+0
178	1+2	10+1	19+0	6-9	13-10	20-11				23-6	12-1	18-3
179	5-1	14-2	23-3	3-12	10+11	17+10					3-2	18-3
180	9-4	18-5		0+9	7+8	14+7	21+6				9-4	23-5
181	3-6	13-7	22-8	4+5	11+4	18+3				7-7	14-6	20-8
182	8-9	17-10		1+2	8+1	15+0	22-1				5-7	20-8
183	2-11	12-12	21+11	5-2	12-3	19-4				14-8	10-9	16-11
184	6+10	16+9		2-5	9-0	16-7	23-8				1-10	21+11
185	1+8	10+7	20+6	6-9	13-10	20-11				21-9	7-12	
186	5+5	15+4		3-12	10+11	17+10					12+10	18+8
187	0+3	9+2	19+1	0+9	7+8	14+7	21+6				3+9	23+6
188	4+0	13-1	23-2	4+5	11+4	18+3				4-10	9+7	23+6
189	8-3	18-4		1+2	8+1	15+0	22-1				14+5	20+3
190	3-5	12-6	22-7	5-2	12-3	19-4				11-11	5+4	
191	7-8	16-9		2-5	9-0	16-7	23-8				10+2	16+0
192	2-10	11-11	20-12	6-9	13-10	20-11				18-12	1+1	21-2
193	6+11	15+10		3-12	10+11	17+10					7-1	18-5
194	1+9	10+8	19+7	0+9	7+8	14+7	21+6				12-3	
195	5+6	14+5	23+4	4+5	11+4	18+3				1+11	3-4	23-5
196	9+3	18+2		1+2	8+1	15+0	22-1				8-6	23-7
197	3+1	13+0	22-1	5-2	12-3	19-4				8+10	14-8	20-10
198	8-2	17-3		2-5	9-0	16-7	23-8	21+7	21-7		5-9	10-11
199	2-4	12-5	21-6	6-9	13-10	20-11				15+9	10-11	16+11
200	6-7	16-8		3-12	10+11	17+10					1-12	
201	1-9	11-10	20-11	0+9	7+8	14+7	21+6			23+8	7+10	21+9
202	5-12	15+11		4+5	11+4	18+3					12+8	18+6
203	0+10	9+9	19+8	1+2	8+1	15+0	22-1				3+7	23+4
204	4+7	13+6	23+5	4-3	11+3	18-4				6+7	8+5	23+4
205	8+4	18+3		1-5	8-0	15-7	22-8				14+3	
206	3+2	12+1	22+0	5-9	12-10	19-11				13+6	5+2	19+1
207	7-1	16-2		2-12	9+11	16+10	23+9				10+0	16-2
208	2-3	11-4	21-5	6+8	13+7	20+6				20+5	1-1	21-4
209	6-0	15-7		3+5	10+4	17+3					7-3	21-4
210	1-8	10-9	19-10	0+2	7+1	14+0	21-1				12-5	
211	5-11	14-12	23+11	4-2	11-3	18-4				3+4	3-6	18-7
212	9+10	18+9		1-5	8-0	15-7	22-8				8-8	23-9
213	4+8	13+7	22+6	5-9	12-10	19-11				10+3	14-10	19-12
214	8+5	17+4		2-12	9+11	16+10	23+9				5-11	19+10
215	2+3	12+2	21+1	6+8	13+7	20+6				17+2	10+11	
216	6+0	16-1		3+5	10+4	17+3					1+10	16+9
217	1-2	11-3	20-4	0+2	7+1	14+0	21-1				7+8	21+7
218	5-5	15-6		4-3	11-3	18-4				0+1	12+6	18+4
219	0-7	9-8	19-9	1-5	8-0	15-7	22-8				3+5	23+2
220	4-10	14-11	23-12	5-9	12-10	19-11				7+0	8+3	
221	8+11	18+10		2-12	9+11	16+10	23+9				14+1	19-1
222	3+9	12+8	22+7	6+8	13+7	20+6				15-1	5+0	19-1
223	7+6	16+5		3+5	10+4	17+3					10-2	16-4
224	2+4	11+3	21+2	0+2	7+1	14+0	21-1			22-2	1-3	21-6
225	6+1	15+0		4-2	11-3	18-4					6-5	
226	1-1	10-2	19-3	1-5	8-0	15-7	22-8				12-7	18-9
227	5-4	14-5		5-9	12-10	19-11				5-3	3-8	23-11
228	0-6	9-7	18-8	2-12	9+11	16+10	23+9				8-10	19+10
229	4-9	13-10	22-11	6+8	13+7	20+6		7+8	7-8	12-4	14-12	
230	8-12	17+11		3+5	10+4	17+3					5+11	19+10
231	2+10	12+9	21+8	0+2	7+1	14+0	21-1			19-5	10+9	16+7
232	7+7	16+6		4-2	11-3	18-4					1+8	21+5
233	1+5	11+4	20+3	1-5	8-0	15-7	22-8				6+6	17+2
234	5+2	15+1		5-9	12-10	19-11				2-6	12+4	
235	0+0	9-1	19-2	2-12	9+11	16+10	23+9				3+3	23+0
236	4-3	14-4	23-5	5+8	12+7	19+6				9-7	8+1	14-1
237	8-0	18-7		2+5	9+4	16+3	23+2				5-2	19-3
238	3-8	12-9	22-10	6+1	13+0	20-1				16-8	10-4	16-6
239	7-11	17-12		3-2	10-3	17-4					1-5	
240	2+11	11+10	21+9	0-5	7-0	14-7	21-8			23-9		

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	Q			2 Q			R	T	λ	μ or 2 MS	
241	6+8 ←	15+7 ↑	4-9 ↑	11-10 ↑	18-11 ↑				6-7 ↑	21-8 ↑
242	1+6 ←	10+5 ↑	19+4 ↑	1-12 ↑	8+11 ↑	15+10 ↑	22+9 ↑			12-9 ←
243	5+3 ←	14+2 ↑	5+8 ↑	12+7 ↑	19+6 ↑			3-10 ←	17-11 ↑
244	0+1 ←	9+0 ←	18-1 ↑	2+5 ↑	9+4 ↑	16+3 ↑	23+2 ↑		7-10 ←	8-12 ↑	23+11 ←
245	4-2 ←	13-3 ↑	22-4 ↑	6+1 ↑	13+0 ↑	20-1 ↑		14-11 ←	14+10 ←
246	8-5 ←	7-6 ↑	3-2 ↑	10-3 ↑	17-4 ↑			4+9 ↑	19+8 ↑
247	2-7 ↑	12-8 ←	21-9 ↑	0-5 ↑	7-8 ↑	14-7 ↑	21-8 ↑		21-12 ←	10+7 ↑
248	7-10 ←	16-11 ←	4-9 ↑	11-10 ↑	18-11 ↑			1+6 ←	16+5 ←
249	1-12 ↑	11+11 ←	20+10 ↑	1-12 ↑	8+11 ↑	15+10 ↑	22+9 ↑			6+4 ↑	21+3 ↑
250	5+9 ↑	15+8 ↑	5+8 ↑	12+7 ↑	19+6 ↑		4+11 ←	12+2 ←
251	0+7 ↑	10+6 ←	19+5 ←	2+5 ←	9+4 ←	16+3 ←	23+2 ←			3+1 ↑	17+0 ↑
252	4+4 ↑	14+3 ←	23+2 ↑	6+1 ←	13+0 ←	20-1 ←		11+10 ←	8-1 ↑	23-2 ←
253	8+1 ↑	18+0 ←	3-2 ↑	10-3 ↑	17-4 ↑			14-3 ←
254	3-1 ↑	12-2 ↑	22-3 ←	0-5 ←	7-8 ←	14-7 ←	21-8 ←		18+9 ↑	4-4 ↑	19-5 ↑
255	7-4 ↑	17-5 ←	4-9 ↑	11-10 ←	18-11 ←			10-6 ←
256	2-6 ←	11-7 ↑	21-8 ←	1-12 ←	8+11 ←	15+10 ←	22+9 ←			1-7 ←	15-8 ↑
257	6-9 ↑	15-10 ↑	5+8 ↑	12+7 ↑	19+6 ↑		1+8 ↑	6-9 ↑	21-10 ↑
258	1-11 ←	10-12 ↑	20+11 ←	2+5 ←	9+4 ←	16+3 ←	23+2 ←			12-11 ←
259	5+10 ←	14+9 ↑	6+1 ←	13+0 ←	20-1 ←	18+9 ↑	18-9 ←	8+7 ↑	17+11 ↑
260	0+8 ←	9+7 ←	18+6 ↑	3-2 ←	10-3 ←	17-4 ←			8+10 ↑	23+9 ←
261	4+5 ←	13+4 ↑	22+3 ↑	0-5 ←	7-6 ←	14-7 ←	21-8 ←		15+6 ↑	14+8 ←
262	8+2 ↑	17+1 ↑	4-9 ↑	11-10 ←	18-11 ←			4+7 ↑	19+6 ↑
263	3+0 ←	12-1 ←	21-2 ↑	1-12 ←	8+11 ←	15+10 ←	22+9 ←		22+5 ↑	10+5 ←
264	7-3 ↑	16-4 ↑	5+8 ↑	12+7 ↑	19+6 ↑			1+4 ←	15+3 ↑
265	1-5 ↑	11-6 ←	20-7 ↑	2+5 ←	9+4 ←	16+3 ←	23+2 ←			6+2 ↑	21+1 ←
266	5-8 ↑	15-9 ←	6+1 ←	13+0 ←	19-1 ←			6+4 ←	12+0 ←
267	0-10 ↑	10-11 ←	19-12 ←	2-2 ↑	9-3 ↑	16-4 ↑	23-5 ↑			2-1 ↑	17-2 ↑
268	4+11 ↑	14+10 ←	23+9 ↑	6-6 ↑	13-7 ↑	20-8 ↑		13+3 ←	8-3 ↑	23-4 ←
269	8+8 ↑	18+7 ←	3-9 ↑	10-10 ↑	17-11 ↑			14-5 ←
270	3+6 ↑	13+5 ←	22+4 ←	0-12 ↑	7+11 ↑	14+10 ↑	21+9 ↑		20+2 ←	4-6 ↑	19-7 ↑
271	7+3 ↑	17+2 ←	4+8 ↑	11+7 ↑	18+6 ↑			10-8 ←
272	2+1 ←	11+0 ↑	21-1 ←	1+5 ↑	8+4 ↑	15+3 ↑	22+2 ↑			1-9 ←	15-10 ↑
273	6-2 ↑	15-3 ↑	5+1 ↑	12+0 ↑	19-1 ↑		3+1 ←	6-11 ↑	21-12 ←
274	1-4 ←	10-3 ↑	20-6 ←	2-2 ↑	9-3 ↑	16-4 ↑	23-5 ↑			12+11 ←
275	5-7 ←	14-8 ↑	6-6 ↑	13-7 ↑	20-8 ↑		10+0 ↑	2+10 ↑	17+9 ↑
276	0-9 ←	9-10 ↑	18-11 ↑	3-9 ↑	10-10 ↑	17-11 ↑			8+8 ←	23+7 ←
277	4-12 ←	13+11 ↑	23+10 ←	0-12 ↑	7+11 ↑	14+10 ↑	21+9 ↑		17-1 ↑	13+6 ↑
278	8+9 ↑	17+8 ↑	4+8 ↑	11+7 ↑	18+6 ↑			4+5 ↑	19+4 ↑
279	3+7 ←	16+6 ←	21+5 ↑	1+5 ↑	8+4 ↑	15+3 ↑	22+2 ↑			10+3 ←
280	7+4 ←	16+3 ↑	5+1 ↑	12+0 ↑	19-1 ↑		0-2 ↑	1+2 ←	15+1 ↑
281	1+2 ↑	11+1 ←	20+0 ↑	2-2 ↑	9-3 ↑	16-4 ↑	23-5 ↑			6+0 ↑	21-1 ←
282	6-1 ←	15-2 ←	6-6 ↑	13-7 ↑	20-8 ↑			12-2 ←
283	0-3 ↑	10-4 ←	19-5 ↑	3-9 ↑	10-10 ←	17-11 ←		7-3 ↑	2-3 ↑	17-4 ↑
284	4-8 ↑	14-7 ←	23-8 ↑	0-12 ←	7+11 ←	14+10 ←	21+9 ←		14-4 ↑	8-5 ←	23-6 ←
285	8-9 ↑	18-10 ←	4+8 ↑	11+7 ↑	18+6 ↑			13-7 ↑
286	3-11 ↑	13-12 ←	22+11 ←	1+5 ←	8+4 ←	15+3 ←	22+2 ←		22-5 ←	4-8 ↑	19-9 ←
287	7+10 ↑	17+9 ↑	5+1 ↑	12+0 ↑	19-1 ↑			10-10 ←
288	2+8 ↑	11+7 ↑	21+6 ←	2-2 ↑	9-3 ↑	16-4 ↑	23-5 ←			1-11 ←	15-12 ↑
289	6+5 ↑	16+4 ←	6-6 ↑	13-7 ↑	20-8 ↑		5-6 ←	6+11 ↑	21-10 ←
290	1+3 ←	10+2 ↑	20+1 ←	3-9 ↑	10-10 ←	17-11 ←	4+10 ←	4-10 ↑	12+9 ←
291	5+0 ←	14-1 ↑	0-12 ←	7+11 ←	14+10 ←	21+9 ←			2+8 ↑	17+7 ↑
292	0-2 ←	9-3 ↑	18-4 ↑	4+8 ↑	11+7 ↑	18+6 ↑		12-7 ←	8+6 ←	23+5 ←
293	4-5 ←	13-6 ↑	23-7 ←	1+5 ↑	8+4 ↑	15+3 ↑	22+2 ←		19-8 ←	13+4 ↑
294	8-8 ↑	17-9 ↑	5+1 ↑	12+0 ↑	19-1 ↑			4+3 ↑	19+2 ←
295	3-10 ←	12-11 ↑	21-12 ↑	2-2 ↑	9-3 ↑	16-4 ↑	23-5 ←			10+1 ←
296	7+11 ←	16+10 ↑	6-6 ↑	13-7 ↑	20-8 ↑		2-9 ↑	0+0 ↑	15-1 ↑
297	1+9 ↑	11+8 ↑	20+7 ↑	3-9 ↑	10-10 ←	17-11 ←	23-12 ↑			6-2 ←	21-3 ←
298	6+6 ←	15+5 ←	6+11 ↑	13+10 ↑	20+9 ↑		9-10 ↑	12+4 ↑
299	0+4 ↑	10+3 ↑	19+2 ↑	3+8 ↑	10+7 ↑	17+6 ↑			2-5 ↑	17-6 ↑
300	4+1 ↑	14+0 ←	23-1 ↑	0+5 ↑	7+4 ↑	14+3 ↑	21+2 ↑		16-11 ↑	8-7 ←	23-8 ←
301	9-2 ←	18-3 ←	4+1 ↑	11+0 ↑	18-1 ↑			13-9 ↑
302	3-4 ↑	13-5 ←	22-6 ←	1-2 ↑	8-3 ↑	15-4 ↑	22-5 ↑		23-12 ↑	4-10 ↑	19-11 ←
303	7-7 ↑	17-8 ↑	5-6 ↑	12-7 ↑	19-8 ↑			10-12 ←
304	2-9 ↑	11-10 ↑	21-11 ←	2-9 ↑	9-10 ↑	16-11 ↑	23-12 ↑			0+11 ↑	15+10 ↑
305	6-12 ↑	16+11 ←	6+11 ↑	13+10 ↑	20+9 ↑		6+11 ↑	6+9 ↑	21+8 ←
306	1+10 ←	10+9 ↑	20+8 ←	3+8 ↑	10+7 ↑	17+6 ↑			11+7 ↑
307	5+7 ↑	14+6 ↑	0+5 ↑	7+4 ↑	14+3 ↑	21+2 ↑		14+10 ←	2+6 ↑	17+5 ←
308	0+5 ←	9+4 ↑	19+3 ←	4+1 ↑	11+0 ↑	18-1 ↑			8+4 ↑	23+3 ←
309	4+2 ←	13+1 ↑	23+0 ←	1-2 ↑	8-3 ↑	15-4 ↑	22-5 ↑		21+9 ←	13+2 ↑
310	8-1 ←	17-2 ↑	5-6 ↑	12-7 ↑	19-8 ↑			4+1 ↑	19+0 ←
311	3-3 ←	12-4 ↑	21-5 ↑	2-9 ↑	9-10 ↑	16-11 ↑	23-12 ↑			10-1 ←
312	7-6 ←	16-7 ↑	6+11 ↑	13+10 ↑	20+9 ↑		4+8 ←	0-2 ↑	15-3 ↑
313	2-8 ←	11-9 ↑	20-10 ↑	3+8 ↑	10+7 ↑	17+6 ↑			6-4 ↑	21-5 ←
314	6-11 ←	15-12 ↑	0+5 ↑	7+4 ↑	14+3 ↑	21+2 ←		11+7 ←	11-6 ↑
315	0+11 ↑	10+10 ←	19+9 ↑	4+1 ↑	11+0 ←	18-1 ←			2-7 ↑	17-8 ←
316	4+8 ↑	14+7 ←	23+6 ↑	1-2 ←	8-3 ←	15-4 ←	22-5 ←		18+6 ↑	8-9 ←	22-10 ↑
317	9+5 ←	18+4 ←	5-6 ←	12-7 ←	19-8 ←			13-11 ↑
318	3+3 ↑	13+2 ←	22+1 ↑	2-9 ←	9-10 ←	16-11 ←	23-12 ←			4-12 ↑	19+11 ←
319	7+0 ↑	17-1 ←	6+11 ←	13+10 ←	20+9 ←	15+11 ↑	15-11 ←	10+10 ←
320	2-2 ↑	12-3 ←	21-4 ←	3+8 ←	10+7 ←	17+6 ←			0+9 ↑	15+8 ↑

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series	Q			2 Q				R	T	Λ	μ or 2 MS	
321	6-5 ↑	16-6 ←	0+5 ←	7+4 ←	14+3 ←	21+2 ←	8+4 ↑	6+7 ←	21+6 ←
322	1-7 ←	10-8 ↑	20-9 ←	4+1 ←	11+0 ←	18-1 ←	11+5 ↑
323	5-10 ↑	14-11 ↑	1-2 ←	8-3 ←	15-4 ←	22-5 ←	15+3 ↑	2+4 ↑	17+3 ←
324	0-12 ←	9+11 ↑	19+10 ←	5-6 ←	12-7 ←	19-8 ←	8+2 ←	22+1 ↑
325	4+9 ←	13+8 ↑	23+7 ←	2-9 ←	9-10 ←	16-11 ←	23-12 ←	22+2 ↑	13+0 ↑
326	8+6 ↑	17+5 ↑	6+11 ←	13+10 ←	20+9 ←	4-1 ←	19-2 ←
327	3+4 ←	12+3 ↑	21+2 ↑	3+8 ←	10+7 ←	17+6 ←	9-3 ←
328	7+1 ←	16+0 ↑	0+5 ←	7+4 ←	14+3 ←	21+2 ←	6+1 ←	0-4 ←	15-5 ↑
329	2-1 ←	11-2 ←	20-3 ↑	4+1 ←	10+0 ↑	17-1 ↑	6-6 ←	21-7 ←
330	6-4 ←	15-5 ↑	0-2 ↑	7-3 ↑	14-4 ↑	21-5 ↑	13+0 ←	11-8 ↑
331	0-6 ↑	10-7 ←	19-8 ↑	4-6 ↑	11-7 ↑	18-8 ↑	2-9 ↑	17-10 ←
332	5-9 ←	14-10 ←	23-11 ↑	1-9 ↑	8-10 ↑	15-11 ↑	22-12 ↑	20-1 ←	8-11 ←	22-12 ↑
333	9-12 ←	18+11 ↑	5+11 ↑	12+10 ↑	19+9 ↑	13+11 ↑
334	3+10 ↑	13+9 ↑	22+8 ↑	2+8 ↑	9+7 ↑	16+6 ↑	23+5 ↑	4+10 ←	19+9 ←
335	7+7 ↑	17+6 ↑	6+4 ↑	13+3 ↑	20+2 ↑	3-2 ←	9+8 ↑
336	2+5 ↑	12+4 ↑	21+3 ←	3+1 ↑	10+0 ↑	17-1 ↑	0+7 ↑	15+6 ←
337	6+2 ↑	16+1 ↑	0-2 ↑	7-3 ↑	14-4 ↑	21-5 ↑	10-3 ←	6+5 ↑	20+4 ↑
338	1+0 ↑	10-1 ↑	20-2 ←	4-6 ↑	11-7 ↑	18-8 ↑	11+3 ↑
339	5-3 ←	15-4 ↑	1-9 ↑	8-10 ↑	15-11 ↑	22-12 ↑	17-4 ↑	2+2 ↑	17+1 ←
340	0-5 ←	9-6 ↑	19-7 ←	5+11 ↑	12+10 ↑	19+9 ↑	8+0 ←	22-1 ↑
341	4-8 ←	13-9 ↑	23-10 ←	2+8 ↑	9+7 ↑	16+6 ↑	23+5 ↑	13-2 ↑
342	8-11 ↑	17-12 ↑	6+4 ↑	13+3 ↑	20+2 ↑	0-5 ↑	4-3 ↑	19-4 ←
343	3+11 ←	12+10 ↑	22+9 ←	3+1 ↑	10+0 ↑	17-1 ↑	9-5 ↑
344	7+8 ←	16+7 ↑	0-2 ↑	7-3 ↑	14-4 ↑	21-5 ↑	7-6 ↑	0-6 ↑	15-7 ←
345	2+6 ←	11+5 ↑	20+4 ↑	4-6 ←	11-7 ←	18-8 ←	6-8 ←	20-9 ↑
346	6+3 ↑	15+2 ↑	1-9 ←	8-10 ←	15-11 ←	22-12 ←	14-7 ↑	11-10 ↑
347	0+1 ↑	10+0 ↑	19-1 ↑	5+11 ←	12+10 ←	19+9 ←	2-11 ←	17-12 ←
348	5-2 ←	14-3 ←	23-4 ↑	2+8 ←	9+7 ←	16+6 ←	23+5 ←	22-8 ←	8+11 ←	22+10 ↑
349	9-5 ←	18-6 ↑	6+4 ←	13+3 ←	20+2 ←	13+9 ↑
350	3-7 ↑	13-8 ←	22-9 ↑	3+1 ←	10+0 ←	17-1 ←	4+8 ←	19+7 ←
351	8-10 ←	17-11 ←	0-2 ←	7-3 ←	14-4 ←	21-5 ←	1+12 ←	1-12 ↑	5-9 ←	9+6 ↑
352	2-12 ↑	12+11 ←	21+10 ←	4-6 ←	11-7 ←	18-8 ←	0+5 ↑	15+4 ←
353	6+9 ↑	16+8 ↑	1-9 ←	8-10 ←	15-11 ←	22-12 ←	12-10 ←	6+3 ↑	20+2 ↑
354	1+7 ↑	10+6 ↑	20+5 ←	5+11 ←	12+10 ←	19+9 ←	11+1 ↑
355	5+4 ↑	15+3 ←	2+8 ←	9+7 ←	16+6 ←	23+5 ←	19-11 ←	2+0 ←	17-1 ←
356	0+2 ←	9+1 ↑	19+0 ←	6+4 ←	13+3 ←	20+2 ←	7-2 ↑	22-3 ↑
357	4-1 ↑	13-2 ↑	23-3 ←	3+1 ↑	10+0 ↑	17-1 ↑	13-4 ↑
358	8-4 ↑	18-5 ←	0-2 ↑	7-3 ↑	14-4 ↑	21-5 ↑	2-12 ←	4-5 ←	19-6 ←
359	3-6 ←	12-7 ↑	22-8 ←	4-6 ←	11-7 ←	18-8 ←	9-7 ↑
360	7-9 ←	16-10 ↑	1-9 ←	8-10 ←	14-11 ↑	21-12 ↑	9+11 ↑	0-8 ↑	15-9 ←
361	2-11 ←	11-12 ↑	20+11 ↑	4+11 ↑	11+10 ↑	18+9 ↑	6-10 ←	20-11 ↑
362	6+10 ←	15+9 ↑	1+8 ↑	8+7 ↑	15+6 ↑	22+5 ↑	16+10 ↑	11-12 ↑
363	1+8 ←	10+7 ←	19+6 ↑	5+4 ↑	12+3 ↑	19+2 ↑	2+11 ←	17+10 ←
364	5+5 ←	14+4 ↑	23+3 ↑	2+1 ↑	9+0 ↑	16-1 ↑	23-2 ↑	23+9 ↑	7+9 ↑	22+8 ↑
365	9+2 ←	18+1 ↑	6-3 ↑	13-4 ↑	20-5 ↑	13+7 ↑
366	3+0 ↑	13-1 ←	22-2 ↑	3-6 ↑	10-7 ↑	17-8 ↑	4+6 ←	18+5 ↑
367	8-3 ←	17-4 ←	0-9 ↑	7-10 ↑	14-11 ↑	21-12 ↑	6+8 ↑	9+4 ↑
368	2-5 ↑	12-6 ←	21-7 ↑	4+11 ↑	11+10 ↑	18+9 ↑	0+3 ←	15+2 ←
369	6-8 ↑	16-9 ←	1+8 ↑	8+7 ↑	15+6 ↑	22+5 ↑	14+7 ←	6+1 ←	20+0 ↑
370	1-10 ↑	11-11 ←	20-12 ←	5+4 ↑	12+3 ↑	19+2 ↑	11-1 ↑
371	5+11 ↑	15+10 ←	2+1 ↑	9+0 ↑	16-1 ↑	23-2 ↑	21+6 ←	2-2 ←	17-3 ←

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	ν	ρ	MK	2MK	MN	MS	2SM
1	11-1 ←	5-1 ↑	15-2 ↑	11-1 ↑	15+1 ←
2	7-2 ←	1-3 ↑	11-4 ←	21-5 ←	0-1 ←	21+2 ↑
3	3-3 ←	6-6 ←	16-7 ↑	7-3 ←
4	19-5 ↑	2-8 ↑	12-9 ←	22-10 ←	4-4 ↑	2+3 ←
5	15-6 ↑	8-11 ←	17-12 ↑	2-5 ↑	8+4 ↑
6	12-7 ←	3+11 ↑	13+10 ←	23+9 ←	0-6 ←	22-7 ←	13+5 ←
7	8-8 ←	9+8 ↑	18+7 ↑	19-8 ↑	19+6 ↑
8	4-9 ←	4+6 ↑	14+5 ↑	17-9 ↑
9	0-10 ↑	0+4 ↑	10+3 ←	19+2 ↑	16-5 ↑	0+7 ←
10	16-12 ↑	5+1 ↑	15+0 ↑	12-11 ↑	6+8 ↑
11	13+11 ←	1-1 ←	11-2 ←	21-3 ←	10-12 ↑	12+9 ↑
12	9+10 ←	6-4 ↑	16-5 ↑	8+11 ←	17+10 ←
13	5+9 ←	2-6 ↑	12-7 ←	22-8 ←	6+10 ←	2-5 ↑	23+11 ↑
14	1+8 ←	7-9 ↑	17-10 ↑	3+9 ↑
15	17+6 ↑	3-11 ↑	13-12 ←	23+11 ←	1-8 ←	13-6 ↑	4+12 ←
16	14+5 ←	9+10 ←	18+9 ↑	2+8 ↑	10-11 ↑
17	10+4 ←	4+8 ↑	14+7 ↑	1+7 ↑	15-10 ←
18	6+3 ←	0+6 ↑	10+5 ↑	19+4 ↑	0+6 ↑	23+5 ↑	21-9 ↑
19	2+2 ←	5+3 ↑	15+2 ↑	14+3 ↑
20	18+0 ↑	1+1 ↑	11+0 ↑	20-1 ↑	11+2 ↑	11-8 ↑	2-8 ←
21	15-1 ←	6-2 ↑	16-3 ↑	9+1 ←	21+2 ↑	8-7 ↑
22	11-2 ←	2-4 ↑	12-5 ←	22-6 ←	7+0 ↑	21+1 ↑	13-6 ←
23	7-3 ←	7-7 ↑	17-8 ↑	4-1 ↑	20+0 ↑	19-5 ↑
24	3-4 ↑	3-9 ↑	13-10 ←	23-11 ←	2-2 ↑	19-1 ↑	10-10 ←
25	19-6 ↑	8-12 ↑	18+11 ↑	0-3 ←	18-2 ↑	0-4 ←
26	16-7 ←	4+10 ↑	14+9 ↑	19-5 ↑	6-3 ↑
27	12-8 ←	0+8 ↑	9+7 ↑	19+6 ↑	17-6 ↑	11-2 ←
28	8-9 ←	5+5 ↑	15+4 ↑	15-7 ↑	17-1 ↑
29	4-10 ↑	1+3 ↑	11+2 ↑	20+1 ↑	12-8 ↑	8-12 ←
30	0-11 ↑	6+0 ↑	16-1 ↑	10-9 ↑	22-0 ←
31	17+11 ←	2-2 ↑	12-3 ←	21-4 ↑	8-10 ←	4+1 ↑
32	13+10 ←	7-5 ↑	17-6 ↑	5-11 ↑	10+2 ↑
33	9+9 ↑	3-7 ↑	13-8 ↑	22-9 ↑	3-12 ↑	15+3 ↑
34	5+8 ↑	8-10 ↑	18-11 ↑	1+11 ←	23+10 ←	21+4 ↑
35	1+7 ↑	4-12 ↑	14+11 ←	20-9 ↑
36	18+5 ←	0+10 ←	9+9 ↑	19+8 ↑	18-8 ↑	2+5 ←
37	14+4 ←	5+7 ↑	15+6 ↑	16+7 ↑	8+6 ↑
38	10+3 ↑	1+5 ↑	10+4 ↑	20+3 ↑	13+6 ↑	13+7 ↑
39	6+2 ↑	6+2 ↑	16+1 ↑	11+5 ↑	19+8 ↑
40	2+1 ↑	2+0 ↑	12-1 ↑	21-2 ↑	9+4 ←
41	19-1 ←	7-3 ↑	17-4 ←	7+3 ←	0+9 ←
42	15-2 ←	3-5 ↑	13-6 ↑	22-7 ↑	4+2 ↑	6+10 ↑
43	11-3 ↑	8-8 ↑	18-9 ↑	2+1 ↑	11+11 ←
44	7-4 ↑	4-10 ↑	14-11 ↑	23-12 ↑	0+0 ↑	21-1 ↑	17+12 ↑
45	3-5 ↑	9+11 ↑	19+10 ↑	19-2 ↑	22-11 ←
46	0-6 ←	5+9 ↑	15+8 ↑	17-3 ←
47	16-8 ←	1+7 ↑	10+6 ↑	20+5 ↑	15-4 ↑	4-10 ↑
48	12-9 ↑	6+4 ↑	16+3 ↑	12-5 ↑	9-9 ↑
49	8-10 ↑	2+2 ↑	11+1 ↑	21+0 ↑	10-6 ↑	15-8 ↑
50	4-11 ↑	7-1 ↑	17-2 ↑	8-7 ↑	20-7 ←
51	1-12 ←	3-3 ↑	12-4 ↑	22-5 ↑	5-8 ↑
52	17+10 ←	8-6 ↑	18-7 ↑	3-9 ↑	2-6 ←
53	13+9 ↑	4-8 ↑	14-9 ↑	23-10 ↑	1-10 ←	22-11 ↑	8-5 ↑
54	9+8 ↑	9-11 ↑	19-12 ↑	20-12 ↑	13-4 ←
55	5+7 ↑	5+11 ↑	15+10 ↑	18+11 ←	19-3 ↑
56	2+6 ←	0+9 ↑	10+8 ↑	20+7 ↑	16+10 ←
57	18+4 ←	6+6 ↑	16+5 ↑	13+9 ↑	0-2 ←
58	14+3 ↑	2+4 ↑	11+3 ↑	21+2 ↑	11+8 ↑	6-1 ↑
59	10+2 ↑	7+1 ↑	17+0 ↑	9+7 ↑	11-0 ←
60	6+1 ↑	3-1 ↑	12-2 ↑	22-3 ↑	6+6 ↑	17+1 ↑
61	3+0 ←	8-4 ↑	18-5 ↑	4+5 ↑	22+2 ←
62	19-2 ←	4-6 ↑	13-7 ↑	23-8 ↑	2+4 ↑
63	15-3 ↑	9-9 ↑	19-10 ↑	0+3 ←	21+2 ↑	4+3 ↑
64	11-4 ↑	5-11 ↑	15-12 ↑	19+1 ↑	9+4 ↑
65	7-5 ↑	0+11 ↑	10+10 ↑	20+9 ↑	17+0 ←	15+5 ↑
66	4-6 ←	6+8 ↑	16+7 ↑	14-1 ↑	20+6 ←
67	0-7 ←	1+6 ↑	11+5 ↑	21+4 ↑	12-2 ↑
68	16-9 ↑	7+3 ↑	17+2 ↑	10-3 ↑	2+7 ↑
69	12-10 ↑	2+1 ↑	12+0 ↑	22-1 ↑	8-4 ↑	7+8 ↑
70	8-11 ↑	8-2 ↑	18-3 ↑	5-5 ↑	13+9 ↑
71	5-12 ←	4-4 ↑	13-5 ↑	23-6 ↑	3-6 ←
72	1+11 ←	9-7 ↑	19-8 ↑	1-7 ←	22-8 ↑
73	17+9 ↑	5-9 ↑	14-10 ↑	20-9 ↑	0+11 ←
74	13+8 ↑	0-11 ↑	10-12 ↑	20+11 ←	18-10 ←	6+12 ↑
75	9+7 ↑	6+10 ↑	15+9 ↑	16-11 ←	11-11 ←
76	6+6 ←	1+8 ↑	11+7 ↑	21+6 ←	13-12 ↑	17-10 ↑
77	2+5 ←	7+5 ↑	17+4 ↑	11+11 ←	22-9 ←
78	18+3 ↑	2+3 ↑	12+2 ↑	22+1 ←	9+10 ←
79	14+2 ↑	8+0 ↑	18-1 ↑	6+9 ↑	4-8 ↑
80	10+1 ↑	3-2 ↑	13-3 ↑	23-4 ↑	4+8 ↑	9-7 ←

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	ν	ρ	MK	2MK	MN	MS	2SM
81	7+0 ←	9-5 ←	19-6 ←	18+5 ↑	2+7 ←	2-11 ←	15-6 ↑
82	3-1 ←	5-7 ←	14-8 ↑	21+5 ↑	1-12 ↑	0-9 ←	20-5 ←
83	19-3 ↑	0-9 ↑	10-10 ←	19+4 ←	1+11 ←	11-10 ↑	2-4 ↑
84	15-4 ↑	6-12 ←	15+11 ↑	17+4 ←	0+10 ←	23+9 ↑	7-3 ←
85	12-5 ←	1+10 ↑	11+9 ↑	14+2 ↑	22+8 ↑	22-11 ↑	13-2 ↑
86	8-6 ←	7+7 ←	16+6 ↑	15+3 ←	12+1 ↑	22+7 ←	18-1 ←
87	4-7 ←	2+5 ↑	12+4 ↑	13+2 ←	10+0 ←	21+6 ←	17+3 ↑
88	0-8 ←	8+2 ←	18+1 ↑	11+1 ↑	7-1 ↑	20+5 ↑	22+4 ←
89	16-10 ↑	3+0 ↑	13-1 ↑	5-2 ↑	20+4 ←	9-12 ↑	0-0 ↑
90	13-11 ←	9-3 ←	19-4 ←	3-3 ←	19+3 ←	20+11 ↑	5-1 ←
91	9-12 ←	4-5 ↑	14-6 ↑	9+0 ↑	1-4 ←	18+2 ↑	11+2 ↑
92	5+11 ←	0-7 ↑	10-8 ←	20-6 ←	9-11 ←	17+1 ↑	17+3 ↑
93	1+10 ↑	5-10 ↑	15-11 ↑	18-7 ↑	17+0 ←	7+10 ↑	22+4 ←
94	17+8 ↑	1-12 ↑	11+11 ←	7-1 ↑	15-8 ↑	16-1 ↑	4+5 ↑
95	14+7 ←	7+9 ←	16+8 ↑	13-9 ↑	15-2 ↑	18+9 ↑	9+6 ←
96	10+6 ←	2+7 ↑	12+6 ↑	6-2 ←	11-10 ←	15-3 ←	15+7 ↑
97	6+5 ←	8+4 ←	17+3 ↑	4-3 ←	9-11 ←	14-4 ←	20+8 ←
98	2+4 ←	3+2 ↑	13+1 ↑	2-4 ←	13-5 ↑	5+8 ↑	2+9 ↑
99	18+2 ↑	9-1 ←	18-2 ↑	4+11 ←	12-6 ↑	16+7 ↑	9+6 ←
100	15+1 ←	4-3 ↑	14-4 ↑	2+10 ←	12-7 ←	2+9 ↑	15+7 ↑
101	11+0 ←	0-5 ←	10-6 ←	21+8 ↑	11-8 ↑	10-9 ↑	13+11 ↑
102	7-1 ←	5-8 ↑	15-9 ↑	0-5 ↑	10-9 ↑	10-10 ←	18+12 ←
103	3-2 ←	1-10 ↑	11-11 ←	22-6 ↑	16+6 ↑	4+6 ←	0-11 ↑
104	19-4 ↑	6+11 ↑	16+10 ↑	14+5 ←	9-11 ←	15+5 ←	5-10 ←
105	16-5 ←	2+9 ↑	12+8 ↑	20-7 ↑	8-12 ↑	11-9 ↑	16-8 ↑
106	12-6 ←	8+6 ←	17+5 ↑	10+3 ←	8+11 ←	5+7 ←	22-7 ↑
107	8-7 ←	3+4 ↑	13+3 ↑	7+2 ↑	7+10 ←	2+4 ←	11-0 ↑
108	4-8 ←	9+1 ↑	18+0 ↑	5+1 ↑	6+9 ↑	13+3 ←	16-8 ↑
109	0-9 ←	4-1 ↑	14-2 ↑	3+0 ←	5+8 ↑	13+3 ←	22-7 ↑
110	17-11 ←	0-3 ←	10-4 ←	0-1 ↑	5+7 ←	13+3 ←	3-6 ←
111	13-12 ←	5-6 ↑	15-7 ↑	15-10 ←	4+6 ←	3+5 ↑	9-5 ←
112	9+11 ←	1-8 ↑	11-9 ↑	18-4 ←	3+4 ↑	0+2 ←	15-4 ↑
113	5+10 ↑	6-11 ↑	16-12 ↑	13-11 ←	15-5 ↑	11+1 ←	20-3 ←
114	1+9 ↑	2+11 ←	12+10 ←	13-6 ←	2+3 ←	11+1 ←	2-2 ↑
115	18+7 ↑	7+8 ↑	17+7 ↑	11-12 ↑	1+2 ↑	22+0 ←	7-1 ←
116	14+6 ←	3+6 ↑	13+5 ↑	8-8 ↑	0+1 ↑	22+0 ←	13-0 ↑
117	10+5 ←	8+3 ↑	18+2 ↑	6-9 ↑	0+0 ←	23-1 ←	18+1 ←
118	6+4 ←	4+1 ↑	14+0 ←	4-10 ←	22-2 ↑	9-1 ←	2-2 ↑
119	2+3 ↑	0-1 ←	10-2 ←	2-11 ←	22-3 ←	9-1 ←	7+10 ↑
120	19+1 ←	5-4 ↑	15-5 ←	21+11 ←	21-4 ←	9-1 ←	13+0 ↑
121	15+0 ←	1-6 ←	11-7 ←	6+9 ←	19+10 ←	20-5 ↑	0+2 ↑
122	11-1 ←	6-9 ↑	16-10 ↑	16+8 ↑	19-6 ↑	20-2 ↑	5+3 ↑
123	7-2 ↑	2-11 ←	12-12 ←	14+8 ↑	19-7 ↑	11+4 ↑	16+5 ↑
124	3-3 ↑	7+10 ↑	17+9 ↑	12+7 ↑	18-8 ↑	7-3 ↑	22+6 ↑
125	20-5 ←	3+8 ←	13+7 ←	2+7 ←	10+6 ←	17-9 ↑	22+6 ↑
126	16-6 ←	8+5 ↑	18+4 ↑	0+6 ↑	7+5 ↑	17-10 ←	3+7 ↑
127	12-7 ←	4+3 ↑	14+2 ↑	22+5 ↑	5+4 ←	16-11 ←	9+8 ↑
128	8-8 ←	0+1 ←	9+0 ↑	22+5 ↑	3+3 ←	15-12 ↑	14+9 ↑
129	4-9 ←	5-2 ↑	15-3 ←	20+4 ↑	0+2 ←	14+11 ↑	20+10 ↑
130	0-10 ↑	1-4 ←	11-5 ←	20+4 ↑	20+0 ←	14+10 ←	20+10 ↑
131	17-12 ←	6-7 ↑	16-8 ←	17-1 ↑	13+9 ↑	16-6 ↑	2+11 ↑
132	13+11 ←	2-9 ↑	12-10 ←	15-2 ↑	12+8 ↑	12+7 ↑	7+12 ↑
133	9+10 ↑	7-12 ↑	17+11 ↑	13-3 ←	12+6 ←	11+6 ←	13-11 ↑
134	5+9 ↑	3+10 ←	13+9 ←	11-4 ↑	11+5 ↑	3-7 ↑	18-10 ←
135	1+8 ↑	8+7 ↑	18+6 ↑	8-5 ↑	10+5 ↑	3-7 ↑	18-10 ←
136	18+6 ←	4+5 ←	14+4 ←	15+1 ←	6-6 ↑	10+4 ←	0-9 ↑
137	14+5 ←	0+3 ←	9+2 ↑	13+0 ↑	4-7 ←	9+3 ↑	5-8 ←
138	10+4 ↑	5+0 ←	15-1 ←	21-10 ←	1-8 ↑	8+2 ↑	11-7 ↑
139	6+3 ↑	1-2 ←	10-3 ↑	11-1 ↑	7+1 ↑	7+0 ←	16-6 ←
140	3+2 ←	6-5 ↑	16-6 ←	11-1 ↑	19-11 ←	1-9 ↑	22-5 ↑
141	19+0 ←	2-7 ←	11-8 ↑	16-12 ↑	6-1 ←	5-2 ↑	13-10 ←
142	15-1 ↑	7-10 ↑	17-11 ←	9-2 ↑	14+11 ←	5-3 ↑	3-4 ↑
143	11-2 ↑	3-12 ←	13+11 ←	12+10 ←	9+9 ↑	4-4 ←	9-3 ↑
144	7-3 ↑	8+9 ↑	18+8 ↑	8-3 ←	7+8 ↑	3-5 ↑	14-2 ←
145	4-4 ←	4+7 ←	14+6 ←	23+5 ↑	4-7 ←	14-8 ↑	0-9 ↑
146	0-5 ←	9+4 ↑	19+3 ↑	6-4 ←	5+7 ←	2-6 ↑	20-1 ↑
147	16-7 ↑	5+2 ←	15+1 ↑	4-5 ←	3+6 ←	2-7 ←	1-0 ←
148	12-8 ↑	1+0 ←	10-1 ↑	20-2 ↑	20+5 ↑	1-8 ←	7+1 ↑
149	8-9 ↑	6-3 ←	16-4 ←	2-6 ↑	0-9 ↑	22+11 ←	12+2 ←
150	5-10 ←	2-5 ←	11-6 ↑	21-7 ↑	17+2 ↑	0-10 ←	18+3 ↑
151	1-11 ←	7-8 ↑	17-9 ↑	0-7 ↑	15+1 ↑	22-12 ↑	21+11 ↑
152	17+11 ↑	3-10 ←	12-11 ↑	22-12 ↑	13+0 ←	21+10 ←	9+10 ←
153	13+10 ↑	8+11 ↑	18+10 ←	22-8 ↑	10-1 ↑	20+9 ↑	0+4 ↑
154	9+9 ↑	4+9 ←	14+8 ←	23+7 ↑	8-2 ↑	20+9 ↑	5+5 ↑
155	6+8 ←	9+6 ↑	19+5 ←	21-9 ←	6-3 ←	19+8 ↑	11+6 ↑
156	2+7 ←	5+4 ←	15+3 ↑	19-10 ←	4-4 ←	19+7 ←	16+7 ↑
157	18+5 ↑	0+2 ↑	10+1 ↑	23-6 ↑	1-5 ↑	18+6 ←	22+8 ↑
158	14+4 ↑	6-1 ←	16-2 ←	21-7 ↑	18-8 ↑	17+5 ↑	3+9 ←
159	10+3 ↑	1-3 ↑	11-4 ↑	21-5 ↑	16-9 ↑	16+4 ←	9+10 ↑
160	7+2 ←	7-6 ←	17-7 ←	17-11 ←	16-9 ↑	16+3 ←	9+10 ↑

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series	v	p	MK	2 M K	M N	MS	2 S M
161	3+1 ←	23+0 ←	3-8 ←	12-9 ↑	22-10 ↑	15-12 ↑	14-11 ←
162	19-1 ↑	8-11 ←	18-12 ←	13+11 ↑	12-11 ←	14+1 ↑	20+12 ↑
163	15-2 ↑	4+11 ←	13+10 ↑	9-12 ↑	14+0 ←	5+6 ↑	1-11 ←
164	11-3 ↑	9+8 ↑	19+7 ←	7+11 ←	13-1 ←	16+5 ↑	7-10 ↑
165	8-4 ←	5+6 ←	14+5 ↑	11+10 ↑	12-2 ↑		
166	4-5 ←	0+4 ↑	10+3 ↑	20+2 ←	12-3 ←		12-9 ←
167	0-6 ←	6+1 ←	16+0 ←	10+9 ←	11-4 ←	3+4 ↑	18-8 ↑
168	16-8 ↑	1-1 ↑	11-2 ↑	21-3 ←	10-5 ↑		23-7 ←
169	12-9 ↑	7-4 ↑	17-5 ←	8+8 ←	9-6 ↑	14+3 ↑	5-6 ↑
170	9-10 ←	2-6 ↑	12-7 ↑	22-8 ↑	9-7 ←		
171	5-11 ←	8-9 ←	18-10 ←	6+7 ←	13+3 ←	8-8 ←	10-5 ←
172	1-12 ←	4-11 ←	13-12 ↑	23+11 ↑	10+2 ↑	7-9 ↑	16-4 ←
173	17+10 ↑	9+10 ←	19+9 ←	4+6 ↑	8+1 ↑	1+2 ↑	22-3 ↑
174	13+9 ↑	5+8 ←	14+7 ↑	2+5 ↑	6+0 ←	12+1 ↑	3-2 ←
175	10+8 ←	0+6 ↑	10+5 ↑		4-1 ←	5-12 ↑	
176	6+7 ←	6+3 ←	15+2 ↑	1-2 ↑	23-3 ←	4+11 ↑	9-1 ↑
177	2+6 ←	1+1 ↑	11+0 ↑	0+4 ↑	21-4 ←	4+10 ←	14-0 ←
178	18+4 ↑	7-2 ←	17-3 ←	22+3 ↑	18-5 ↑	3+9 ←	20+1 ↑
179	14+3 ↑	2-4 ↑	12-5 ↑	22-6 ←	16-6 ↑	2+8 ↑	10-1 ↑
180	11+2 ←	8-7 ↑	18-8 ←	21+2 ←	14-7 ←	2+7 ←	1+2 ←
181	7+1 ←	3-9 ↑	13-10 ↑	23-11 ↑	11-8 ↑	1+6 ←	7+3 ↑
182	3+0 ←	9-12 ←	19+11 ←	19+1 ←	9-9 ↑	0+5 ↑	12+4 ↑
183	19-2 ↑	4+10 ↑	14+9 ↑	7-10 ←	23+3 ←	23+4 ↑	18+5 ↑
184	15-3 ↑	0+8 ↑	10+7 ←	20+6 ←	5-11 ←	22+2 ↑	23+6 ←
185	12-4 ←	6+5 ←	15+4 ↑		2-12 ↑	21+1 ↑	
186	8-5 ←	1+3 ↑	11+2 ↑	21+1 ←	0+11 ↑	22+10 ←	5+7 ↑
187	4-6 ←	7+0 ←	16-1 ↑	15-1 ↑	19+9 ↑	21+0 ←	10+8 ↑
188	0-7 ↑	2-2 ↑	12-3 ↑	13-2 ↑	17+8 ↑	20-1 ←	16+9 ↑
189	16-9 ↑	8-5 ↑	17-8 ↑	11-3 ↑	15+7 ←	18-3 ↑	21+10 ←
190	13-10 ←	3-7 ↑	13-8 ↑	23-9 ←	13+6 ←	18-4 ←	
191	9-11 ←	9-10 ←	19-11 ←		10+5 ↑	17-5 ↑	18-6 ←
192	5-12 ←	4-12 ↑	14+11 ↑	10-4 ←	8+4 ←	16-6 ←	3+11 ↑
193	1+11 ↑	0+10 ↑	10+9 ←	20+8 ←	6+3 ←	16-7 ←	9-12 ↑
194	17+9 ↑	5+7 ↑	15+6 ↑	8-5 ←	3+2 ↑	15-8 ←	14-11 ←
195	14+8 ←	1+5 ↑	11+4 ←	21+3 ←	1+1 ↑	14-9 ↑	20-10 ↑
196	10+7 ←	7+2 ←	16+1 ↑	6-6 ←	21-1 ←	14-10 ←	1-9 ←
197	6+6 ←	2+0 ↑	12-1 ↑	22-2 ←	18-2 ↑	13-11 ←	7-8 ↑
198	2+5 ↑	8-3 ←	17-4 ↑	4-7 ↑	16-3 ←	12-12 ↑	12-7 ↑
199	19+3 ←	3-5 ↑	13-6 ↑	23-7 ←	14-4 ←	11+11 ↑	18-6 ↑
200	15+2 ←	9-8 ↑	18-9 ↑	2-8 ↑	11-5 ↑	11+10 ←	23-5 ←
201	11+1 ←	4-10 ↑	14-11 ↑		9-6 ↑	10+9 ←	14-10 ←
202	7+0 ↑	0-12 ←	10+11 ←	20+10 ←	7-7 ←	9+8 ↑	5-4 ↑
203	3-1 ↑	5+9 ↑	15-8 ↑	0-9 ↑	4-8 ↑	10-3 ↑	10-3 ↑
204	20-3 ←	1+7 ←	11+6 ←	21+5 ←	2-9 ↑	8+6 ←	1-11 ↑
205	16-4 ←	6+4 ↑	16+3 ↑	21-11 ←	0-10 ←	7+5 ↑	16-2 ↑
206	12-5 ←	2+2 ↑	12+1 ↑	22+0 ←	19-12 ↑	6+4 ↑	21-1 ←
207	8-6 ←	7-1 ↑	17-2 ↑	19-12 ←	17+11 ↑	6+3 ←	3-0 ↑
208	4-7 ↑	3-3 ↑	13-4 ←	23-5 ←	15+10 ←	5+2 ↑	8+1 ↑
209	0-8 ↑	9-8 ↑	18-7 ↑	17+11 ↑	12+9 ↑	4+1 ↑	14+2 ↑
210	17-10 ←	4-8 ↑	14-9 ←		10+8 ↑	4+0 ←	19+3 ↑
211	13-11 ←	0-10 ←	10-11 ←	19-12 ↑	8+7 ←	3-1 ←	10+10 ↑
212	9-12 ↑	5+11 ↑	15+10 ↑	13+9 ↑	6+6 ←	2-2 ↑	1+4 ↑
213	5+11 ↑	1+9 ←	11+8 ←	20+7 ↑	3+5 ↑	1-3 ↑	7+5 ↑
214	1+10 ↑	6+6 ↑	16+5 ↑	22+2 ←	1+4 ←	1-4 ←	12+6 ↑
215	18+8 ←	2+4 ←	12+3 ←	12+8 ←	20+2 ↑	0-5 ↑	18+7 ↑
216	14+7 ←	7+1 ↑	17+0 ↑		18+1 ↑	23-7 ←	23+8 ←
217	10+6 ↑	3-1 ↑	13-2 ↑	23-3 ←	16+0 ←	22-8 ←	5+9 ↑
218	6+5 ↑	8-4 ↑	18-5 ↑	10+7 ←	14-1 ←	21-9 ↑	16+10 ←
219	2+4 ↑	4-6 ↑	14-7 ←	8+6 ←	11-2 ↑	20-10 ↑	16+11 ↑
220	19+2 ←	0-8 ←	10-9 ←	19-10 ↑	9-3 ←	20-11 ←	
221	15+1 ←	5-11 ↑	15-12 ←	6+5 ↑	7-4 ←	19-12 ↑	6+6 ↑
222	11+0 ↑	1+11 ↑	11+10 ←	20+9 ↑	4-5 ↑	18+11 ↑	21+12 ←
223	7-1 ↑	6+8 ↑	16+7 ↑	4+4 ↑	2-6 ↑	18+10 ←	3-11 ↑
224	3-2 ↑	2+6 ←	12+5 ←	21+4 ↑	0-7 ←	17+9 ←	8-10 ←
225	0-3 ←	7+3 ↑	17+2 ↑	2+3 ↑	19-9 ↑	16+8 ↑	14-9 ↑
226	16-5 ←	3+1 ←	13+0 ←	23-1 ←	17-10 ←	16+7 ←	19-8 ←
227	12-6 ↑	8-2 ↑	18-3 ↑	1+2 ←	15-11 ←	15+6 ←	1-7 ↑
228	8-7 ↑	4-4 ←	14-5 ←	23+1 ←	12-12 ↑	14+5 ↑	6-6 ←
229	4-8 ↑	0-6 ←	9-7 ↑	19-8 ↑	10+11 ↑	13+4 ←	12-5 ↑
230	1-9 ↑	5-9 ↑	15-10 ←	21+0 ←	8+10 ←	13+3 ←	
231	17-11 ←	1-11 ←	10-12 ↑	20+11 ↑	5+9 ↑	12+2 ←	17-4 ←
232	13-12 ↑	6+10 ↑	16+9 ←	19-1 ↑	3+8 ↑	11+1 ↑	23-3 ←
233	9+11 ↑	2+8 ←	12+7 ←	21+6 ↑	1+7 ←	11+0 ←	
234	5+10 ↑	7+5 ↑	17+4 ←	17-2 ↑	20+5 ↑	10-1 ←	5-2 ↑
235	2+9 ←	3+3 ←	13+2 ←	22+1 ↑	18+4 ↑	9-2 ↑	10-1 ←
236	18+7 ←	8+0 ↑	18-1 ↑	15-3 ↑	16+3 ←	8-3 ↑	16-0 ↑
237	14+6 ↑	4-2 ←	14-3 ←	14-4 ←	13+2 ↑	8-4 ↑	21+1 ←
238	10+5 ↑	0-4 ←	9-5 ↑	19-6 ↑	11+1 ↑	7-5 ↑	12-1 ←
239	6+4 ↑	5-7 ←	15-8 ←	12-5 ←	9+0 ←	6-8 ↑	3+2 ↑
240	3+3 ←	1-9 ←	10-10 ↑		7-1 ←	6-7 ←	8+3 ←

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	ν	ρ	MK	2MK	MN	MS	2SM
241	19+1 ←	6-12 ↑	16+11 ←	4-2 ↑	5-8 ←		14+4 ↑
242	15+0 ↑	2+10 ←	11+9 ↑	10-6 ←	4-9 ↑		19+5 ←
243	11-1 ↑	7+7 ↑	17+6 ←	0-4 ←	3-10 ↑	10-3 ↑	1+6 ↑
244	7-2 ↑	3+5 ←	13+4 ←	8-7 ←	3-11 ←		6+7 ←
245	4-3 ←	8+2 ↑	18+1 ←	17-7 ←	2-12 ↑	21-4 ↑	
246	0-4 ←	4+0 ←	14-1 ←	6-8 ↑	1+11 ↑		12+8 ↑
247	16-6 ↑	9-3 ↑	19-4 ↑	12-9 ↑	1+10 ←		17+9 ←
248	12-7 ↑	5-5 ←	15-6 ←	4-9 ↑	0+9 ←	23+8 ↑	8-5 ↑
249	8-8 ↑	0-7 ↑	10-8 ↑	20-9 ↑	22+7 ↑		23+10 ↑
250	5-9 ←	6-10 ←	16-11 ←	2-10 ↑	22+6 ←	19-6 ↑	4+11 ←
251	1-10 ←	2-12 ←	11+11 ↑	21+10 ↑	3+11 ↑		10+12 ↑
252	17-12 ↑	7+9 ←	17+8 ←	1-11 ←	1+10 ←	22+9 ↑	16-11 ↑
253	13+11 ↑	3+7 ←	12+6 ↑	22+5 ↑	20+3 ←		21-10 ←
254	10+10 ←	8+4 ↑	18+3 ↑	23-12 ←	18+7 ←	6-7 ↑	
255	6+9 ←	4+2 ↑	13+1 ↑	23+0 ↑	16+6 ←	17-8 ↑	3-9 ↑
256	2+8 ←	9-1 ↑	19-2 ←		13+5 ↑		8-8 ←
257	18+6 ↑	5-3 ←	15-4 ←	19+10 ↑	11+4 ↑		14-7 ↑
258	14+5 ↑	0-5 ←	10-6 ←	20-7 ←	9+3 ←		19-6 ↑
259	11+4 ←	6-8 ←	16-9 ←	17+9 ↑	6+2 ↑	4-9 ↑	
260	7+3 ←	1-10 ↑	11-11 ↑	21-12 ↑	4+1 ↑	15-4 ←	1-5 ↑
261	3+2 ←	7+11 ←	17+10 ←	15+8 ↑	2+0 ←	14-5 ←	6-4 ←
262	19+0 ↑	3+9 ←	12+8 ↑	22+7 ↑	0-1 ←	13-6 ↑	12-3 ↑
263	15-1 ↑	8+6 ←	18+5 ←	14+7 ←	19-3 ↑	13-7 ←	17-2 ↑
264	12-2 ←	4+4 ←	13+3 ↑	23+2 ↑	17-4 ←	12-8 ←	23-1 ↑
265	8-3 ←	9+1 ↑	19+0 ←	12+6 ←	14-5 ↑	11-6 ↑	
266	4-4 ←	5-1 ←	14-2 ↑		12-6 ↑	10-10 ↑	4-0 ←
267	0-5 ↑	0-3 ↑	10-4 ↑	20-5 ←	10-7 ←	10-11 ←	10+1 ↑
268	16-7 ↑	6-6 ←	16-7 ←	10+5 ←	8-8 ←		15+2 ←
269	13-8 ←	1-8 ↑	11-9 ↑	21-10 ←	5-9 ↑	1+11 ←	21+3 ↑
270	9-9 ←	7-11 ←	17-12 ←	8+4 ↑	3-10 ←	8+10 ←	
271	5-10 ←	2+11 ↑	12+10 ↑	22+9 ↑	6+3 ↑	7+9 ↑	2+4 ↑
272	1-11 ↑	8+8 ←	18+7 ←	6+8 ←	20+11 ↑	6+8 ←	8+5 ↑
273	17+11 ↑	3+6 ←	13+5 ↑	23+4 ↑	4+2 ↑	18+10 ←	14+6 ↑
274	14+10 ←	9+3 ←	19+2 ←	16+9 ←	5+7 ↑	5+7 ↑	19+7 ↑
275	10+9 ←	5+1 ←	14+0 ↑	3+1 ←	13+8 ↑	4+5 ↑	
276	6+8 ←	0-1 ↑	10-2 ←	20-3 ←	11+7 ←	3+4 ↑	1+8 ↑
277	2+7 ←	6-4 ←	15-5 ↑	21-6 ←	9+6 ←	3+3 ←	6+9 ←
278	18+5 ↑	1-6 ↑	11-7 ↑	21-8 ←	6+5 ↑	2+2 ←	12+10 ↑
279	15+4 ←	7-9 ←	16-10 ↑	22-11 ←	4+4 ↑	1+1 ↑	17+11 ↑
280	11+3 ←	2-11 ↑	12-12 ↑	22+11 ←	2+3 ←	0+0 ↑	23+12 ↑
281	7+2 ←	8+10 ←	18+9 ←	23+6 ↑	21+1 ↑	0-1 ←	23-2 ↑
282	3+1 ↑	3+8 ↑	13+7 ↑	19+3 ↑	19+0 ←	22-3 ↑	19+5 ↑
283	19-1 ↑	9+5 ←	19+4 ←	17-4 ↑	17-1 ←	22-4 ←	4-11 ←
284	16-2 ←	4+3 ↑	14+2 ↑	20-1 ←	14-2 ↑	21-5 ←	10-10 ↑
285	12-3 ←	0+1 ↑	10+0 ←	17-4 ↑	12-3 ↑	20-6 ↑	15-9 ↑
286	8-4 ←	6-2 ←	15-3 ↑	16-5 ←	10-4 ←	20-7 ←	21-8 ↑
287	4-5 ↑	1-4 ↑	11-5 ←	21-6 ←	7-5 ←	19-8 ←	2-7 ←
288	0-6 ↑	7-7 ←	16-8 ↑	14-6 ←	5-6 ↑	18-9 ↑	8-6 ↑
289	17-8 ↑	2-9 ↑	12-10 ↑	22-11 ←	3-7 ←	17-10 ↑	13-5 ←
290	13-9 ←	8-12 ←	17+11 ↑	12-7 ←	1-8 ←	17-11 ←	19-4 ↑
291	9-10 ←	3+10 ↑	13+9 ↑	23+8 ←	20-10 ←	16-12 ←	
292	5-11 ↑	9+7 ←	19+6 ←	10-8 ↑	18-11 ←	15+11 ↑	0-3 ←
293	1-12 ↑	4+5 ↑	14+4 ↑	8-9 ↑	15-12 ↑	15+10 ←	6-2 ←
294	18+10 ←	0+3 ←	10+2 ←	20+1 ←	13+11 ↑	14+9 ←	12-1 ↑
295	14+9 ←	5+0 ↑	15-1 ↑		11+10 ←	13+8 ↑	17-0 ←
296	10+8 ←	1-2 ↑	11-3 ←	21-4 ←	9+9 ←	12+7 ↑	23+1 ↑
297	6+7 ↑	6-5 ↑	16-6 ↑	6-10 ↑	6+8 ↑	12+6 ←	
298	2+6 ↑	2-7 ↑	12-8 ←	22-9 ←	4+7 ←	11+5 ↑	4+2 ←
299	19+4 ←	8-10 ←	17-11 ↑	5-11 ←	2+6 ←	10+4 ↑	10+3 ↑
300	15+3 ←	3-12 ↑	13+11 ←	23+10 ←	3-12 ←	10+3 ←	15+4 ←
301	11+2 ←	9+9 ←	18+8 ↑		19+3 ←	9+2 ↑	21+5 ↑
302	7+1 ↑	4+7 ↑	14+6 ↑	1+11 ←	16+2 ↑	8+1 ↑	11-3 ↑
303	3+0 ↑	0+5 ←	10+4 ←	19+3 ↑	14+1 ↑	7+0 ↑	2+6 ←
304	20-2 ↑	5+2 ↑	15+1 ↑	23+10 ↑	12+0 ←	7-1 ↑	8+7 ↑
305	16-3 ←	1+0 ←	11-1 ←	21-2 ←	10-1 ←	6-2 ↑	13+8 ←
306	12-4 ↑	6-3 ↑	16-4 ↑		7-2 ↑	5-3 ↑	19+9 ↑
307	8-5 ↑	2-5 ↑	12-6 ←	22-7 ←	5-3 ←	5-4 ←	10-5 ←
308	4-6 ↑	7-8 ↑	17-9 ↑		3-4 ←	4-5 ←	0+10 ←
309	1-7 ↑	3-10 ↑	13-11 ←	23-12 ←	0-5 ↑	3-6 ↑	6+11 ↑
310	17-9 ←	9+11 ←	18+10 ↑		20-7 ←	2-7 ↑	11+12 ←
311	13-10 ↑	4+9 ↑	14+8 ←	16+6 ←	18-8 ←	2-8 ←	17-11 ↑
312	9-11 ↑	0+7 ←	10+6 ←	19+5 ↑	15-9 ↑	1-9 ↑	23-10 ↑
313	5-12 ↑	5+4 ↑	15+3 ↑		13-10 ↑	0-10 ↑	
314	2+11 ←	1+2 ↑	11+1 ↑	20+0 ↑	11-11 ←	0-11 ←	4-9 ←
315	18+9 ←	6-1 ↑	16-2 ↑	12+4 ←	8-12 ↑	22+11 ↑	10-8 ↑
316	14+8 ↑	2-3 ←	12-4 ←	22-5 ←	6+11 ↑	22+10 ←	15-7 ←
317	10+7 ↑	7-6 ↑	17-7 ↑		4+10 ←	21+9 ←	21-6 ↑
318	6+6 ↑	3-8 ←	13-9 ←	23-10 ←	2+9 ←	20+8 ↑	
319	3+5 ↑	8-11 ↑	18-12 ↑		21+7 ←	19+7 ↑	2-5 ←
320	19+3 ←	4+11 ↑	14+10 ←	8+2 ↑	19+6 ←	19+6 ←	8-4 ↑

TABLE 42.—Component hours derived from solar hours—Continued.

Day of series.	ν	ρ	MK	2MK	MN	MS	2SM
321	15+2 ↑	0+9 ← 9+8 ↑ 19+7 ↑	6+1 ↑	16+5 ↑	18+5 ←		13-3 ←
322	11+1 ↑	5+6 ↑ 15+5 ←		14+4 ↑	17+4 ↑	4-11 ←	19-2 ↑
323	7+0 ↑	1+4 ← 11+3 ←	5+0 ←	12+3 ←	17+3 ←		
324	4-1 ←	6+1 ↑ 16+0 ←		10+2 ←	16+2 ←	15-12 ↑	0-1 ←
325	0-2 ← 20-3 ←	2-1 ← 12-2 ←	3-1 ←	7+1 ↑	15+1 ↑		6-0 ↑
326	16-4 ↑	7-4 ↑ 17-5 ↑		5+0 ←	14+0 ↑		11+1 ←
327	12-5 ↑	3-6 ← 13-7 ←	1-2 ←	3-1 ←	14-1 ←	2+11 ↑	17+2 ↑
328	8-6 ↑	8-9 ↑ 18-10 ↑	23-3 ↑	0-2 ↑ 22-3 ↑	13-2 ↑		22+3 ↑
329	5-7 ←	4-11 ← 14-12 ←		20-4 ←	12-3 ↑	13+10 ↑	
330	1-8 ← 21-9 ←	0+11 ← 9+10 ↑	21-4 ↑	17-5 ↑	12-4 ←		4+4 ↑
331	17-10 ↑	5+8 ↑ 15+7 ←		15-6 ↑	11-5 ←		9+5 ←
332	13-11 ↑	1+6 ← 10+5 ↑	19-5 ↑	13-7 ←	10-6 ←	0+9 ↑	15+6 ↑
333	9-12 ↑	6+3 ↑ 16+2 ←		11-8 ←	9-7 ↑		21+7 ↑
334	6+11 ←	2+1 ← 12+0 ←	18-6 ←	8-9 ↑	9-8 ←	11+8 ↑	
335	2+10 ← 22+9 ←	7-2 ↑ 17-3 ←		6-10 ↑	8-9 ↑		2+8 ←
336	18+8 ↑	3-4 ← 13-5 ←	22-6 ↑	16-7 ←	4-11 ←	22+7 ↑	8+9 ↑
337	14+7 ↑	8-7 ↑ 18-8 ↑		1-12 ↑ 23+11 ↑	7-11 ←		13+10 ←
338	10+6 ↑	4-9 ← 14-10 ←	14-8 ←	21+10 ←	6-12 ←		19+11 ↑
339	7+5 ←	9-12 ↑ 19+11 ↑		19+9 ←	5+11 ↑	9+6 ↑	
340	3+4 ← 23+3 ←	5+10 ← 15+9 ←	12-9 ↑	16+8 ↑	5+10 ←		0+12 ←
341	19+2 ↑	1+8 ← 10+7 ↑	20+6 ↑	14+7 ←	4+9 ←	20+5 ↑	6-11 ↑
342	15+1 ↑	6+5 ← 16+4 ←	10-10 ↑	12+6 ←	3+8 ↑		11-10 ←
343	11+0 ↑	2+3 ← 11+2 ↑	21+1 ↑	9+5 ↑	2+7 ↑		17-9 ↑
344	8-1 ←	7+0 ↑ 17-1 ←	8-11 ↑	7+4 ↑	2+6 ←	8+4 ←	22-8 ←
345	4-2 ←	3-2 ← 12-3 ↑	22-4 ↑	5+3 ←	1+5 ←		
346	0-3 ← 20-4 ↑	8-5 ↑ 18-6 ←	7-12 ←	3+2 ←	0+4 ↑	19+3 ←	4-7 ↑
347	16-5 ↑	4-7 ↑ 14-8 ←	23-9 ↑	0+1 ↑ 22+0 ←	0+3 ← 23+2 ←		9-6 ←
348	12-6 ↑	9-10 ↑ 19-11 ←	5+11 ←	20-1 ←	22+1 ↑		15-5 ↑
349	9-7 ←	5-12 ← 15+11 ←		17-2 ↑	21+0 ↑	6+2 ←	20-4 ←
350	5-8 ←	0+10 ↑ 10+9 ↑	3+10 ←	15-3 ↑	21-1 ←		
351	1-9 ← 21-10 ↑	6+7 ← 16+6 ←		13-4 ←	20-2 ←	17+1 ←	2-3 ↑
352	17-11 ↑	2+5 ← 11+4 ↑	21+3 ↑	1+9 ↑	19-3 ↑		7-2 ←
353	13-12 ↑	7+2 ← 17+1 ←	23+8 ↑	10-5 ↑	19-4 ↑		13-1 ←
354	10+11 ←	3+0 ← 12-1 ↑	22-2 ↑	6-7 ←	18-5 ←	4+0 ←	19-0 ↑
355	6+10 ←	8-3 ↑ 18-4 ←	21+7 ↑	4-8 ←	17-6 ←		
356	2+9 ← 22+8 ↑	4-5 ← 13-6 ↑	23-7 ↑	1-9 ↑ 23-10 ↑	16-7 ↑	15-1 ←	0+1 ←
357	18+7 ↑	9-8 ↑ 19-9 ←	20+6 ←	21-11 ←	16-8 ←		6+2 ↑
358	14+6 ↑	5-10 ← 15-11 ←		18-12 ↑	15-9 ↑		11+3 ←
359	11+5 ←	0-12 ↑ 10+11 ↑	20+10 ←	16+11 ↑	14-10 ↑	2-2 ←	17+4 ↑
360	7+4 ←	6+9 ← 16+8 ←		14+10 ←	14-11 ←		22+5 ←
361	3+3 ← 23+2 ↑	1+7 ↑ 11+6 ↑	21+5 ↑	16+4 ←	13-12 ←	13-3 ←	
362	19+1 ↑	7+4 ← 17+3 ←		9+8 ↑	12+11 ↑		4+6 ↑
363	16+0 ↑	2+2 ← 12+1 ↑	22+0 ↑	14+3 ↑	11+10 ↑		9+7 ←
364	12-1 ←	8-1 ← 18-2 ←		5+6 ←	11+9 ←	0-4 ↑	15+8 ↑
365	8-2 ←	4-3 ← 13-4 ↑	23-5 ↑	12+2 ↑	10+8 ↑		20+9 ↑
366	4-3 ↑	9-6 ← 19-7 ←		0+4 ↑ 22+3 ←	9+7 ↑	11-5 ↑	
367	0-4 ↑ 20-5 ↑	5-8 ← 14-9 ↑	10+1 ↑	20+2 ←	9+6 ←		2+10 ↑
368	17-6 ↑	0-10 ↑ 10-11 ↑	20-12 ←	17+1 ↑	8+5 ←	22-6 ↑	7+11 ←
369	13-7 ←	6+11 ← 15+10 ↑	9+0 ←	15+0 ←	7+4 ↑		13+12 ↑
370	9-8 ←	1+9 ↑ 11+8 ↑	21+7 ←	13-1 ←	7+3 ←		18-11 ←
371	5-9 ↑	7+6 ← 17+5 ←	7-1 ←	10-2 ↑	6+2 ←	9-7 ↑	

Where one, and only one, hourly height is to go on each component hour, the arrow is used to indicate which hourly height to use. A horizontal arrow indicates that the hourly height belonging to the solar hour written is the one to be taken; an arrow pointing upward indicates that the hourly height belonging to the solar hour next preceding the solar hour written is the one to be taken. For the components J, K, OO, R, and 2 SM the value thus indicated is to be used twice. The group covered is obviously a solar and not a component hour. See §§ 53, 57, Part II.

Rules for constructing or verifying this table.

$$\text{Left-hand part of tabular value} = 1 + (d-1) \frac{15}{15 \sim c_1},$$

discarding the decimal even if it exceed 0.5; d is an integer such that

$$d = 24 \left[(\text{day of series} - 1) \div \frac{15}{15 \sim c_1} \right] \mp \text{right-hand part of tabular value, including sign.}$$

The quotient in the brackets is taken to the nearest integer generally. The upper sign is used when $c_1 < 15$; the lower, when $c_1 > 15$.

$c_1 < 15$. If the decimal of solar hour in the first equation above fall between 0.0 and 0.5, the arrow should be horizontal; if between 0.5 and 1.0, vertical.

$c_1 > 15$. Reverse this rule.

The speeds used are those derived from the mean motions given in § 13.

Instead of the above rules, the following may be used:

Suppose all solar hours of the series to have been converted into component hours; in each doubtful case mark that solar hour which lies nearest the component hour thus considered.

TABLE 43.—For the summation of long-period tides.

Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa
1	0	0	0	0	76	18	13←	18		151	12	2	11	
2	1	1	1		77	19	14	19←		152	13	3	12	
3	2	2	2		78	20	15	19		153	14	4	13	
4	3	3	3		79	21	16	20		154	15	5	14	
5	4	4←	4		80	22	17←	21		155	16	6	15	
6	5	4	5		81	23	17	22		156	17←	6←	15←	
7	6	5	6		82	24	18	23		157	18	7	16	
8	7←	6	7		83	0	19	0		158	19	8	17	
9	7	7	7←	0 1	84	1	20	1		159	20	9	18	10
10	8	8	8		85	2	21←	2←	6 5	160	21	10←	19	11
11	9	9	9		86	3	21	2		161	22	11	20	
12	10	9←	10		87	4	22	3		162	23	12	21	
13	11	10	11		88	5	23	4		163	24	13	22	
14	12	11	12		89	6	0	5		164	0←	14←	23	
15	13	12	13←		90	7←	1	6		165	1	15	0	
16	14←	13	14		91	8	2	7		166	2	16	1	
17	14	13←	15		92	9	3	8		167	3	17	2	
18	15	14	16		93	10	4	9←		168	4	18	3	
19	16	15	17		94	11	5	10		169	5	19	4←	
20	17	16	18		95	12	6←	11		170	6	20	5	
21	18	17←	19		96	13	7	12		171	7	21	6	
22	19	18	20←	1	97	14	8	13		172	8	22	7	
23	20	19	21	2	98	15	9	14		173	9	23	8	11
24	21	20	22		99	16	10←	15	6 7	174	10	24	9	12
25	22	21	23		100	17	11	16		175	11	25	10	
26	22←	22	23		101	18	12	17		176	12	26	11	
27	23	23	0		102	19	13	18		177	13	27	12	
28	0	23←	1		103	20	14	19		178	14	28	13	
29	1	23	2		104	21	15	20		179	15	29	14	
30	2	0	3		105	22	16	21		180	16	30	15	
31	3	1	3←		106	23	17	22		181	17	31	16	
32	4	2	4		107	24	18	23		182	18	32	17	
33	5	3	5		108	25	19	24		183	19	33	18	
34	5←	4	6		109	26	20	25		184	20	34	19	
35	6	5	7		110	27	21	26		185	21	35	20	
36	7	6←	8		111	28	22	27		186	22	36	21	
37	8	7	9		112	29	23	28		187	23	37	22	
38	9	8	10	2	113	30	24	29		188	24	38	23	
39	10	9	11	3	114	31	25	30	7 8	189	25	39	24	12
40	11	10	12		115	32	26	31		190	26	40	25	13
41	12	11	13		116	33	27	32		191	27	41	26	
42	12←	12	14		117	34	28	33		192	28	42	27	
43	13	13	15		118	35	29	34		193	29	43	28	
44	14	14	16		119	36	30	35		194	30	44	29	
45	15	15	17		120	37	31	36		195	31	45	30	
46	16	16	18		121	38	32	37		196	32	46	31	
47	17	17	19		122	39	33	38		197	33	47	32	
48	18	18	20		123	40	34	39		198	34	48	33	
49	19←	19←	21		124	41	35	40		199	35	49	34	
50	19	20	22		125	42	36	41		200	36	50	35	
51	20	21	23		126	43	37	42		201	37	51	36	
52	21	22	24	3	127	44	38	43		202	38	52	37	
53	22	23	25	4	128	45	39	44		203	39	53	38	
54	23	24	26		129	46	40	45		204	40	54	39	13
55	0	25	27		130	47	41	46	8 9	205	41	55	40	14
56	1	26	28		131	48	42	47		206	42	56	41	
57	2	27	29		132	49	43	48		207	43	57	42	
58	3	28	30		133	50	44	49		208	44	58	43	
59	4	29	31		134	51	45	50		209	45	59	44	
60	5	30	32		135	52	46	51		210	46	60	45	
61	6	31	33		136	53	47	52		211	47	61	46	
62	7	32	34		137	54	48	53		212	48	62	47	
63	8	33	35		138	55	49	54		213	49	63	48	
64	9	34	36		139	56	50	55		214	50	64	49	
65	10	35	37		140	57	51	56		215	51	65	50	
66	11	36	38		141	58	52	57		216	52	66	51	
67	12	37	39		142	59	53	58		217	53	67	52	
68	13	38	40		143	60	54	59		218	54	68	53	
69	14	39	41	4	144	61	55	60	9	219	55	69	54	
70	15	40	42	5	145	62	56	61	10	220	56	70	55	14
71	16	41	43		146	63	57	62		221	57	71	56	
72	17	42	44		147	64	58	63		222	58	72	57	15
73	18	43	45		148	65	59	64		223	59	73	58	
74	19	44	46		149	66	60	65		224	60	74	59	
75	20	45	47		150	67	61	66		225	61	75	60	

This table gives the nearest component "hour" (i. e., 24th of monthly or yearly period) for each day (11:30 a. m.) of the series.

In Mf, MSf, and Mm two days sometimes fall upon the same "hour." The arrow is used to indicate the one making the closer coincidence. Consequently the one so marked, or rather the corresponding daily height, is the one to be taken in preference to the other. See note given below Table 38.

TABLE 43.—For the summation of long-period tides—Continued.

Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa
226	6	15	4←		311	9	12←	6←	20	396	11←	9	8	
227	7	16	5		312	10←	13	7	21	397	12	10	9	
228	8	17	6		313	10	14	8		398	13	11	10	
229	9	18	7		314	11	15	9		399	14	12	11	
230	10←	19	8		315	12	16	10		400	15	13←	12	
231	10	19←	9		316	13	16←	11		401	16	13	13	
232	11	20	10←		317	14	17	12		402	17	14	14	
233	12	21	10		318	15	18	13		403	18	15	15	
234	13	22	11		319	16	19	13←		404	18←	16	15←	
235	14	23	12	15	320	17	20←	14		405	19	17	16	
236	15	23←	13	16	321	18	20	15		406	20	18	17	
237	16	0	14		322	18←	21	16		407	21	18←	18	
238	17←	1	15		323	19	22	17		408	22	19	19	
239	17	2	16		324	20	23	18		409	23	20	20	
240	18	3←	17←		325	21	0	19←		410	0	21	21	
241	19	3	17		326	22	1	19	21	411	1←	22←	22	
242	20	4	18		327	23	1←	20	22	412	1	22	22←	
243	21	5	19		328	0	2	21		413	2	23	23	
244	22	6	20		329	1	3	22		414	3	0	0	
245	23	7	21		330	1←	4	23		415	4	1	1	
246	0	8	22		331	2	5	0		416	5	2←	2	
247	1	8←	23		332	3	5←	1		417	6	3	3	
248	1←	9	0		333	4	6	2←		418	7	4	4←	
249	2	10	0←		334	5	7	2		419	8←	5	5	3
250	3	11	1	16	335	6	8	3		420	8	6	6	4
251	4	12	2	17	336	7	9←	4		421	9	7	7	
252	5	12←	3		337	8←	9	5		422	10	8	8	
253	6	13	4		338	9	10	6		423	11	9	9	
254	7	14	5		339	10	11	7		424	12	10	10	
255	8	15	6		340	11	12	8		425	13	11	11	
256	8←	16←	7		341	12	13	9	22	426	14	12	12	
257	9	16	7←		342	13	14	9←	23	427	15	13	13	
258	10	17	8		343	14	15	10		428	16	14	14	
259	11	18	9		344	15	16	11		429	17	15	15	
260	12	19	10		345	16	17	12		430	18	16	16	
261	13	20	11		346	17	18	13		431	19	17	17	
262	14	21	12		347	18	19	14		432	20	18	18	
263	15←	21←	13←		348	19	20	15		433	21	19	19	
264	15	22	13		349	20	21	16		434	22	20	20	
265	16	23	14	17	350	21	22	16←		435	23	21	21	4
266	17	0	15	18	351	22	23	17		436	24	22	22	5
267	18	1	16		352	23	24	18		437	25	23	23	
268	19	14←	17		353	24	25	19		438	26	24	24	
269	20	2	18		354	25	26	20		439	27	25	25	
270	21	3	19		355	26	27	21		440	28	26	26	
271	22←	4	20←		356	27	28	22←	23	441	29	27	27	
272	22	5←	20		357	28	29	23	0	442	30	28	28	
273	23	6	21		358	29	30	24		443	31	29	29	
274	0	7	22		359	30	31	25		444	32	30	30	
275	1	8	23		360	31	32	26		445	33	31	31	
276	2	9	0		361	32	33	27		446	34	32	32	
277	3	10	1		362	33	34	28		447	35	33	33	
278	4	11	2		363	34	35	29		448	36	34	34	
279	5	12	3		364	35	36	30		449	37	35	35	
280	6	13	4	18	365	36	37	31		450	38	36	36	
281	6←	14	5	19	366	37	38	32		451	39	37	37	
282	7	15	6		367	38	39	33		452	40	38	38	
283	8	16	7		368	39	40	34		453	41	39	39	
284	9	17	8		369	40	41	35		454	42	40	40	
285	10	18	9		370	41	42	36		455	43	41	41	
286	11	19	10		371	42	43	37		456	44	42	42	
287	12	20	11		372	43	44	38	0	457	45	43	43	
288	13	21	12		373	44	45	39	1	458	46	44	44	
289	13←	22	13		374	45	46	40		459	47	45	45	
290	14	23	14		375	46	47	41		460	48	46	46	
291	15	24	15		376	47	48	42		461	49	47	47	
292	16	25	16		377	48	49	43		462	50	48	48	
293	17	26	17		378	49	50	44		463	51	49	49	
294	18	27	18		379	50	51	45		464	52	50	50	
295	19	28	19		380	51	52	46		465	53	51	51	
296	20	0	20	19	381	52	53	47		466	54	52	52	
297	20←	1	21	20	382	53	54	48		467	55	53	53	
298	21	2	22		383	54	55	49		468	56	54	54	
299	22	3	23		384	55	56	50		469	57	55	55	
300	23	4	24		385	56	57	51		470	58	56	56	
301	0	5	25		386	57	58	52		471	59	57	57	
302	1	6	26		387	58	59	53		472	60	58	58	
303	2	7	27		388	59	60	54		473	61	59	59	
304	3←	8	28		389	60	61	55		474	62	60	60	
305	3	9	29		390	61	62	56		475	63	61	61	
306	4	10	30		391	62	63	57		476	64	62	62	
307	5	11	31		392	63	64	58		477	65	63	63	
308	6	12	32		393	64	65	59		478	66	64	64	
309	7	13	33		394	65	66	60		479	67	65	65	
310	8	14	34		395	66	67	61		480	68	66	66	

TABLE 43.—For the summation of long-period tides—Continued.

Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa
481	14	6	10		566	17	4	13		651	19←	1←	15←	
482	15	7	11		567	18←	4←	13←		652	20	1	15	
483	16	8	12		568	18	5	14		653	21	2	16	
484	17	9	13		569	19	6	15		654	22	3	17	
485	18←	10	14		570	20	7	16	13	655	23	4	18	
486	18	11	15		571	21	8←	17	14	656	0	5	19	
487	19	11←	16		572	22	8	18		657	1	6	20	
488	20	12	17←		573	23	9	19		658	2	6←	21	
489	21	13	17		574	0	10	20		659	2←	7	22	
490	22	14	18		575	1	11	20←		660	3	8	22←	
491	23	15←	19		576	2	12	21		661	4	9	23	19
492	0	15	20		577	2←	13	22		662	5	10←	0	20
493	1←	16	21		578	3	13←	23		663	6	10	1	
494	1	17	22	8	579	4	14	0		664	7	11	2	
495	2	18	23	9	580	5	15	1		665	8	12	3	
496	3	19←	0		581	6	16	2←		666	9←	13	4	
497	4	19	0←		582	7	17	2		667	9	14←	5	
498	5	20	1		583	8	17←	3		668	10	14	5←	
499	6	21	2		584	9	18	4		669	11	15	6	
500	7	22	3		585	9←	19	5	14	670	12	16	7	
501	8	23	4		586	10	20	6	15	671	13	17	8	
502	9	0	5		587	11	21←	7		672	14	18	9	
503	9←	0←	6		588	12	21	8		673	15	19	10	
504	10	1	7		589	13	22	9←		674	16←	19←	11←	
505	11	2	7←		590	14	23	9		675	16	20	11	
506	12	3	8		591	15	0	10		676	17	21	12	20
507	13	4←	9		592	16←	1	11		677	18	22	13	21
508	14	4	10		593	16	2	12		678	19	23←	14	
509	15	5	11	9	594	17	2←	13		679	20	23	15	
510	16	6	12	10	595	18	3	14		680	21	0	16	
511	16←	7	13←		596	19	4	15		681	22	1	17	
512	17	8	13		597	20	5	16		682	23	2	18←	
513	18	9	14		598	21	6	16←		683	0	3←	18	
514	19	9←	15		599	22	6←	17		684	0←	3	19	
515	20	10	16		600	23←	7	18	15	685	1	4	20	
516	21	11	17		601	23	8	19	16	686	2	5	21	
517	22	12	18		602	0	9	20		687	3	6	22	
518	23←	13	19		603	1	10←	21		688	4	7	23	
519	23	13←	20←		604	2	10	22		689	5	8	0	
520	0	14	20		605	3	11	23		690	6	8←	1	
521	1	15	21		606	4	12	23←		691	7	9	1←	21
522	2	16	22		607	5	13	0		692	7←	10	2	22
523	3	17←	23		608	6	14	1		693	8	11	3	
524	4	17	0	10	609	7	15	2		694	9	12←	4	
525	5	18	1	11	610	7←	15←	3		695	10	12	5	
526	6←	19	2		611	8	16	4		696	11	13	6	
527	6	20	3		612	9	17	5←		697	12	14	7	
528	7	21	3←		613	10	18	5		698	13	15	8	
529	8	22	4		614	11	19	6		699	14←	16←	8←	
530	9	22←	5		615	12	19←	7	16	700	14	16	9	
531	10	23	6		616	13	20	8	17	701	15	17	10	
532	11	0	7		617	14	21	9		702	16	18	11	
533	12	1	8		618	14←	22	10		703	17	19	12	
534	13	2	9		619	15	23←	11		704	18	20	13	
535	14	2←	10		620	16	23	12←		705	19	21	14←	
536	14←	3	10←		621	17	0	12		706	20	21←	14	
537	15	4	11		622	18	1	13		707	21←	22	15	22
538	16	5	12		623	19	2	14		708	21	23	16	23
539	17	6←	13	11	624	20	3	15		709	22	0	17	
540	18	6	14	12	625	21	4	16		710	23	1←	18	
541	19	7	15		626	21←	4←	17		711	0	1	19	
542	20	8	16←		627	22	5	18		712	1	2	20	
543	21	9	16		628	23	6	19		713	2	3	21←	
544	21←	10	17		629	0	7	19←		714	3	4	21	
545	22	11	18		630	1	8	20		715	4←	5←	22	
546	23	11←	19		631	2	8←	21	17	716	4	5	23	
547	0	12	20		632	3	9	22	18	717	5	6	0	
548	1	13	21		633	4←	10	23		718	6	7	1	
549	2	14	22		634	4	11	0		719	7	8	2	
550	3	15	23←		635	5	12←	1		720	8	9	3	
551	4	15←	23		636	6	12	2		721	9	10	4	
552	4←	16	0		637	7	13	2←		722	10	10←	4←	23
553	5	17	1	12	638	8	14	3		723	11	11	5	0
554	6	18	2	13	639	9	15	4		724	12	12	6	
555	7	19←	3		640	10	16	5		725	12←	13	7	
556	8	19	4		641	11←	17	6		726	13	14←	8	
557	9	20	5		642	11	17←	7		727	14	14	9	
558	10	21	6		643	12	18	8←		728	15	15	10	
559	11←	22	6←		644	13	19	8		729	16	16	11	
560	11	23	7		645	14	20	9		730	17	17	11←	
561	12	0	8		646	15	21	10	18	731	18	18←	12	
562	13	0←	9		647	16	21←	11	19	732	19	18	13	
563	14	1	10		648	17	22	12		733	19←	19	14	
564	15	2	11		649	18	23	13		734	20	20	15	
565	16	3	12		650	19	0	14		735	21	21	16	

TABLE 43.—For the summation of long-period tides—Continued.

Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa	Day of series.	Mf	MSf	Mm	Sa
736	22	22	17←		821	1	19	19		906	3←	16	21	12
737	23	23	17	0	822	2←	20←	20		907	4	17	22	
738	0	23←	18	1	823	2	20	20←		908	5	18	22←	
739	1	0	19		824	3	21	21		909	6	18←	23	
740	2←	1	20		825	4	22	22		910	7	19	0	
741	2	2	21		826	5	23	23		911	8	20	1	
742	3	3←	22		827	6	0	0		912	9	21	2	
743	4	3	23		828	7	1	1	6	913	10	22	3	
744	5	4	0←		829	8	1←	2←	7	914	10←	22←	4	
745	6	5	0		830	9	2	2		915	11	23	5	
746	7	6	1		831	10	3	3		916	12	0	5←	
747	8	7←	2		832	10←	4	4		917	13	1	6	
748	9	7	3		833	11	5	5		918	14	2←	7	
749	9	8	4		834	12	5←	6		919	15	2	8	
750	10	9	5		835	13	6	7		920	16	3	9	12
751	11	10	6		836	14	7	8		921	17←	4	10	13
752	12	11	7	1	837	15	8	9←		922	17	5	11←	
753	13	12	7←	2	838	16	9←	9		923	18	6	11	
754	14	12←	8		839	17	9	10		924	19	7	12	
755	15	13	9		840	17←	10	11		925	20	7←	13	
756	16	14	10		841	18	11	12		926	21	8	14	
757	17	15	11		842	19	12	13		927	22	9	15	
758	17←	16←	12		843	20	13	14		928	23	10	16	
759	18	16	13		844	21	14	15	7	929	0←	11	17	
760	19	17	14		845	22	14←	16	8	930	0	11←	18←	
761	20	18	14←		846	23	15	16←		931	1	12	18	
762	21	19	15		847	0←	16	17		932	2	13	19	
763	22	20←	16		848	0	17	18		933	3	14	20	
764	23	20	17		849	1	18	19		934	4	15←	21	
765	0	21	18		850	2	18←	20		935	5	15	22	13
766	0←	22	19		851	3	19	21		936	6	16	23	14
767	1	23	20←		852	4	20	22		937	7	17	0	
768	2	0	20	2	853	5	21	23		938	8	18	1	
769	3	1	21	3	854	6	22←	23←		939	8←	19	1←	
770	4	1←	22		855	7←	22	0		940	9	20	2	
771	5	2	23		856	7	23	1		941	10	20←	3	
772	6	3	0		857	8	0	2		942	11	21	4	
773	7←	4	1		858	9	1	3		943	12	22	5	
774	7	5←	2		859	10	2	4	8	944	13	23	6	
775	8	5	3←		860	11	3	5←	9	945	14	0←	7	
776	9	6	3		861	12	3←	5		946	15	0	8	
777	10	7	4		862	13	4	6		947	15←	1	8←	
778	11	8	5		863	14	5	7		948	16	2	9	
779	12	9←	6		864	15	6	8		949	17	3	10	
780	13	9	7		865	15←	7	9		950	18	4←	11	14
781	14←	10	8		866	16	7←	10		951	19	4	12	15
782	14	11	9		867	17	8	11		952	20	5	13	
783	15	12	10	3	868	18	9	12←		953	21	6	14←	
784	16	13	10←	4	869	19	10	12		954	22←	7	14	
785	17	14	11		870	20	11←	13		955	22	8	15	
786	18	14←	12		871	21	11	14		956	23	9	16	
787	19	15	13		872	22	12	15		957	0	9←	17	
788	20	16	14		873	22←	13	16		958	1	10	18	
789	21←	17	15		874	23	14	17	9	959	2	11	19	
790	21	18←	16		875	0	15	18	10	960	3	12	20	
791	22	18	17		876	1	16	19		961	4	13←	21←	
792	23	19	17←		877	2	16←	19←		962	5←	13	21	
793	0	20	18		878	3	17	20		963	5	14	22	
794	1	21	19		879	4	18	21		964	6	15	23	15
795	2	22←	20		880	5	19	22		965	7	16	0	
796	3	22	21		881	5←	20	23		966	8	17←	1	16
797	4	23	22		882	6	20←	0		967	9	17	2	
798	5	0	23←	4	883	7	21	1		968	10	18	3	
799	5←	1	23	5	884	8	22	2		969	11	19	4	
800	6	2	0		885	9	23	2←		970	12←	20	4←	
801	7	3	1		886	10	0←	3		971	12	21	5	
802	8	3←	2		887	11	0	4		972	13	22	6	
803	9	4	3		888	12←	1	5		973	14	22←	7	
804	10	5	4		889	12	2	6	10	974	15	23	8	
805	11	6	5		890	13	3	7	11	975	16	0	9	
806	12	7←	6←		891	14	4	8←		976	17	1	10	
807	12←	7	6		892	15	5	8		977	18	2←	11	
808	13	8	7		893	16	5←	9		978	19	2	11←	
809	14	9	8		894	17	6	10		979	20	3	12	
810	15	10	9		895	18	7	11		980	20←	4	13	
811	16	11	10		896	19←	8	12		981	21	5	14	16
812	17	12	11		897	19	9	13		982	22	6←	15	17
813	18	12←	12		898	20	9←	14		983	23	6	16	
814	19←	13	13	5	899	21	10	15←		984	0	7	17←	
815	19	14	13←		900	22	11	15		985	1	8	17	
816	20	15	14		901	23	12	16		986	2	9	18	
817	21	16	15		902	0	13←	17		987	3	10	19	
818	22	16←	16		903	1	13	18		988	3←	11	20	
819	23	17	17		904	2	14	19		989	4	11←	21	
820	0	18	18		905	3	15	20	11	990	5	12	22	

TABLE 43.—For the summation of long-period tides—Continued.

Day of series	Mf	MSf	Mm	Sa	Day of series	Mf	MSf	Mm	Sa	Day of series	Mf	MSf	Mm	Sa
991	6	13	23		1071	4	6	20		1151	3	23	18	
992	7	14	0		1072	5	7	21	22	1152	3	0	19	
993	8	15	0		1073	6	8	22	23	1153	4	1	20	
994	9	15	1		1074	7	8	23		1154	5	1	21	
995	10	16	2		1075	8	9	0		1155	6	2	22	
996	10	17	3	17	1076	9	10	1		1156	7	3	22	
997	11	18	4	18	1077	10	11	2		1157	8	4	23	
998	12	19	5		1078	10	12	2		1158	9	5	0	
999	13	19	6		1079	11	12	3		1159	10	6	1	
1000	14	20	7		1080	12	13	4		1160	11	6	2	
1001	15	21	7		1081	13	14	5		1161	11	7	3	
1002	16	22	8		1082	14	15	6		1162	12	8	4	
1003	17	23	9		1083	15	16	7		1163	13	9	5	4
1004	17	0	10		1084	16	17	8		1164	14	10	5	5
1005	18	0	11		1085	17	17	9		1165	15	10	6	
1006	19	1	12		1086	18	18	9		1166	16	11	7	
1007	20	2	13		1087	18	19	10	23	1167	17	12	8	
1008	21	3	14		1088	19	20	11	0	1168	18	13	9	
1009	22	4	14		1089	20	21	12		1169	18	14	10	
1010	23	4	15		1090	21	21	13		1170	19	14	11	
1011	0	5	16	18	1091	22	22	14		1171	20	15	11	
1012	1	6	17	19	1092	23	23	15		1172	21	16	12	
1013	1	7	18		1093	0	0	16		1173	22	17	13	
1014	2	8	19		1094	1	1	16		1174	23	18	14	
1015	3	8	20		1095	1	2	17		1175	0	19	15	
1016	4	9	20		1096	2	2	18		1176	1	19	16	
1017	5	10	21		1097	3	3	19		1177	1	20	17	
1018	6	11	22		1098	4	4	20		1178	2	21	18	
1019	7	12	23		1099	5	5	21		1179	3	22	18	5
1020	8	13	0		1100	6	6	22		1180	4	23	19	
1021	8	13	1		1101	7	6	23	0	1181	5	23	20	
1022	9	14	2		1102	7	7	23	1	1182	6	0	21	
1023	10	15	3		1103	8	8	0		1183	7	1	22	
1024	11	16	3		1104	9	9	1		1184	8	2	23	
1025	12	17	4		1105	10	10	2		1185	8	3	0	
1026	13	17	5	19	1106	11	10	3		1186	9	4	1	
1027	14	18	6	20	1107	12	11	4		1187	10	4	1	
1028	15	19	7		1108	13	12	5		1188	11	5	2	
1029	15	20	8		1109	14	13	5		1189	12	6	3	
1030	16	21	9		1110	15	14	6		1190	13	7	4	
1031	17	21	10		1111	15	15	7		1191	14	8	5	
1032	18	22	10		1112	16	15	8		1192	15	8	6	
1033	19	23	11		1113	17	16	9		1193	16	9	7	
1034	20	0	12		1114	18	17	10		1194	16	10	8	
1035	21	1	13		1115	19	18	11		1195	17	11	8	
1036	22	2	14		1116	20	19	12		1196	18	12	9	
1037	22	2	15		1117	21	19	12	1	1197	19	12	10	
1038	23	3	16		1118	22	20	13	2	1198	20	13	11	
1039	0	4	17		1119	23	21	14		1199	21	14	12	
1040	1	5	17		1120	23	22	15		1200	22	15	13	
1041	2	6	18	20	1121	0	23	16		1201	23	16	14	
1042	3	6	19	21	1122	1	23	17		1202	23	16	14	
1043	4	7	20		1123	2	0	18		1203	0	17	15	
1044	5	8	21		1124	3	1	19		1204	1	18	16	
1045	5	9	22		1125	4	2	19		1205	2	19	17	
1046	6	10	23		1126	5	3	20		1206	3	20	18	
1047	7	10	23		1127	6	4	21		1207	4	21	19	
1048	8	11	0		1128	6	4	22		1208	5	21	20	
1049	9	12	1		1129	7	5	23		1209	6	22	21	
1050	10	13	2		1130	8	6	0		1210	6	23	21	
1051	11	14	3		1131	9	7	1		1211	7	0	22	
1052	12	15	4		1132	10	8	2		1212	8	1	23	
1053	13	15	5		1133	11	8	2	2	1213	9	1	0	
1054	13	16	6		1134	12	9	3	3	1214	10	2	1	
1055	14	17	6		1135	13	10	4		1215	11	3	2	
1056	15	18	7		1136	13	11	5		1216	12	4	3	
1057	16	19	8	21	1137	14	12	6		1217	13	5	4	
1058	17	19	9	22	1138	15	12	7		1218	13	5	4	
1059	18	20	10		1139	16	13	8		1219	14	6	5	
1060	19	21	11		1140	17	14	8		1220	15	7	6	
1061	20	22	12		1141	18	15	9		1221	16	8	7	
1062	20	23	13		1142	19	16	10		1222	17	9	8	
1063	21	23	13		1143	20	17	11		1223	18	10	9	
1064	22	0	14		1144	20	17	12		1224	19	10	10	
1065	23	1	15		1145	21	18	13		1225	20	11	11	
1066	0	2	16		1146	22	19	14		1226	20	12	11	
1067	1	3	17		1147	23	20	15		1227	21	13	12	
1068	2	4	18		1148	0	21	15		1228	22	14	13	
1069	3	4	19		1149	1	21	16		1229	23	14	14	
1070	3	5	20		1150	2	22	17	4	1230	0	15	15	

TABLE 43.—For the summation of long-period tides—Continued.

Day of series	Mf	MSf	Mm	Sa	Day of series	Mf	MSf	Mm	Sa	Day of series	Mf	MSf	Mm	Sa
1231	1	16	16		1311	23	9	13←		1391	21←	2	11	19
1232	2	17	17←		1312	0	10	14		1392	22	3	12	20
1233	3	18←	17		1313	1	11←	15		1393	23	4	13	
1234	4	18	18		1314	2	11	16	14	1394	0	5	14	
1235	4←	19	19		1315	3	12	17	15	1395	1	5←	15←	
1236	5	20	20		1316	4	13	18		1396	2	6	15	
1237	6	21	21		1317	4←	14	19		1397	3	7	16	
1238	7	22	22		1318	5	15	20		1398	4←	8	17	
1239	8	23	23	9	1319	6	16	20←		1399	4	9	18	
1240	9	23←	0←	10	1320	7	16←	21		1400	5	9←	19	
1241	10	0	0		1321	8	17	22		1401	6	10	20	
1242	11	1	1		1322	9	18	23		1402	7	11	21	
1243	11←	2	2		1323	10	19	0		1403	8	12	22	
1244	12	3←	3		1324	11←	20←	1		1404	9	13←	22←	
1245	13	3	4		1325	11	20	2←		1405	10	13	23	
1246	14	4	5		1326	12	21	2		1406	11←	14	0	20
1247	15	5	6		1327	13	22	3		1407	11	15	1	21
1248	16	6	7		1328	14	23	4		1408	12	16	2	
1249	17	7←	7←		1329	15	0←	5		1409	13	17	3	
1250	18←	7	8		1330	16	0	6	15	1410	14	18	4	
1251	18	8	9		1331	17	1	7	16	1411	15	18←	5	
1252	19	9	10		1332	18	2	8		1412	16	19	5←	
1253	20	10	11		1333	19	3	9←		1413	17	20	6	
1254	21	11	12	10	1334	19←	4	9		1414	18	21	7	
1255	22	12	13	11	1335	20	5	10		1415	19	22	8	
1256	23	12←	14		1336	21	5←	11		1416	19←	22←	9	
1257	0	13	14←		1337	22	6	12		1417	20	23	10	
1258	14←	14	15		1338	23	7	13		1418	21	0	11←	
1259	1	15	16		1339	0	8	14		1419	22	1	11	
1260	2	16←	17		1340	1	9←	15		1420	23	2←	12	
1261	3	16	18		1341	2	9	16		1421	0	2	13	21
1262	4	17	19		1342	2←	10	16←		1422	1	3	14	22
1263	5	18	20←		1343	3	11	17		1423	2	4	15	
1264	6	19	20		1344	4	12	18		1424	2←	5	16	
1265	7	20←	21		1345	5	13←	19	16	1425	3	6	17	
1266	8	20	22		1346	6	13	20	17	1426	4	7	18←	
1267	9	21	23		1347	7	14	21		1427	5	7←	18	
1268	9←	22	0		1348	8	15	22		1428	6	8	19	
1269	10	23	1	11	1349	9	16	23		1429	7	9	20	
1270	11	0	2	12	1350	9←	17	23←		1430	8	10	21	
1271	12	1	3←		1351	10	18	0		1431	9←	11	22	
1272	13	1←	3		1352	11	18←	1		1432	9	11←	23	
1273	14	2	4		1353	12	19	2		1433	10	12	0	
1274	15	3	5		1354	13	20	3		1434	11	13	1	
1275	16	4	6		1355	14	21	4		1435	12	14	1←	
1276	16←	5←	7		1356	15	22←	5←		1436	13	15←	2	22
1277	17	5	8		1357	16←	22	5		1437	14	15	3	23
1278	18	6	9		1358	16	23	6		1438	15	16	4	
1279	19	7	10		1359	17	0	7		1439	16←	17	5	
1280	20	8	10←		1360	18	1	8	17	1440	16	18	6	
1281	21	9←	11		1361	19	2←	9	18	1441	17	19	7	
1282	22	9	12		1362	20	2	10		1442	18	20	8	
1283	23←	10	13		1363	21	3	11		1443	19	20←	8←	
1284	23	11	14	12	1364	22	4	12←		1444	20	21	9	
1285	0	12	15	13	1365	23←	5	12		1445	21	22	10	
1286	1	13	16		1366	23	6	13		1446	22	23	11	
1287	2	14	17		1367	0	7	14		1447	23	0	12	
1288	3	14←	17←		1368	1	7←	15		1448	0	0←	13	
1289	4	15	18		1369	2	8	16		1449	0←	1	14←	
1290	5	16	19		1370	3	9	17		1450	1	2	14	
1291	6←	17	20		1371	4	10	18		1451	2	3	15	23
1292	6	18←	21		1372	5	11←	19		1452	3	4←	16	0
1293	7	18	22		1373	6	11	19←		1453	4	4	17	
1294	8	19	23←		1374	7	12	20		1454	5	5	18	
1295	9	20	23		1375	7←	13	21	18	1455	6	6	19	
1296	10	21	0		1376	8	14	22	19	1456	7	7	20	
1297	11	22←	1		1377	9	15	23		1457	7←	8	21←	
1298	12	22	2		1378	10	16	0		1458	8	9	21	
1299	13←	23	3	13	1379	11	16←	1		1459	9	9←	22	
1300	13	0	4	14	1380	12	17	2		1460	10	10	23	
1301	14	1	5		1381	13	18	2←		1461	11	11	0	
1302	15	2	6←		1382	14	19	3						
1303	16	3	6		1383	14←	20	4						
1304	17	3←	7		1384	15	20←	5						
1305	18	4	8		1385	16	21	6						
1306	19	5	9		1386	17	22	7						
1307	20	6	10		1387	18	23	8←						
1308	21	7←	11		1388	19	0←	8						
1309	21←	7	12		1389	20	0	9						
1310	22	8	13		1390	21	1	10						

TABLE 44.—Acceleration in HW and LW of

[The amplitude of the semidiurnal wave is taken as unity.]

HW phase.* LW phase.*	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /
0.0	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
0.1	0 00	0 29	0 57	1 24	1 48	2 10	2 27	2 40	2 49	2 52
0.2	0 00	0 57	1 52	2 45	3 33	4 15	4 50	5 19	5 36	5 44
0.3	0 00	1 23	2 45	4 02	5 14	6 17	7 11	7 53	8 22	8 36
0.4	0 00	1 49	3 35	5 16	6 51	8 15	9 28	10 26	11 07	11 29
0.5	0 00	2 13	4 23	6 28	8 24	10 10	11 41	12 56	13 50	14 22
0.6	0 00	2 36	5 09	7 37	9 55	12 03	13 52	15 24	16 33	17 15
0.7	0 00	2 58	5 53	8 43	11 23	13 50	16 00	17 50	19 15	20 09
0.8	0 00	3 19	6 36	9 46	12 48	15 35	18 06	20 14	21 56	23 04
0.9	0 00	3 40	7 17	10 48	14 09	17 18	20 09	22 37	24 36	26 00
1.0	0 00	4 00	7 56	11 47	15 29	18 58	22 09	24 57	27 16	28 57
1.1	0 00	4 18	8 34	12 44	16 46	20 35	24 06	27 15	29 55	31 55
1.2	0 00	4 36	9 10	13 39	18 00	22 09	26 01	29 32	32 33	34 55
1.3	0 00	4 54	9 46	14 33	19 12	23 41	27 54	31 46	35 11	37 56
1.4	0 00	5 11	10 20	15 24	20 22	25 10	29 44	34 00	37 48	40 58
1.5	0 00	5 27	10 52	16 14	21 30	26 37	31 33	36 11	40 24	44 03
1.6	0 00	5 43	11 24	17 02	22 36	28 02	33 18	38 20	43 00	47 09
1.7	0 00	5 58	11 53	17 49	23 40	29 25	35 02	40 28	45 36	50 18
1.8	0 00	6 12	12 24	18 34	24 41	30 45	36 43	42 34	48 11	53 29
1.9	0 00	6 26	12 52	19 18	25 42	32 04	38 23	44 38	50 46	56 43
2.0	0 00	6 40	13 20	20 00	26 40	33 20	40 00	46 40	53 20	60 00
2.1	0 00	6 53	13 47	20 41	27 37	34 34	41 35	48 40	55 54	63 20
2.2	0 00	7 06	14 13	21 21	28 32	35 47	43 08	50 39	58 27	66 44
2.3	0 00	7 18	14 38	22 00	29 25	36 57	44 39	52 36	60 59	70 12
2.4	0 00	7 30	15 02	22 37	30 17	38 06	46 08	54 32	63 31	73 44
2.5	0 00	7 42	15 26	23 13	31 08	39 13	47 36	56 25	66 03	77 22
2.6	0 00	7 53	15 48	23 49	31 57	40 18	49 01	58 16	68 33	81 05
2.7	0 00	8 04	16 11	24 23	32 45	41 22	50 24	60 08	71 03	84 54
2.8	0 00	8 15	16 32	24 56	33 32	42 22	51 45	61 54	73 32	88 51
2.9	0 00	8 25	16 53	25 29	34 17	43 25	53 05	63 59	76 00	92 56
3.0	0 00	8 35	17 14	26 00	35 01	44 22	54 22	65 23	78 26	97 11
3.1	0 00	8 44	17 33	26 31	35 44	45 21	55 38	67 05	80 52	101 36
3.2	0 00	8 54	17 52	27 01	36 26	46 17	56 53	68 45	83 16	106 15
3.3	0 00	9 03	18 11	27 30	37 06	47 12	58 05	70 23	85 39	111 11
3.4	0 00	9 12	18 30	27 56	37 46	48 05	59 16	71 58	87 59	116 25
3.5	0 00	9 21	18 47	28 26	38 25	48 57	60 25	73 32	90 19	122 05
3.6	0 00	9 29	19 05	28 53	39 01	49 47	61 30	75 04	92 35	128 19
3.7	0 00	9 38	19 22	29 19	39 39	50 37	62 38	76 34	94 49	135 21
3.8	0 00	9 46	19 38	29 44	40 14	51 25	63 42	78 02	97 01	143 37
3.9	0 00	9 53	19 54	30 09	40 49	52 12	64 44	79 27	99 10	154 19
4.0	0 00	10 01	20 09	30 33	41 23	52 58	65 45	80 50	101 16	180 00
HW Phase. LW Phase.	360° 180	350° 170	340° 160	330° 150	320° 140	310° 130	300° 120	290° 110	280° 100	270° 90

*I. e., the phase of the diurnal wave (*B*) at the time of HW or LW of the semidiurnal wave (*A*).

HW phase = (time of HW of *A* - time of HW of *B*) *b*.
 LW phase = (time of LW of *A* - time of HW of *B*) *b*.

If one of the speeds be somewhat variable, the resultant times and heights will be given more accurately by keeping the phases between -90° and $+90^\circ$. If they do not fall within these limits, this and the following table may be entered with the phases:

HW phase = (time of LW of *A* - time of LW of *B*) *b*.
 LW phase = (time of HW of *A* - time of LW of *B*) *b*:

the resultant heights must, however, have their signs changed. For tropic tides

$$\text{HW phase} = n\pi + \frac{1}{2}A^\circ - B^\circ, \text{ LW phase} = n\pi \pm \frac{\pi}{2} + \frac{1}{2}A^\circ - B^\circ,$$

n being an integer. (See §§ 5, 20, Part III.)

a semidiurnal wave due to a diurnal wave.

[The amplitude of the semidiurnal wave is taken as unity.]

HW Phase. LW Phase.	100° 280	110° 290	120° 300	130° 310	140° 320	150° 330	160° 340	170° 350	180° 360	
	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	0 /	
0.0	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	
0.1	2 50	2 43	2 31	2 14	1 53	1 28	1 00	0 31	0 00	
0.2	5 42	5 29	5 05	4 32	3 50	3 00	2 03	1 03	0 00	
0.3	8 35	8 18	7 44	6 55	5 52	4 36	3 09	1 37	0 00	
0.4	11 30	11 10	10 28	9 24	7 59	6 17	4 20	2 12	0 00	
0.5	14 28	14 06	13 16	11 58	10 12	8 02	5 33	2 50	0 00	
0.6	17 27	17 06	16 10	14 38	12 31	9 54	6 51	3 30	0 00	
0.7	20 29	20 10	19 09	17 24	14 57	11 51	8 13	4 13	0 00	
0.8	23 34	23 19	22 14	20 18	17 30	13 55	9 40	4 58	0 00	
0.9	26 42	26 33	25 27	23 20	20 12	16 06	11 13	5 45	0 00	
1.0	29 53	29 52	28 47	26 31	23 03	18 26	12 52	6 36	0 00	
1.1	33 07	33 18	32 16	29 53	26 04	20 54	14 37	7 31	0 00	
1.2	36 26	36 51	35 56	33 27	29 18	23 34	16 30	8 29	0 00	
1.3	39 49	40 32	39 47	37 15	32 46	26 26	18 32	9 32	0 00	
1.4	43 17	44 23	43 52	41 21	36 33	29 32	20 43	10 40	0 00	
1.5	46 51	48 24	48 15	45 48	40 40	32 56	23 07	11 53	0 00	
1.6	50 31	52 40	52 59	50 43	45 17	36 43	25 44	13 13	0 00	
1.7	54 19	57 11	58 12	56 17	50 32	40 59	28 38	14 40	0 00	
1.8	58 15	62 03	64 05	62 53	56 48	45 55	31 53	16 16	0 00	
1.9	62 21	67 22	71 01	71 19	64 57	51 56	35 39	18 02	0 00	
2.0	66 40	73 20	80 00	86 40	80 00	60 00	40 00	20 00	0 00	
2.1	71 14	80 18					45 24	22 13	0 00	
2.2	76 07	89 14					52 48	24 45	0 00	
2.3	81 25							27 44	0 00	
2.4	87 18							31 17	0 00	
2.5	94 08							35 47	0 00	
2.6	102 48							42 08	0 00	
2.7									0 00	
2.8									0 00	
2.9									0 00	
3.0									0 00	
3.1									0 00	
3.2									0 00	
3.3									0 00	
3.4									0 00	
3.5									0 00	
3.6									0 00	
3.7									0 00	
3.8									0 00	
3.9									0 00	
4.0									0 00	
HW Phase. LW Phase.	260° 80	250° 70	240° 60	230° 50	220° 40	210° 30	200° 20	190° 10	180° 0	

When the top argument is used the tabular values are positive; when the bottom argument, they are negative. To express the acceleration in time divide by a , the speed of the semidiurnal component.

To find the acceleration when b is not exactly equal to $\frac{1}{2}a$, multiply the tabular values by $\frac{2b}{a}$.

This acceleration is directly expressed in time by multiplying the tabular values by $\frac{2b}{a}$.

Table 17 is a graphic form of this table.

Rollet de l'Isle has given in the Annales Hydrographique for 1896 (p. 248) a graphic table serving the purpose of Tables 17, 18, or Tables 44, 45, and which he calls an abacus. It is really the inverse of Tables 17, 18.

TABLE 45.—*Height of HW and LW for a tide*

[The amplitude of the semidiurnal wave is taken as unity.]

HW Phase.* LW Phase.*	0° 180	10° 190	20° 200	30° 210	40° 220	50° 230	60° 240	70° 250	80° 260	90° 270
Amplitude of diurnal wave.	0°0	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000
	0°1	1°1000	1°0985	1°0941	1°0869	1°0771	1°0650	1°0509	1°0353	1°0186
	0°2	1°2000	1°1971	1°1885	1°1744	1°1552	1°1314	1°1039	1°0727	1°0395
	0°3	1°3000	1°2958	1°2831	1°2624	1°2342	1°1991	1°1581	1°1123	1°0629
	0°4	1°4000	1°3945	1°3780	1°3510	1°3141	1°2681	1°2143	1°1539	1°0885
	0°5	1°5000	1°4932	1°4731	1°4401	1°3948	1°3384	1°2721	1°1975	1°1165
	0°6	1°6000	1°5921	1°5684	1°5296	1°4763	1°4098	1°3314	1°2430	1°1467
	0°7	1°7000	1°6908	1°6639	1°6195	1°5585	1°4822	1°3922	1°2904	1°1792
	0°8	1°8000	1°7899	1°7596	1°7099	1°6415	1°5558	1°4545	1°3397	1°2139
	0°9	1°9000	1°8888	1°8555	1°8006	1°7253	1°6304	1°5182	1°3908	1°2508
	1°0	2°0000	1°9878	1°9515	1°8917	1°8094	1°7059	1°5832	1°4436	1°2898
	1°1	2°1000	2°0869	2°0477	1°9832	1°8942	1°7824	1°6496	1°4982	1°3309
	1°2	2°2000	2°1860	2°1440	2°0749	1°9797	1°8598	1°7172	1°5544	1°3741
	1°3	2°3000	2°2851	2°2405	2°1670	2°0656	1°9380	1°7860	1°6122	1°4194
	1°4	2°4000	2°3842	2°3371	2°2594	2°1522	2°0170	1°8560	1°6716	1°4668
	1°5	2°5000	2°4834	2°4339	2°3520	2°2392	2°0970	1°9271	1°7325	1°5161
	1°6	2°6000	2°5826	2°5307	2°4450	2°3266	2°1774	1°9993	1°7949	1°5673
	1°7	2°7000	2°6818	2°6276	2°5382	2°4145	2°2586	2°0725	1°8588	1°6206
	1°8	2°8000	2°7811	2°7247	2°6316	2°5029	2°3406	2°1468	1°9241	1°6757
	1°9	2°9000	2°8804	2°8219	2°7252	2°5917	2°4232	2°2220	1°9908	1°7327
	2°0	3°0000	2°9797	2°9191	2°8191	2°6809	2°5065	2°2981	2°0587	1°7915
	2°1	3°1000	3°0790	3°0165	2°9132	2°7704	2°5903	2°3752	2°1280	1°8521
	2°2	3°2000	3°1784	3°1139	3°0074	2°8604	2°6747	2°4531	2°1985	1°9144
	2°3	3°3000	3°2778	3°2114	3°1019	2°9506	2°7597	2°5318	2°2702	1°9785
	2°4	3°4000	3°3772	3°3090	3°1965	3°0412	2°8452	2°6114	2°3431	2°0443
	2°5	3°5000	3°4766	3°4067	3°2913	3°1321	2°9312	2°6917	2°4171	2°1117
	2°6	3°6000	3°5760	3°5044	3°3863	3°2233	3°0177	2°7728	2°4922	2°1808
	2°7	3°7000	3°6755	3°6023	3°4814	3°3148	3°1047	2°8545	2°5683	2°2514
	2°8	3°8000	3°7749	3°7002	3°5767	3°4065	3°1921	2°9370	2°6455	2°3234
	2°9	3°9000	3°8744	3°7981	3°6721	3°4985	3°2800	3°0201	2°7236	2°3970
	3°0	4°0000	3°9739	3°8961	3°7677	3°5908	3°3682	3°1038	2°8027	2°4721
	3°1	4°1000	4°0734	3°9941	3°8634	3°6833	3°4569	3°1882	2°8827	2°5485
	3°2	4°2000	4°1730	4°0922	3°9592	3°7760	3°5459	3°2731	2°9636	2°6262
	3°3	4°3000	4°2725	4°1904	4°0552	3°8690	3°6353	3°3586	3°0452	2°7053
	3°4	4°4000	4°3720	4°2886	4°1512	3°9622	3°7250	3°4446	3°1277	2°7856
	3°5	4°5000	4°4716	4°3869	4°2474	4°0556	3°8151	3°5311	3°2110	2°8671
	3°6	4°6000	4°5712	4°4852	4°3437	4°1492	3°9055	3°6181	3°2950	2°9500
	3°7	4°7000	4°6708	4°5836	4°4401	4°2429	3°9962	3°7056	3°3797	3°0334
	3°8	4°8000	4°7704	4°6820	4°5366	4°3369	4°0872	3°7936	3°4652	3°1182
	3°9	4°9000	4°8699	4°7804	4°6331	4°4310	4°1785	3°8820	3°5512	3°2039
	4°0	5°0000	4°9695	4°8789	4°7298	4°5253	4°2701	3°9708	3°6379	3°2906
HW Phase. LW Phase.	360° 180°	350° 170°	340° 160°	330° 150°	320° 140°	310° 130°	300° 120°	290° 110°	280° 100°	270° 90°

* See footnote, preceding table.

For high waters use the tabular values as given; but for low waters, alter their signs.

composed of a diurnal and semidiurnal wave.

[The amplitude of the semidiurnal wave is taken as unity.]

HW Phase. LW Phase.	100° 280	110° 290	120° 300	130° 310	140° 320	150° 330	160° 340	170° 350	180° 360	Mean value.†
Amplitude of diurnal wave.	0°0	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000	1°0000
	0°1	0°9839	0°9669	0°9510	0°9365	0°9238	0°9137	0°9062	0°9016	1°0006
	0°2	0°9702	0°9361	0°9038	0°8745	0°8489	0°8281	0°8127	0°8032	1°0025
	0°3	0°9590	0°9076	0°8587	0°8141	0°7751	0°7432	0°7195	0°7049	1°0056
	0°4	0°9503	0°8815	0°8158	0°7554	0°7025	0°6591	0°6267	0°6067	1°0100
	0°5	0°9442	0°8578	0°7750	0°6986	0°6313	0°5758	0°5342	0°5087	1°0157
	0°6	0°9406	0°8367	0°7365	0°6436	0°5614	0°4933	0°4423	0°4107	1°0226
	0°7	0°9397	0°8181	0°7004	0°5906	0°4932	0°4118	0°3508	0°3129	1°0308
	0°8	0°9415	0°8023	0°6667	0°5397	0°4262	0°3314	0°2598	0°2152	1°0404
	0°9	0°9460	0°7891	0°6357	0°4911	0°3612	0°2521	0°1693	0°1174	1°0513
	1°0	0°9532	0°7789	0°6074	0°4448	0°2980	0°1739	0°0794	0°0202	1°0635
	1°1	0°9632	0°7715	0°5819	0°4012	0°2368	0°0971	—0°0098	—0°0770	1°0772
	1°2	0°9760	0°7672	0°5595	0°3602	0°1778	0°0213	—0°0982	—0°1740	1°0921
	1°3	0°9919	0°7661	0°5403	0°3223	0°1212	—0°0520	—0°1859	—0°2709	1°1088
	1°4	1°0105	0°7682	0°5245	0°2874	0°0672	—0°1239	—0°2727	—0°3674	1°1268
	1°5	1°0322	0°7738	0°5123	0°2561	0°0161	—0°1939	—0°3584	—0°4637	1°1465
	1°6	1°0569	0°7830	0°5041	0°2287	—0°0326	—0°2616	—0°4429	—0°5597	1°1677
	1°7	1°0848	0°7959	0°5002	0°2058	—0°0756	—0°3266	—0°5264	—0°6554	1°1909
	1°8	1°1159	0°8129	0°5012	0°1879	—0°1151	—0°3886	—0°6082	—0°7506	1°2160
	1°9	1°1504	0°8343	0°5077	0°1765	—0°1487	—0°4468	—0°6882	—0°8454	1°2433
	2°0	1°1882	0°8604	0°5210	0°1744	—0°1736	—0°5000	—0°7660	—0°9397	1°2732
	2°1	1°2296	0°8919					—0°8412	—1°0333	
	2°2	1°2748	0°9298					—0°9126	—1°1262	
	2°3	1°3238							—1°2182	
	2°4	1°3770							—1°3090	
	2°5	1°4348							—1°3984	
	2°6	1°4979							—1°4855	
	2°7								—1°5000	
	2°8								—1°6000	
	2°9								—1°7000	
	3°0								—1°8000	
	3°1								—1°9000	
	3°2								—2°0000	
	3°3								—2°1000	
	3°4								—2°2000	
	3°5								—2°3000	
	3°6								—2°4000	
	3°7								—2°5000	
	3°8								—2°6000	
	3°9								—2°7000	
	4°0								—2°8000	
									—2°9000	
									—3°0000	
HW Phase. LW Phase.	260° 80°	250° 70°	240° 60°	230° 50°	220° 40°	210° 30°	200° 20°	190° 10°	180° 0°	

† When δ is not exactly equal to $\frac{1}{2}\alpha$, mean value = $1 + (\text{tabular value} - 1) \frac{4\delta^2}{\alpha^2}$.

Table 18 is a graphic form of this table.

The above column of mean values may be compared with expression (29), Part III, and with the last column of Table 21.

TABLE 46.—Hyperbolic functions.

u		v		θ	$\sinh u$	$\cosh u$	$\tanh u$	e^u	e^{-u}
In degrees.		In degrees.			$= \tan v$	$= \sec v$	$= \sin v$		
	°		°	°					
0'00	0'000000	0'0000	0'0000	0'000	0'0000	1'0000	0'0000	1'0000	1'0000
0'02	1'1459156	0'0200	1'1458	1'145	0'0200	1'0002	0'0200	1'0202	0'9802
0'04	2'291831	0'0400	2'2912	2'288	0'0400	1'0008	0'0400	1'0408	0'9608
0'06	3'437747	0'0600	3'4357	3'428	0'0600	1'0018	0'0599	1'0618	0'9418
0'08	4'58366	0'0799	4'5788	4'561	0'0801	1'0032	0'0798	1'0833	0'9231
0'10	5'72958	0'0998	5'720	5'693	0'1002	1'0050	0'0997	1'1052	0'9048
0'12	6'87549	0'1197	6'859	6'811	0'1203	1'0072	0'1194	1'1275	0'8869
0'14	8'02141	0'1395	7'995	7'917	0'1405	1'0098	0'1391	1'1503	0'8694
0'16	9'16732	0'1593	9'128	9'011	0'1607	1'0128	0'1586	1'1735	0'8521
0'18	10'3132	0'1790	10'258	10'100	0'1810	1'0162	0'1781	1'1972	0'8353
0'20	11'4592	0'1987	11'384	11'167	0'2013	1'0201	0'1974	1'2214	0'8187
0'22	12'6051	0'2183	12'505	12'216	0'2218	1'0243	0'2165	1'2461	0'8025
0'24	13'7510	0'2377	13'621	13'254	0'2423	1'0289	0'2355	1'2712	0'7866
0'26	14'8969	0'2571	14'732	14'271	0'2629	1'0340	0'2543	1'2969	0'7711
0'28	16'0428	0'2764	15'837	15'265	0'2837	1'0395	0'2729	1'3231	0'7558
0'30	17'1887	0'2956	16'937	16'245	0'3045	1'0453	0'2913	1'3499	0'7408
0'32	18'3346	0'3147	18'030	17'197	0'3255	1'0516	0'3095	1'3771	0'7261
0'34	19'4806	0'3336	19'116	18'134	0'3466	1'0584	0'3275	1'4049	0'7118
0'36	20'6265	0'3525	20'195	19'045	0'3678	1'0655	0'3452	1'4333	0'6977
0'38	21'7724	0'3712	21'267	19'935	0'3892	1'0731	0'3627	1'4623	0'6839
0'40	22'9183	0'3894	22'331	20'801	0'4108	1'0811	0'3799	1'4918	0'6703
0'42	24'0642	0'4082	23'386	21'648	0'4325	1'0895	0'3969	1'5220	0'6570
0'44	25'2101	0'4264	24'434	22'470	0'4543	1'0984	0'4136	1'5527	0'6440
0'46	26'3561	0'4446	25'473	23'275	0'4764	1'1077	0'4301	1'5841	0'6313
0'48	27'5020	0'4626	26'503	24'045	0'4986	1'1174	0'4462	1'6161	0'6188
0'50	28'6479	0'4804	27'524	24'803	0'5211	1'1276	0'4621	1'6487	0'6065
0'52	29'7938	0'4980	28'535	25'533	0'5438	1'1383	0'4777	1'6820	0'5945
0'54	30'9397	0'5155	29'537	26'245	0'5666	1'1494	0'4930	1'7160	0'5827
0'56	32'0856	0'5328	30'529	26'930	0'5897	1'1609	0'5080	1'7507	0'5712
0'58	33'2316	0'5500	31'511	27'595	0'6131	1'1730	0'5227	1'7860	0'5599
0'60	34'3775	0'5669	32'483	28'237	0'6367	1'1855	0'5370	1'8221	0'5488
0'62	35'5234	0'5837	33'444	28'861	0'6605	1'1984	0'5511	1'8589	0'5379
0'64	36'6693	0'6003	34'395	29'462	0'6846	1'2119	0'5649	1'8965	0'5273
0'66	37'8152	0'6167	35'336	30'045	0'7090	1'2258	0'5784	1'9348	0'5169
0'68	38'9611	0'6329	36'265	30'604	0'7336	1'2402	0'5915	1'9739	0'5066
0'70	40'1070	0'6489	37'183	31'149	0'7586	1'2552	0'6044	2'0138	0'4966
0'72	41'2530	0'6648	38'091	31'670	0'7838	1'2706	0'6169	2'0544	0'4868
0'74	42'3989	0'6804	38'987	32'174	0'8094	1'2865	0'6291	2'0959	0'4771
0'76	43'5448	0'6958	39'872	32'663	0'8353	1'3030	0'6411	2'1383	0'4677
0'78	44'6907	0'7111	40'746	33'132	0'8615	1'3199	0'6527	2'1815	0'4584
0'80	45'8366	0'7261	41'608	33'587	0'8881	1'3374	0'6640	2'2255	0'4493
0'82	46'9825	0'7412	42'460	34'025	0'9150	1'3555	0'6751	2'2705	0'4404
0'84	48'1285	0'7557	43'299	34'446	0'9423	1'3740	0'6858	2'3164	0'4317
0'86	49'2744	0'7702	44'128	34'848	0'9700	1'3932	0'6963	2'3632	0'4232
0'88	50'4203	0'7844	44'944	35'238	0'9981	1'4128	0'7064	2'4109	0'4148
0'90	51'5662	0'7985	45'750	35'613	1'0265	1'4331	0'7163	2'4596	0'4066
0'92	52'7121	0'8123	46'544	35'976	1'0554	1'4539	0'7259	2'5093	0'3985
0'94	53'8580	0'8260	47'326	36'323	1'0847	1'4753	0'7352	2'5600	0'3906
0'96	55'0039	0'8394	48'097	36'660	1'1144	1'4973	0'7443	2'6117	0'3829
0'98	56'1499	0'8528	48'857	36'983	1'1446	1'5199	0'7531	2'6645	0'3753
1'00	57'2958	0'8658	49'605	37'293	1'1752	1'5431	0'7616	2'7183	0'3679
1'02	58'4417	0'8787	50'343	37'593	1'2063	1'5669	0'7699	2'7732	0'36059
1'04	59'5876	0'8913	51'069	37'880	1'2379	1'5913	0'7779	2'8292	0'35345
1'06	60'7335	0'9038	51'783	38'158	1'2700	1'6164	0'7857	2'8864	0'34646
1'08	61'8794	0'9160	52'485	38'423	1'3025	1'6421	0'7932	2'9447	0'33960
1'10	63'0254	0'9281	53'178	38'677	1'3356	1'6685	0'8005	3'0042	0'33287
1'12	64'1713	0'9400	53'860	38'924	1'3693	1'6956	0'8076	3'0649	0'32628
1'14	65'3172	0'9518	54'531	39'160	1'4035	1'7233	0'8144	3'1268	0'31982
1'16	66'4631	0'9632	55'189	39'387	1'4382	1'7517	0'8210	3'1899	0'31349
1'18	67'6090	0'9745	55'837	39'607	1'4735	1'7808	0'8275	3'2544	0'30728
1'20	68'7549	0'9857	56'476	39'817	1'5095	1'8107	0'8337	3'3201	0'30119

θ = the angle at the center of the hyperbola made by any secant line and the transverse axis of the hyperbola.

u = twice the area of the hyperbolic sector thus determined, the length of the semiaxis being unity.

$\tan \theta = \tanh u$.

v = an auxiliary angle called the *gudermanian*,* such that the equations of the hyperbola are $x = \sec v$, $y = \tan v$.

* For representations of this angle and for further particulars concerning hyperbolic functions see Chapter IV, by James McMahon, in Merriman and Woodward's Higher Mathematics; and Hoüel, Recueil de Formules et de Tables numérique. Newman and Glaisher have tabulated e^x and e^{-x} in the Transactions of the Cambridge Phil. Soc., Vol. 13 (1883), III.

TABLE 46.—*Hyperbolic functions*—Continued.

u		v		θ	$\sinh u$	$\cosh u$	$\tanh u$	e^u	e^{-u}
In degrees.		In degrees.			$= \tan v$	$= \sec v$	$= \sin v$		
	°		°	°					
1'22	69'9009	0'9967	57'103	40'023	1'5460	1'8412	0'8397	3'3872	0'29523
1'24	71'0468	1'0074	57'721	40'215	1'5831	1'8725	0'8455	3'4556	0'28938
1'26	72'1927	1'0180	58'328	40'401	1'6209	1'9045	0'8511	3'5254	0'28365
1'28	73'3386	1'0284	58'925	40'582	1'6593	1'9373	0'8565	3'5966	0'27804
1'30	74'4845	1'0387	59'511	40'753	1'6984	1'9709	0'8617	3'6693	0'27253
1'32	75'6304	1'0490	60'087	40'920	1'7381	2'0053	0'8668	3'7434	0'26714
1'34	76'7763	1'0586	60'654	41'080	1'7786	2'0404	0'8717	3'8190	0'26185
1'36	77'9223	1'0684	61'212	41'232	1'8198	2'0764	0'8764	3'8962	0'25666
1'38	79'0682	1'0779	61'758	41'380	1'8617	2'1132	0'8810	3'9749	0'25158
1'40	80'2141	1'0873	62'295	41'523	1'9043	2'1509	0'8854	4'0552	0'24650
1'42	81'3600	1'0965	62'823	41'657	1'9477	2'1894	0'8896	4'1371	0'24171
1'44	82'5059	1'1055	63'343	41'788	1'9919	2'2288	0'8937	4'2207	0'23693
1'46	83'6518	1'1145	63'851	41'915	2'0369	2'2691	0'8977	4'3060	0'23224
1'48	84'7978	1'1231	64'351	42'034	2'0827	2'3103	0'9015	4'3929	0'22764
1'50	85'9437	1'1317	64'843	42'148	2'1293	2'3524	0'9051	4'4817	0'22313
1'52	87'0896	1'1402	65'327	42'261	2'1768	2'3955	0'9087	4'5722	0'21861
1'54	88'2355	1'1484	65'800	42'370	2'2251	2'4395	0'9121	4'6646	0'21438
1'56	89'3814	1'1566	66'265	42'473	2'2743	2'4845	0'9154	4'7588	0'21014
1'58	90'5273	1'1646	66'728	42'571	2'3245	2'5305	0'9186	4'8550	0'20598
1'60	91'6732	1'1724	67'171	42'668	2'3756	2'5775	0'9217	4'9530	0'20190
1'62	92'8192	1'1800	67'612	42'756	2'4276	2'6255	0'9246	5'0531	0'19790
1'64	93'9651	1'1876	68'045	42'846	2'4806	2'6746	0'9275	5'1552	0'19398
1'66	95'1110	1'1953	68'469	42'930	2'5346	2'7247	0'9302	5'2593	0'19014
1'68	96'2569	1'2023	68'885	43'013	2'5896	2'7760	0'9329	5'3656	0'18637
1'70	97'4028	1'2094	69'294	43'090	2'6456	2'8283	0'9354	5'4739	0'18268
1'72	98'5487	1'2164	69'696	43'166	2'7027	2'8818	0'9379	5'5845	0'17907
1'74	99'6947	1'2233	70'091	43'233	2'7609	2'9364	0'9402	5'6973	0'17552
1'76	100'8406	1'2300	70'476	43'303	2'8202	2'9922	0'9425	5'8124	0'17204
1'78	101'9865	1'2366	70'856	43'373	2'8806	3'0492	0'9447	5'9299	0'16864
1'80	103'1324	1'2432	71'228	43'433	2'9422	3'1075	0'9468	6'0496	0'16530
1'82	104'2783	1'2495	71'593	43'497	3'0049	3'1669	0'9488	6'1719	0'16203
1'84	105'4242	1'2559	71'952	43'556	3'0689	3'2277	0'9508	6'2965	0'15882
1'86	106'5702	1'2619	72'303	43'615	3'1340	3'2897	0'9527	6'4237	0'15567
1'88	107'7161	1'2680	72'649	43'666	3'2005	3'3530	0'9545	6'5535	0'15259
1'90	108'8620	1'2739	72'987	43'720	3'2682	3'4177	0'9562	6'6859	0'14957
1'92	110'0079	1'2797	73'319	43'770	3'3372	3'4838	0'9579	6'8210	0'14661
1'94	111'1538	1'2854	73'645	43'816	3'4075	3'5512	0'9595	6'9588	0'14370
1'96	112'2997	1'2910	73'966	43'864	3'4792	3'6201	0'9611	7'0993	0'14086
1'98	113'4456	1'2964	74'274	43'910	3'5523	3'6904	0'9626	7'2427	0'13807
2'00	114'5916	1'3017	74'584	43'950	3'6269	3'7622	0'9640	7'3891	0'13534
2'02	115'7375	1'3070	74'886	43'993	3'7028	3'8355	0'9654	7'5383	0'13266
2'04	116'8834	1'3122	75'183	44'032	3'7803	3'9103	0'9667	7'6906	0'13003
2'06	118'0293	1'3173	75'472	44'070	3'8593	3'9867	0'9680	7'8460	0'12745
2'08	119'1752	1'3222	75'758	44'108	3'9398	4'0647	0'9693	8'0045	0'12493
2'10	120'3211	1'3271	76'037	44'145	4'0219	4'1443	0'9705	8'1662	0'12246
2'12	121'4671	1'3319	76'311	44'177	4'1055	4'2256	0'9716	8'3311	0'12003
2'14	122'6130	1'3365	76'578	44'208	4'1909	4'3085	0'9727	8'4994	0'11765
2'16	123'7589	1'3412	76'843	44'239	4'2779	4'3932	0'9737	8'6711	0'11533
2'18	124'9048	1'3457	77'102	44'270	4'3666	4'4797	0'9748	8'8463	0'11304
2'20	126'0507	1'3501	77'354	44'297	4'4571	4'5679	0'9757	9'0250	0'11080
2'22	127'1966	1'3544	77'603	44'327	4'5494	4'6580	0'9767	9'2073	0'10861
2'24	128'3425	1'3587	77'848	44'352	4'6434	4'7499	0'9776	9'3933	0'10646
2'26	129'4885	1'3628	78'084	44'378	4'7394	4'8437	0'9785	9'5831	0'10435
2'28	130'6344	1'3669	78'320	44'402	4'8372	4'9395	0'9793	9'7767	0'10228
2'30	131'7803	1'3710	78'549	44'425	4'9370	5'0372	0'9801	9'9742	0'10026
2'32	132'9262	1'3748	78'773	44'449	5'0387	5'1370	0'9809	10'1757	0'09827
2'34	134'0721	1'3787	78'996	44'469	5'1424	5'2388	0'9816	10'3812	0'09633
2'36	135'2180	1'3825	79'212	44'490	5'2483	5'3427	0'9823	10'5909	0'09442
2'38	136'3640	1'3862	79'425	44'511	5'3562	5'4487	0'9830	10'8049	0'09255
2'40	137'5099	1'3899	79'633	44'532	5'4662	5'5569	0'9837	11'0232	0'09072
2'42	138'6558	1'3934	79'836	44'549	5'5785	5'6674	0'9843	11'2459	0'08892
2'44	139'8017	1'3969	80'037	44'565	5'6929	5'7801	0'9849	11'4730	0'08716
2'46	140'9476	1'4003	80'233	44'582	5'8097	5'8951	0'9855	11'7048	0'08543
2'48	142'0935	1'4037	80'426	44'598	5'9288	6'0125	0'9861	11'9413	0'08371
2'50	143'2394	1'4070	80'615	44'616	6'0502	6'1323	0'9866	12'1825	0'08208
2'60	148'9690	1'4227	81'504	44'683	6'6947	6'7690	0'9890	13'4637	0'07427
2'70	154'6986	1'4366	82'310	44'741	7'4063	7'4735	0'9910	14'8797	0'06721
2'80	160'4282	1'4493	83'040	44'787	8'1919	8'2527	0'9926	16'4446	0'06081
2'90	166'1578	1'4609	83'701	44'828	9'0596	9'1146	0'9940	18'1741	0'05502

TABLE 46.—Hyperbolic functions—Continued.

u		v		θ	$\sinh u$	$\cosh u$	$\tanh u$	e^u	e^{-u}
	In degrees.		In degrees.		$= \tan v$	$= \sec v$	$= \sin v$		
	0		0	0					
3'00	171'8873	1'4713	84'301	44'861	10'0179	10'0677	0'9951	20'0855	0'04979
3'10	177'6169	1'4808	84'841	44'883	11'0765	11'1215	0'9959	22'1980	0'04505
3'20	183'3465	1'4894	85'331	44'906	12'2459	12'2866	0'9967	24'5325	0'04076
3'30	189'0761	1'4971	85'775	44'925	13'5379	13'5748	0'9973	27'1126	0'03688
3'40	194'8057	1'5041	86'177	44'936	14'9654	14'9987	0'9978	29'9641	0'03337
3'50	200'5352	1'5104	86'541	44'948	16'5426	16'5728	0'9982	33'1155	0'03020
3'60	206'2648	1'5162	86'870	44'961	18'2854	18'3128	0'9985	36'5982	0'02732
3'70	211'9944	1'5214	87'168	44'966	20'2113	20'2360	0'9988	40'4473	0'02472
3'80	217'7240	1'5261	87'445	44'971	22'3394	22'3618	0'9990	44'7012	0'02237
3'90	223'4535	1'5303	87'681	44'975	24'6911	24'7113	0'9992	49'4024	0'02024
4'00	229'1831	1'5342	87'901	44'980	27'2899	27'3082	0'9993	54'5981	0'01832
5'00	286'4789	1'5573	89'227	44'989	74'202	74'208	0'9999	148'41	0'006738
6'00	343'7747	1'5658	89'716	44'993	201'71	201'72	0'9999	403'43	0'002479
7'00	401'0705	1'5690	89'895	45'000	548'35	548'35	1'0000	1096'6	0'000912
8'00	458'3662	1'5701	89'960	45'000	1490'5	1490'5	1'0000	2981'0	0'000335
9'00	515'6620	1'5705	89'986	45'000	4051'6	4051'6	1'0000	8103'1	0'000123
10'00	572'9578	1'5706	89'995	45'000	11013'2	11013'2	1'0000	22026'5	0'000045
(10	00	1'5708	90'000	45'000	∞	∞	1'0000	∞	0

$$\sinh u = \frac{e^u - e^{-u}}{2}, \cosh u = \frac{e^u + e^{-u}}{2}, \tanh u = \frac{\sinh u}{\cosh u} = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

TABLE 47.—Period of a wave.

Depth of water (h).	Length of wave in feet (λ).								
	1	10	100	1 000	10 000	100 000	1 000 000	10 000 000	100 000 000
Feet.	Seconds.	Seconds.	Seconds.	Seconds.	Seconds.	Seconds.	Seconds.	Seconds.	Seconds.
1	0'442	1'873	17'641	176'29	1762'9	17629	176295	1762947	17629473
10	0'442	1'398	5'922	55'789	557'51	5575'1	55751	557508	5575085
100	0'442	1'398	4'419	18'726	176'41	1762'9	17629	176295	1762947
1 000	0'442	1'398	4'419	13'975	59'218	557'89	5575'1	55751	557508
10 000	0'442	1'398	4'419	13'975	44'192	187'26	1764'1	17629	176295
100 000	0'442	1'398	4'419	13'975	44'192	139'75	592'18	5579	55751

The period (τ) of a wave is determined by the equation

$$\tau = \frac{2\pi\lambda}{g} \left/ \tanh \frac{2\pi h}{\lambda} \right., = \frac{0.1953 \lambda}{\tanh 6.283185 \frac{h}{\lambda}}$$

where g is taken equal to 32.1722 feet per second, as in this table; or

$$\tau = \frac{0.195373 \lambda}{\tanh 6.283185 \frac{h}{\lambda}}$$

if g is taken equal to 32.16.

TABLE 48.—Wave velocity.

Depth of water. (<i>h</i>).	Length of wave in feet (λ).									
	1	10	100	1 000	10 000	100 000	1 000 000	10 000 000	100 000 000	Infinite.
<i>Feet.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>	<i>Ft./ sec.</i>
1	2'262	5'340	5'668	5'672	5'672	5'672	5'672	5'672	5'672	5'672
10	2'262	7'156	16'89	17'92	17'94	17'94	17'94	17'94	17'94	17'94
100	2'262	7'156	22'63	53'40	56'68	56'72	56'72	56'72	56'72	56'72
1 000	2'262	7'156	22'63	71'56	168'9	179'2	179'4	179'4	179'4	179'4
10 000	2'262	7'156	22'63	71'56	226'3	534'0	566'8	567'2	567'2	567'2
100 000	2'262	7'156	22'63	71'56	226'3	715'6	1689	1793	1794	1794

The wave velocity, i. e., velocity of propagation, is

$$\lambda/\tau.$$

Tables 47 and 48 are adapted from Airy's Tides and Waves.

TABLE 49.—Ratio of vertical to horizontal axes of elliptic orbits of water particles.

$\frac{y}{\lambda}$	$2\pi\frac{y}{\lambda}$	Ratio of axes.	$\frac{y}{\lambda}$	$2\pi\frac{y}{\lambda}$	Ratio of axes.	$\frac{y}{\lambda}$	$2\pi\frac{y}{\lambda}$	Ratio of axes.
0'00	0'0000	0'0000	0'10	0'6283	0'5568	0'40	2'5133	0'9869
0'01	0'0628	0'0627	0'12	0'7540	0'6375	0'50	3'1416	0'9962
0'02	0'1257	0'1250	0'14	0'8796	0'7062	0'60	3'7699	0'9989
0'03	0'1885	0'1863	0'16	1'0053	0'7638	0'70	4'3982	0'9995
0'04	0'2513	0'2461	0'18	1'1310	0'8113	0'80	5'0265	0'9999
0'05	0'3142	0'3042	0'20	1'2566	0'8501	0'90	5'6549	0'9999
0'06	0'3770	0'3601				1'00	6'2832	0'9999
0'07	0'4398	0'4134	0'25	1'5708	0'9171			
0'08	0'5027	0'4642	0'30	1'8850	0'9549	10'00	62'8319	1'0000
0'09	0'5655	0'5120	0'35	2'1991	0'9757	∞	∞	1'0000

The maximum horizontal displacement at the bottom (where $y=0$) being A , and $2A$, the distance between the foci of the elliptic orbits, the maximum displacements for other depths are:

$$x = A \cosh ly = A \cosh 2\pi \frac{y}{\lambda},$$

$$y = A \sinh ly = A \sinh 2\pi \frac{y}{\lambda};$$

$$\therefore \frac{y}{x} = \tanh 2\pi \frac{y}{\lambda},$$

which is the ratio tabulated above.

TABLE 50.—Propagation of a free tide wave along a uniform channel.

Depths.		Velocity of propagation.			Wave length, statute miles.	Time required to travel			Difference in phase of tide wave		
Fathoms.	Feet.	Feet per second.	Knots, or nautical miles, per hour.	Statute miles per hour.		1 foot.	1 naut. mile.	1 stat. mile.	per statute mile.	per foot.	
						s.	h.	h.	°	°	Radians.
0	0	0'000	0'000	0'000	0'00					[0'00	[0'0000
	1	5'672	3'358	3'867	48'03	0'1763	0'2978	0'2586	7'4953	14196	2478
	2	8'022	4'750	5'469	67'93	0'1247	0'2105	0'1828	5'2996	10037	1752
	3	9'824	5'817	6'698	83'20	0'1018	0'1719	0'1493	4'3269	08195	1430
	4	11'344	6'717	7'735	96'07	0'08818	0'1489	0'1293	3'7472	07097	1239
	5	12'683	7'510	8'648	107'41	0'07886	0'1332	0'1156	3'3517	06348	1108
1	6	13'894	8'226	9'473	117'66	0'07199	0'1216	0'1056	3'0597	05795	1011
	7	15'007	8'886	10'232	127'09	0'06662	0'1125	0'09775	2'8327	05365	0936
	8	16'043	9'499	10'938	135'86	0'06234	0'1053	0'09141	2'6498	05019	0876
	9	17'016	10'075	11'602	144'10	0'05877	0'09921	0'08621	2'4983	04732	0826
	10	17'937	10'620	12'230	151'90	0'05574	0'09416	0'08177	2'3700	04489	0783
	11	18'812	11'139	12'826	159'31	0'05316	0'08985	0'07794	2'2597	04280	0747
2	12	19'649	11'634	13'397	166'40	0'05089	0'08598	0'07463	2'1635	04098	0715
	13	20'451	12'109	13'944	173'19	0'04890	0'08258	0'07174	2'0785	03937	0687
	14	21'223	12'566	14'470	179'73	0'04713	0'07955	0'06911	2'0030	03794	0662
	15	21'968	13'007	14'978	186'04	0'04552	0'07686	0'06676	1'9351	03665	0640
	16	22'688	13'434	15'469	192'14	0'04407	0'07446	0'06464	1'8737	03549	0619
	17	23'387	13'847	15'945	198'05	0'04275	0'07220	0'06270	1'8177	03443	0601
3	18	24'065	14'249	16'408	203'79	0'04156	0'07018	0'06094	1'7664	03345	0584
4	24	27'787	16'453	18'946	235'32	0'03598	0'06079	0'05277	1'5298	02897	0506
5	30	31'067	18'395	21'182	263'09	0'03219	0'05435	0'04721	1'3683	02591	0452
6	36	34'032	20'151	23'204	288'21	0'02939	0'04963	0'04310	1'2491	02366	0413
7	42	36'759	21'765	25'063	311'30	0'02720	0'04593	0'03990	1'1564	02190	0382
8	48	39'297	23'268	26'794	332'79	0'02545	0'04297	0'03733	1'0818	02049	0358
9	54	41'681	24'680	28'419	352'99	0'02399	0'04052	0'03519	1'0199	01932	0337
10	60	43'936	26'014	29'956	372'07	0'02276	0'03845	0'03338	0'9675	01832	0320
15	90	53'810	31'861	36'688	455'69	0'01858	0'03139	0'02726	0'7900	01496	0261
20	120	62'134	36'790	42'364	526'19	0'01610	0'02718	0'02361	0'6842	01296	0226
30	180	76'099	45'058	51'885	644'45	0'01314	0'02219	0'01927	0'5586	01058	0185
40	240	87'871	52'029	59'912	744'14	0'01138	0'01922	0'01669	0'4838	00916	0160
50	300	98'243	58'170	66'984	831'98	0'01018	0'01719	0'01493	0'4327	00820	0143
60	360	107'620	63'722	73'377	911'39	0'009294	0'01569	0'01363	0'3950	00748	0131
70	420	116'243	68'828	79'256	984'41	0'008606	0'01453	0'01262	0'3657	00693	0121
80	480	124'268	73'580	84'728	1052'38	0'008045	0'01359	0'01180	0'3421	00648	0113
90	540	131'807	78'043	89'868	1116'22	0'007587	0'01281	0'01113	0'3225	00611	0107
100	600	138'936	82'265	94'729	1176'60	0'007199	0'01216	0'01056	0'3060	00579	0101
150	900	170'162	100'754	116'019	1441'03	0'005877	0'00992	0'00862	0'2498	00473	0083
200	1200	196'486	116'340	133'968	1663'96	0'005089	0'008598	0'007463	0'2163	00410	0072
300	1800	240'645	142'487	164'076	2037'92	0'004156	0'007018	0'006094	0'1766	00334	0058
400	2400	277'873	164'530	189'459	2353'19	0'003598	0'006079	0'005277	0'1530	00290	0051
500	3000	310'671	183'950	211'821	2630'95	0'003219	0'005435	0'004721	0'1368	00259	0045
600	3600	340'323	201'507	232'039	2882'06	0'002939	0'004963	0'004310	0'1249	00237	0041
700	4200	367'591	217'653	250'630	3112'98	0'002720	0'004593	0'003990	0'1156	00219	0038
800	4800	392'971	232'680	267'935	3327'91	0'002545	0'004297	0'003733	0'1082	00205	0036
900	5400	416'809	246'795	284'188	3529'79	0'002399	0'004052	0'003519	0'1020	00193	0034
1000	6000	439'356	260'145	299'561	3720'72	0'002276	0'003845	0'003338	0'0967	00183	0032
1500	9000	538'098	318'611	366'885	4556'94	0'001858	0'003139	0'002726	0'0790	00150	0026
2000	12000	621'342	367'900	423'643	5261'90	0'001610	0'002718	0'002361	0'0684	00130	0023
3000	18000	760'986	450'584	518'854	6444'48	0'001314	0'002219	0'001927	0'0559	00106	0019
4000	24000	878'711	520'289	599'121	7441'44	0'001138	0'001922	0'001669	0'0484	00092	0016
5000	30000	982'428	581'701	669'838	8319'78	0'001018	0'001719	0'001493	0'0433	00082	0014
6000	36000	1076'20	637'222	733'770	9113'87	0'000929	0'001569	0'001363	0'0395	00075	0013
7000	42000	1162'43	688'278	792'563	9844'10	0'000861	0'001453	0'001262	0'0366	00069	0012
8000	48000	1242'68	735'800	847'283	10523'79	0'000804	0'001359	0'001180	0'0342	00065	0011
9000	54000	1318'07	780'434	898'682	11162'17	0'000759	0'001281	0'001113	0'0323	00061	0011
10000	60000	1389'36	822'650	947'294	11765'96	0'000720	0'001216	0'001056	0'0306	00058	0010

$$\text{Velocity} = \sqrt{gh}$$

where h = the undisturbed depth in feet and g = the acceleration of gravity, assumed to be 32.1722 feet per second in this computation.

If

τ = the periodic time (= 12.4206012 solar hours or = $\frac{1}{2}$ lunar day for the tide wave), then

λ , or wave-length, = $\tau \sqrt{gh}$ feet = $\frac{\tau}{5280} \sqrt{gh}$ miles.

Difference in phase = $\frac{360^\circ}{\lambda}$.

The nautical mile is taken as 6080 feet.

TABLE 51.—*Velocity and length of tide wave.*

Depth.	Velocity of propagation.						Wave length.					
	Per lunar hour.			Per solar hour.			Lunar.			Solar.		
Fath.	Degs.	Sea miles.	Stat. miles.	Degs.	Sea miles.	Stat. miles.	Degs.	Sea miles.	Stat. miles.	Degs.	Sea miles.	Stat. miles.
25	0°7096	42°57	49°02	0°6855	41°13	47°36	8°515	510°9	588°3	8°226	493°6	568°4
50	1°0035	60°21	69°33	°9695	58°17	66°99	12°042	722°5	832°0	11°634	698°1	803°8
100	1°4191	85°15	98°05	1°3711	82°26	94°73	17°030	1 021°8	1 176°6	16°453	987°2	1 136°8
200	2°0070	120°42	138°66	1°9390	116°34	133°97	24°084	1 445°0	1 664°0	23°268	1 396°1	1 607°6
500	3°1733	190°40	219°24	3°0658	183°95	211°82	38°079	2 284°8	2 630°9	36°790	2 207°4	2 541°8
1 000	4°4877	269°26	310°06	4°3358	260°15	299°56	53°853	3 231°2	3 720°7	52°029	3 121°8	3 594°7
1 500	5°4963	329°78	379°74	5°3102	318°61	366°88	65°955	3 957°3	4 556°9	63°722	3 823°3	4 402°6
2 000	6°3467	380°80	438°49	6°1317	367°90	423°64	76°160	4 569°6	5 261°9	73°580	4 414°8	5 083°7
2 100	6°5033	390°20	449°32	6°2831	376°99	434°10	78°040	4 682°4	5 391°8	75°397	4 523°8	5 209°3
2 200	6°6563	399°38	459°89	6°4309	385°86	444°32	79°876	4 792°6	5 518°7	77°171	4 630°3	5 331°8
2 300	6°8059	408°36	470°23	6°5754	394°53	454°30	81°671	4 900°3	5 642°7	78°905	4 734°3	5 451°6
2 400	6°9523	417°14	480°34	6°7169	403°01	464°08	83°428	5 005°7	5 764°1	80°603	4 836°2	5 568°9
2 500	7°0957	425°74	490°25	6°8554	411°32	473°64	85°148	5 108°9	5 883°0	82°265	4 935°9	5 683°7
2 600	7°2362	434°17	499°96	6°9912	419°47	483°03	86°834	5 210°1	5 999°5	83°894	5 033°6	5 796°3
2 700	7°3740	442°44	509°48	7°1243	427°46	492°23	88°485	5 309°3	6 113°8	85°492	5 129°5	5 906°7
2 800	7°5094	450°56	518°83	7°2551	435°31	501°26	90°113	5 406°8	6 226°0	87°061	5 223°7	6 015°1
2 900	7°6423	458°54	528°01	7°3835	443°01	510°13	91°708	5 502°5	6 336°1	88°602	5 316°1	6 121°6
3 000	7°7729	466°38	537°04	7°5097	450°58	518°85	93°275	5 596°5	6 444°5	90°117	5 407°0	6 226°2
3 100	7°9014	474°09	545°92	7°6339	458°03	527°43	94°817	5 689°0	6 551°0	91°600	5 496°4	6 329°2
3 200	8°0278	481°67	554°65	7°7560	465°36	535°87	96°334	5 780°0	6 655°8	93°072	5 584°3	6 430°4
3 300	8°1523	489°14	563°25	7°8762	472°57	544°18	97°828	5 869°7	6 759°0	94°515	5 670°9	6 530°1
3 400	8°2750	496°50	571°72	7°9947	479°68	552°36	99°299	5 958°0	6 860°7	95°937	5 756°2	6 628°4
3 500	8°3957	503°74	580°07	8°1114	486°69	560°43	100°749	6 044°9	6 960°8	97°337	5 840°2	6 725°1
Feet.												
100	0°5794	34°76	40°03	0°5598	33°58	38°67	6°952	417°1	480°3	6°717	403°0	464°1
500	1°2955	77°72	89°51	1°2516	75°10	86°48	15°546	932°8	1 074°1	15°019	901°2	1 037°7
1 000	1°8321	109°93	126°58	1°7701	106°20	122°30	21°985	1 319°1	1 519°0	21°241	1 274°4	1 467°6
2 000	2°5910	155°46	179°01	2°5032	150°19	172°95	31°091	1 865°5	2 148°1	30°038	1 082°3	2 075°4
3 000	3°1733	190°40	219°24	3°0658	183°95	211°82	38°080	2 284°8	2 630°9	36°790	2 207°4	2 541°8
4 000	3°6642	219°46	253°16	3°5401	212°41	244°59	43°971	2 638°3	3 038°0	42°482	2 548°9	2 935°1
5 000	4°0968	245°80	283°05	3°9580	237°48	273°46	49°161	2 949°7	3 396°6	47°496	2 849°8	3 281°5
6 000	4°4878	269°27	310°06	4°3358	260°15	299°56	53°853	3 231°2	3 720°7	52°029	3 121°8	3 594°8
7 000	4°8473	290°84	334°90	4°6831	280°99	323°56	58°168	3 490°1	4 018°8	56°195	3 371°9	3 882°7
8 000	5°1820	310°92	358°03	5°0065	300°39	345°90	62°184	3 731°0	4 296°3	60°078	3 604°7	4 150°8
9 000	5°4963	329°78	379°74	5°3102	318°61	366°88	65°955	3 957°3	4 556°9	63°722	3 823°3	4 402°6
10 000	5°7936	347°62	400°29	5°5974	335°84	386°73	69°524	4 171°4	4 803°4	67°169	4 030°1	4 640°8
11 000	6°0764	364°58	419°82	5°8706	352°24	405°61	72°917	4 375°0	5 037°9	70°448	4 226°9	4 867°3
12 000	6°3466	380°80	438°49	6°1317	367°90	423°64	76°159	4 569°6	5 261°9	73°580	4 414°8	5 083°7
13 000	6°6058	396°34	456°40	6°3821	382°92	440°94	79°269	4 756°1	5 476°8	76°585	4 595°1	5 291°3
14 000	6°8551	411°31	473°62	6°6230	397°38	457°59	82°261	4 935°7	5 683°5	79°476	4 768°5	5 491°0
15 000	7°0957	425°74	490°25	6°8554	411°32	473°64	85°148	5 108°9	5 883°0	82°265	4 935°9	5 683°7
16 000	7°3284	439°70	506°32	7°0802	424°81	489°18	87°941	5 276°4	6 075°9	84°963	5 097°8	5 870°2
17 000	7°5540	453°24	521°91	7°2982	437°89	504°23	90°647	5 438°8	6 262°9	87°578	5 254°7	6 050°8
18 000	7°7729	466°38	537°04	7°5097	450°58	518°85	93°275	5 596°5	6 444°5	90°116	5 407°0	6 226°2
19 000	7°9859	479°16	551°75	7°7155	462°93	533°07	95°831	5 749°9	6 621°0	92°586	5 555°2	6 396°8
20 000	8°1934	491°60	566°09	7°9159	474°96	546°92	98°320	5 899°2	6 793°0	94°991	5 699°5	6 563°0

TABLE 52.—Periodic time of oscillations in relatively deep water.

h	$\frac{1}{2} \lambda$, in inches.										Great length.
	2	4	6	8	10	12	15	20	25	30	
<i>Inches.</i>											
0.1	6465	1'2849	1'9323	2'5758	3.2196	3'8633	4'8289	6'4383	8'0478	9'6573	16095A
0.2	4626	9142	1'3682	1'8228	2'2777	2'7327	3'4153	4'5532	5'6911	6'8291	11381A
0.3	3851	7503	1'1197	1'4903	1'8613	2'2325	2'7996	3'7184	4'6474	5'5765	99293A
0.4	3419	6542	9732	1'2929	1'6138	1'9349	2'4171	3'2212	4'0255	4'8300	88048A
0.5	3151	5903	8731	1'1588	1'4455	1'7324	2'1633	2'8822	3'6014	4'3208	77198A
0.6	2973	5445	8012	1'0610	1'3218	1'5835	1'9764	2'6322	3'2885	3'9451	66571A
0.7	2851	5101	7460	9855	1'2264	1'4682	1'8316	2'4383	3'0456	3'6533	66083A
0.8	2768	4835	7024	9252	1'1498	1'3756	1'7150	2'2822	2'8500	3'4183	55691A
0.9	2707	4625	6669	8759	1'0872	1'2995	1'6189	2'1532	2'6883	3'2234	55365A
1.0	2665	4455	6375	8349	1'0344	1'2352	1'5383	2'0443	2'5516	3'0594	55090A
1.1	2632	4318	6130	7997	9896	1'1807	1'4685	1'9506	2'4336	2'9182	48553A
1.2	2611	4205	5922	7701	9507	1'1331	1'4082	1'8695	2'3320	2'7951	46464A
1.3	2594	4111	5744	7441	9171	1'0916	1'3556	1'7979	2'2419	2'6867	44644A
1.4	2581	4034	5590	7214	8873	1'0551	1'3086	1'7344	2'1619	2'5902	43022A
1.5	2575	3968	5457	7014	8610	1'0225	1'2669	1'6776	2'0899	2'5037	41565A
1.6	2569	3913	5341	6839	8374	9930	1'2293	1'6264	2'0252	2'4254	40224A
1.7	2563	3868	5239	6681	8163	9669	1'1954	1'5799	1'9666	2'3541	39044A
1.8	2560	3828	5150	6541	7973	9431	1'1644	1'5375	1'9128	2'2894	37944A
1.9	2557	3796	5071	6415	7801	9215	1'1364	1'4985	1'8636	2'2298	36922A
2.0	2556	3768	5002	6302	7646	9016	1'1106	1'4629	1'8180	2'1752	35999A
2.1	2555	3744	4940	6200	7503	8836	1'0868	1'4299	1'7763	2'1242	35122A
2.2	2554	3724	4886	6107	7373	8669	1'0647	1'3994	1'7373	2'0768	34322A
2.3	2553	3707	4836	6023	7255	8517	1'0444	1'3709	1'7011	2'0328	33565A
2.4	2553	3692	4793	5947	7146	8376	1'0256	1'3446	1'6674	1'9919	32855A
2.5	2553	3680	4755	5878	7045	8244	1'0080	1'3200	1'6356	1'9531	32199A
2.6	2552	3670	4720	5814	6953	8123	9918	1'2970	1'6057	1'9174	31577A
2.7	2552	3661	4689	5756	6868	8011	9765	1'2752	1'5777	1'8830	30988A
2.8	2552	3652	4661	5704	6789	7906	9623	1'2549	1'5516	1'8510	30422A
2.9	2552	3646	4637	5656	6716	7809	9489	1'2356	1'5268	1'8206	29899A
3.0	2552	3641	4615	5612	6649	7718	9364	1'2176	1'5033	1'7917	29388A
3.1	2552	3636	4595	5571	6586	7633	9245	1'2005	1'4812	1'7646	28911A
3.2	2552	3631	4577	5534	6528	7554	9135	1'1842	1'4601	1'7385	28454A
3.3	2552	3629	4561	5501	6474	7479	9031	1'1690	1'4400	1'7140	28022A
3.4	2552	3626	4546	5469	6424	7410	8932	1'1544	1'4210	1'6905	27604A
3.5	2552	3623	4534	5441	6378	7345	8839	1'1407	1'4030	1'6682	27211A
3.6	2552	3621	4522	5414	6334	7284	8752	1'1276	1'3857	1'6469	26833A
3.7	2552	3619	4512	5390	6294	7226	8669	1'1150	1'3691	1'6265	26464A
3.8	2552	3618	4502	5368	6255	7172	8590	1'1033	1'3535	1'6071	26122A
3.9	2552	3616	4494	5347	6220	7121	8516	1'0920	1'3385	1'5885	25777A
4.0	2552	3615	4487	5328	6187	7073	8445	1'0813	1'3240	1'5705	25454A
Great depth	2552	3609	4420	5103	5706	6250	6948	8069	9021	9882	

The time is expressed in seconds.

$$\text{General formula } \tau^2 = \frac{2\pi\lambda}{g} \tanh \frac{2\pi h}{\lambda} \quad \tau = \frac{\lambda}{gh} \left[1 + \frac{1}{6} \left(\frac{2\pi h}{\lambda} \right)^2 - \frac{1}{40} \left(\frac{2\pi h}{\lambda} \right)^4 + \dots \right]$$

g is taken at 386.0664 inches (32.1722 feet); $\sqrt{g} = 19.6473$.

For lengths great in comparison with the depth, use last column, or $\tau = \lambda : \sqrt{gh}$.

For lengths small in comparison with the depth, use last line, or $\tau = (2\pi) \sqrt{\frac{\lambda}{g}}$.

See Tables 47-50.

TABLE 53.—*For converting solar into lunar time.*

Solar hours.	Lunar hours.	Solar minutes.	Lunar hours.	Solar minutes.	Lunar hours.	Solar minutes.	Lunar hours.	Solar hours.	Lunar hours.
0	0'000	0	0'000	20	0'322	40	0'644	0'00	0'000
1	0'966	1	0'016	21	0'338	41	0'660	0'01	0'010
2	1'932	2	0'032	22	0'354	42	0'676	0'02	0'019
3	2'898	3	0'048	23	0'370	43	0'692	0'03	0'029
4	3'865	4	0'064	24	0'386	44	0'709	0'04	0'039
5	4'831	5	0'081	25	0'403	45	0'725	0'05	0'048
6	5'797	6	0'097	26	0'419	46	0'741	0'06	0'058
7	6'763	7	0'113	27	0'435	47	0'757	0'07	0'068
8	7'729	8	0'129	28	0'451	48	0'773	0'08	0'077
9	8'695	9	0'145	29	0'467	49	0'789	0'09	0'087
10	9'661	10	0'161	30	0'483	50	0'805	0'10	0'097
11	10'628	11	0'177	31	0'499	51	0'821		
12	11'594	12	0'193	32	0'515	52	0'837	0'0	0'000
13	12'560	13	0'209	33	0'531	53	0'853	0'1	0'097
14	13'526	14	0'225	34	0'547	54	0'870	0'2	0'193
15	14'492	15	0'242	35	0'564	55	0'886	0'3	0'290
16	15'458	16	0'258	36	0'580	56	0'902	0'4	0'386
17	16'424	17	0'274	37	0'596	57	0'918	0'5	0'483
18	17'390	18	0'290	38	0'612	58	0'934	0'6	0'580
19	18'357	19	0'306	39	0'628	59	0'950	0'7	0'676
20	19'323					60	0'966	0'8	0'773
21	20'289							0'9	0'870
22	21'255							1'0	0'966
23	22'221								
24	23'187								
25	24'153								

TABLE 54.—*For converting lunar into solar time.*

Lunar hours.	Solar hours and minutes.	Solar hours and decimals.	Lunar hours.	Solar minutes.	Lunar hours.	Solar minutes.	Lunar hours.	Solar minutes.	Lunar hours.	Solar minutes.
0	0 00'0	0'000	0'00	0'00	0'25	15'53	0'50	31'05	0'75	46'58
1	1 02'1	1'035	0'01	0'62	0'26	16'15	0'51	31'67	0'76	47'20
2	2 04'2	2'070	0'02	1'24	0'27	16'77	0'52	32'29	0'77	47'82
3	3 06'3	3'105	0'03	1'86	0'28	17'39	0'53	32'91	0'78	48'44
4	4 08'4	4'140	0'04	2'48	0'29	18'01	0'54	33'53	0'79	49'06
5	5 10'5	5'175	0'05	3'10	0'30	18'63	0'55	34'16	0'80	49'68
6	6 12'6	6'210	0'06	3'73	0'31	19'25	0'56	34'78	0'81	50'30
7	7 14'7	7'245	0'07	4'35	0'32	19'87	0'57	35'40	0'82	50'92
8	8 16'8	8'280	0'08	4'97	0'33	20'49	0'58	36'02	0'83	51'54
9	9 18'9	9'315	0'09	5'59	0'34	21'11	0'59	36'64	0'84	52'16
10	10 21'0	10'350	0'10	6'21	0'35	21'74	0'60	37'26	0'85	52'79
11	11 23'1	11'386	0'11	6'83	0'36	22'36	0'61	37'88	0'86	53'41
12	12 25'2	12'421	0'12	7'45	0'37	22'98	0'62	38'50	0'87	54'03
13	13 27'3	13'456	0'13	8'07	0'38	23'60	0'63	39'12	0'88	54'65
14	14 29'4	14'491	0'14	8'69	0'39	24'22	0'64	39'74	0'89	55'27
15	15 31'5	15'526	0'15	9'32	0'40	24'84	0'65	40'37	0'90	55'89
16	16 33'6	16'561	0'16	9'94	0'41	25'46	0'66	40'99	0'91	56'51
17	17 35'7	17'596	0'17	10'56	0'42	26'08	0'67	41'61	0'92	57'13
18	18 37'9	18'631	0'18	11'18	0'43	26'70	0'68	42'23	0'93	57'75
19	19 40'0	19'666	0'19	11'80	0'44	27'32	0'69	42'85	0'94	58'37
20	20 42'1	20'701	0'20	12'42	0'45	27'94	0'70	43'47	0'95	59'00
21	21 44'2	21'736	0'21	13'04	0'46	28'57	0'71	44'09	0'96	59'62
22	22 46'3	22'771	0'22	13'66	0'47	29'19	0'72	44'71	0'97	60'24
23	23 48'4	23'806	0'23	14'28	0'48	29'81	0'73	45'33	0'98	60'86
24	24 50'5	24'841	0'24	14'90	0'49	30'43	0'74	45'95	0'99	61'48
25	25 52'6	25'876							1'00	62'10

AUXILIARY TABLES FOR THE REDUCTION AND PREDICTION OF TIDES

[Tables 1 to 54 are appended to Part III, Appendix No. 7, Report for 1894,
to Part II, Appendix No. 8, Report for 1897, and to
Part IV, Appendix No. 7, 1900]

TABLE 55.—*For clearing one component of the effects of the others.**

[Length of series, 14 days.]

Component sought (<i>A</i>)	Disturbing components (<i>B, C, etc.</i>)									
	<i>J</i> ₁	<i>K</i> ₁	<i>M</i> ₁	<i>O</i> ₁	<i>OO</i>	<i>P</i> ₁	<i>Q</i> ₁	<i>2Q</i>	<i>S</i> ₁	<i>P</i> ₁
<i>J</i> ₁	0.6263	0.020	0.2065	0.615	0.525	0.020	0.123	0.576	0.014
	269	356	264	93	255	353	261	262	185
<i>K</i> ₁	0.626	0.621	0.0242	0.024	0.990	0.207	0.020	0.998	0.216
	91	268	356	4	346	264	353	353	276
<i>M</i> ₁	0.020	0.6208	0.6208	0.206	0.716	0.020	0.207	0.669	0.050
	4	92	268	97	78	356	265	85	189
<i>O</i> ₁	0.207	0.0242	0.621	0.024	0.054	0.626	0.016	0.014	0.710
	96	4	92	9	171	269	357	178	281
<i>OO</i>	0.615	0.0242	0.206	0.0241	0.091	0.122	0.021	0.059	0.128
	267	356	263	351	342	260	348	349	272
<i>P</i> ₁	0.525	0.9904	0.716	0.0544	0.091	0.217	0.018	0.998	0.215
	105	14	282	189	18	278	186	7	290
<i>Q</i> ₁	0.020	0.2065	0.020	0.6263	0.122	0.217	0.626	0.213	0.992
	7	96	4	91	100	82	269	89	12
<i>2Q</i>	0.123	0.0200	0.207	0.0159	0.021	0.018	0.626	0.001	0.537
	99	7	95	3	12	174	91	0	104
<i>S</i> ₁	0.576	0.9976	0.669	0.0137	0.059	0.998	0.213	0.001	0.217
	98	7	275	182	11	353	271	360	283
<i>P</i> ₁	0.014	0.2160	0.050	0.7105	0.128	0.215	0.992	0.537	0.217
	175	84	171	79	88	70	348	256	77

* This is a continuation of Table 41.

TABLE 55.—For clearing one component of the effects of the others*—Continued.

[Length of series, 15 days.]

Component sought (<i>A</i>)	Disturbing components (<i>B</i> , <i>C</i> , etc.)											
	<i>K</i> ₁	<i>L</i> ₁	<i>M</i> ₁	<i>N</i> ₁	<i>zN</i>	<i>R</i> ₁	<i>S</i> ₁	<i>T</i> ₁	<i>λ</i> ₁	<i>μ</i> ₁	<i>ν</i> ₁	<i>zSM</i>
<i>K</i> ₁ 260	0.567 342	0.0879 244	0.175 326	0.081 353	0.997 345	0.9889 338	0.975 247	0.468 339	0.053 257	0.198 168	0.070 168
<i>L</i> ₁	0.567 100 262	0.5790 344	0.080 246	0.178 344	0.620 92	0.6724 85	0.722 77	0.991 347	0.200 259	0.016 357	0.214 88
<i>M</i> ₁	0.088 18	0.579 98 262	0.579 344	0.080 10	0.054 3	0.0156 175	0.026 85	0.672 357	0.016 275	0.672 275	0.016 6
<i>N</i> ₁	0.175 116	0.080 16	0.5790 98 262	0.579 108	0.189 101	0.2004 93	0.209 3	0.016 275	0.672 13	0.991 13	0.120 104
<i>zN</i>	0.081 34	0.178 114	0.0804 16	0.579 98 26	0.066 19	0.0488 11	0.031 101	0.200 13	0.991 111	0.481 111	0.038 22
<i>R</i> ₁	0.997 7	0.620 268	0.0536 350	0.189 252	0.066 334 353	0.9972 345	0.989 255	0.524 255	0.035 347	0.207 265	0.026 175
<i>S</i> ₁	0.989 15	0.672 275	0.0156 357	0.200 259	0.049 341	0.997 7 353	0.997 262	0.579 262	0.016 354	0.214 272	0.016 3
<i>T</i> ₁	0.975 22	0.722 283	0.0258 185	0.209 267	0.031 349	0.989 15	0.9972 7 269	0.632 182	0.005 280	0.217 280	0.054 10
<i>λ</i> ₁	0.468 113	0.991 13	0.6724 275	0.016 357	0.200 259	0.524 105	0.5790 98	0.632 91 272	0.214 190	0.060 190	0.200 101
<i>μ</i> ₁	0.053 21	0.200 101	0.0156 3	0.672 85	0.991 347	0.035 13	0.0156 6	0.005 178	0.214 88 98	0.579 98	0.016 9
<i>ν</i> ₁	0.198 103	0.016 3	0.6724 85	0.991 347	0.481 249	0.207 95	0.2138 88	0.217 80	0.060 170	0.579 262 262	0.127 91
<i>zSM</i>	0.070 192	0.214 272	0.0156 354	0.120 256	0.038 338	0.026 185	0.0156 357	0.054 350	0.200 259	0.016 351	0.127 269 269

*This is a continuation of Table 41.

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 58 days.]

Component sought (<i>A</i>)	Disturbing components (<i>B, C, etc.</i>)									
	<i>J</i> ₁	<i>K</i> ₁	<i>M</i> ₁	<i>O</i> ₁	<i>OO</i>	<i>P</i> ₁	<i>Q</i> ₁	<i>2Q</i>	<i>S</i> ₁	<i>ρ</i> ₁
<i>J</i> ₁0489	.049	.0447	.064	.128	.037	.030	.104	.020
		341	319	297	25	284	278	259	313	329
<i>K</i> ₁	.049056	.0523	.052	.842	.045	.037	.959	.011
	19		338	316	44	303	297	278	331	348
<i>M</i> ₁	.049	.05650565	.046	.101	.049	.043	.018	.014
	41	22		338	66	145	319	300	174	190
<i>O</i> ₁	.045	.0523	.056037	.018	.049	.046	.021	.092
	63	44	22		88	167	341	322	16	212
<i>OO</i>	.064	.0523	.046	.0375068	.029	.020	.069	.026
	335	316	294	272		259	253	234	287	303
<i>P</i> ₁	.128	.8421	.101	.0181	.068005	.016	.959	.039
	76	57	215	193	101		354	335	29	225
<i>Q</i> ₁	.037	.0447	.049	.0489	.029	.005049	.029	.875
	82	63	41	19	107	6		341	35	51
<i>2Q</i>	.030	.0373	.043	.0463	.020	.016	.049031	.125
	101	82	60	38	126	25	19		53	70
<i>S</i> ₁	.104	.9591	.018	.0210	.069	.959	.029	.031015
	47	29	186	344	73	331	325	307		196
<i>ρ</i> ₁	.020	.0113	.014	.0920	.026	.039	.875	.125	.015
	31	12	170	148	57	135	309	290	164

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 58 days.]

Component sought (<i>A</i>)	Disturbing components (<i>B, C, etc.</i>)											
	<i>K</i> ₂	<i>L</i> ₂	<i>M</i> ₂	<i>N</i> ₂	2 <i>N</i>	<i>R</i> ₂	<i>S</i> ₂	<i>T</i> ₂	<i>λ</i> ₂	<i>μ</i> ₂	<i>ν</i> ₂	2 <i>SM</i>
<i>K</i> ₂064	.0523	.045	.037	.959	.8421	.666	.128	.020	.011	.083
	335	316	297	275	331	303	274	284	329	348	110
<i>L</i> ₂	.0640489	.046	.042	.099	.0920	.166	.875—	.005	.018	.039
	25	341	322	303	177	148	120	309	354	193	135
<i>M</i> ₂	.052	.049049	.046	.021	.0181	.056	.092	.018	.092	.018
	44	19	341	322	16	167	138	148	193	212	154
<i>N</i> ₂	.045	.046	.0489049	.029	.0055—	.021	.018	.092	.875	.004
	63	38	19	341	35	6	157	167	212	51	173
2 <i>N</i>	.037	.042	.0463	.049031	.0104	.003	.005	.875	.125	.005
	82	57	38	19	53	25	176	6	51	70	12
<i>R</i> ₂	.959	.009	.0210	.029	.0319591	.842	.104	.002	.015	.056
	29	183	344	325	307	331	303	313	357	196	138
<i>S</i> ₂	.842	.092	.0181	.005	.016	.959959	.049	.018	.039	.018
	57	212	193	354	335	29	331	341	206	225	167
<i>T</i> ₂	.666	.166	.0560	.021	.003	.842	.9591028	.034	.055	.021
	86	240	222	203	184	57	29	190	234	253	16
<i>λ</i> ₂	.128	.875—	.0920	.018	.005	.104	.0489	.028039	.078	.005
	76	51	212	193	354	47	19	170	225	244	6
<i>μ</i> ₂	.020	.005	.0181	.092	.875	.002	.0177	.034	.039049	.017
	31	6	167	148	309	3	154	126	135	19	141
<i>ν</i> ₂	.011	.018	.0920	.875	.125	.015	.0390	.055	.078	.049028
	12	167	148	309	290	164	135	107	116	341	122
2 <i>SM</i>	.083	.039	.0177	.004	.005	.056	.0181	.021	.005	.017	.028
	250	225	206	187	348	222	193	344	354	219	238

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 87 days.]

Component sought (A)	Disturbing components (B, C, etc.)									
	J ₁	K ₁	M ₁	O ₁	OO	P ₁	Q ₁	2Q	S ₁	P ₁
J ₁0478	.044	.0333	.061	.080	.021	.010	.089	.019
	332	298	265	38	246	237	209	289	313
K ₁	.046055	.0458	.046	.666	.033	.021	.909	.011
	28	327	294	66	274	265	237	317	341
M ₁	.044	.05470547	.033	.093	.044	.034	.018	.014
	62	33	327	100	127	298	270	170	195
O ₁	.033	.0458	.055018	.018	.048	.042	.021	.086
	95	66	33	133	161	332	303	23	228
OO	.061	.0458	.033	.0184022	.007	.003	.045	.020
	322	294	260	227	208	199	351	251	275
P ₁	.080	.6663	.093	.0180	.022005	.016	.909	.034
	114	86	233	199	152	351	323	43	247
Q ₁	.021	.0333	.044	.0478	.007	.005048	.027	.731
	123	95	62	28	161	9	332	52	76
2Q	.010	.0210	.034	.0421	.003	.016	.048025	.086
	151	123	90	57	9	37	28	80	104
S ₁	.089	.9092	.018	.0207	.045	.909	.027	.025015
	71	43	190	337	109	317	308	280	204
P ₁	.019	.0112	.4	.0861	.020	.034	.731	.086	.015
	47	19	165	132	85	113	284	256	156

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series 87 days.]

Component sought (<i>A</i>)	Disturbing components (<i>B, C, etc.</i>)											
	<i>K</i> ₁	<i>L</i> ₁	<i>M</i> ₁	<i>N</i> ₁	2 <i>N</i>	<i>R</i> ₁	<i>S</i> ₁	<i>T</i> ₁	<i>λ</i> ₁	<i>μ</i> ₁	<i>ν</i> ₁	2 <i>SM</i> .
<i>K</i> ₁061	.0458	.033	.021	.909	.6663	.348	.080	.019	.011	.057
	322	.294	265	237	317	274	231	246	313	341	75
<i>L</i> ₁	.0610478	.042	.033	.009	.0861	.127	.731	.005	.018	.034
	38	332	303	275	175	132	89	284	351	199	113
<i>M</i> ₁	.046	.048048	.042	.021	.0180	.050	.086	.018	.086	.017
	66	28	322	303	23	161	118	132	199	228	141
<i>N</i> ₁	.033	.042	.0478048	.027	.0055	.020	.018	.086	.731	.004
	95	57	28	332	52	9	146	161	228	76	170
2 <i>N</i>	.021	.033	.0421	.048025	.0158	.003	.005	.731	.086	.005
	123	85	57	28	80	37	174	9	76	104	18
<i>R</i> ₁	.909	.009	.0207	.027	.0259092	.666	.089	.002	.015	.050
	43	185	337	308	280	317	274	289	356	204	118
<i>S</i> ₁	.666	.086	.0180	.005	.016	.909909	.048	.017	.034	.018
	86	228	199	351	323	43	317	332	219	247	161
<i>T</i> ₁	.348	.127	.0498	.020	.003	.666	.9092027	.027	.036	.021
	129	271	242	214	186	86	43	195	262	290	23
<i>λ</i> ₁	.080	.731	.0861	.018	.005	.089	.0478	.027034	.058	.005
	114	76	228	199	351	71	28	165	247	275	9
<i>μ</i> ₁	.019	.005	.0180	.086	.731	.002	.0169	.027	.034048	.015
	47	9	161	132	284	4	141	98	113	28	122
<i>ν</i> ₁	.011	.018	.0861	.731	.086	.015	.0340	.036	.058	.048022
	19	161	132	284	256	156	113	70	85	332	93
2 <i>SM</i>	.057	.034	.0169	.004	.005	.050	.0180	.021	.005	.015	.022
	285	247	219	190	342	242	199	337	351	238	267

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series 104½ days.]

Component sought (A)	Disturbing components (B, C, etc.)									
	J ₁	K ₁	M ₁	O ₁	OO	P ₁	Q ₁	2Q	S ₁	P ₁
J ₁0509	.039	.0274	.036	.066	.014	.001	.019	.016
	217	249	280	154	294	318	355	346	229
K ₁	.051044	.0371	.037	.542	.027	.014	.871	.006
	143	212	243	117	257	280	318	308	192
M ₁	.039	.04350435	.028	.070	.039	.027	.089	.015
	111	148	212	85	45	249	286	97	340
O ₁	.027	.0371	.044017	.011	.051	.040	.039	.076
	80	117	148	54	14	217	255	65	309
OO	.036	.0371	.028	.0168025	.005	.005	.008	.017
	206	243	275	306	320	343	.201	192	255
P ₁	.066	.5420	.070	.0108	.025012	.019	.871	.028
	66	103	315	346	40	203	241	52	295
Q ₁	.014	.0274	.039	.0509	.005	.012051	.013	.627
	42	80	111	143	17	157	217	28	91
2Q	.001	.0140	.027	.0405	.005	.019	.051003	.060
	5	42	74	105	159	119	143	171	54
S ₁	.019	.8707	.089	.0393	.008	.871	.013	.003027
	14	52	263	295	168	308	332	189	243
P ₁	.016	.0059	.015	.0756	.017	.028	.627	.060	.027
	131	168	20	51	105	65	269	306	117

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 104½ days.]

Component sought (A)	Disturbing components (B, C, etc.)											
	K ₁	L ₁	M ₁	N ₁	2N	R ₁	S ₁	T ₁	λ ₁	μ ₁	ν ₁	2SM
K ₁036	.0371	.027	.014	.871	.5420	.160	.066	.016	.006	.049
	206	243	280	318	308	257	205	294	229	192	91
L ₁	.0360509	.040	.026	.087	.0756	.000	.627	.012	.011	.028
	154	217	255	292	103	51	180	269	203	346	65
M ₁	.037	.051051	.040	.039	.0108	.029	.076	.011	.076	.011
	117	143	217	255	65	14	142	51	346	309	28
N ₁	.027	.040	.0509051	.013	.0116	.029	.011	.076	.627	.003
	80	105	143	217	28	157	105	14	309	91	171
2N	.014	.026	.0405	.051003	.0189	.020	.012	.627	.060	.011
	42	68	105	143	171	119	68	157	91	54	133
R ₁	.871	.087	.0393	.013	.0038707	.542	.019	.022	.027	.029
	52	257	295	332	189	308	257	346	281	243	142
S ₁	.542	.076	.0108	.012	.019	.871871	.051	.011	.028	.011
	103	309	346	203	241	52	308	217	332	295	14
T ₁	.160	.000	.0286	.029	.020	.542	.8707091	.009	.007	.039
	155	180	218	255	292	103	52	269	204	346	65
λ ₁	.066	.627	.0756	.011	.012	.019	.0509	.091028	.047	.012
	66	91	309	346	203	14	143	91	295	257	157
μ ₁	.016	.012	.0108	.076	.627	.022	.0105	.009	.028051	.010
	131	157	14	51	269	79	28	156	65	143	42
ν ₁	.006	.011	.0756	.627	.060	.027	.0279	.007	.047	.051018
	168	14	51	269	306	117	65	14	103	217	79
2SM	0.49	.028	.0105	.003	.011	.029	.0108	.039	.012	.010	.018
	269	295	332	189	227	218	346	295	203	318	281

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TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 134 days.]

Component sought (A)	Disturbing component (B, C, etc.)									
	J ₁	K ₁	M ₁	O ₁	OO	P ₁	Q ₁	2Q	S ₁	ρ ₁
J ₁0273	.022	.0186	.011	.054	.016	.012	.040	.006
	205	222	239	170	253	264	288	319	201
K ₁	.027019	.0183	.018	.322	.019	.016	.793	.002
	155	197	214	146	228	239	264	294	356
M ₁	.022	.01920192	.017	.039	.022	.020	.070	.013
	138	163	197	128	31	222	246	97	339
O ₁	.019	.0183	.019015	.008	.027	.025	.033	.047
	121	146	163	111	14	205	229	80	322
OO	.011	.0183	.017	.0151030	.013	.010	.016	.007
	190	214	232	249	262	273	298	328	211
P ₁	.054	.3219	.039	.0082	.030004	.010	.793	.019
	107	132	329	346	98	191	216	66	308
Q ₁	.016	.0186	.022	.0273	.013	.004027	.018	.435
	96	121	138	155	87	169	205	55	117
2Q	.012	.0162	.020	.0248	.010	.010	.027008	.058
	72	96	114	131	62	144	155	30	92
S ₁	.040	.7928	.070	.0331	.016	.793	.018	.008021
	41	66	263	280	32	294	305	330	242
ρ ₁	.006	.0015	.013	.0467	.007	.019	.435	.058	.021
	159	4	21	38	149	52	243	268	118

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 134 days.]

Component sought (A)	Disturbing component (B, C, etc.)											
	K ₂	L ₂	M ₂	N ₂	2N	R ₂	S ₂	T ₂	λ ₂	μ ₂	ν ₂	2SM
K ₂011	.0183	.019	.016	.793	.3219	.090	.054	.006	.002	.034
	190	214	239	264	294	228	342	253	0201	356	61
L ₂	.0110273	.025	.021	.067	.0467	.039	.435	.004	.008	.019
	170	205	229	254	104	38	152	243	191	346	52
M ₂	.018	.027027	.025	.033	.0082	.029	.047	.008	.047	.008
	146	155	205	229	80	14	128	38	346	322	27
N ₂	.019	.025	.0273027	.018	.0044	.023	.008	.047	.435	.001
	121	131	155	205	55	169	103	14	322	117	2
2N	.016	.021	.0248	.027008	.0099	.017	.004	.435	.058	.004
	96	106	131	155	30	144	78	169	117	32	158
R ₂	.793	.067	.0331	.018	.0087928	.322	.040	.017	.021	.029
	66	256	280	305	330	204	228	319	267	242	128
S ₂	.322	.047	.0082	.004	.010	.793793	.027	.008	.019	.008
	132	322	346	191	216	66	294	205	333	308	14
T ₂	.090	.039	.0290	.023	.017	.322	.7928071	.011	.006	.033
	18	208	232	257	282	132	66	271	219	194	80
λ ₂	.054	.435	.0467	.008	.004	.040	.0273	.071019	.037	.004
	107	117	322	346	191	41	155	89	308	284	169
μ ₂	.006	.004	.0082	.047	.435	.017	.0080	.011	.019027	.008
	159	169	14	38	243	93	27	141	52	155	41
ν ₂	.002	.008	.0467	.435	.058	.021	.0188	.006	.037	.027013
	4	14	38	243	268	118	52	166	76	205	65
2SM	.034	.019	.0080	.001	.004	.029	.0082	.033	.004	.008	.013
	299	308	333	358	202	232	346	280	191	319	295

TABLE 55—*For clearing one component of the effects of the others—Continued.*

[Length of series, 162½ days.]

Component sought (<i>A</i>)	Disturbing components (<i>B, C, etc.</i>)									
	<i>J</i> ₁	<i>K</i> ₁	<i>M</i> ₁	<i>O</i> ₁	<i>OO</i>	<i>P</i> ₁	<i>Q</i> ₁	<i>2Q</i>	<i>S</i> ₁	<i>P</i> ₁
<i>J</i> ₁0171	.013	.0108	.000	.029	.011	.010	.044	.004
	198	208	217	180	218	236	254	298	198
<i>K</i> ₁	.017009	.0086	.009	.121	.011	.011	.705	.000
	162	189	199	161	200	217	236	280	359
<i>M</i> ₁	.013	.00880088	.008	.011	.013	.013	.058	.005
	152	171	189	152	10	208	226	90	350
<i>O</i> ₁	.011	.0086	.009008	.001	.017	.016	.027	.021
	143	161	171	142	1	198	217	81	341
<i>OO</i>	.000	.0086	.008	.0082016	.009	.009	.023	.003
	190	199	208	218	219	236	255	299	198
<i>P</i> ₁	.029	.1213	.011	.0005	.016006	.008	.705	.007
	142	160	350	359	141	197	216	80	340
<i>Q</i> ₁	.011	.0108	.013	.0171	.009	.006017	.016	.248
	124	143	152	162	124	163	198	63	142
<i>2Q</i>	.010	.0111	.013	.0162	.009	.008	.017010	.040
	106	124	134	143	105	144	162	44	124
<i>S</i> ₁	.044	.7048	.058	.0275	.023	.705	.016	.010019
	62	80	270	279	61	280	297	316	259
<i>P</i> ₁	.004	.0002	.005	.0208	.003	.007	.248	.040	.019
	162	1	10	19	162	20	218	236	101

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 162½ days.]

Component sought (<i>A</i>)	Disturbing components (<i>B</i> , <i>C</i> , etc.)											
	<i>K</i> ₂	<i>L</i> ₂	<i>M</i> ₂	<i>N</i> ₂	2 <i>N</i>	<i>R</i> ₂	<i>S</i> ₂	<i>T</i> ₂	<i>λ</i> ₂	<i>μ</i> ₂	<i>ν</i> ₂	2 <i>SM</i>
<i>K</i> ₂000	.0086	.011	.011	.705	.1213	.207	.029	.004	.000	.011
	180	199	217	236	280	200	300	218	198	359	21
<i>L</i> ₂	.0000171	.016	.015	.057	.0208	.059	.248	.006	.001	.007
	180	198	217	235	100	19	119	218	197	359	20
<i>M</i> ₂	.009	.017017	.016	.027	.0005	.030	.021	.001	.021	.001
	161	162	198	217	81	1	101	19	359	341	2
<i>N</i> ₂	.011	.016	.0171017	.016	.0057	.019	.001	.021	.248	.003
	143	143	162	198	63	163	82	1	341	142	164
2 <i>N</i>	.011	.015	.0162	.017010	.0082	.013	.006	.248	.040	.005
	124	125	143	162	44	144	64	163	142	124	145
<i>R</i> ₂	.705	.057	.0275	.016	.0107048	.121	.044	.014	.019	.030
	80	260	279	297	316	280	200	298	278	259	101
<i>S</i> ₂	.121	.021	.0005	.006	.008	.705705	.017	.001	.007	.001
	160	341	359	197	216	80	280	198	358	340	1
<i>T</i> ₂	.207	.059	.0296	.019	.013	.121	.7048058	.014	.018	.027
	60	241	259	278	296	160	80	279	258	240	81
<i>λ</i> ₂	.029	.248	.0208	.001	.006	.044	.0171	.058007	.020	.006
	142	142	341	359	197	62	162	81	340	321	163
<i>μ</i> ₂	.004	.006	.0005	.021	.248	.014	.0005	.014	.007017	.001
	162	163	1	19	218	82	2	102	20	162	3
<i>ν</i> ₂	.000	.001	.0208	.248	.040	.019	.0069	.018	.020	.017004
	1	1	19	218	236	101	20	120	39	198	21
2 <i>SM</i>	.011	.007	.0005	.003	.005	.030	.0005	.027	.006	.001	.004
	339	340	358	196	215	259	359	279	197	357	339

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 191½ days.]

Component sought (<i>A</i>)	Disturbing Components (<i>B, C, etc.</i>)									
	<i>J</i> ₁	<i>K</i> ₁	<i>M</i> ₁	<i>O</i> ₁	<i>OO</i>	<i>P</i> ₁	<i>Q</i> ₁	<i>2Q</i>	<i>S</i> ₁	<i>ρ</i> ₁
<i>J</i> ₁0072	.003	.0015	.010	.000	.003	.004	.042	.000
	189	187	186	12	180	195	204	275	182
<i>K</i> ₁	.007001	.0013	.001	.046	.002	.003	.605	.002
	171	358	357	3	351	186	195	266	353
<i>M</i> ₁	.003	.00130013	.001	.007	.003	.004	.049	.002
	173	2	358	5	173	187	196	87	355
<i>O</i> ₁	.002	.0013	.001001	.002	.007	.007	.024	.003
	174	3	2	7	175	189	198	89	356
<i>OO</i>	.010	.0013	.001	.0013004	.000	.002	.022	.002
	348	357	355	353	348	182	191	262	350
<i>P</i> ₁	.000	.0462	.007	.0023	.004004	.005	.605	.001
	180	9	187	185	12	194	204	94	182
<i>Q</i> ₁	.003	0.015	.003	.0072	.000	.004007	.015	.075
	165	174	173	171	178	166	189	80	167
<i>2Q</i>	.004	.0029	.004	.0071	.002	.005	.007011	.015
	156	165	164	162	169	156	171	71	158
<i>S</i> ₁	.042	.6053	.049	.0236	.022	.605	.015	.011016
	85	94	273	271	98	266	280	289	268
<i>ρ</i> ₁	.000	.0019	.002	.0033	.002	.001	.075	.015	.016
	178	7	5	4	10	178	193	202	92

TABLE 55.—*For clearing one component of the effects of the others—Continued.*

[Length of series, 191½ days.]

Component sought (<i>A</i>)	Disturbing Components (<i>B, C, etc.</i>)											
	K_2	L_2	M_2	N_2	$2N$	R_2	S_2	T_2	λ_2	μ_2	ν_2	$2SM$
K_2010	.0013	.002	.003	.605	.0462	.197	.000	.000	.002	.007
	348	357	186	195	266	351	257	180	182	353	166
L_2	.0100072	.007	.007	.048	.0033	.058	.075	.004	.002	.001
	12	189	198	207	98	4	89	193	194	185	178
M_2	.001	.007007	.007	.024	.0023	.025	.003	.002	.003	.002
	3	171	189	198	89	175	80	4	185	356	169
N_2	.002	.007	.0072007	.015	.0040	.016	.002	.003	.075	.003
	174	162	171	189	80	166	71	175	356	167	160
$2N$.003	.007	.0071	.007011	.0047	.011	.004	.075	.015	.004
	165	153	162	171	71	156	62	166	167	158	151
R_2	.605	.048	.0236	.015	.0116053	.046	.042	.012	.016	.025
	94	262	271	280	289	266	351	275	277	268	80
S_2	.046	.003	.0023	.004	.005	.605605	.007	.002	.001	.002
	9	356	185	194	204	94	266	189	191	182	175
T_2	.197	.058	.0252	.016	.011	.046	.6053048	.012	.017	.024
	103	271	280	289	298	9	94	283	285	276	89
λ_2	.000	.075	.0033	.002	.004	.042	.0072	.048001	.003	.004
	180	167	356	185	194	85	171	77	182	353	166
μ_2	.000	.004	.0023	.003	.075	.012	.0023	.012	.001007	.002
	178	166	175	4	193	83	169	75	178	171	164
ν_2	.002	.002	.0033	.075	.015	.016	.0006	.017	.003	.007001
	7	175	4	193	202	92	178	84	7	189	173
$2SM$.007	.001	.0023	.003	.004	.025	.0023	.024	.004	.002	.001
	194	182	191	200	209	280	185	271	194	196	187

TABLE 56.—For the summation of the annual tide.

Day of series	Sa.	Day of series	Sa.	Day of series	Sa.	Day of series	Sa.	Day of series	Sa.	Day of series	Sa.	Day of series	Sa.
1	0	206	14	420	4	633	18	846	8	1 059	22	1 272	12
8	0	221	14	434	4	647	18	860	8	1 073	22	1 286	12
9	1	222	15	435	5	648	19	861	9	1 074	23	1 287	13
23	1	236	15	449	5	662	19	875	9	1 088	23	1 301	13
24	2	237	16	450	6	663	20	876	10	1 089	0	1 302	14
38	2	251	16	464	6	677	20	890	10	1 103	0	1 316	14
39	3	252	17	465	7	678	21	891	11	1 104	1	1 317	15
53	3	266	17	479	7	692	21	906	11	1 119	1	1 332	15
54	4	267	18	480	8	693	22	907	12	1 120	2	1 333	16
69	4	282	18	495	8	708	22	921	12	1 134	2	1 347	16
70	5	283	19	496	9	709	23	922	13	1 135	3	1 348	17
84	5	297	19	510	9	723	23	936	13	1 149	3	1 362	17
85	6	298	20	511	10	724	0	937	14	1 150	4	1 363	18
99	6	312	20	525	10	738	0	951	14	1 164	4	1 377	18
100	7	313	21	526	11	739	1	952	15	1 165	5	1 378	19
114	7	327	21	540	11	753	1	966	15	1 179	5	1 393	19
115	8	328	22	541	12	754	2	967	16	1 180	6	1 394	20
129	8	342	22	555	12	769	2	982	16	1 195	6	1 408	20
130	9	343	23	556	13	770	3	983	17	1 196	7	1 409	21
145	9	358	23	571	13	784	3	997	17	1 210	7	1 423	21
146	10	359	0	572	14	785	4	998	18	1 211	8	1 424	22
160	10	373	0	586	14	799	4	1 012	18	1 225	8	1 438	22
161	11	374	1	587	15	800	5	1 013	19	1 226	9	1 439	23
175	11	388	1	601	15	814	5	1 027	19	1 240	9	1 453	23
176	12	389	2	602	16	815	6	1 028	20	1 241	10	1 454	0
190	12	403	2	616	16	829	6	1 042	20	1 256	10	1 469	0
191	13	404	3	617	17	830	7	1 043	21	1 257	11	1 470	1
205	13	419	3	632	17	845	7	1 058	21	1 271	11

This table gives the nearest component "hour" (i. e., 24th of monthly or yearly period) for each day (11.30 a. m.) of the series.

The values here given replace those given in the last column of Table 43 (Part II), which were found to be slightly in error.

TABLE 57.—Days having similar tides.*

Day of—		29 days		162.5 days		191.5 days		355 days		384 days	
Year	Month	Before	After	Before	After	Before	After	Before	After	Before	After
1	Jan. 1	Dec. 3	Jan. 30	July 22.5	June 12.5	June 23.5	July 11.5	Jan. 11	Dec. 22	Dec. 13	Jan. 20
2	2	4	31	23.5	13.5	24.5	12.5	12	23	14	21
3	3	5	Feb. 1	24.5	14.5	25.5	13.5	13	24	15	22
4	4	6	2	25.5	15.5	26.5	14.5	14	25	16	23
5	5	7	3	26.5	16.5	27.5	15.5	15	26	17	24
6	6	8	4	27.5	17.5	28.5	16.5	16	27	18	25
7	7	9	5	28.5	18.5	29.5	17.5	17	28	19	26
8	8	10	6	29.5	19.5	30.5	18.5	18	29	20	27
9	9	11	7	30.5	20.5	July 1.5	19.5	19	30	21	28
10	10	12	8	31.5	21.5	2.5	20.5	20	31	22	29
11	11	13	9	Aug. 1.5	22.5	3.5	21.5	21	Jan. 1	23	30
12	12	14	10	2.5	23.5	4.5	22.5	22	2	24	31
13	13	15	11	3.5	24.5	5.5	23.5	23	3	25	Feb. 1
14	14	16	12	4.5	25.5	6.5	24.5	24	4	26	2
15	15	17	13	5.5	26.5	7.5	25.5	25	5	27	3
16	16	18	14	6.5	27.5	8.5	26.5	26	6	28	4
17	17	19	15	7.5	28.5	9.5	27.5	27	7	29	5
18	18	20	16	8.5	29.5	10.5	28.5	28	8	30	6
19	19	21	17	9.5	30.5	11.5	29.5	29	9	31	7
20	20	22	18	10.5	July 1.5	12.5	30.5	30	10	Jan. 1	8
21	21	23	19	11.5	2.5	13.5	31.5	31	11	2	9
22	22	24	20	12.5	3.5	14.5	Aug. 1.5	Feb. 1	12	3	10
23	23	25	21	13.5	4.5	15.5	2.5	2	13	4	11
24	24	26	22	14.5	5.5	16.5	3.5	3	14	5	12
25	25	27	23	15.5	6.5	17.5	4.5	4	15	6	13
26	26	28	24	16.5	7.5	18.5	5.5	5	16	7	14
27	27	29	25	17.5	8.5	19.5	6.5	6	17	8	15
28	28	30	26	18.5	9.5	20.5	7.5	7	18	9	16
29	29	31	27	19.5	10.5	21.5	8.5	8	19	10	17
30	30	Jan. 1	28	20.5	11.5	22.5	9.5	9	20	11	18
31	31	2	Mar. 1	21.5	12.5	23.5	10.5	10	21	12	19
32	Feb. 1	3	2	22.5	13.5	24.5	11.5	11	22	13	20
33	2	4	3	23.5	14.5	25.5	12.5	12	23	14	21
34	3	5	4	24.5	15.5	26.5	13.5	13	24	15	22
35	4	6	5	25.5	16.5	27.5	14.5	14	25	16	23
36	5	7	6	26.5	17.5	28.5	15.5	15	26	17	24
37	6	8	7	27.5	18.5	29.5	16.5	16	27	18	25
38	7	9	8	28.5	19.5	30.5	17.5	17	28	19	26
39	8	10	9	29.5	20.5	31.5	18.5	18	29	20	27
40	9	11	10	30.5	21.5	Aug. 1.5	19.5	19	30	21	28
41	10	12	11	31.5	22.5	2.5	20.5	20	31	22	Mar. 1
42	11	13	12	Sept. 1.5	23.5	3.5	21.5	21	Feb. 1	23	2
43	12	14	13	2.5	24.5	4.5	22.5	22	2	24	3
44	13	15	14	3.5	25.5	5.5	23.5	23	3	25	4
45	14	16	15	4.5	26.5	6.5	24.5	24	4	26	5
46	15	17	16	5.5	27.5	7.5	25.5	25	5	27	6

* For explanation of this table see sec. 142.

TABLE 57.—Days having similar tides—Continued.

Day of—		29 days		162.5 days		191.5 days		355 days		384 days	
Year	Month.	Before	After	Before	After	Before	After	Before	After	Before	After
47	Feb. 16	Jan. 18	Mar. 17	Sept. 6.5	July 28.5	Aug. 8.5	Aug. 26.5	Feb. 26	Feb. 6	Jan. 28	Mar. 7
48	17	19	18	7.5	29.5	9.5	27.5	27	7	29	8
49	18	20	19	8.5	30.5	10.5	28.5	28	8	30	9
50	19	21	20	9.5	31.5	11.5	29.5	Mar. 1	9	31	10
51	20	22	21	10.5	Aug. 1.5	12.5	30.5	2	10	Feb. 1	11
52	21	23	22	11.5	2.5	13.5	31.5	3	11	2	12
53	22	24	23	12.5	3.5	14.5	Sept. 1.5	4	12	3	13
54	23	25	24	13.5	4.5	15.5	2.5	5	13	4	14
55	24	26	25	14.5	5.5	16.5	3.5	6	14	5	15
56	25	27	26	15.5	6.5	17.5	4.5	7	15	6	16
57	26	28	27	16.5	7.5	18.5	5.5	8	16	7	17
58	27	29	28	17.5	8.5	19.5	6.5	9	17	8	18
59	28	30	29	18.5	9.5	20.5	7.5	10	18	9	19
60	Mar. 1	31	30	19.5	10.5	21.5	8.5	11	19	10	20
61	2	Feb. 1	31	20.5	11.5	22.5	9.5	12	20	11	21
62	3	2	Apr. 1	21.5	12.5	23.5	10.5	13	21	12	22
63	4	3	2	22.5	13.5	24.5	11.5	14	22	13	23
64	5	4	3	23.5	14.5	25.5	12.5	15	23	14	24
65	6	5	4	24.5	15.5	26.5	13.5	16	24	15	25
66	7	6	5	25.5	16.5	27.5	14.5	17	25	16	26
67	8	7	6	26.5	17.5	28.5	15.5	18	26	17	27
68	9	8	7	27.5	18.5	29.5	16.5	19	27	18	28
69	10	9	8	28.5	19.5	30.5	17.5	20	28	19	29
70	11	10	9	29.5	20.5	31.5	18.5	21	Mar. 1	20	30
71	12	11	10	30.5	21.5	Sept. 1.5	19.5	22	2	21	31
72	13	12	11	Oct. 1.5	22.5	2.5	20.5	23	3	22	Apr. 1
73	14	13	12	2.5	23.5	3.5	21.5	24	4	23	2
74	15	14	13	3.5	24.5	4.5	22.5	25	5	24	3
75	16	15	14	4.5	25.5	5.5	23.5	26	6	25	4
76	17	16	15	5.5	26.5	6.5	24.5	27	7	26	5
77	18	17	16	6.5	27.5	7.5	25.5	28	8	27	6
78	19	18	17	7.5	28.5	8.5	26.5	29	9	28	7
79	20	19	18	8.5	29.5	9.5	27.5	30	10	Mar. 1	8
80	21	20	19	9.5	30.5	10.5	28.5	31	11	2	9
81	22	21	20	10.5	31.5	11.5	29.5	Apr. 1	12	3	10
82	23	22	21	11.5	Sept. 1.5	12.5	30.5	2	13	4	11
83	24	23	22	12.5	2.5	13.5	Oct. 1.5	3	14	5	12
84	25	24	23	13.5	3.5	14.5	2.5	4	15	6	13
85	26	25	24	14.5	4.5	15.5	3.5	5	16	7	14
86	27	26	25	15.5	5.5	16.5	4.5	6	17	8	15
87	28	27	26	16.5	6.5	17.5	5.5	7	18	9	16
88	29	28	27	17.5	7.5	18.5	6.5	8	19	10	17
89	30	Mar. 1	28	18.5	8.5	19.5	7.5	9	20	11	18
90	31	2	29	19.5	9.5	20.5	8.5	10	21	12	19
91	Apr. 1	3	30	20.5	10.5	21.5	9.5	11	22	13	20
92	2	4	May 1	21.5	11.5	22.5	10.5	12	23	14	21
93	3	5	2	22.5	12.5	23.5	11.5	13	24	15	22

TABLE 57.—*Days having similar tides*—Continued.

Day of—		29 days		162.5 days		101.5 days		355 days		384 days	
Year	Month	Before	After	Before	After	Before	After	Before	After	Before	After
94	Apr. 4	Mar. 6	May 3	Oct. 23.5	Sept. 13.5	Sept. 24.5	Oct. 12.5	Apr. 14	Mar. 25	Mar. 16	Apr. 23
95	5	7	4	24.5	14.5	25.5	13.5	15	26	17	24
96	6	8	5	25.5	15.5	26.5	14.5	16	27	18	25
97	7	9	6	26.5	16.5	27.5	15.5	17	28	19	26
98	8	10	7	27.5	17.5	28.5	16.5	18	29	20	27
99	9	11	8	28.5	18.5	29.5	17.5	19	30	21	28
100	10	12	9	29.5	19.5	30.5	18.5	20	31	22	29
101	11	13	10	30.5	20.5	Oct. 1.5	19.5	21	Apr. 1	23	30
102	12	14	11	31.5	21.5	2.5	20.5	22	2	24	May 1
103	13	15	12	Nov. 1.5	22.5	3.5	21.5	23	3	25	2
104	14	16	13	2.5	23.5	4.5	22.5	24	4	26	3
105	15	17	14	3.5	24.5	5.5	23.5	25	5	27	4
106	16	18	15	4.5	25.5	6.5	24.5	26	6	28	5
107	17	19	16	5.5	26.5	7.5	25.5	27	7	29	6
108	18	20	17	6.5	27.5	8.5	26.5	28	8	30	7
109	19	21	18	7.5	28.5	9.5	27.5	29	9	31	8
110	20	22	19	8.5	29.5	10.5	28.5	30	10	Apr. 1	9
111	21	23	20	9.5	30.5	11.5	29.5	May 1	11	2	10
112	22	24	21	10.5	Oct. 1.5	12.5	30.5	2	12	3	11
113	23	25	22	11.5	2.5	13.5	31.5	3	13	4	12
114	24	26	23	12.5	3.5	14.5	Nov. 1.5	4	14	5	13
115	25	27	24	13.5	4.5	15.5	2.5	5	15	6	14
116	26	28	25	14.5	5.5	16.5	3.5	6	16	7	15
117	27	29	26	15.5	6.5	17.5	4.5	7	17	8	16
118	28	30	27	16.5	7.5	18.5	5.5	8	18	9	17
119	29	31	28	17.5	8.5	19.5	6.5	9	19	10	18
120	30	Apr. 1	29	18.5	9.5	20.5	7.5	10	20	11	19
121	May 1	2	30	19.5	10.5	21.5	8.5	11	21	12	20
122	2	3	31	20.5	11.5	22.5	9.5	12	22	13	21
123	3	4	June 1	21.5	12.5	23.5	10.5	13	23	14	22
124	4	5	2	22.5	13.5	24.5	11.5	14	24	15	23
125	5	6	3	23.5	14.5	25.5	12.5	15	25	16	24
126	6	7	4	24.5	15.5	26.5	13.5	16	26	17	25
127	7	8	5	25.5	16.5	27.5	14.5	17	27	18	26
128	8	9	6	26.5	17.5	28.5	15.5	18	28	19	27
129	9	10	7	27.5	18.5	29.5	16.5	19	29	20	28
130	10	11	8	28.5	19.5	30.5	17.5	20	30	21	29
131	11	12	9	29.5	20.5	31.5	18.5	21	May 1	22	30
132	12	13	10	30.5	21.5	Nov. 1.5	19.5	22	2	23	31
133	13	14	11	Dec. 1.5	22.5	2.5	20.5	23	3	24	June 1
134	14	15	12	2.5	23.5	3.5	21.5	24	4	25	2
135	15	16	13	3.5	24.5	4.5	22.5	25	5	26	3
136	16	17	14	4.5	25.5	5.5	23.5	26	6	27	4
137	17	18	15	5.5	26.5	6.5	24.5	27	7	28	5
138	18	19	16	6.5	27.5	7.5	25.5	28	8	29	6
139	19	20	17	7.5	28.5	8.5	26.5	29	9	30	7
140	20	21	18	8.5	29.5	9.5	27.5	30	10	May 1	8
141	21	22	19	9.5	30.5	10.5	28.5	31	11	2	9

TABLE 57.—Days having similar tides—Continued.

Day of—		29 days		162.5 days		101.5 days		355 days		384 days	
Year	Month	Before	After	Before	After	Before	After	Before	After	Before	After
142	May 22	Apr. 23	June 20	Dec. 10.5	Oct. 31.5	Nov. 11.5	Nov. 29.5	June 1	May 12	May 3	June 10
143	23	24	21	11.5	Nov. 1.5	12.5	30.5	2	13	4	11
144	24	25	22	12.5	2.5	13.5	Dec. 1.5	3	14	5	12
145	25	26	23	13.5	3.5	14.5	2.5	4	15	6	13
146	26	27	24	14.5	4.5	15.5	3.5	5	16	7	14
147	27	28	25	15.5	5.5	16.5	4.5	6	17	8	15
148	28	29	26	16.5	6.5	17.5	5.5	7	18	9	16
149	29	30	27	17.5	7.5	18.5	6.5	8	19	10	17
150	30	May 1	28	18.5	8.5	19.5	7.5	9	20	11	18
151	31	2	29	19.5	9.5	20.5	8.5	10	21	12	19
152	June 1	3	30	20.5	10.5	21.5	9.5	11	22	13	20
153	2	4	July 1	21.5	11.5	22.5	10.5	12	23	14	21
154	3	5	2	22.5	12.5	23.5	11.5	13	24	15	22
155	4	6	3	23.5	13.5	24.5	12.5	14	25	16	23
156	5	7	4	24.5	14.5	25.5	13.5	15	26	17	24
157	6	8	5	25.5	15.5	26.5	14.5	16	27	18	25
158	7	9	6	26.5	16.5	27.5	15.5	17	28	19	26
159	8	10	7	27.5	17.5	28.5	16.5	18	29	20	27
160	9	11	8	28.5	18.5	29.5	17.5	19	30	21	28
161	10	12	9	29.5	19.5	30.5	18.5	20	31	22	29
162	11	13	10	30.5	20.5	Dec. 1.5	19.5	21	June 1	23	30
163	12	14	11	31.5	21.5	2.5	20.5	22	2	24	July 1
164	13	15	12	Jan. 1.5	22.5	3.5	21.5	23	3	25	2
165	14	16	13	2.5	23.5	4.5	22.5	24	4	26	3
166	15	17	14	3.5	24.5	5.5	23.5	25	5	27	4
167	16	18	15	4.5	25.5	6.5	24.5	26	6	28	5
168	17	19	16	5.5	26.5	7.5	25.5	27	7	29	6
169	18	20	17	6.5	27.5	8.5	26.5	28	8	30	7
170	19	21	18	7.5	28.5	9.5	27.5	29	9	31	8
171	20	22	19	8.5	29.5	10.5	28.5	30	10	June 1	9
172	21	23	20	9.5	30.5	11.5	29.5	July 1	11	2	10
173	22	24	21	10.5	Dec. 1.5	12.5	30.5	2	12	3	11
174	23	25	22	11.5	2.5	13.5	31.5	3	13	4	12
175	24	26	23	12.5	3.5	14.5	Jan. 1.5	4	14	5	13
176	25	27	24	13.5	4.5	15.5	2.5	5	15	6	14
177	26	28	25	14.5	5.5	16.5	3.5	6	16	7	15
178	27	29	26	15.5	6.5	17.5	4.5	7	17	8	16
179	28	30	27	16.5	7.5	18.5	5.5	8	18	9	17
180	29	31	28	17.5	8.5	19.5	6.5	9	19	10	18
181	30	June 1	29	18.5	9.5	20.5	7.5	10	20	11	19
182	July 1	2	30	19.5	10.5	21.5	8.5	11	21	12	20
183	2	3	31	20.5	11.5	22.5	9.5	12	22	13	21
184	3	4	Aug. 1	21.5	12.5	23.5	10.5	13	23	14	22
185	4	5	2	22.5	13.5	24.5	11.5	14	24	15	23
186	5	6	3	23.5	14.5	25.5	12.5	15	25	16	24
187	6	7	4	24.5	15.5	26.5	13.5	16	26	17	25
188	7	8	5	25.5	16.5	27.5	14.5	17	27	18	26
189	8	9	6	26.5	17.5	28.5	15.5	18	28	19	27
190	9	10	7	27.5	18.5	29.5	16.5	19	29	20	28

TABLE 57.—Days having similar tides—Continued.

Day of—		29 days		162.5 days		191.5 days		355 days		384 days	
Year	Month	Before	After	Before	After	Before	After	Before	After	Before	After
191	July 10	June 11	Aug. 8	Jan. 28.5	Dec. 19.5	Dec. 30.5	Jan. 17.5	July 20	June 30	June 21	Jul. 29
192	11	12	9	29.5	20.5	31.5	18.5	21	July 1	22	30
193	12	13	10	30.5	21.5	Jan. 1.5	19.5	22	2	23	31
194	13	14	11	31.5	22.5	2.5	20.5	23	3	24	Aug. 1
195	14	15	12	Feb. 1.5	23.5	3.5	21.5	24	4	25	2
196	13	16	13	2.5	24.5	4.5	22.5	25	5	26	3
197	16	17	14	3.5	25.5	5.5	23.5	26	6	27	4
198	17	18	15	4.5	26.5	6.5	24.5	27	7	28	5
199	18	19	16	5.5	27.5	7.5	25.5	28	8	29	6
200	19	20	17	6.5	28.5	8.5	26.5	29	9	30	7
201	20	21	18	7.5	29.5	9.5	27.5	30	10	July 1	8
202	21	22	19	8.5	30.5	10.5	28.5	31	11	2	9
203	22	23	20	9.5	31.5	11.5	29.5	Aug. 1	12	3	10
204	23	24	21	10.5	Jan. 1.5	12.5	30.5	2	13	4	11
205	24	25	22	11.5	2.5	13.5	31.5	3	14	5	12
206	25	26	23	12.5	3.5	14.5	Feb. 1.5	4	15	6	13
207	26	27	24	13.5	4.5	15.5	2.5	5	16	7	14
208	27	28	25	14.5	5.5	16.5	3.5	6	17	8	15
209	28	29	26	15.5	6.5	17.5	4.5	7	18	9	16
210	29	30	27	16.5	7.5	18.5	5.5	8	19	10	17
211	30	July 1	28	17.5	8.5	19.5	6.5	9	20	11	18
212	31	2	29	18.5	9.5	20.5	7.5	10	21	12	19
213	Aug. 1	3	30	19.5	10.5	21.5	8.5	11	22	13	20
214	2	4	31	20.5	11.5	22.5	9.5	12	23	14	21
215	3	5	Sept. 1	21.5	12.5	23.5	10.5	13	24	15	22
216	4	6	2	22.5	13.5	24.5	11.5	14	25	16	23
217	5	7	3	23.5	14.5	25.5	12.5	15	26	17	24
218	6	8	4	24.5	15.5	26.5	13.5	16	27	18	25
219	7	9	5	25.5	16.5	27.5	14.5	17	28	19	26
220	8	10	6	26.5	17.5	28.5	15.5	18	29	20	27
221	9	11	7	27.5	18.5	29.5	16.5	19	30	21	28
222	10	12	8	28.5	19.5	30.5	17.5	20	31	22	29
223	11	13	9	Mar. 1.5	20.5	31.5	18.5	21	Aug. 1	23	30
224	12	14	10	2.5	21.5	Feb. 1.5	19.5	22	2	24	31
225	13	15	11	3.5	22.5	2.5	20.5	23	3	25	Sept. 1
226	14	16	12	4.5	23.5	3.5	21.5	24	4	26	2
227	15	17	13	5.5	24.5	4.5	22.5	25	5	27	3
228	16	18	14	6.5	25.5	5.5	23.5	26	6	28	4
229	17	19	15	7.5	26.5	6.5	24.5	27	7	29	5
230	18	20	16	8.5	27.5	7.5	25.5	28	8	30	6
231	19	21	17	9.5	28.5	8.5	26.5	29	9	31	7
232	20	22	18	10.5	29.5	9.5	27.5	30	10	Aug. 1	8
233	21	23	19	11.5	30.5	10.5	28.5	31	11	2	9
234	22	24	20	12.5	31.5	11.5	Mar. 1.5	Sept. 1	12	3	10
235	23	25	21	13.5	Feb. 1.5	12.5	2.5	2	13	4	11
236	24	26	22	14.5	2.5	13.5	3.5	3	14	5	12

TABLE 57.—*Days having similar tides*—Continued.

Day of—		29 days		162.5 days		191.5 days		355 days		384 days	
Year	Month	Before	After	Before	After	Before	After	Before	After	Before	After
237	Aug. 25	July 27	Sept. 23	Mar. 15.5	Feb. 3.5	Feb. 14.5	Mar. 4.5	Sept. 4	Aug. 15	Aug. 6	Sept. 13
238	26	28	24	16.5	4.5	15.5	5.5	5	16	7	14
239	27	29	25	17.5	5.5	16.5	6.5	6	17	8	15
240	28	30	26	18.5	6.5	17.5	7.5	7	18	9	16
241	29	31	27	19.5	7.5	18.5	8.5	8	19	10	17
242	30	Aug. 1	28	20.5	8.5	19.5	9.5	9	20	11	18
243	31	2	29	21.5	9.5	20.5	10.5	10	21	12	19
244	Sept. 1	3	30	22.5	10.5	21.5	11.5	11	22	13	20
245	2	4	Oct. 1	23.5	11.5	22.5	12.5	12	23	14	21
246	3	5	2	24.5	12.5	23.5	13.5	13	24	15	22
247	4	6	3	25.5	13.5	24.5	14.5	14	25	16	23
248	5	7	4	26.5	14.5	25.5	15.5	15	26	17	24
249	6	8	5	27.5	15.5	26.5	16.5	16	27	18	25
250	7	9	6	28.5	16.5	27.5	17.5	17	28	19	26
251	8	10	7	29.5	17.5	28.5	18.5	18	29	20	27
252	9	11	8	30.5	18.5	Mar. 1.5	19.5	19	30	21	28
253	10	12	9	31.5	19.5	2.5	20.5	20	31	22	29
254	11	13	10	Apr. 1.5	20.5	3.5	21.5	21	Sept. 1	23	30
255	12	14	11	2.5	21.5	4.5	22.5	22	2	24	Oct. 1
256	13	15	12	3.5	22.5	5.5	23.5	23	3	25	2
257	14	16	13	4.5	23.5	6.5	24.5	24	4	26	3
258	15	17	14	5.5	24.5	7.5	25.5	25	5	27	4
259	16	18	15	6.5	25.5	8.5	26.5	26	6	28	5
260	17	19	16	7.5	26.5	9.5	27.5	27	7	29	6
261	18	20	17	8.5	27.5	10.5	28.5	28	8	30	7
262	19	21	18	9.5	28.5	11.5	29.5	29	9	31	8
263	20	22	19	10.5	Mar. 1.5	12.5	30.5	30	10	Sept. 1	9
264	21	23	20	11.5	2.5	13.5	31.5	Oct. 1	11	2	10
265	22	24	21	12.5	3.5	14.5	Apr. 1.5	2	12	3	11
266	23	25	22	13.5	4.5	15.5	2.5	3	13	4	12
267	24	26	23	14.5	5.5	16.5	3.5	4	14	5	13
268	25	27	24	15.5	6.5	17.5	4.5	5	15	6	14
269	26	28	25	16.5	7.5	18.5	5.5	6	16	7	15
270	27	29	26	17.5	8.5	19.5	6.5	7	17	8	16
271	28	30	27	18.5	9.5	20.5	7.5	8	18	9	17
272	29	31	28	19.5	10.5	21.5	8.5	9	19	10	18
273	30	Sept. 1	29	20.5	11.5	22.5	9.5	10	20	11	19
274	Oct. 1	2	30	21.5	12.5	23.5	10.5	11	21	12	20
275	2	3	31	22.5	13.5	24.5	11.5	12	22	13	21
276	3	4	Nov. 1	23.5	14.5	25.5	12.5	13	23	14	22
277	4	5	2	24.5	15.5	26.5	13.5	14	24	15	23
278	5	6	3	25.5	16.5	27.5	14.5	15	25	16	24
279	6	7	4	26.5	17.5	28.5	15.5	16	26	17	25
280	7	8	5	27.5	18.5	29.5	16.5	17	27	18	26
281	8	9	6	28.5	19.5	30.5	17.5	18	28	19	27
282	9	10	7	29.5	20.5	31.5	18.5	19	29	20	28

TABLE 57.—Days having similar tides—Continued.

Day of		29 days		162.5 days		191.5 days		355 days		384 days	
Year	Month	Before	After	Before	After	Before	After	Before	After	Before	After
283	Oct. 10	Sep. 11	Nov. 8	Apr. 30.5	Mar. 21.5	Apr. 1.5	Apr. 19.5	Oct. 20	Sep. 30	Sep. 21	Oct. 29
284	11	12	9	May 1.5	22.5	2.5	20.5	21	Oct. 1	22	30
285	12	13	10	2.5	23.5	3.5	21.5	22	2	23	31
286	13	14	11	3.5	24.5	4.5	22.5	23	3	24	Nov. 1
287	14	15	12	4.5	25.5	5.5	23.5	24	4	25	2
288	15	16	13	5.5	26.5	6.5	24.5	25	5	26	3
289	16	17	14	6.5	27.5	7.5	25.5	26	6	27	4
290	17	18	15	7.5	28.5	8.5	26.5	27	7	28	5
291	18	19	16	8.5	29.5	9.5	27.5	28	8	29	6
292	19	20	17	9.5	30.5	10.5	28.5	29	9	30	7
293	20	21	18	10.5	31.5	11.5	29.5	30	10	Oct. 1	8
294	21	22	19	11.5	Apr. 1.5	12.5	30.5	31	11	2	9
295	22	23	20	12.5	2.5	13.5	May 1.5	Nov. 1	12	3	10
296	23	24	21	13.5	3.5	14.5	2.5	2	13	4	11
297	24	25	22	14.5	4.5	15.5	3.5	3	14	5	12
298	25	26	23	15.5	5.5	16.5	4.5	4	15	6	13
299	26	27	24	16.5	6.5	17.5	5.5	5	16	7	14
300	27	28	25	17.5	7.5	18.5	6.5	6	17	8	15
301	28	29	26	18.5	8.5	19.5	7.5	7	18	9	16
302	29	30	27	19.5	9.5	20.5	8.5	8	19	10	17
303	30	Oct. 1	28	20.5	10.5	21.5	9.5	9	20	11	18
304	31	2	29	21.5	11.5	22.5	10.5	10	21	12	19
305	Nov. 1	3	30	22.5	12.5	23.5	11.5	11	22	13	20
306	2	4	Dec. 1	23.5	13.5	24.5	12.5	12	23	14	21
307	3	5	2	24.5	14.5	25.5	13.5	13	24	15	22
308	4	6	3	25.5	15.5	26.5	14.5	14	25	16	23
309	5	7	4	26.5	16.5	27.5	15.5	15	26	17	24
310	6	8	5	27.5	17.5	28.5	16.5	16	27	18	25
311	7	9	6	28.5	18.5	29.5	17.5	17	28	19	26
312	8	10	7	29.5	19.5	30.5	18.5	18	29	20	27
313	9	11	8	30.5	20.5	May 1.5	19.5	19	30	21	28
314	10	12	9	31.5	21.5	2.5	20.5	20	31	22	29
315	11	13	10	June 1.5	22.5	3.5	21.5	21	Nov. 1	23	30
316	12	14	11	2.5	23.5	4.5	22.5	22	2	24	Dec. 1
317	13	15	12	3.5	24.5	5.5	23.5	23	3	25	2
318	14	16	13	4.5	25.5	6.5	24.5	24	4	26	3
319	15	17	14	5.5	26.5	7.5	25.5	25	5	27	4
320	16	18	15	6.5	27.5	8.5	26.5	26	6	28	5
321	17	19	16	7.5	28.5	9.5	27.5	27	7	29	6
322	18	20	17	8.5	29.5	10.5	28.5	28	8	30	7
323	19	21	18	9.5	30.5	11.5	29.5	29	9	31	8
324	20	22	19	10.5	May 1.5	12.5	30.5	30	10	Nov. 1	9
325	21	23	20	11.5	2.5	13.5	31.5	Dec. 1	11	2	10
326	22	24	21	12.5	3.5	14.5	June 1.5	2	12	3	11
327	23	25	22	13.5	4.5	15.5	2.5	3	13	4	12
328	24	26	23	14.5	5.5	16.5	3.5	4	14	5	13
329	25	27	24	15.5	6.5	17.5	4.5	5	15	6	14
330	26	28	25	16.5	7.5	18.5	5.5	6	16	7	15
331	27	29	26	17.5	8.5	19.5	6.5	7	17	8	16

TABLE 57.—*Days having similar tides*—Continued.

Day of		29 days		162.5 days		191.5 days		355 days		384 days	
Year	Month	Before	After	Before	After	Before	After	Before	After	Before	After
332	Nov. 28	Oct. 30	Dec. 27	June 18.5	May 9.5	May 20.5	June 7.5	Dec. 8	Nov. 18	Nov. 9	Dec. 17
333	29	31	28	19.5	10.5	21.5	8.5	9	19	10	18
334	30	Nov. 1	29	20.5	11.5	22.5	9.5	10	20	11	19
335	Dec. 1	2	30	21.5	12.5	23.5	10.5	11	21	12	20
336	2	3	31	22.5	13.5	24.5	11.5	12	22	13	21
337	3	4	Jan. 1	23.5	14.5	25.5	12.5	13	23	14	22
338	4	5	2	24.5	15.5	26.5	13.5	14	24	15	23
339	5	6	3	25.5	16.5	27.5	14.5	15	25	16	24
340	6	7	4	26.5	17.5	28.5	15.5	16	26	17	25
341	7	8	5	27.5	18.5	29.5	16.5	17	27	18	26
342	8	9	6	28.5	19.5	30.5	17.5	18	28	19	27
343	9	10	7	29.5	20.5	31.5	18.5	19	29	20	28
344	10	11	8	30.5	21.5	June 1.5	19.5	20	30	21	29
345	11	12	9	July 1.5	22.5	2.5	20.5	21	Dec. 1	22	30
346	12	13	10	2.5	23.5	3.5	21.5	22	2	23	31
347	13	14	11	3.5	24.5	4.5	22.5	23	3	24	Jan. 1
348	14	15	12	4.5	25.5	5.5	23.5	24	4	25	2
349	15	16	13	5.5	26.5	6.5	24.5	25	5	26	3
350	16	17	14	6.5	27.5	7.5	25.5	26	6	27	4
351	17	18	15	7.5	28.5	8.5	26.5	27	7	28	5
352	18	19	16	8.5	29.5	9.5	27.5	28	8	29	6
353	19	20	17	9.5	30.5	10.5	28.5	29	9	30	7
354	20	21	18	10.5	31.5	11.5	29.5	30	10	Dec. 1	8
355	21	22	19	11.5	June 1.5	12.5	30.5	31	11	2	9
356	22	23	20	12.5	2.5	13.5	July 1.5	Jan. 1	12	3	10
357	23	24	21	13.5	3.5	14.5	2.5	2	13	4	11
358	24	25	22	14.5	4.5	15.5	3.5	3	14	5	12
359	25	26	23	15.5	5.5	16.5	4.5	4	15	6	13
360	26	27	24	16.5	6.5	17.5	5.5	5	16	7	14
361	27	28	25	17.5	7.5	18.5	6.5	6	17	8	15
362	28	29	26	18.5	8.5	19.5	7.5	7	18	9	16
363	29	30	27	19.5	9.5	20.5	8.5	8	19	10	17
364	30	Dec. 1	28	20.5	10.5	21.5	9.5	9	20	11	18
365	31	2	29	21.5	11.5	22.5	10.5	10	21	12	19

TABLE 58.—*Greenwich mean civil times of mean perigee and apogee, 1850–1950.*

1850	1851	1852	1853	1854	1855
hr.	hr.	hr.	hr.	hr.	hr.
A Jan. 12 12.0	A Jan. 5 17.0	P Jan. 12 16.7	P Jan. 4 21.7	A Jan. 11 21.4	A Jan. 5 2.4
P Jan. 26 6.7	P Jan. 19 11.7	A Jan. 26 11.4	A Jan. 18 16.4	P Jan. 25 16.1	P Jan. 18 21.1
A Feb. 9 1.3	A Feb. 2 6.3	P Feb. 9 6.0	P Feb. 1 11.0	A Feb. 8 10.7	A Feb. 1 15.7
P Feb. 22 20.0	P Feb. 16 1.0	A Feb. 23 0.7	A Feb. 15 5.7	P Feb. 22 5.4	P Feb. 15 10.4
A Mar. 8 14.6	A Mar. 1 19.7	P Mar. 7 19.3	P Mar. 1 0.3	A Mar. 8 0.0	A Mar. 1 5.0
P Mar. 22 9.3	P Mar. 15 14.3	A Mar. 21 14.0	A Mar. 14 19.0	P Mar. 21 18.7	P Mar. 14 23.7
A Apr. 5 3.9	A Mar. 29 9.0	P Apr. 4 8.6	P Mar. 28 13.7	A Apr. 4 13.3	A Mar. 28 18.4
P Apr. 18 22.6	P Apr. 12 3.6	A Apr. 18 3.3	A Apr. 11 8.3	P Apr. 18 8.0	P Apr. 11 13.0
A May 2 17.3	A Apr. 25 22.3	P May 1 21.9	P Apr. 25 3.0	A May 2 2.6	A Apr. 25 7.7
P May 16 11.9	P May 9 16.9	A May 15 16.6	A May 8 21.6	P May 15 21.3	P May 9 2.3
A May 30 6.6	A May 23 11.6	P May 29 11.3	P May 22 16.3	A May 29 16.0	A May 22 21.0
P June 13 1.2	P June 6 6.2	A June 12 5.9	A June 5 10.9	P June 12 10.6	P June 5 15.6
A June 26 19.9	A June 20 0.9	P June 26 0.6	P June 19 5.6	A June 26 5.3	A June 19 10.3
P July 10 14.5	P July 3 19.5	A July 9 19.2	A July 3 0.2	P July 9 23.9	P July 3 4.9
A July 24 9.2	A July 17 14.2	P July 23 13.9	P July 16 18.9	A July 23 18.6	A July 16 23.6
P Aug. 7 3.8	P July 31 8.9	A Aug. 6 8.5	A July 30 13.5	P Aug. 6 13.2	P July 30 18.2
A Aug. 20 22.5	A Aug. 14 3.5	P Aug. 20 3.2	P Aug. 13 8.2	A Aug. 20 7.9	A Aug. 13 12.9
P Sept. 3 17.1	P Aug. 27 22.2	A Sept. 2 21.8	A Aug. 27 2.9	P Sept. 3 2.5	P Aug. 27 7.6
A Sept. 17 11.8	A Sept. 10 16.8	P Sept. 16 16.5	P Sept. 9 21.5	A Sept. 16 21.2	A Sept. 10 2.2
P Oct. 1 6.5	P Sept. 24 11.5	A Sept. 30 11.1	A Sept. 23 16.2	P Sept. 30 15.8	P Sept. 23 20.9
A Oct. 15 1.1	A Oct. 8 6.1	P Oct. 14 5.8	P Oct. 7 10.8	A Oct. 14 10.5	A Oct. 7 15.5
P Oct. 28 19.8	P Oct. 22 0.8	A Oct. 28 0.5	A Oct. 21 5.5	P Oct. 28 5.2	P Oct. 21 10.2
A Nov. 11 14.4	A Nov. 4 19.4	P Nov. 10 19.1	P Nov. 4 0.1	A Nov. 10 23.8	A Nov. 4 4.8
P Nov. 25 9.1	P Nov. 18 14.1	A Nov. 24 13.8	A Nov. 17 18.8	P Nov. 24 18.5	P Nov. 17 23.5
A Dec. 9 3.7	A Dec. 2 8.7	P Dec. 8 8.4	P Dec. 1 13.4	A Dec. 8 13.1	A Dec. 1 18.1
P Dec. 22 22.4	P Dec. 16 3.4	A Dec. 22 3.1	A Dec. 15 8.1	P Dec. 22 7.8	P Dec. 15 12.8
	A Dec. 29 22.1		P Dec. 29 2.7		A Dec. 29 7.4
1856	1857	1858	1859	1860	1861
hr.	hr.	hr.	hr.	hr.	hr.
P Jan. 12 2.1	P Jan. 4 7.1	A Jan. 11 6.8	A Jan. 4 11.8	P Jan. 11 11.5	P Jan. 3 16.5
A Jan. 25 20.8	A Jan. 18 1.8	P Jan. 25 1.4	P Jan. 18 6.5	A Jan. 25 6.1	A Jan. 17 11.2
P Feb. 8 15.4	P Jan. 31 20.4	A Feb. 7 20.1	A Feb. 1 1.1	P Feb. 8 0.8	P Jan. 31 5.8
A Feb. 22 10.1	A Feb. 14 15.1	P Feb. 21 14.8	P Feb. 14 19.8	A Feb. 21 19.5	A Feb. 14 0.5
P Mar. 7 4.7	P Feb. 28 9.7	A Mar. 7 9.4	A Feb. 28 14.4	P Mar. 6 14.1	P Feb. 27 19.1
A Mar. 20 23.4	A Mar. 14 4.4	P Mar. 21 4.1	P Mar. 14 9.1	A Mar. 20 8.8	A Mar. 13 13.8
P Apr. 3 18.0	P Mar. 27 23.0	A Apr. 3 22.7	A Mar. 28 3.7	P Apr. 3 3.4	P Mar. 27 8.4
A Apr. 17 12.7	A Apr. 10 17.7	P Apr. 17 17.4	P Apr. 10 22.4	A Apr. 16 22.1	A Apr. 10 3.1
P May 1 7.3	P Apr. 24 12.4	A May 1 12.0	A Apr. 24 17.0	P Apr. 30 16.7	P Apr. 23 21.7
A May 15 2.0	A May 8 7.0	P May 15 6.7	P May 8 11.7	A May 14 11.4	A May 7 16.4
P May 28 20.6	P May 22 1.7	A May 29 1.3	A May 22 6.4	P May 28 6.0	P May 21 11.1
A June 11 15.3	A June 4 20.3	P June 11 20.0	P June 5 1.0	A June 11 0.7	A June 4 5.7
P June 25 10.0	P June 18 15.0	A June 25 14.6	A June 18 19.7	P June 24 19.3	P June 18 0.4
A July 9 4.6	A July 2 9.6	P July 9 9.3	P July 2 14.3	A July 8 14.0	A July 1 19.0
P July 22 23.3	P July 16 4.3	A July 23 4.0	A July 16 9.0	P July 22 8.7	P July 15 13.7
A Aug. 5 17.9	A July 29 22.9	P Aug. 5 22.6	P July 30 3.6	A Aug. 5 3.3	A July 29 8.3
P Aug. 19 12.6	P Aug. 12 17.6	A Aug. 19 17.3	A Aug. 12 22.3	P Aug. 18 22.0	P Aug. 12 3.0
A Sept. 2 7.2	A Aug. 26 12.2	P Sept. 2 11.9	P Aug. 26 16.9	A Sept. 1 16.6	A Aug. 25 21.6
P Sept. 16 1.9	P Sept. 9 6.9	A Sept. 16 6.6	A Sept. 9 11.6	P Sept. 15 11.3	P Sept. 8 16.3
A Sept. 29 20.5	A Sept. 23 1.6	P Sept. 30 1.2	P Sept. 23 6.2	A Sept. 29 5.9	A Sept. 22 10.9
P Oct. 13 15.2	P Oct. 6 20.2	A Oct. 13 19.9	A Oct. 7 0.9	P Oct. 13 0.6	P Oct. 6 5.6
A Oct. 27 9.8	A Oct. 20 14.9	P Oct. 27 14.5	P Oct. 20 19.6	A Oct. 26 19.2	A Oct. 20 0.3
P Nov. 10 4.5	P Nov. 3 9.5	A Nov. 10 9.2	A Nov. 3 14.2	P Nov. 9 13.9	P Nov. 2 18.9
A Nov. 23 23.2	P Nov. 17 4.2	P Nov. 24 3.8	P Nov. 17 8.9	A Nov. 23 8.5	A Nov. 16 13.6
P Dec. 7 17.8	A Dec. 14 17.5	A Dec. 7 22.5	A Dec. 1 3.5	P Dec. 7 3.2	P Nov. 30 8.2
A Dec. 21 12.5	P Dec. 28 12.1	P Dec. 21 17.2	P Dec. 14 22.2	A Dec. 20 21.9	A Dec. 14 2.9
			A Dec. 28 16.8		P Dec. 27 21.5

TABLE 58.—*Greenwich mean civil times of mean perigee and apogee, 1850-1950—Con.*

1862	1863	1864	1865	1866	1867
hr.	hr.	hr.	hr.	hr.	hr.
A Jan. 10 16.2	A Jan. 3 21.2	P Jan. 10 20.9	P Jan. 3 1.9	A Jan. 10 1.6	A Jan. 3 6.6
P Jan. 24 10.8	P Jan. 17 15.9	A Jan. 24 15.5	A Jan. 16 20.5	P Jan. 23 20.2	P Jan. 17 1.2
A Feb. 7 5.5	A Jan. 31 10.5	P Feb. 7 10.2	P Jan. 30 15.2	A Feb. 6 14.9	A Jan. 30 19.9
P Feb. 21 0.1	P Feb. 14 5.2	A Feb. 21 4.8	A Feb. 13 9.9	P Feb. 20 9.5	P Feb. 13 14.6
A Mar. 6 18.8	A Feb. 27 23.8	P Mar. 5 23.5	P Feb. 27 4.5	A Mar. 6 4.2	A Feb. 27 9.2
P Mar. 20 13.5	P Mar. 13 18.5	A Mar. 19 18.1	A Mar. 12 23.2	P Mar. 19 22.8	P Mar. 13 3.9
A Apr. 3 8.1	A Mar. 27 13.1	P Apr. 2 12.8	P Mar. 26 17.8	A Apr. 2 17.5	A Mar. 26 22.5
P Apr. 17 2.8	P Apr. 10 7.8	A Apr. 16 7.5	A Apr. 9 12.5	P Apr. 16 12.2	P Apr. 9 17.2
A Apr. 30 21.4	A Apr. 24 2.4	P Apr. 30 2.1	P Apr. 23 7.1	A Apr. 30 6.8	A Apr. 23 11.8
P May 14 16.1	P May 7 21.1	A May 13 20.8	A May 7 1.8	P May 14 1.5	P May 7 6.5
A May 28 10.7	A May 21 15.7	P May 27 15.4	P May 20 20.4	A May 27 20.1	A May 21 1.1
P June 11 5.4	P June 4 10.4	A June 10 10.1	A June 3 15.1	P June 10 14.8	P June 3 19.8
A June 25 0.0	A June 18 5.1	P June 24 4.7	P June 17 9.7	A June 24 9.4	A June 17 14.4
P July 8 18.7	P July 1 23.7	A July 7 23.4	A July 1 4.4	P July 8 4.1	P July 1 9.1
A July 22 13.3	A July 15 18.4	P July 21 18.0	P July 14 23.1	A July 21 22.7	A July 15 3.8
P Aug. 5 8.0	P July 29 13.0	A Aug. 4 12.7	A July 28 17.7	P Aug. 4 17.4	P July 28 22.4
A Aug. 19 2.7	A Aug. 12 7.7	P Aug. 18 7.3	P Aug. 11 12.4	A Aug. 18 12.0	A Aug. 11 17.1
P Sept. 1 21.3	P Aug. 26 2.3	A Sept. 1 2.0	A Aug. 25 7.0	P Sept. 1 6.7	P Aug. 25 11.7
A Sept. 15 16.0	A Sept. 8 21.0	P Sept. 14 20.7	P Sept. 8 1.7	A Sept. 15 1.4	A Sept. 8 6.4
P Sept. 29 10.6	P Sept. 22 15.6	A Sept. 28 15.3	A Sept. 21 20.3	P Sept. 28 20.0	P Sept. 22 1.0
A Oct. 13 5.3	A Oct. 6 10.3	P Oct. 12 10.0	P Oct. 5 15.0	A Oct. 12 14.7	A Oct. 5 19.7
P Oct. 27 0.0	P Oct. 20 4.9	A Oct. 26 4.6	A Oct. 19 9.6	P Oct. 26 9.3	P Oct. 19 14.3
A Nov. 9 18.6	A Nov. 2 23.6	P Nov. 8 23.3	P Nov. 2 4.3	A Nov. 9 4.0	A Nov. 2 9.0
P Nov. 23 13.2	P Nov. 16 18.2	A Nov. 22 17.9	A Nov. 15 22.9	P Nov. 22 22.6	P Nov. 16 3.6
A Dec. 7 7.9	A Nov. 30 12.9	P Dec. 6 12.6	P Nov. 29 17.6	A Dec. 6 17.3	A Nov. 29 22.3
P Dec. 21 2.5	P Dec. 14 7.6	A Dec. 20 7.2	A Dec. 13 12.3	P Dec. 20 11.9	P Dec. 13 17.0
	A Dec. 28 2.2		P Dec. 27 6.9		A Dec. 27 11.6
1868	1869	1870	1871	1872	1873
hr.	hr.	hr.	hr.	hr.	hr.
P Jan. 10 6.3	P Jan. 2 11.3	A Jan. 9 11.0	A Jan. 2 16.0	P Jan. 9 15.7	P Jan. 1 20.7
A Jan. 24 0.9	A Jan. 16 5.9	P Jan. 23 5.6	P Jan. 16 10.6	A Jan. 23 10.3	A Jan. 15 15.3
P Feb. 6 19.6	P Jan. 30 0.6	A Feb. 6 0.3	A Jan. 30 5.3	P Feb. 6 5.0	P Jan. 29 10.0
A Feb. 20 14.2	A Feb. 12 19.2	P Feb. 19 18.9	P Feb. 12 23.9	A Feb. 19 23.6	A Feb. 12 4.6
P Mar. 5 8.9	P Feb. 26 13.9	A Mar. 5 13.6	A Feb. 26 18.6	P Mar. 4 18.3	P Feb. 25 23.3
A Mar. 19 3.5	A Mar. 12 8.6	P Mar. 19 8.2	P Mar. 12 13.2	A Mar. 18 12.9	A Mar. 11 17.9
P Apr. 1 22.2	P Mar. 26 3.2	A Apr. 2 2.9	A Mar. 26 7.9	P Apr. 1 7.6	P Mar. 25 12.6
A Apr. 15 16.8	A Apr. 8 21.9	P Apr. 15 21.5	P Apr. 9 2.6	A Apr. 15 2.2	A Apr. 8 7.3
P Apr. 29 11.5	P Apr. 22 16.5	A Apr. 29 16.2	A Apr. 22 21.2	P Apr. 28 20.9	P Apr. 22 1.9
A May 13 6.2	A May 6 11.2	P May 13 10.8	P May 6 15.9	A May 12 15.5	A May 5 20.6
P May 27 0.8	P May 20 5.8	A May 27 5.5	A May 20 10.5	P May 26 10.2	P May 19 15.2
A June 9 19.5	A June 3 0.5	P June 10 0.2	P June 3 5.2	A June 9 4.9	A June 2 9.9
P June 23 14.1	P June 16 19.1	A June 23 18.8	A June 16 23.8	P June 22 23.5	P June 16 4.5
A July 7 8.8	A June 30 13.8	P July 7 13.5	P June 30 18.5	A July 6 18.2	A June 29 23.2
P July 21 3.4	P July 14 8.4	A July 21 8.1	A July 14 13.1	P July 20 12.8	P July 13 17.8
A Aug. 3 22.1	A July 28 3.1	P Aug. 4 2.8	P July 28 7.8	A Aug. 3 7.5	A July 27 12.5
P Aug. 17 16.7	P Aug. 10 21.8	A Aug. 17 21.4	A Aug. 11 2.4	P Aug. 17 2.1	P Aug. 10 7.1
A Aug. 31 11.4	A Aug. 24 16.4	P Aug. 31 16.2	P Aug. 24 21.1	A Aug. 30 20.8	A Aug. 24 1.8
P Sept. 14 6.0	P Sept. 7 11.1	A Sept. 14 10.7	A Sept. 7 15.8	P Sept. 13 15.4	P Sept. 6 20.5
A Sept. 28 0.7	A Sept. 21 5.7	P Sept. 28 5.4	P Sept. 21 10.4	A Sept. 27 10.1	A Sept. 20 15.1
P Oct. 11 19.4	P Oct. 5 0.4	A Oct. 12 0.0	A Oct. 5 5.1	P Oct. 11 4.7	P Oct. 4 9.8
A Oct. 25 14.0	A Oct. 18 19.0	P Oct. 25 18.7	P Oct. 18 23.7	A Oct. 24 23.4	A Oct. 18 4.4
P Nov. 8 8.7	P Nov. 1 13.7	A Nov. 8 13.4	A Nov. 1 18.4	P Nov. 7 18.1	P Oct. 31 23.1
A Nov. 22 3.3	A Nov. 15 8.3	P Nov. 22 8.0	P Nov. 15 13.0	A Nov. 21 12.7	A Nov. 14 17.7
P Dec. 5 22.0	P Nov. 29 3.0	A Dec. 6 2.7	A Nov. 29 7.7	P Dec. 5 7.4	P Nov. 28 12.4
A Dec. 19 16.6	A Dec. 12 21.6	P Dec. 19 21.3	P Dec. 13 2.3	A Dec. 19 2.0	A Dec. 12 7.0
	P Dec. 26 16.3		A Dec. 26 21.0		P Dec. 26 1.7

TABLE 58.—*Greenwich mean civil times of mean perigee and apogee, 1850-1950—Con.*

1874	1875	1876	1877	1878	1879
<i>hr.</i>	<i>hr.</i>	<i>hr.</i>	<i>hr.</i>	<i>hr.</i>	<i>hr.</i>
A Jan. 8 20.3	A Jan. 2 1.4	P Jan. 9 1.0	P Jan. 1 6.1	A Jan. 8 5.7	A Jan. 1 10.8
P Jan. 22 15.0	P Jan. 15 20.0	A Jan. 22 19.7	A Jan. 15 0.7	P Jan. 22 0.4	P Jan. 15 5.4
A Feb. 5 9.7	A Jan. 29 14.7	P Feb. 5 14.3	P Jan. 28 19.4	A Feb. 4 19.0	A Jan. 29 0.1
P Feb. 19 4.3	P Feb. 12 9.3	A Feb. 19 9.0	A Feb. 11 14.0	P Feb. 18 13.7	P Feb. 11 18.7
A Mar. 4 23.0	A Feb. 26 4.0	P Mar. 4 3.7	P Feb. 25 8.7	A Mar. 4 8.4	A Feb. 25 13.4
P Mar. 18 17.6	P Mar. 11 22.6	A Mar. 17 22.3	A Mar. 11 3.3	P Mar. 18 3.0	P Mar. 11 8.0
A Apr. 1 12.3	A Mar. 25 17.3	P Mar. 31 17.0	P Mar. 24 22.0	A Mar. 31 21.7	A Mar. 25 2.8
P Apr. 15 6.9	P Apr. 8 11.9	A Apr. 14 11.6	A Apr. 7 16.6	P Apr. 14 16.3	P Apr. 7 21.3
A Apr. 29 1.6	A Apr. 22 6.6	P Apr. 28 6.3	P Apr. 21 11.3	A Apr. 28 11.0	A Apr. 21 16.0
P May 12 20.2	P May 6 1.3	A May 12 0.9	A May 5 5.9	P May 12 5.6	P May 5 10.6
A May 26 14.9	A May 19 19.9	P May 25 19.6	P May 19 0.6	A May 26 0.3	A May 19 5.3
P June 9 9.5	P June 2 14.6	A June 8 14.2	A June 1 19.3	P June 8 18.9	P June 2 0.0
A June 23 4.2	A June 16 9.2	P June 22 8.9	P June 15 13.9	A June 22 13.6	A June 15 18.6
P July 6 22.9	P June 30 3.9	A July 6 3.5	A June 29 8.6	P July 6 8.2	P June 29 13.3
A July 20 17.5	A July 13 22.5	P July 19 22.2	P July 13 3.2	A July 20 2.9	A July 13 7.9
P Aug. 3 12.2	P July 27 17.2	A Aug. 2 16.9	A July 26 21.9	P Aug. 2 21.6	P July 27 2.6
A Aug. 17 6.8	A Aug. 10 11.8	P Aug. 16 11.5	P Aug. 9 16.5	A Aug. 16 16.2	A Aug. 9 21.2
P Aug. 31 1.5	P Aug. 24 6.5	A Aug. 30 6.2	A Aug. 23 11.2	P Aug. 30 10.9	P Aug. 23 15.9
A Sept. 13 20.1	A Sept. 7 1.1	P Sept. 13 0.8	P Sept. 6 5.8	A Sept. 13 5.5	A Sept. 6 10.5
P Sept. 27 14.8	P Sept. 20 19.8	A Sept. 26 19.5	A Sept. 20 0.5	P Sept. 27 0.2	P Sept. 20 5.2
A Oct. 11 9.4	A Oct. 4 14.5	P Oct. 10 14.1	P Oct. 3 19.1	A Oct. 10 18.8	A Oct. 3 23.8
P Oct. 25 4.1	P Oct. 18 9.1	A Oct. 24 8.8	A Oct. 17 13.8	P Oct. 24 13.5	P Oct. 17 18.5
A Nov. 7 22.7	A Nov. 1 3.8	P Nov. 7 3.4	P Oct. 31 8.5	A Nov. 7 8.1	A Oct. 31 13.2
P Nov. 21 17.4	P Nov. 14 22.4	A Nov. 20 22.1	A Nov. 14 3.1	P Nov. 21 2.8	P Nov. 14 7.8
A Dec. 5 12.1	A Nov. 28 17.1	P Dec. 4 16.7	P Nov. 27 21.8	A Dec. 4 21.4	A Nov. 28 2.5
P Dec. 19 6.7	P Dec. 12 11.7	A Dec. 18 11.4	A Dec. 11 16.4	P Dec. 18 16.1	P Dec. 11 21.1
	A Dec. 26 6.4		P Dec. 25 11.1		A Dec. 25 15.8
1880	1881	1882	1883	1884	1885
<i>hr.</i>	<i>hr.</i>	<i>hr.</i>	<i>hr.</i>	<i>hr.</i>	<i>hr.</i>
P Jan. 8 10.4	A Jan. 14 10.1	A Jan. 7 15.1	P Jan. 14 14.8	P Jan. 7 19.8	A Jan. 13 19.5
A Jan. 22 5.1	P Jan. 28 4.8	P Jan. 21 9.8	A Jan. 28 9.4	A Jan. 21 14.5	P Jan. 27 14.1
P Feb. 4 23.7	A Feb. 10 23.4	A Feb. 4 4.4	P Feb. 11 4.1	P Feb. 4 9.1	A Feb. 10 8.8
A Feb. 18 18.4	P Feb. 24 18.1	P Feb. 17 23.0	A Feb. 24 22.8	A Feb. 18 3.8	P Feb. 24 3.5
P Mar. 3 13.0	A Mar. 10 12.7	A Mar. 3 17.7	P Mar. 10 17.4	P Mar. 2 22.4	A Mar. 9 22.1
A Mar. 17 7.7	P Mar. 24 7.4	P Mar. 17 12.4	A Mar. 24 12.1	A Mar. 16 17.1	P Mar. 23 16.8
P Mar. 31 2.4	A Apr. 7 2.0	A Mar. 31 7.0	P Apr. 7 6.7	P Mar. 30 11.7	A Apr. 6 11.4
A Apr. 13 21.0	P Apr. 20 20.7	P Apr. 14 1.7	A Apr. 21 1.4	A Apr. 13 6.4	P Apr. 20 6.1
P Apr. 27 15.7	A May 4 15.3	A Apr. 27 20.4	P May 4 20.0	P Apr. 27 1.1	A May 4 0.7
A May 11 10.3	P May 18 10.0	P May 11 15.0	A May 18 14.7	A May 10 19.7	P May 17 19.4
P May 25 5.0	A June 1 4.6	A May 25 9.7	P June 1 9.3	P May 24 14.4	A May 31 14.0
A June 7 23.6	P June 14 23.3	P June 8 4.3	A June 15 4.0	A June 7 9.0	P June 14 8.7
P June 21 18.3	A June 28 18.0	A June 21 23.0	P June 28 22.6	P June 21 3.7	A June 28 3.3
A July 5 12.9	P July 12 12.6	P July 5 17.6	A July 12 17.3	A July 4 22.3	P July 11 22.0
P July 19 7.6	A July 26 7.3	A July 19 12.3	P July 26 12.0	P July 18 17.0	A July 25 16.7
A Aug. 2 2.2	P Aug. 9 1.9	P Aug. 2 6.9	A Aug. 9 6.6	A Aug. 1 11.6	P Aug. 8 11.3
P Aug. 15 20.9	A Aug. 22 20.6	A Aug. 16 1.6	P Aug. 23 1.3	P Aug. 15 6.3	A Aug. 22 6.0
A Aug. 29 15.6	P Sept. 5 15.2	P Aug. 29 20.2	A Sept. 5 19.9	A Aug. 29 0.9	P Sept. 5 0.6
P Sept. 12 10.2	A Sept. 19 9.9	A Sept. 12 14.9	P Sept. 19 14.6	P Sept. 11 19.6	A Sept. 18 19.3
A Sept. 26 4.9	P Oct. 3 4.5	P Sept. 26 9.6	A Oct. 3 9.2	A Sept. 25 14.3	P Oct. 2 13.9
P Oct. 9 23.5	A Oct. 16 23.2	A Oct. 10 4.2	P Oct. 17 3.9	P Oct. 9 8.9	A Oct. 16 8.6
A Oct. 23 18.2	P Oct. 30 17.8	P Oct. 23 22.9	A Oct. 30 22.5	A Oct. 23 3.6	P Oct. 30 3.2
P Nov. 6 12.8	A Nov. 13 12.5	A Nov. 6 17.5	P Nov. 13 17.2	P Nov. 5 22.2	A Nov. 12 21.9
A Nov. 20 7.5	P Nov. 27 7.2	P Nov. 20 12.2	A Nov. 27 11.9	A Nov. 19 16.9	P Nov. 26 16.5
P Dec. 4 2.1	A Dec. 11 1.8	A Dec. 4 6.8	P Dec. 11 6.5	P Dec. 3 11.5	A Dec. 10 11.1
A Dec. 17 20.8	P Dec. 24 20.5	P Dec. 18 1.5	A Dec. 25 1.2	A Dec. 17 6.2	P Dec. 24 5.9
P Dec. 31 15.4		A Dec. 31 20.1		P Dec. 31 0.8	

TABLE 58—Greenwich mean civil times of mean perigee and apogee, 1850-1950—Con.

1886	1887	1888	1889	1890	1891
hr.	hr.	hr.	hr.	hr.	hr.
A Jan. 7 0.5	P Jan. 14 0.2	P Jan. 7 5.2	A Jan. 13 4.9	A Jan. 6 9.9	P Jan. 13 9.6
P Jan. 20 19.2	A Jan. 27 18.8	A Jan. 20 23.9	P Jan. 26 23.5	P Jan. 20 4.6	A Jan. 27 4.2
A Feb. 3 13.8	P Feb. 10 13.5	P Feb. 3 18.5	A Feb. 9 18.2	A Feb. 2 23.2	P Feb. 9 22.9
P Feb. 17 8.5	A Feb. 24 8.1	A Feb. 17 13.2	P Feb. 23 12.8	P Feb. 16 17.9	A Feb. 23 17.5
A Mar. 3 3.1	P Mar. 10 2.8	P Mar. 2 7.8	A Mar. 9 7.5	A Mar. 2 12.5	P Mar. 9 12.2
P Mar. 16 21.8	A Mar. 23 21.5	A Mar. 16 2.5	P Mar. 23 2.1	P Mar. 16 7.2	A Mar. 23 6.8
A Mar. 30 16.4	P Apr. 6 16.1	P Mar. 29 21.1	A Apr. 5 20.8	A Mar. 30 1.8	P Apr. 6 1.5
P Apr. 13 11.1	A Apr. 20 10.8	A Apr. 12 15.8	P Apr. 19 15.5	P Apr. 12 20.5	A Apr. 19 20.2
A Apr. 27 5.7	P May 4 5.4	P Apr. 26 10.4	A May 3 10.1	A Apr. 26 15.1	P May 3 14.8
P May 11 0.4	A May 18 0.1	A May 10 5.1	P May 17 4.8	P May 10 9.8	A May 17 9.5
A May 24 19.1	P May 31 18.7	P May 23 23.7	A May 30 23.4	A May 24 4.4	P May 31 4.1
P June 7 13.7	A June 14 13.4	A June 6 18.4	P June 13 18.1	P June 6 23.1	A June 13 22.8
A June 21 8.4	P June 28 8.0	P June 20 13.1	A June 27 12.7	A June 20 17.8	P June 27 17.4
P July 5 3.0	A July 12 2.7	A July 4 7.7	P July 11 7.4	P July 4 12.4	A July 11 12.1
A July 18 21.7	P July 25 21.3	P July 18 2.4	A July 25 2.0	A July 18 7.1	P July 25 6.7
P Aug. 1 16.3	A Aug. 8 16.0	A July 31 21.0	P Aug. 7 20.7	P Aug. 1 1.7	A Aug. 8 1.4
A Aug. 15 11.0	P Aug. 22 10.7	P Aug. 14 15.7	A Aug. 21 15.3	A Aug. 14 20.4	P Aug. 21 20.0
P Aug. 29 5.6	A Sept. 5 5.3	A Aug. 28 10.3	P Sept. 4 10.0	P Aug. 28 15.0	A Sept. 4 14.7
A Sept. 12 0.3	P Sept. 19 0.0	P Sept. 11 5.0	A Sept. 18 4.7	A Sept. 11 9.7	P Sept. 18 9.4
P Sept. 25 18.9	A Oct. 2 18.6	A Sept. 24 23.6	P Oct. 1 23.3	P Sept. 25 4.3	A Oct. 2 4.0
A Oct. 9 13.6	P Oct. 16 13.3	P Oct. 8 18.3	A Oct. 15 18.0	A Oct. 8 23.0	P Oct. 15 22.7
P Oct. 23 8.3	A Oct. 30 7.9	A Oct. 22 12.9	P Oct. 29 12.6	P Oct. 22 17.6	A Oct. 29 17.3
A Nov. 6 2.9	P Nov. 13 2.6	P Nov. 5 7.6	A Nov. 12 7.3	A Nov. 5 12.3	P Nov. 12 12.0
P Nov. 19 21.6	A Nov. 26 21.2	A Nov. 19 2.3	P Nov. 26 1.9	P Nov. 19 7.0	A Nov. 26 6.6
A Dec. 3 16.2	P Dec. 10 15.9	P Dec. 2 20.9	A Dec. 9 20.6	A Dec. 3 1.6	P Dec. 10 1.2
P Dec. 17 10.9	A Dec. 24 10.5	A Dec. 16 15.6	P Dec. 23 15.2	P Dec. 16 20.3	A Dec. 23 19.9
A Dec. 31 5.5		P Dec. 30 10.2		A Dec. 30 14.9	
1892	1893	1894	1895	1896	1897
hr.	hr.	hr.	hr.	hr.	hr.
P Jan. 6 14.6	A Jan. 12 14.3	A Jan. 5 19.3	P Jan. 12 19.0	P Jan. 6 0.0	A Jan. 11 23.7
A Jan. 20 9.2	P Jan. 26 8.9	P Jan. 19 13.9	A Jan. 26 13.6	A Jan. 19 18.6	P Jan. 25 18.3
P Feb. 3 3.9	A Feb. 9 3.6	A Feb. 2 8.6	P Feb. 9 8.3	P Feb. 2 13.3	A Feb. 8 13.0
A Feb. 16 22.6	P Feb. 22 22.2	P Feb. 16 3.2	A Feb. 23 2.9	A Feb. 16 7.9	P Feb. 22 7.6
P Mar. 1 17.2	A Mar. 8 16.9	A Mar. 1 21.9	P Mar. 8 21.6	P Mar. 1 2.6	A Mar. 8 2.3
A Mar. 15 11.9	P Mar. 22 11.5	P Mar. 15 16.6	A Mar. 22 16.2	A Mar. 14 21.3	P Mar. 21 20.9
P Mar. 29 6.5	A Apr. 5 6.2	A Mar. 29 11.2	P Apr. 5 10.9	P Mar. 28 15.9	A Apr. 4 15.6
A Apr. 12 1.2	P Apr. 19 0.8	P Apr. 12 5.9	A Apr. 19 5.5	A Apr. 11 10.6	P Apr. 18 10.2
P Apr. 25 19.8	A May 2 19.5	A Apr. 26 0.5	P May 3 0.2	P Apr. 25 5.2	A May 2 4.9
A May 9 14.5	P May 16 14.2	P May 9 19.2	A May 16 18.8	A May 8 23.9	P May 15 23.5
P May 23 9.1	A May 30 8.8	A May 23 13.8	P May 30 13.5	P May 22 18.5	A May 29 18.2
A June 6 3.8	P June 13 3.5	P June 6 8.5	A June 13 8.2	A June 5 13.2	P June 12 12.9
P June 19 22.4	A June 26 22.1	A June 20 3.1	P June 27 2.8	P June 19 7.8	A June 26 7.5
A July 3 17.1	P July 10 16.8	P July 3 21.8	A July 10 21.5	A July 3 2.5	P July 10 2.2
P July 17 11.8	A July 24 11.4	A July 17 16.4	P July 24 16.1	P July 16 21.1	A July 23 20.8
A July 31 6.4	P Aug. 7 6.1	P July 31 11.1	A Aug. 7 10.8	A July 30 15.8	P Aug. 6 15.5
P Aug. 14 1.1	A Aug. 21 0.7	A Aug. 14 5.8	P Aug. 21 5.4	P Aug. 13 10.5	A Aug. 20 10.1
A Aug. 27 19.7	P Sept. 3 19.4	P Aug. 28 0.4	A Sept. 4 0.1	A Aug. 27 5.1	P Sept. 3 4.8
P Sept. 10 14.4	A Sept. 17 14.0	A Sept. 10 19.1	P Sept. 17 18.7	P Sept. 9 23.8	A Sept. 16 23.4
A Sept. 24 9.0	P Oct. 1 8.7	P Sept. 24 13.7	A Oct. 1 13.4	A Sept. 23 18.4	P Sept. 30 18.1
P Oct. 8 3.7	A Oct. 15 3.4	A Oct. 8 8.4	P Oct. 15 8.0	P Oct. 7 13.1	A Oct. 14 12.7
A Oct. 21 22.3	P Oct. 28 22.0	P Oct. 22 3.0	A Oct. 29 2.7	A Oct. 21 7.7	P Oct. 28 7.4
P Nov. 4 17.0	A Nov. 11 16.7	A Nov. 4 21.7	P Nov. 11 21.4	P Nov. 4 2.4	A Nov. 11 2.1
A Nov. 18 11.6	P Nov. 25 11.3	P Nov. 18 16.3	A Nov. 25 16.0	A Nov. 17 21.0	P Nov. 24 20.7
P Dec. 2 6.3	A Dec. 9 6.0	A Dec. 2 11.0	P Dec. 9 10.7	P Dec. 1 15.7	A Dec. 8 15.4
A Dec. 16 1.0	P Dec. 23 0.6	P Dec. 16 5.6	A Dec. 23 5.3	A Dec. 15 10.3	P Dec. 22 10.0
P Dec. 29 19.6		A Dec. 30 0.3		P Dec. 29 5.0	

TABLE 58.—*Greenwich mean civil times of mean perigee and apogee, 1850–1950—Con.*

1898	1899	1900	1901	1902	1903
hr.	hr.	hr.	hr.	hr.	hr.
A Jan. 5 4.7	P Jan. 12 4.3	P Jan. 5 9.4	A Jan. 12 9.0	A Jan. 5 4.1	P Jan. 12 13.7
P Jan. 18 23.3	A Jan. 25 23.0	A Jan. 19 4.0	P Jan. 26 3.7	P Jan. 19 8.7	A Jan. 26 8.4
A Feb. 1 18.0	P Feb. 8 17.7	P Feb. 1 22.7	A Feb. 8 22.3	A Feb. 2 3.4	P Feb. 9 3.0
P Feb. 15 12.6	A Feb. 22 12.3	A Feb. 15 17.3	P Feb. 22 17.0	P Feb. 15 22.0	A Feb. 22 21.7
A Mar. 1 7.3	P Mar. 8 7.0	P Mar. 1 12.0	A Mar. 8 11.7	A Mar. 1 16.7	P Mar. 8 16.4
P Mar. 15 1.9	A Mar. 22 1.6	A Mar. 15 6.6	P Mar. 22 6.3	P Mar. 15 11.3	A Mar. 22 11.0
A Mar. 28 20.6	P Apr. 4 20.3	P Mar. 29 1.3	A Apr. 5 1.0	A Mar. 29 6.0	P Apr. 5 5.7
P Apr. 11 15.3	A Apr. 18 14.9	A Apr. 11 19.9	P Apr. 18 19.6	P Apr. 12 0.6	A Apr. 19 0.3
A Apr. 25 9.9	P May 2 9.6	P Apr. 25 14.6	A May 2 14.3	A Apr. 25 19.3	P May 2 19.0
P May 9 4.6	A May 16 4.2	A May 9 9.3	P May 16 8.9	P May 9 14.0	A May 16 13.6
A May 22 23.2	P May 29 22.9	P May 23 3.9	A May 30 3.6	A May 23 8.6	P May 30 8.3
P June 5 17.9	A June 12 17.5	A June 5 22.6	P June 12 22.2	P June 6 3.3	A June 13 2.9
A June 19 12.5	P June 26 12.2	P June 19 17.2	A June 26 16.9	A June 19 21.9	P June 26 21.6
P July 3 7.2	A July 10 6.9	A July 3 11.9	P July 10 11.6	P July 3 16.6	A July 10 16.2
A July 17 1.8	P July 24 1.5	P July 17 6.5	A July 24 6.2	A July 17 11.2	P July 24 10.9
P July 30 20.5	A Aug. 6 20.2	A July 31 1.2	P Aug. 7 0.9	P July 31 5.9	A Aug. 7 5.6
A Aug. 13 15.1	P Aug. 20 14.8	P Aug. 13 19.8	A Aug. 20 19.5	A Aug. 14 0.5	P Aug. 21 0.2
P Aug. 27 9.8	A Sept. 3 9.5	A Aug. 27 14.5	P Sept. 3 14.2	P Aug. 27 19.2	A Sept. 3 18.9
A Sept. 10 4.5	P Sept. 17 4.1	P Sept. 10 9.1	A Sept. 17 8.8	A Sept. 10 13.8	P Sept. 17 13.5
P Sept. 23 23.1	A Sept. 30 22.8	A Sept. 24 3.8	P Oct. 1 3.5	P Sept. 24 8.5	A Oct. 1 8.2
A Oct. 7 17.6	P Oct. 14 17.4	P Oct. 7 22.5	A Oct. 14 22.1	A Oct. 8 3.2	P Oct. 15 2.8
P Oct. 21 12.4	A Oct. 28 12.1	A Oct. 21 17.1	P Oct. 28 16.8	P Oct. 21 21.8	A Oct. 28 21.5
A Nov. 4 7.1	P Nov. 11 6.7	P Nov. 4 11.8	A Nov. 11 11.4	A Nov. 4 16.5	P Nov. 11 16.1
P Nov. 18 1.7	A Nov. 25 1.4	A Nov. 18 6.4	P Nov. 25 6.1	P Nov. 18 11.1	A Nov. 25 10.8
A Dec. 1 20.4	P Dec. 8 20.1	P Dec. 2 1.1	A Dec. 9 0.8	A Dec. 2 5.8	P Dec. 9 5.4
P Dec. 15 15.0	A Dec. 22 14.7	A Dec. 15 19.7	P Dec. 22 19.4	P Dec. 16 0.4	A Dec. 23 0.1
A Dec. 29 9.7		P Dec. 29 14.4		A Dec. 29 19.1	
1904	1905	1906	1907	1908	1909
hr.	hr.	hr.	hr.	hr.	hr.
P Jan. 5 18.8	A Jan. 11 18.4	A Jan. 4 23.4	P Jan. 11 23.1	P Jan. 5 4.1	A Jan. 11 3.8
A Jan. 19 13.4	P Jan. 25 13.1	P Jan. 18 18.1	A Jan. 25 17.8	A Jan. 18 22.8	P Jan. 24 22.5
P Feb. 2 8.1	A Feb. 8 7.7	A Feb. 1 12.8	P Feb. 8 12.4	P Feb. 1 17.5	A Feb. 7 17.1
A Feb. 16 2.7	P Feb. 22 2.4	P Feb. 15 7.4	A Feb. 22 7.1	A Feb. 15 12.1	P Feb. 21 11.8
P Feb. 29 21.4	A Mar. 7 21.0	A Mar. 1 2.1	P Mar. 8 1.7	P Feb. 29 6.8	A Mar. 7 6.4
A Mar. 14 16.0	P Mar. 21 15.7	P Mar. 14 20.7	A Mar. 21 20.4	A Mar. 14 1.4	P Mar. 21 1.1
P Mar. 28 10.7	A Apr. 4 10.4	A Mar. 28 15.4	P Apr. 4 15.0	P Mar. 27 20.1	A Apr. 3 19.7
A Apr. 11 5.3	P Apr. 18 5.0	P Apr. 11 10.0	A Apr. 18 9.7	A Apr. 10 14.7	P Apr. 17 14.4
P Apr. 25 0.0	A May 1 23.7	A Apr. 25 4.7	P May 2 4.4	P Apr. 24 9.4	A May 1 9.1
A May 8 18.6	P May 15 18.3	P May 8 23.3	A May 15 23.0	A May 8 4.0	P May 15 3.7
P May 22 13.3	A May 29 13.0	A May 22 18.0	P May 29 17.7	P May 21 22.7	A May 28 22.4
A June 5 8.0	P June 12 7.6	P June 5 12.6	A June 12 12.3	A June 4 17.3	P June 11 17.0
P June 19 2.6	A June 26 2.3	A June 19 7.3	P June 26 7.0	P June 18 12.0	A June 25 11.7
A July 2 21.3	P July 9 20.9	P July 3 2.0	A July 10 1.6	A July 2 6.7	P July 9 6.3
P July 16 15.9	A July 23 15.6	A July 16 20.6	P July 23 20.3	P July 16 1.3	A July 23 1.0
A July 30 10.6	P Aug. 6 10.2	P July 30 15.3	A Aug. 6 14.9	A July 29 20.0	P Aug. 5 19.6
P Aug. 13 5.2	A Aug. 20 4.9	A Aug. 13 9.9	P Aug. 20 9.6	A Aug. 12 14.6	A Aug. 19 14.3
A Aug. 26 23.9	P Sept. 2 23.6	P Aug. 27 4.6	A Sept. 3 4.3	A Aug. 26 9.3	P Sept. 2 8.9
P Sept. 9 18.5	A Sept. 16 18.2	A Sept. 9 23.2	P Sept. 16 22.9	P Sept. 9 3.9	A Sept. 16 3.6
A Sept. 23 13.2	P Sept. 30 12.9	P Sept. 23 17.9	A Sept. 30 17.6	A Sept. 22 22.6	P Sept. 29 22.3
P Oct. 7 7.8	A Oct. 14 7.5	A Oct. 7 12.5	P Oct. 14 12.2	P Oct. 6 17.2	A Oct. 13 16.9
A Oct. 21 2.5	P Oct. 28 2.2	P Oct. 21 7.2	A Oct. 28 6.9	A Oct. 20 11.9	P Oct. 27 11.6
P Nov. 3 21.2	A Nov. 10 20.8	A Nov. 4 1.8	P Nov. 11 1.5	P Nov. 3 6.5	A Nov. 10 6.2
A Nov. 17 15.8	P Nov. 24 15.5	P Nov. 17 20.5	A Nov. 24 20.2	A Nov. 17 1.2	P Nov. 24 0.9
P Dec. 1 10.5	A Dec. 8 10.1	A Dec. 1 15.2	P Dec. 8 14.8	P Nov. 30 19.9	A Dec. 7 19.5
A Dec. 15 5.1	P Dec. 22 4.8	P Dec. 15 9.8	A Dec. 22 9.5	A Dec. 14 14.5	P Dec. 21 14.2
P Dec. 28 23.8		A Dec. 29 4.5		P Dec. 28 9.2	

TABLE 58.—*Greenwich mean civil times of mean perigee and apogee, 1850-1950—Con.*

1910		1911		1912		1913		1914		1915	
	hr.		hr.		hr.		hr.		hr.		hr.
A Jan.	4 8.8	P Jan.	11 8.5	P Jan.	4 13.5	A Jan.	10 13.2	A Jan.	3 18.2	P Jan.	10 17.9
P Jan.	18 3.5	A Jan.	25 3.2	A Jan.	18 8.2	P Jan.	24 7.9	P Jan.	17 12.9	A Jan.	24 12.6
A Jan.	31 22.1	P Feb.	7 21.8	P Feb.	1 2.8	A Feb.	7 2.5	A Jan.	31 7.5	P Feb.	7 7.2
P Feb.	14 16.8	A Feb.	21 16.5	A Feb.	14 21.5	P Feb.	20 21.2	P Feb.	14 2.2	A Feb.	21 1.9
A Feb.	28 11.5	P Mar.	7 11.1	P Feb.	28 16.1	A Mar.	6 15.8	A Feb.	27 20.8	P Mar.	6 20.5
P Mar.	14 6.1	A Mar.	21 5.8	A Mar.	13 10.8	P Mar.	20 10.5	P Mar.	13 15.5	A Mar.	20 15.2
A Mar.	28 0.8	P Apr.	4 0.4	P Mar.	27 5.5	A Apr.	3 5.1	A Mar.	27 10.2	P Apr.	3 9.8
P Apr.	10 19.4	A Apr.	17 19.1	A Apr.	10 0.1	P Apr.	16 23.8	P Apr.	10 4.8	A Apr.	17 4.5
A Apr.	24 14.1	P May	1 13.7	P Apr.	23 18.8	A Apr.	30 18.4	A Apr.	23 23.5	P Apr.	30 23.1
P May	8 8.7	A May	15 8.4	A May	7 13.4	P May	14 13.1	P May	7 18.1	A May	14 17.8
A May	22 3.4	P May	29 3.1	P May	21 8.1	A May	28 7.8	A May	21 12.8	P May	28 12.4
P June	4 22.0	A June	11 21.7	A June	4 2.7	P June	11 2.4	P June	4 7.4	A June	11 7.1
A June	18 16.7	P June	25 16.4	P June	17 21.4	A June	24 21.1	A June	18 2.1	P June	25 1.8
P July	2 11.3	A July	9 11.0	A July	1 16.0	P July	8 15.7	P July	1 20.7	A July	8 20.4
A July	16 6.0	P July	23 5.7	P July	15 10.7	A July	22 10.4	A July	15 15.4	P July	22 15.1
P July	30 0.7	A Aug.	6 0.3	A July	29 5.3	P Aug.	5 5.0	P July	29 10.0	A Aug.	5 9.7
A Aug.	12 19.3	P Aug.	19 19.0	P Aug.	12 0.0	A Aug.	18 23.7	A Aug.	12 4.7	P Aug.	19 4.4
P Aug.	26 14.0	A Sept.	2 13.6	A Aug.	25 18.7	P Sept.	1 18.3	P Aug.	25 23.4	A Sept.	1 23.0
A Sept.	9 8.6	P Sept.	16 8.3	P Sept.	8 13.3	A Sept.	15 13.0	A Sept.	8 18.0	P Sept.	15 17.7
P Sept.	23 3.3	A Sept.	30 2.9	A Sept.	22 8.0	P Sept.	29 7.6	P Sept.	22 12.7	A Sept.	29 12.3
A Oct.	6 21.9	P Oct.	13 21.6	P Oct.	6 2.6	A Oct.	13 2.3	A Oct.	6 7.3	P Oct.	13 7.0
P Oct.	20 16.6	A Oct.	27 16.3	A Oct.	19 21.3	P Oct.	26 21.0	P Oct.	20 2.0	A Oct.	27 1.6
A Nov.	3 11.2	P Nov.	10 10.9	P Nov.	2 15.9	A Nov.	9 15.6	A Nov.	2 20.6	P Nov.	9 20.3
P Nov.	17 5.9	A Nov.	24 5.6	A Nov.	16 10.6	P Nov.	23 10.3	P Nov.	16 15.3	A Nov.	23 15.0
A Dec.	1 0.5	P Dec.	8 0.2	P Nov.	30 5.2	A Dec.	7 4.9	A Nov.	30 9.9	P Dec.	7 9.6
P Dec.	14 19.2	A Dec.	21 18.9	A Dec.	13 23.9	P Dec.	20 23.6	P Dec.	14 4.6	A Dec.	21 4.3
A Dec.	28 13.9			P Dec.	27 18.6			A Dec.	27 23.2		
1916		1917		1918		1919		1920		1921	
	hr.		hr.		hr.		hr.		hr.		hr.
P Jan.	3 22.9	A Jan.	9 22.6	A Jan.	3 3.6	P Jan.	10 3.3	P Jan.	3 8.3	A Jan.	9 8.0
A Jan.	17 17.6	P Jan.	23 17.2	P Jan.	16 22.3	A Jan.	23 21.9	A Jan.	17 3.0	P Jan.	23 2.6
P Jan.	31 12.2	A Feb.	6 11.9	A Jan.	30 16.9	P Feb.	6 16.6	P Jan.	30 21.6	A Feb.	5 21.3
A Feb.	14 6.9	P Feb.	20 6.6	P Feb.	13 11.6	A Feb.	20 11.2	A Feb.	13 16.3	P Feb.	19 15.9
P Feb.	28 1.5	A Mar.	6 1.2	A Feb.	27 6.2	P Mar.	6 5.9	P Feb.	27 10.9	A Mar.	5 10.6
A Mar.	12 20.2	P Mar.	19 19.9	P Mar.	13 0.9	A Mar.	20 0.6	A Mar.	12 5.6	P Mar.	19 5.3
P Mar.	26 14.8	A Apr.	2 14.5	A Mar.	26 19.5	P Apr.	2 19.2	P Mar.	26 0.2	A Apr.	1 23.9
A Apr.	9 9.5	P Apr.	16 9.2	P Apr.	9 14.2	A Apr.	16 13.9	A Apr.	8 18.9	P Apr.	15 18.6
P Apr.	23 4.2	A Apr.	30 3.8	A Apr.	23 8.8	P Apr.	30 8.5	P Apr.	22 13.5	A Apr.	29 13.2
A May	6 22.8	P May	13 22.5	P May	7 3.5	A May	14 3.2	A May	6 8.2	P May	13 7.9
P May	20 17.4	A May	27 17.1	A May	20 22.2	P May	27 21.8	P May	20 2.9	A May	27 2.5
A June	3 12.1	P June	10 11.8	P June	3 16.8	A June	10 16.5	A June	2 21.5	P June	9 21.2
P June	17 6.8	A June	24 6.4	A June	17 11.5	P June	24 11.1	P June	16 16.2	A June	23 15.8
A July	1 1.4	P July	8 1.1	P July	1 6.1	A July	8 5.8	A June	30 10.8	P July	7 10.5
P July	14 20.1	A July	21 19.8	A July	15 0.8	P July	22 0.5	P July	14 5.5	A July	21 5.1
A July	28 14.7	P Aug.	4 14.4	P July	28 19.4	A Aug.	4 19.1	A July	28 0.1	P Aug.	3 23.8
P Aug.	11 9.4	A Aug.	18 9.1	A Aug.	11 14.1	P Aug.	18 13.8	P Aug.	10 18.8	A Aug.	17 18.5
A Aug.	25 4.0	P Sept.	1 3.7	P Aug.	25 8.7	A Sept.	1 8.4	A Aug.	24 13.4	P Aug.	31 13.1
P Sept.	7 22.7	A Sept.	14 22.4	A Sept.	8 3.4	P Sept.	15 3.1	P Sept.	7 8.1	A Sept.	14 7.8
A Sept.	21 17.4	P Sept.	28 17.0	P Sept.	21 22.0	A Sept.	28 21.7	A Sept.	21 2.7	P Sept.	28 2.4
P Oct.	5 12.0	A Oct.	12 11.7	A Oct.	5 16.7	P Oct.	12 16.4	P Oct.	4 21.4	A Oct.	11 21.1
A Oct.	19 6.7	P Oct.	26 6.3	P Oct.	19 11.4	A Oct.	26 11.0	A Oct.	18 16.1	P Oct.	25 15.7
P Nov.	2 1.3	A Nov.	9 1.0	A Nov.	2 6.0	P Nov.	9 5.7	P Nov.	1 10.7	A Nov.	8 10.4
A Nov.	15 20.0	P Nov.	22 19.6	P Nov.	16 0.7	A Nov.	23 0.3	A Nov.	15 5.4	P Nov.	22 5.0
P Nov.	29 14.6	A Dec.	6 14.3	A Nov.	29 19.3	P Dec.	6 19.0	P Nov.	29 0.0	A Dec.	5 23.7
A Dec.	13 9.3	P Dec.	20 9.0	A Dec.	13 14.0	A Dec.	20 13.6	A Dec.	12 18.7	P Dec.	19 18.3
P Dec.	27 3.9			A Dec.	27 8.6			P Dec.	26 13.3		

TABLE 58.—*Greenwich mean civil times of mean perigee and apogee, 1850-1950—Con.*

1922	1923	1924	1925	1926	1927
hr.	hr.	hr.	hr.	hr.	hr.
A Jan. 2 13.0	P Jan. 9 12.7	P Jan. 2 17.7	A Jan. 8 17.4	A Jan. 1 22.4	P Jan. 8 22.1
P Jan. 16 7.7	A Jan. 23 7.3	A Jan. 16 12.3	P Jan. 22 12.0	P Jan. 15 17.0	A Jan. 22 16.7
A Jan. 30 2.3	P Feb. 6 2.0	P Jan. 30 7.0	A Feb. 5 6.7	A Jan. 29 11.7	P Feb. 5 11.4
P Feb. 12 21.0	A Feb. 19 20.6	A Feb. 13 1.7	P Feb. 19 1.3	P Feb. 12 6.4	A Feb. 19 6.0
A Feb. 26 15.6	P Mar. 5 15.3	P Feb. 26 20.3	A Mar. 4 20.0	A Feb. 26 1.0	P Mar. 5 0.7
P Mar. 12 10.3	A Mar. 19 9.9	A Mar. 11 15.0	P Mar. 18 14.6	P Mar. 11 19.7	A Mar. 18 19.3
A Mar. 26 4.9	P Apr. 2 4.6	P Mar. 25 9.6	A Apr. 1 9.3	A Mar. 25 14.3	P Apr. 1 14.0
P Apr. 8 23.6	A Apr. 15 23.3	A Apr. 8 4.3	P Apr. 15 3.9	P Apr. 8 9.0	A Apr. 15 8.6
A Apr. 22 18.2	P Apr. 29 17.9	P Apr. 21 22.9	A Apr. 28 22.6	A Apr. 22 3.6	P Apr. 29 3.3
P May 6 12.9	A May 13 12.6	A May 5 17.6	P May 12 17.3	P May 5 22.3	A May 12 22.0
A May 20 7.5	P May 27 7.2	P May 19 12.2	A May 26 11.9	A May 19 16.9	P May 26 16.6
P June 3 2.2	A June 10 1.9	A June 2 6.9	P June 9 6.6	P June 2 11.6	A June 9 11.3
A June 16 20.9	P June 23 20.5	P June 16 1.5	A June 23 1.2	A June 16 6.2	P June 23 5.9
P June 30 15.5	A July 7 15.2	A June 29 20.2	P July 6 19.9	P June 30 0.9	A July 7 0.6
A July 14 10.2	P July 21 9.8	P July 13 14.9	A July 20 14.5	A July 13 19.6	P July 20 19.2
P July 28 4.8	A Aug. 4 4.5	A July 27 9.5	P Aug. 3 9.2	P July 27 14.2	A Aug. 3 13.9
A Aug. 10 23.5	P Aug. 17 23.1	P Aug. 10 4.2	A Aug. 17 3.8	A Aug. 10 8.9	P Aug. 17 8.5
P Aug. 24 18.1	A Aug. 31 17.8	A Aug. 23 22.8	P Aug. 30 22.5	P Aug. 24 3.5	A Aug. 31 3.2
A Sept. 7 12.8	P Sept. 14 12.5	P Sept. 6 17.5	A Sept. 13 17.2	A Sept. 6 22.2	P Sept. 13 21.8
P Sept. 21 7.4	A Sept. 28 7.1	A Sept. 20 12.1	P Sept. 27 11.8	P Sept. 20 16.8	A Sept. 27 16.5
A Oct. 5 2.1	P Oct. 12 1.8	P Oct. 4 6.8	A Oct. 11 6.5	A Oct. 4 11.5	P Oct. 11 11.2
P Oct. 18 20.7	A Oct. 25 20.4	A Oct. 18 1.4	P Oct. 25 1.1	P Oct. 18 6.1	A Oct. 25 5.8
A Nov. 1 15.4	P Nov. 8 15.1	P Oct. 31 20.1	A Nov. 7 19.8	A Nov. 1 0.8	P Nov. 8 0.5
P Nov. 15 10.1	A Nov. 22 9.7	A Nov. 14 14.7	P Nov. 21 14.4	P Nov. 14 19.4	A Nov. 21 19.1
A Nov. 29 4.7	P Dec. 6 4.4	P Nov. 28 9.4	A Dec. 5 9.1	A Nov. 28 14.1	P Dec. 5 13.8
P Dec. 12 23.4	A Dec. 19 23.0	A Dec. 12 4.1	P Dec. 19 3.7	P Dec. 12 8.8	A Dec. 19 8.4
A Dec. 26 18.0		P Dec. 25 22.7		A Dec. 26 3.4	
1928	1929	1930	1931	1932	1933
hr.	hr.	hr.	hr.	hr.	hr.
P Jan. 2 3.1	A Jan. 8 2.8	A Jan. 1 7.8	P Jan. 8 7.4	P Jan. 1 12.5	A Jan. 7 12.1
A Jan. 15 21.7	P Jan. 21 21.4	P Jan. 15 2.4	A Jan. 22 2.1	A Jan. 15 7.1	P Jan. 21 6.8
P Jan. 29 16.4	A Feb. 4 16.1	A Jan. 28 21.1	P Feb. 4 20.8	P Jan. 29 1.8	A Feb. 4 1.5
A Feb. 12 11.0	P Feb. 18 10.7	P Feb. 11 15.7	A Feb. 18 15.4	A Feb. 11 20.4	P Feb. 17 20.1
P Feb. 26 5.7	A Mar. 4 5.4	A Feb. 25 10.4	P Mar. 4 10.1	P Feb. 25 15.1	A Mar. 3 14.8
A Mar. 11 0.4	P Mar. 18 0.0	P Mar. 11 5.0	A Mar. 18 4.7	A Mar. 10 9.7	P Mar. 17 9.4
P Mar. 24 19.0	A Mar. 31 18.7	A Mar. 24 23.7	P Mar. 31 23.4	P Mar. 24 4.4	A Mar. 31 4.1
A Apr. 7 13.7	P Apr. 14 13.3	P Apr. 7 18.4	A Apr. 14 18.0	A Apr. 6 23.1	P Apr. 13 22.7
P Apr. 21 8.3	A Apr. 28 8.0	A Apr. 21 13.0	P Apr. 28 12.7	P Apr. 20 17.7	A Apr. 27 17.4
A May 5 3.0	P May 12 2.6	P May 5 7.7	A May 12 7.3	A May 4 12.4	P May 11 12.0
P May 18 21.6	A May 25 21.3	A May 19 2.3	P May 26 2.0	P May 18 7.0	A May 25 6.7
A June 1 16.3	P June 8 16.0	P June 1 21.0	A June 8 20.7	A June 1 1.7	P June 8 1.3
P June 15 10.9	A June 22 10.6	A June 15 15.6	P June 22 15.3	P June 14 20.3	A June 21 20.0
A June 29 5.6	P July 6 5.3	P June 29 10.3	A July 6 10.0	A June 28 15.0	P July 5 14.7
P July 13 0.2	A July 19 23.9	A July 13 4.9	P July 20 4.6	P July 12 9.6	A July 19 9.3
A July 26 18.9	P Aug. 2 18.6	P July 26 23.6	A Aug. 2 23.3	A July 26 4.3	P Aug. 2 4.0
P Aug. 9 13.5	A Aug. 16 13.2	A Aug. 9 18.2	P Aug. 16 17.9	P Aug. 8 22.9	A Aug. 15 22.6
A Aug. 23 8.2	P Aug. 30 7.9	P Aug. 23 12.9	A Aug. 30 12.6	A Aug. 22 17.6	P Aug. 29 17.3
P Sept. 6 2.9	A Sept. 13 2.5	A Sept. 6 7.6	P Sept. 13 7.2	P Sept. 5 12.3	A Sept. 12 11.9
A Sept. 19 21.5	P Sept. 26 21.2	P Sept. 20 2.2	A Sept. 27 1.9	A Sept. 19 6.9	P Sept. 26 6.6
P Oct. 3 16.2	A Oct. 10 15.8	A Oct. 3 20.9	P Oct. 10 20.5	P Oct. 3 1.6	A Oct. 10 1.2
A Oct. 17 10.8	P Oct. 24 10.5	P Oct. 17 15.5	A Oct. 24 15.2	A Oct. 16 20.2	P Oct. 23 19.9
P Oct. 31 5.5	A Nov. 7 5.2	A Oct. 31 10.2	P Nov. 7 9.9	P Oct. 30 14.9	A Nov. 6 14.5
A Nov. 14 0.1	P Nov. 20 23.8	P Nov. 14 4.8	A Nov. 21 4.5	A Nov. 13 9.5	P Nov. 20 9.2
P Nov. 27 18.8	A Dec. 4 18.5	A Nov. 27 23.5	P Dec. 4 23.2	P Nov. 27 4.2	A Dec. 4 3.9
A Dec. 11 13.4	P Dec. 18 13.1	P Dec. 11 18.1	A Dec. 18 17.8	A Dec. 10 22.8	P Dec. 17 22.5
P Dec. 25 8.1		A Dec. 25 12.8		P Dec. 24 17.5	A Dec. 31 17.2

TABLE 58.—Greenwich mean civil times of mean perigee and apogee, 1850-1950—Con.

1934	1935	1936	1937	1938	1939
hr	hr	hr	hr	hr	hr
P Jan. 14 11.8	P Jan. 7 16.8	A Jan. 14 16.5	A Jan. 6 21.5	P Jan. 13 21.2	P Jan. 7 2.2
A Jan. 28 6.5	A Jan. 21 11.5	P Jan. 28 11.2	P Jan. 20 16.2	A Jan. 27 15.9	A Jan. 20 20.9
P Feb. 11 1.1	P Feb. 4 6.1	A Feb. 11 5.8	A Feb. 3 10.8	P Feb. 10 10.5	P Feb. 3 15.5
A Feb. 24 19.8	A Feb. 18 0.8	P Feb. 25 0.5	P Feb. 17 5.5	A Feb. 24 5.2	A Feb. 17 10.2
P Mar. 10 14.4	P Mar. 3 19.5	A Mar. 9 19.1	A Mar. 3 0.2	P Mar. 9 23.8	P Mar. 3 4.8
A Mar. 24 9.1	A Mar. 17 14.1	P Mar. 23 13.8	P Mar. 16 18.8	A Mar. 23 18.5	A Mar. 16 23.5
P Apr. 7 3.7	P Mar. 31 8.8	A Apr. 6 8.4	A Mar. 30 13.5	P Apr. 6 13.1	P Mar. 30 18.2
A Apr. 20 22.4	A Apr. 14 3.4	P Apr. 20 3.1	P Apr. 13 8.1	A Apr. 20 7.8	A Apr. 13 12.8
P May 4 17.1	P Apr. 27 22.1	A May 3 21.7	A Apr. 27 2.8	P May 4 2.4	P Apr. 27 7.5
A May 18 11.7	A May 11 16.7	P May 17 16.4	P May 10 21.4	A May 17 21.1	A May 11 2.1
P June 1 6.4	P May 25 11.4	A May 31 11.1	A May 24 16.1	P May 31 15.8	P May 24 20.8
A June 15 1.0	A June 8 6.0	P June 14 5.7	P June 7 10.7	A June 14 10.4	A June 7 15.4
P June 28 19.7	P June 22 0.7	A June 28 0.4	A June 21 5.4	P June 28 5.1	P June 21 10.1
A July 12 14.3	A July 5 19.3	P July 11 19.0	P July 5 0.0	A July 11 23.7	A July 5 4.7
P July 13 9.0	P July 19 14.0	A July 25 13.7	A July 18 18.7	P July 25 18.4	P July 18 23.4
A Aug. 9 3.6	A Aug. 2 8.7	P Aug. 8 8.3	P Aug. 1 13.4	A Aug. 8 13.0	A Aug. 1 18.0
P Aug. 22 22.3	P Aug. 16 3.3	A Aug. 22 3.0	A Aug. 15 8.0	P Aug. 22 7.7	P Aug. 15 12.7
A Sept. 5 16.9	A Aug. 29 22.0	P Sept. 4 21.6	P Aug. 29 2.7	A Sept. 5 2.3	A Aug. 29 7.4
P Sept. 19 11.6	P Sept. 12 16.6	A Sept. 18 16.3	A Sept. 11 21.3	P Sept. 18 21.0	P Sept. 12 2.0
A Oct. 3 6.3	A Sept. 26 11.3	P Oct. 2 10.9	P Sept. 25 16.0	A Oct. 2 15.7	A Sept. 25 20.7
P Oct. 17 0.9	P Oct. 10 5.9	A Oct. 16 5.6	A Oct. 9 10.6	P Oct. 16 10.3	P Oct. 9 15.3
A Oct. 30 19.6	A Oct. 24 0.6	P Oct. 30 0.3	P Oct. 23 5.3	A Oct. 30 5.0	A Oct. 23 10.0
P Nov. 13 14.2	P Nov. 6 19.2	A Nov. 12 18.9	A Nov. 5 23.9	P Nov. 12 23.6	P Nov. 6 4.7
A Nov. 27 8.9	A Nov. 20 13.9	P Nov. 26 13.6	P Nov. 19 18.6	A Nov. 26 18.3	A Nov. 19 23.3
P Dec. 11 3.5	P Dec. 4 8.5	A Dec. 10 8.2	A Dec. 3 13.2	P Dec. 10 12.9	P Dec. 3 17.8
A Dec. 24 22.2	A Dec. 18 3.2	P Dec. 24 2.9	P Dec. 17 7.9	A Dec. 24 7.6	A Dec. 17 12.6
	P Dec. 31 21.9		A Dec. 31 2.6		P Dec. 31 7.2
1940	1941	1942	1943	1944	1945
hr	hr	hr	hr	hr	hr
A Jan. 14 1.9	A Jan. 6 6.9	P Jan. 13 6.6	P Jan. 6 11.6	A Jan. 13 11.3	A Jan. 5 16.3
P Jan. 27 20.6	P Jan. 20 1.6	A Jan. 27 1.2	A Jan. 20 6.3	P Jan. 27 5.9	P Jan. 19 11.0
A Feb. 10 15.2	A Feb. 2 20.2	P Feb. 9 19.9	P Feb. 3 0.9	A Feb. 10 0.6	A Feb. 2 5.6
P Feb. 24 9.9	P Feb. 16 14.9	A Feb. 23 14.6	A Feb. 16 19.6	P Feb. 23 19.3	P Feb. 16 0.3
A Mar. 9 4.5	A Mar. 2 9.5	P Mar. 9 9.2	P Mar. 2 14.2	A Mar. 8 13.9	A Mar. 1 18.9
P Mar. 22 23.2	P Mar. 16 4.2	A Mar. 23 3.9	A Mar. 16 8.9	P Mar. 22 8.6	P Mar. 15 13.6
A Apr. 5 17.8	A Mar. 29 22.8	P Apr. 5 22.5	P Mar. 30 3.5	A Apr. 5 3.2	A Mar. 29 8.2
P Apr. 19 12.5	P Apr. 12 17.5	A Apr. 19 17.2	A Apr. 12 22.2	P Apr. 18 21.9	P Apr. 12 2.9
A May 3 7.1	A Apr. 26 12.2	P May 3 11.8	P Apr. 26 16.9	A May 2 16.5	A Apr. 25 21.5
P May 17 1.8	P May 10 6.8	A May 17 6.5	A May 10 11.5	P May 16 11.2	P May 9 16.2
A May 30 20.4	A May 24 1.5	P May 31 1.2	P May 24 6.2	A May 30 5.8	A May 23 10.9
P June 13 15.1	P June 6 20.1	A June 13 19.8	A June 7 0.8	P June 13 0.5	P June 6 5.5
A June 27 9.8	A June 20 14.8	P June 27 14.4	P June 20 19.5	A June 26 19.1	A June 20 0.2
P July 11 4.4	P July 4 9.4	A July 11 9.1	A July 4 14.1	P July 10 13.8	P July 3 18.8
A July 24 23.1	A July 18 4.1	P July 25 3.8	P July 18 8.8	A July 24 8.5	A July 17 13.5
P Aug. 7 17.7	P July 31 22.7	A Aug. 7 22.4	A Aug. 1 3.4	P Aug. 7 3.1	P July 31 8.1
A Aug. 21 12.4	A Aug. 14 17.4	P Aug. 21 17.1	P Aug. 14 22.1	A Aug. 20 21.8	A Aug. 14 2.8
P Sept. 4 7.0	P Aug. 28 12.0	A Sept. 4 11.7	A Aug. 28 16.7	P Sept. 3 16.4	P Aug. 27 21.4
A Sept. 18 1.7	A Sept. 11 6.7	P Sept. 18 6.4	P Sept. 11 11.4	A Sept. 17 11.1	A Sept. 10 16.1
P Oct. 1 20.3	P Sept. 25 1.4	A Oct. 2 1.0	A Sept. 25 6.1	P Oct. 1 5.7	P Sept. 24 10.7
A Oct. 15 15.0	A Oct. 8 20.0	P Oct. 15 19.7	P Oct. 9 0.7	A Oct. 15 0.4	A Oct. 8 5.4
P Oct. 29 9.6	P Oct. 22 14.7	A Oct. 29 14.3	A Oct. 22 19.4	P Oct. 28 19.0	P Oct. 22 0.1
A Nov. 12 4.3	A Nov. 5 9.3	P Nov. 12 9.0	P Nov. 5 14.0	A Nov. 11 13.7	A Nov. 4 18.7
P Nov. 25 23.0	P Nov. 19 4.0	A Nov. 26 3.6	A Nov. 19 8.7	P Nov. 25 8.3	P Nov. 18 13.4
A Dec. 9 17.6	A Dec. 2 22.6	P Dec. 9 22.3	P Dec. 3 3.3	A Dec. 9 3.0	A Dec. 2 8.0
P Dec. 23 12.3	P Dec. 16 17.3	A Dec. 23 17.0	A Dec. 16 22.0	P Dec. 22 21.7	P Dec. 16 2.7
	A Dec. 30 11.9		P Dec. 30 16.6		A Dec. 29 21.3

TABLE 58.—*Greenwich mean civil times of mean perigee and apogee, 1850-1950—Con.*

1946		1947		1948		1949		1950	
	hr.		hr.		hr.		hr.		hr.
P Jan. 12	16.0	P Jan. 5	21.0	A Jan. 12	20.7	A Jan. 5	1.7	P Jan. 12	1.4
A Jan. 26	10.6	A Jan. 19	15.7	P Jan. 26	15.3	P Jan. 18	20.4	A Jan. 25	20.0
P Feb. 9	5.3	P Feb. 2	10.3	A Feb. 9	10.0	A Feb. 1	15.0	P Feb. 8	14.7
A Feb. 22	23.9	A Feb. 16	5.0	P Feb. 23	4.6	P Feb. 15	9.7	A Feb. 22	9.3
P Mar. 8	18.6	P Mar. 1	23.6	A Mar. 7	23.3	A Mar. 1	4.3	P Mar. 8	4.0
A Mar. 22	13.3	A Mar. 15	18.3	P Mar. 21	17.9	P Mar. 14	23.0	A Mar. 21	22.6
P Apr. 5	7.9	P Mar. 29	12.9	A Apr. 4	12.6	A Mar. 28	17.7	P Apr. 4	17.3
A Apr. 19	2.6	A Apr. 12	7.6	P Apr. 18	7.3	P Apr. 11	12.3	A Apr. 18	12.0
P May 2	21.2	P Apr. 26	2.2	A May 2	1.9	A Apr. 25	6.9	P May 2	6.6
A May 16	15.9	A May 9	20.9	P May 15	20.6	P May 9	1.6	A May 16	1.3
P May 30	10.5	P May 23	15.5	A May 29	15.2	A May 22	20.2	P May 29	19.9
A June 13	5.2	A June 6	10.2	P June 12	9.9	P June 5	14.9	A June 12	14.6
P June 26	23.8	P June 20	4.9	A June 26	4.5	A June 19	9.6	P June 26	9.2
A July 10	18.5	A July 3	23.5	P July 9	23.2	P July 3	4.2	A July 10	3.9
P July 24	13.1	P July 17	18.2	A July 23	17.8	A July 16	22.9	P July 23	22.5
A Aug. 7	7.8	A July 31	12.8	P Aug. 6	12.5	P July 30	17.5	A Aug. 6	17.2
P Aug. 21	2.5	P Aug. 14	7.5	A Aug. 20	7.1	A Aug. 13	12.2	P Aug. 20	11.8
A Sept. 3	21.1	A Aug. 28	2.1	P Sept. 3	1.8	P Aug. 27	6.8	A Sept. 3	6.5
P Sept. 17	15.8	P Sept. 10	20.8	A Sept. 16	20.5	A Sept. 10	1.5	P Sept. 17	1.2
A Oct. 1	10.4	A Sept. 24	15.4	P Sept. 30	15.1	P Sept. 23	20.1	A Sept. 30	19.8
P Oct. 15	5.1	P Oct. 8	10.1	A Oct. 14	9.8	A Oct. 7	14.8	P Oct. 14	14.5
A Oct. 28	23.7	A Oct. 22	4.7	P Oct. 28	4.4	P Oct. 21	9.4	A Oct. 28	9.1
P Nov. 11	18.4	P Nov. 4	23.4	A Nov. 10	23.1	A Nov. 4	4.1	P Nov. 11	3.8
A Nov. 25	13.0	A Nov. 18	18.1	P Nov. 24	17.7	P Nov. 17	22.8	A Nov. 24	22.4
P Dec. 9	7.7	P Dec. 2	12.7	A Dec. 8	12.4	A Dec. 1	17.4	P Dec. 8	17.1
A Dec. 23	2.3	A Dec. 16	7.4	P Dec. 22	7.0	P Dec. 15	12.1	A Dec. 22	11.7
		P Dec. 30	2.0			A Dec. 29	6.7		

The value of the anomalistic month, of p and of s , used in constructing this table, are given in sections 13 and 21, Part II.

TABLE 59.—*Variation in mean semirange of spring and neap tides due to parallax and evectional waves composed of N_2 , L_2 , $2L$, v_2 , and λ_2 .*

INCREASE IN MEAN SEMIRANGE OF SPRING TIDES.

Time	Length of half group			Time	Length of half group		
	0 tides	2 tides	4 tides		0 tides	2 tides	4 tides
After perigean tides	d. h.			d. h.			
	0 00	+1.11 N_2	+1.11 N_2	0 00	-0.91 N_2	-0.91 N_2	-0.91 N_2
	0 06	+1.11 "	+1.11 "	0 06	-0.91 "	-0.91 "	-0.90 "
	0 12	+1.10 "	+1.10 "	0 12	-0.91 "	-0.91 "	-0.90 "
	0 18	+1.09 "	+1.09 "	0 18	-0.90 "	-0.90 "	-0.90 "
	1 00	+1.07 "	+1.07 "	1 00	-0.90 "	-0.89 "	-0.89 "
	1 06	+1.05 "	+1.05 "	1 06	-0.89 "	-0.88 "	-0.88 "
	1 12	+1.03 "	+1.03 "	1 12	-0.87 "	-0.87 "	-0.87 "
	1 18	+1.00 "	+1.00 "	1 18	-0.86 "	-0.86 "	-0.86 "
	2 00	+0.97 "	+0.97 "	2 00	-0.85 "	-0.84 "	-0.84 "
	2 06	+0.93 "	+0.93 "	2 06	-0.83 "	-0.83 "	-0.82 "
	2 12	+0.89 "	+0.89 "	2 12	-0.81 "	-0.81 "	-0.80 "
	2 18	+0.85 "	+0.85 "	2 18	-0.79 "	-0.78 "	-0.78 "
	3 00	+0.80 "	+0.80 "	3 00	-0.76 "	-0.76 "	-0.75 "
	3 06	+0.75 "	+0.75 "	3 06	-0.73 "	-0.73 "	-0.73 "
	3 12	+0.70 "	+0.70 "	3 12	-0.71 "	-0.70 "	-0.70 "
Before midtime tides	3 12	+0.72 "	+0.72 "	3 12	-0.72 "	-0.72 "	-0.71 "
	3 06	+0.67 "	+0.67 "	3 06	-0.69 "	-0.69 "	-0.68 "
	3 00	+0.61 "	+0.61 "	3 00	-0.66 "	-0.66 "	-0.65 "
	2 18	+0.56 "	+0.56 "	2 18	-0.62 "	-0.62 "	-0.62 "
	2 12	+0.50 "	+0.50 "	2 12	-0.59 "	-0.58 "	-0.58 "
	2 06	+0.44 "	+0.44 "	2 06	-0.55 "	-0.55 "	-0.54 "
	2 00	+0.38 "	+0.38 "	2 00	-0.51 "	-0.50 "	-0.50 "
	1 18	+0.32 "	+0.32 "	1 18	-0.46 "	-0.46 "	-0.46 "
	1 12	+0.26 "	+0.26 "	1 12	-0.42 "	-0.42 "	-0.41 "
	1 06	+0.19 "	+0.19 "	1 06	-0.37 "	-0.37 "	-0.36 "
	1 00	+0.13 "	+0.13 "	1 00	-0.32 "	-0.32 "	-0.31 "
	0 18	+0.07 "	+0.07 "	0 18	-0.27 "	-0.27 "	-0.26 "
	0 12	+0.01 "	+0.01 "	0 12	-0.22 "	-0.21 "	-0.21 "
	0 06	-0.05 "	-0.05 "	0 06	-0.16 "	-0.16 "	-0.16 "
	0 00	-0.10 "	-0.10 "	0 00	-0.10 "	-0.10 "	-0.10 "
After midtime tides	0 00	-0.10 "	-0.10 "	0 00	-0.10 "	-0.10 "	-0.10 "
	0 06	-0.16 "	-0.16 "	0 06	-0.05 "	-0.05 "	-0.04 "
	0 12	-0.22 "	-0.21 "	0 12	+0.01 "	+0.01 "	+0.02 "
	0 18	-0.27 "	-0.27 "	0 18	+0.07 "	+0.07 "	+0.07 "
	1 00	-0.32 "	-0.32 "	1 00	+0.13 "	+0.13 "	+0.13 "
	1 06	-0.37 "	-0.37 "	1 06	+0.19 "	+0.19 "	+0.20 "
	1 12	-0.42 "	-0.42 "	1 12	+0.26 "	+0.26 "	+0.26 "
	1 18	-0.46 "	-0.46 "	1 18	+0.32 "	+0.32 "	+0.32 "
	2 00	-0.51 "	-0.50 "	2 00	+0.38 "	+0.38 "	+0.38 "
	2 06	-0.55 "	-0.55 "	2 06	+0.44 "	+0.44 "	+0.44 "
	2 12	-0.59 "	-0.58 "	2 12	+0.50 "	+0.50 "	+0.50 "
	2 18	-0.62 "	-0.62 "	2 18	+0.56 "	+0.56 "	+0.55 "
	3 00	-0.66 "	-0.66 "	3 00	+0.61 "	+0.61 "	+0.61 "
	3 06	-0.69 "	-0.69 "	3 06	+0.67 "	+0.67 "	+0.66 "
	3 12	-0.72 "	-0.72 "	3 12	+0.72 "	+0.72 "	+0.72 "

TABLE 59.—*Variation in mean semirange of spring and neap tides due to parallax and evectional waves composed of N_2 , L_2 , $2L_1$, ν_2 , and λ_2 —Continued.*

INCREASE IN MEAN SEMIRANGE OF SPRING TIDES—continued.

Time	Length of half group			Time.	Length of half group		
	° tides	2 tides	4 tides		° tide	2 tides	4 tides
Before apogean tides	d. h.			Before perigean tides	d. h.		
	3 12	-0.71 N_2	-0.70 N_2		3 12	+0.70 N_2	+0.69 N_2
	3 06	-0.73 "	-0.73 "		3 06	+0.75 "	+0.74 "
	3 00	-0.76 "	-0.76 "		3 00	+0.80 "	+0.79 "
	2 18	-0.79 "	-0.78 "		2 18	+0.85 "	+0.84 "
	2 12	-0.81 "	-0.81 "		2 12	+0.89 "	+0.88 "
	2 06	-0.83 "	-0.83 "		2 06	+0.93 "	+0.92 "
	2 00	-0.85 "	-0.84 "		2 00	+0.97 "	+0.96 "
	1 18	-0.86 "	-0.86 "		1 18	+1.00 "	+0.99 "
	1 12	-0.87 "	-0.87 "		1 12	+1.03 "	+1.02 "
	1 06	-0.89 "	-0.88 "		1 06	+1.05 "	+1.04 "
	1 00	-0.90 "	-0.89 "		1 00	+1.07 "	+1.06 "
	0 18	-0.90 "	-0.90 "		0 18	+1.09 "	+1.08 "
	0 12	-0.91 "	-0.91 "		0 12	+1.10 "	+1.09 "
	0 06	-0.91 "	-0.91 "		0 06	+1.11 "	+1.10 "
	0 00	-0.91 "	-0.91 "		0 00	+1.11 "	+1.10 "

This table applies best to cases where S_2/M_2 has its theoretical value. If this ratio is small, Tables 25 and 34 may be used.

TABLE 59.—*Variation in mean semirange of spring and neap tides due to parallax and evectational waves composed of N_2 , L_2 , $2N_1$, v_2 and λ_2 —Continued.*

INCREASE IN MEAN SEMIRANGE OF NEAP TIDES.

Time	Length of half group			Time	Length of half group		
	0 tides	2 tides	4 tides		0 tides	2 tides	4 tides
After perigean tides	d. h.			d. h.			
	0 00	+0.73 N_2	+0.73 N_2	0 00	-0.67 N_2	-0.66 N_2	-0.66 N_2
	0 06	+0.73 "	+0.73 "	0 06	-0.66 "	-0.66 "	-0.66 "
	0 12	+0.72 "	+0.72 "	0 12	-0.66 "	-0.66 "	-0.65 "
	0 18	+0.72 "	+0.72 "	0 18	-0.65 "	-0.65 "	-0.65 "
	1 00	+0.71 "	+0.71 "	1 00	-0.65 "	-0.64 "	-0.64 "
	1 06	+0.69 "	+0.69 "	1 06	-0.63 "	-0.63 "	-0.63 "
	1 12	+0.68 "	+0.68 "	1 12	-0.62 "	-0.62 "	-0.62 "
	1 18	+0.66 "	+0.66 "	1 18	-0.61 "	-0.60 "	-0.60 "
	2 00	+0.64 "	+0.64 "	2 00	-0.59 "	-0.59 "	-0.58 "
	2 06	+0.62 "	+0.61 "	2 06	-0.57 "	-0.57 "	-0.57 "
	2 12	+0.59 "	+0.59 "	2 12	-0.55 "	-0.55 "	-0.55 "
	2 18	+0.56 "	+0.56 "	2 18	-0.53 "	-0.53 "	-0.53 "
	3 00	+0.53 "	+0.53 "	3 00	-0.51 "	-0.51 "	-0.50 "
	3 06	+0.50 "	+0.50 "	3 06	-0.48 "	-0.48 "	-0.48 "
	3 12	+0.47 "	+0.47 "	3 12	-0.46 "	-0.46 "	-0.46 "
Before midtime tides	3 12	+0.48 "	+0.48 "	3 12	-0.47 "	-0.47 "	-0.47 "
	3 06	+0.45 "	+0.45 "	3 06	-0.44 "	-0.44 "	-0.44 "
	3 00	+0.41 "	+0.41 "	3 00	-0.42 "	-0.42 "	-0.42 "
	2 18	+0.38 "	+0.38 "	2 18	-0.39 "	-0.39 "	-0.39 "
	2 12	+0.34 "	+0.34 "	2 12	-0.36 "	-0.36 "	-0.36 "
	2 06	+0.30 "	+0.30 "	2 06	-0.34 "	-0.34 "	-0.33 "
	2 00	+0.26 "	+0.26 "	2 00	-0.31 "	-0.31 "	-0.30 "
	1 18	+0.22 "	+0.22 "	1 18	-0.28 "	-0.28 "	-0.27 "
	1 12	+0.18 "	+0.18 "	1 12	-0.25 "	-0.25 "	-0.24 "
	1 06	+0.15 "	+0.15 "	1 06	-0.21 "	-0.21 "	-0.21 "
	1 00	+0.11 "	+0.11 "	1 00	-0.18 "	-0.18 "	-0.18 "
	0 18	+0.07 "	+0.07 "	0 18	-0.15 "	-0.15 "	-0.15 "
	0 12	+0.03 "	+0.03 "	0 12	-0.11 "	-0.11 "	-0.11 "
	0 06	-0.01 "	-0.01 "	0 06	-0.08 "	-0.08 "	-0.08 "
	0 00	-0.04 "	-0.04 "	0 00	-0.04 "	-0.04 "	-0.04 "
After midtime tides	0 00	-0.04 "	-0.04 "	0 00	-0.04 "	-0.04 "	-0.04 "
	0 06	-0.08 "	-0.08 "	0 06	-0.01 "	-0.01 "	0.00 "
	0 12	-0.11 "	-0.11 "	0 12	+0.03 "	+0.03 "	+0.03 "
	0 18	-0.15 "	-0.15 "	0 18	+0.07 "	+0.07 "	+0.07 "
	1 00	-0.18 "	-0.18 "	1 00	+0.11 "	+0.11 "	+0.11 "
	1 06	-0.21 "	-0.21 "	1 06	+0.15 "	+0.15 "	+0.15 "
	1 12	-0.25 "	-0.25 "	1 12	+0.18 "	+0.18 "	+0.18 "
	1 18	-0.28 "	-0.28 "	1 18	+0.22 "	+0.22 "	+0.22 "
	2 00	-0.31 "	-0.31 "	2 00	+0.26 "	+0.26 "	+0.26 "
	2 06	-0.34 "	-0.34 "	2 06	+0.30 "	+0.30 "	+0.30 "
	2 12	-0.36 "	-0.36 "	2 12	+0.34 "	+0.34 "	+0.34 "
	2 18	-0.39 "	-0.39 "	2 18	+0.38 "	+0.38 "	+0.37 "
	3 00	-0.42 "	-0.42 "	3 00	+0.41 "	+0.41 "	+0.41 "
	3 06	-0.44 "	-0.44 "	3 06	+0.45 "	+0.45 "	+0.44 "
	3 12	-0.47 "	-0.47 "	3 12	+0.48 "	+0.48 "	+0.48 "

TABLE 59.—*Variation in mean semirange of spring and neap tides due to parallax and evectional waves composed of N_2 , L_2 , $2N$, v_2 , and λ_2 —Continued.*

INCREASE IN MEAN SEMIRANGE OF NEAP TIDES—continued.

Time	Length of half group			Time	Length of half group		
	° tides	2 tides	4 tides		° tides	2 tides	4 tides
<i>d. h.</i>				<i>d. h.</i>			
Before apogean tides	3 12	-0.46 N_2	-0.46 N_2	3 12	+0.47 N_2	+0.47 N_2	+0.47 N_2
	3 06	-0.48 "	-0.48 "	3 06	+0.50 "	+0.50 "	+0.50 "
	3 00	-0.51 "	-0.51 "	3 00	+0.53 "	+0.53 "	+0.53 "
	2 18	-0.53 "	-0.53 "	2 18	+0.56 "	+0.56 "	+0.56 "
	2 12	-0.55 "	-0.55 "	2 12	+0.59 "	+0.59 "	+0.59 "
	2 06	-0.57 "	-0.57 "	2 06	+0.62 "	+0.61 "	+0.61 "
	2 00	-0.59 "	-0.59 "	2 00	+0.64 "	+0.64 "	+0.63 "
	1 18	-0.61 "	-0.60 "	1 18	+0.66 "	+0.66 "	+0.65 "
	1 12	-0.62 "	-0.62 "	1 12	+0.68 "	+0.68 "	+0.67 "
	1 06	-0.63 "	-0.63 "	1 06	+0.69 "	+0.69 "	+0.69 "
	1 00	-0.65 "	-0.64 "	1 00	+0.71 "	+0.71 "	+0.70 "
	0 18	-0.65 "	-0.65 "	0 18	+0.72 "	+0.72 "	+0.71 "
	0 12	-0.66 "	-0.65 "	0 12	+0.72 "	+0.72 "	+0.72 "
	0 06	-0.66 "	-0.66 "	0 06	+0.73 "	+0.73 "	+0.72 "
	0 00	-0.67 "	-0.66 "	0 00	+0.73 "	+0.73 "	+0.72 "
Before perigean tides	3 12	+0.47 N_2	+0.47 N_2	3 12	+0.47 N_2	+0.47 N_2	+0.47 N_2
	3 06	+0.50 "	+0.50 "	3 06	+0.50 "	+0.50 "	+0.50 "
	3 00	+0.53 "	+0.53 "	3 00	+0.53 "	+0.53 "	+0.53 "
	2 18	+0.56 "	+0.56 "	2 18	+0.56 "	+0.56 "	+0.56 "
	2 12	+0.59 "	+0.59 "	2 12	+0.59 "	+0.59 "	+0.59 "
	2 06	+0.62 "	+0.61 "	2 06	+0.62 "	+0.61 "	+0.61 "
	2 00	+0.64 "	+0.64 "	2 00	+0.64 "	+0.64 "	+0.63 "
	1 18	+0.66 "	+0.66 "	1 18	+0.66 "	+0.66 "	+0.65 "
	1 12	+0.68 "	+0.68 "	1 12	+0.68 "	+0.68 "	+0.67 "
	1 06	+0.69 "	+0.69 "	1 06	+0.69 "	+0.69 "	+0.69 "
	1 00	+0.71 "	+0.71 "	1 00	+0.71 "	+0.71 "	+0.70 "
	0 18	+0.72 "	+0.72 "	0 18	+0.72 "	+0.72 "	+0.71 "
	0 12	+0.72 "	+0.72 "	0 12	+0.72 "	+0.72 "	+0.72 "
	0 06	+0.73 "	+0.73 "	0 06	+0.73 "	+0.73 "	+0.72 "
	0 00	+0.73 "	+0.73 "	0 00	+0.73 "	+0.73 "	+0.72 "

This table applies best to cases where S_2/M_2 has its theoretical value. If this ratio is small, Tables 25 and 34 may be used.

TABLE 60.—*Time of maximum*

Time by which maximum downward slope toward body having

[The amplitude of the larger tide is taken as unity.]

Cot. hour of smaller tide—cot. hour of larger	0	$\frac{1}{2}$	$\frac{2}{3}$	1	$1\frac{1}{2}$	$1\frac{2}{3}$	2	$2\frac{1}{2}$	$2\frac{2}{3}$	3
Amplitude of smaller tide.	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.1	0.000	0.037	0.072	0.104	0.133	0.156	0.174	0.185	0.191	0.190
0.2	0.000	0.083	0.161	0.230	0.288	0.332	0.363	0.380	0.384	0.377
0.3	0.000	0.141	0.271	0.382	0.468	0.530	0.567	0.581	0.577	0.557
0.4	0.000	0.218	0.412	0.567	0.678	0.747	0.781	0.784	0.765	0.727
0.5	0.000	0.323	0.596	0.793	0.917	0.982	1.000	0.985	0.944	0.886
0.6	0.000	0.476	0.840	1.066	1.184	1.227	1.219	1.178	1.114	1.032
0.7	0.000	0.712	1.166	1.388	1.471	1.476	1.433	1.362	1.271	1.167
0.8	0.000	1.107	1.594	1.749	1.767	1.720	1.637	1.533	1.415	1.289
0.9	0.000	1.799	2.113	2.129	2.059	1.952	1.826	1.690	1.547	1.399
1.0	0.000	2.833	2.667	2.500	2.333	2.167	2.000	1.833	1.667	1.500
Cot. hour of larger tide—cot. hour of smaller	0	$\frac{1}{2}$	$\frac{2}{3}$	1	$1\frac{1}{2}$	$1\frac{2}{3}$	2	$2\frac{1}{2}$	$2\frac{2}{3}$	3

Time by which maximum downward slope toward body having

TABLE 61.—*Difference in surface elevation for the two*

[The amplitude of the larger tide is taken as unity.]

Cot. hour of smaller tide—cot. hour of larger	0	$\frac{1}{2}$	$\frac{2}{3}$	1	$1\frac{1}{2}$	$1\frac{2}{3}$	2	$2\frac{1}{2}$	$2\frac{2}{3}$
Amplitude of smaller tide.	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9000	0.9017	0.9067	0.9148	0.9256	0.9389	0.9540	0.9703	0.9875
0.2	0.8000	0.8038	0.8150	0.8329	0.8565	0.8848	0.9166	0.9504	0.9852
0.3	0.7000	0.7065	0.7253	0.7552	0.7940	0.8392	0.8888	0.9407	0.9929
0.4	0.6000	0.6100	0.6389	0.6835	0.7397	0.8036	0.8717	0.9415	1.0104
0.5	0.5000	0.5150	0.5571	0.6197	0.6957	0.7792	0.8660	0.9529	1.0375
0.6	0.4000	0.4222	0.4820	0.5664	0.6639	0.7672	0.8718	0.9744	1.0731
0.7	0.3000	0.3336	0.4176	0.5269	0.6462	0.7682	0.8889	1.0055	1.1167
0.8	0.2000	0.2536	0.3694	0.5044	0.6437	0.7819	0.9165	1.0454	1.1672
0.9	0.1000	0.1933	0.3443	0.5009	0.6566	0.8081	0.9339	1.0929	1.2237
1.0	0.0000	0.1744	0.3472	0.5176	0.6840	0.8452	1.0000	1.1472	1.2856
Cot. hour of larger tide—cot. hour of smaller	0	$\frac{1}{2}$	$\frac{2}{3}$	1	$1\frac{1}{2}$	$1\frac{2}{3}$	2	$2\frac{1}{2}$	$2\frac{2}{3}$

slope of the surface.

smaller tide precedes high water of body having larger tide.

[The amplitude of the larger tide is taken as unity.]

Cot. hour of smaller tide— cot. hour of larger	3½	3¾	4	4¼	4½	5	5¼	5¾	6
Amplitude of smaller tide.	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.1	0.184	0.173	0.157	0.137	0.114	0.088	0.059	0.030	0.000
0.2	0.359	0.333	0.298	0.258	0.212	0.162	0.110	0.056	0.000
0.3	0.523	0.478	0.424	0.363	0.297	0.229	0.153	0.077	0.000
0.4	0.674	0.610	0.537	0.457	0.371	0.282	0.189	0.095	0.000
0.5	0.812	0.729	0.637	0.539	0.436	0.330	0.221	0.111	0.000
0.6	0.938	0.836	0.726	0.612	0.493	0.372	0.249	0.125	0.000
0.7	1.053	0.932	0.806	0.677	0.544	0.410	0.274	0.137	0.000
0.8	1.156	1.018	0.878	0.734	0.589	0.443	0.296	0.148	0.000
0.9	1.249	1.096	0.942	0.787	0.630	0.473	0.316	0.158	0.000
1.0	1.333	1.167	1.000	0.833	0.667	0.500	0.333	0.167	0.000
Cot. hour of larger tide— cot. hour of smaller	3½	3¾	4	4¼	4½	5	5¼	5¾	6

smaller tide follows high water of body having larger tide.

ends of the strait at the time of maximum slope.

[The amplitude of the larger tide is taken at unity.]

Cot. hour of smaller tide— cot. hour of larger	3	3½	3¾	4	4¼	4½	5	5¼	5¾	6
Amplitude of smaller tide.	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>	<i>h</i>
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0050	1.0221	1.0385	1.0536	1.0670	1.0785	1.0877	1.0945	1.0986	1.1000
0.2	1.0198	1.0532	1.0849	1.1135	1.1389	1.1603	1.1775	1.1899	1.1975	1.2000
0.3	1.0440	1.0928	1.1381	1.1790	1.2148	1.2448	1.2687	1.2860	1.2965	1.3000
0.4	1.0770	1.1397	1.1973	1.2490	1.2939	1.3315	1.3611	1.3827	1.3957	1.4000
0.5	1.1180	1.1931	1.2618	1.3228	1.3758	1.4198	1.4546	1.4798	1.4949	1.5000
0.6	1.1662	1.2523	1.3306	1.4000	1.4599	1.5097	1.5490	1.5772	1.5943	1.6000
0.7	1.2207	1.3165	1.4032	1.4798	1.5459	1.6007	1.6439	1.6749	1.6937	1.7000
0.8	1.2806	1.3849	1.4789	1.5620	1.6336	1.6928	1.7395	1.7730	1.7932	1.8000
0.9	1.3453	1.4569	1.5575	1.6463	1.7225	1.7858	1.8354	1.8712	1.8928	1.9000
1.0	1.4142	1.5320	1.6384	1.7320	1.8126	1.8794	1.9318	1.9696	1.9924	2.0000
Cot. hour of larger tide— cot. hour of smaller	3	3½	3¾	4	4¼	4½	5	5¼	5¾	6

TABLE 62.—*Pressure and density of sea water at various depths.*

Depths		Pressures				Height of homogeneous column	Densities	
Fathoms	Feet	Mega-dynes per sq. cm.	Atmospheres	Pounds per sq. in.	Pounds per sq. ft.		Surface=1	Surface=64 pounds per cu. ft.
0	0	0.9997	0.9866	14.500	2 088.0	0.00	1.00000	64.000
	1	1.0303	1.0168	14.944	2 152.0	1.00	000	000
	2	.0610	.0471	15.389	2 216.0	2.00	000	000
	3	.0916	.0773	15.833	2 280.0	3.00	000	000
	4	.1223	.1076	16.278	2 344.0	4.00	1.00001	000
	5	.1529	.1378	16.722	2 408.0	5.00	001	000
1	6	1.1836	1.1681	17.167	2 472.0	6.00	001	000
	7	.2142	.1983	17.611	2 536.0	7.00	001	64.001
	8	.2448	.2286	18.056	2 600.0	8.00	001	001
	9	.2755	.2588	18.500	2 664.0	9.00	001	001
	10	.3061	.2890	18.944	2 728.0	10.00	001	001
	11	.3368	.3193	19.389	2 792.0	11.00	001	001
2	12	1.3674	1.3495	19.833	2 856.0	12.00	1.00002	001
	13	.3981	.3798	20.278	2 920.0	13.00	002	001
	14	.4287	.4100	20.722	2 984.0	14.00	002	001
	15	.4593	.4403	21.167	3 048.0	15.00	002	001
	16	.4900	.4705	21.611	3 112.0	16.00	002	001
	17	.5206	.5008	22.056	3 176.0	17.00	002	001
3	18	1.5513	1.5310	22.500	3 240.0	18.00	002	64.002
	19	.5819	.5613	22.945	3 304.0	19.00	1.00003	002
	20	.6126	.5915	23.389	3 368.0	20.00	003	002
	21	.6432	.6217	23.834	3 432.0	21.00	003	002
	22	.6738	.6520	24.278	3 496.0	22.00	003	002
	23	.7045	.6822	24.722	3 560.0	23.00	003	002
4	24	1.7351	1.7125	25.167	3 624.0	24.00	003	002
5	30	1.9190	1.8939	27.834	4 008.0	30.00	1.00004	64.003
6	36	2.1028	2.0753	30.500	4 392.1	36.00	005	003
7	42	.2867	.2568	33.167	4 776.1	42.00	005	004
8	48	.4706	.4383	35.834	5 160.1	48.00	006	004
9	54	.6544	.6197	38.501	5 544.1	54.00	007	005
10	60	2.8383	2.8012	41.168	5 928.1	60.00	008	005
15	90	3.7577	3.7085	54.502	7 848.3	90.01	1.00012	64.008
20	120	4.6770	4.6160	67.834	9 768.6	120.01	016	010
30	180	6.5159	6.4308	94.510	13 609	180.02	024	015
40	240	8.3550	8.2456	121.18	17 450	240.04	032	020
50	300	10.194	10.061	147.86	21 292	300.06	040	025
60	360	12.034	11.876	174.54	25 133	360.09	047	030
70	420	13.873	13.692	201.22	28 975	420.12	055	035
80	480	15.713	15.507	227.90	32 803	480.15	063	041
90	540	17.552	17.323	254.58	36 646	540.19	071	046
100	600	19.392	19.139	281.27	40 489	600.24	079	051
150	900	28.594	28.220	414.74	59 708	900.53	1.00119	64.076
200	1 200	37.799	37.306	548.26	78 934	1 200.9	158	101
300	1 800	56.220	55.487	815.43	115 840	1 802.1	237	151
400	2 400	74.656	73.677	1082.8	155 930	2 403.8	315	202
500	3 000	93.106	91.886	1 350.4	194 460	3 005.9	394	252
600	3 600	111.57	110.11	1 618.2	233 030	3 608.4	472	302
700	4 200	130.05	128.35	1 886.3	271 620	4 211.5	550	352
800	4 800	148.54	146.60	2 154.5	310 250	4 815.0	627	402
900	5 400	167.05	164.87	2 423.0	348 910	5 419.1	705	451
1 000	6 000	185.57	183.15	2 691.6	387 590	6 023.5	782	501

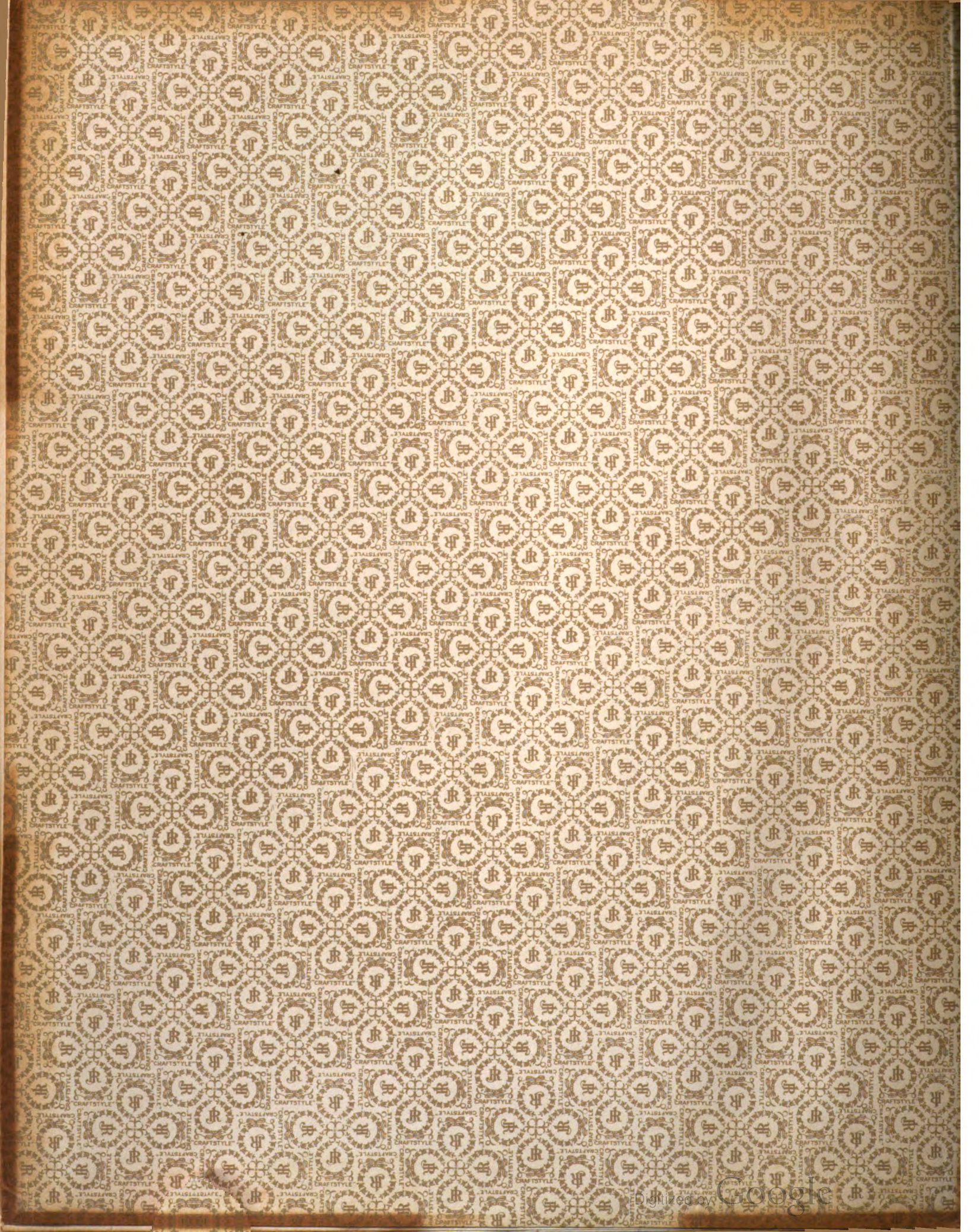
TABLE 62.—*Pressure and density of sea water at various depths—Continued.*

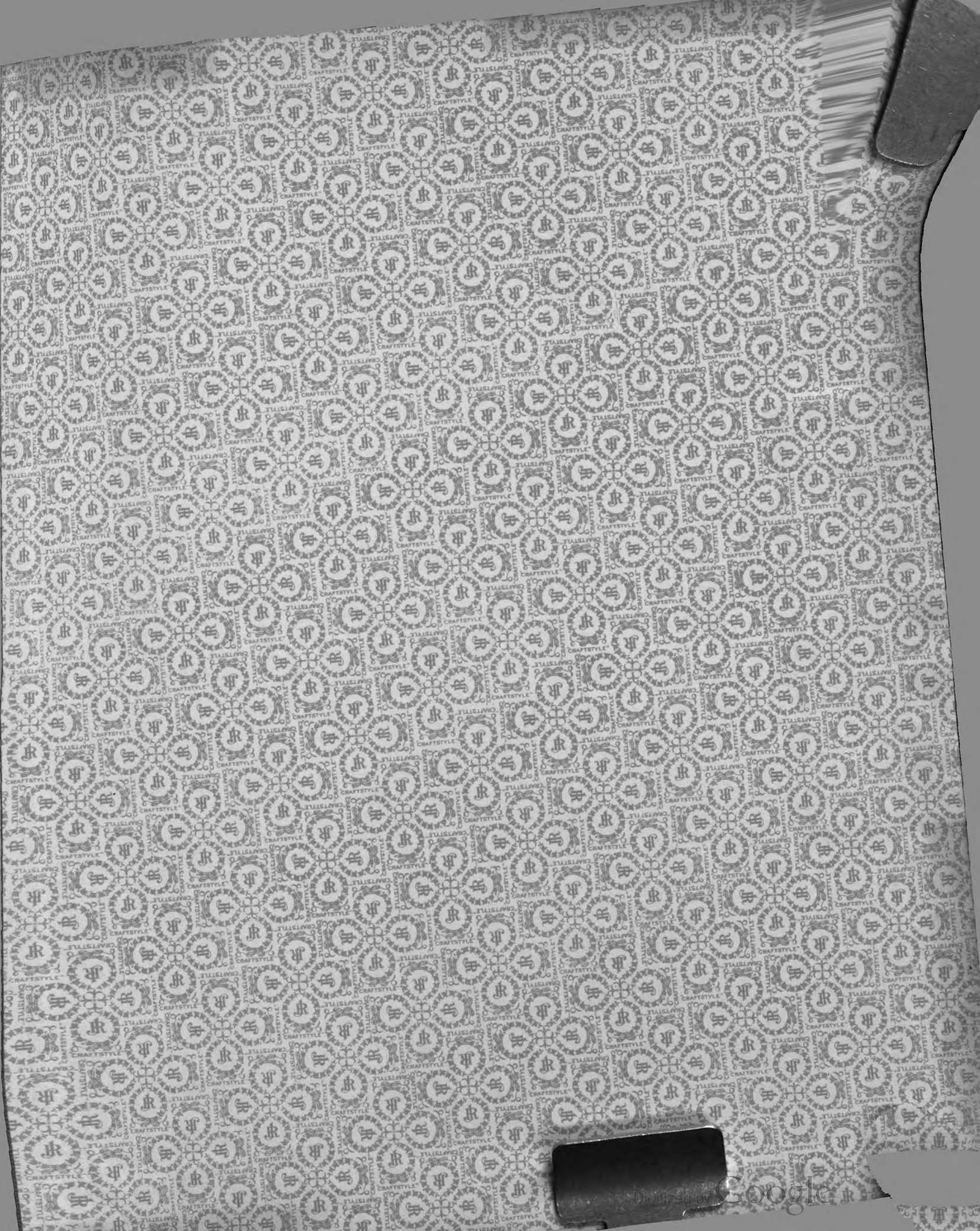
Depths		Pressures				Height of homogeneous column	Densities	
Fathoms	Feet	Mega-dynes per sq. cm.	Atmosphere	Pounds per sq. in.	Pounds per sq. ft.		Surface=1	Surface=64 pounds per cu. ft.
1 500	9 000	278.39	274.75	4 037.9	581 460	9 052.6	1.01167	64.747
2 000	12 000	371.57	366.71	5 389.4	776 070	12 094	546	990
3 000	18 000	558.94	551.65	8 107.3	1 167 400	18 209	1.02293	65.467
4 000	24 000	747.70	737.93	10 845	1 561 700	24 368	1.03023	935
5 000	30 000	937.79	925.53	13 602	1 958 700	30 572	737	66.392
6 000	36 000	1 129.1	1 114.3	16 377	2 358 300	36 816	1.04436	839
7 000	42 000	1 321.8	1 304.5	19 171	2 760 700	43 103	1.05121	67.278
8 000	48 000	1 515.7	1 495.9	21 984	3 165 700	49 431	792	707
9 000	54 000	1 710.8	1 688.4	24 814	3 573 200	55 798	1.06451	68.129
10 000	60 000	1 838.1	1 822.2	27 661	3 983 200	62 206	1.07097	542

1 atmosphere=pressure of 76 cm. of mercury at 0° C. in latitude 45° at sea level; $g=32.1722$ ft. $-\frac{dv}{dp}/v$ has been placed equal to $\frac{a}{1+bp}$ instead of $a(1-bp)$.

This table was computed with Professor Tait's constants, published in his Scientific Papers, Vol. II, p. 27. Approximate expressions are: $p'=64.7y+0.0000422y^2$, $\rho=\rho_0(1+0.0001318y)$, p' denoting water pressure only.

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